# Money, Search and Costly Matchmaking*

Gabriele Camera

Dept. of Economics, Purdue University

West Lafayette, IN 47907-1310

E-mail: Gcamera@mgmt.purdue.edu

## ABSTRACT

I examine the robustness of monetary equilibria in a random matching model where a more efficient mechanism for trade is available. Agents choose between two trading sectors: the search and the intermediated sector. In the former, trade partners arrive randomly and there is a trading externality. In the latter a costly matching technology provides deterministic double-coincidence matches. Multiple equilibria exist with the extent of costly matching endogenously determined. Money and "mediated" trade may coexist. This depends on the size of the probability of a trade, relative to the cost of deterministic matching. This outcome is inferior for an increasing returns externality. Under certain conditions regimes with only costly matching are welfare superior to monetary regimes with random matching.

Keywords: monetary economics, search, multiple equilibria, coordination failures. (JEL C62, D83, E40).

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* This paper is based on material from my Ph.D. dissertation at the University of Iowa. I am indebted to Steve Williamson for valuable comments and support, and to Dean Corbae who suggested the paper's basic idea, and provided thoughtful remarks. The paper has greatly benefited from the comments of three anonymous referees, Andreas Blume, Narayana Kocherlakota, Yiting Li, Hanno Ritter, Alberto Trejos, Randy Wright, seminar participants in the 1996 SEDC and the Midwest Macroeconomics meetings, and at the University of Iowa. Marikah Mancini has provided useful research assistance. All errors are my responsibility.

## 1. Introduction

This paper studies the role of alternative transaction mechanisms on equilibrium patterns of exchange. In particular, it explores the robustness of monetary equilibria to introduction of an improved trade mechanism in a prototypical absence-of-double-coincidence model. This innovation is modeled as a costly matching technology capable of ameliorating the trade frictions by providing deterministic double-coincidence matches. The welfare implications this new trade arrangement has on diverse trading regimes—both monetary and non-monetary—are also examined.

Kiyotaki and Wright (1993) formalize fiat money's medium of exchange function by adopting a search theoretic approach. Money can be endogenously valued if it ameliorates the search frictions stemming from existence of an “imperfect” trading technology, imperfect, that is, when compared to a standard Arrow-Debreu setting. Differentiated goods and pairwise random matching, impair an exchange process which may be thought of as suffering from an extreme degree of spatial separation of spot markets. Since usage of an intrinsically worthless medium of exchange may increase the likelihood of successful exchanges—essentially performing the role of a matchmaker—the model delivers fiat money’s valuation as an *equilibrium* phenomenon. A similar result characterizes the spatial model of Townsend (1980).

One natural question is whether money's value is susceptible to the availability of a costly trade innovation improving the degree of market integration. One suspects the existence of correlation between the degree of interconnectedness of traders in an economy and the types of assets which are used to facilitate exchange (see for example Townsend, 1983). Resorting to fiat money—thus improving on the existing trade technology—naturally has a beneficial welfare effect. May an improved costly trading technology prove superior to strict reliance on a monetary payment system? Townsend (1983) for example points out the existence of a welfare gain when autarky is replaced by a monetary trade regime. However he also notes that taking steps towards a more integrated financial regime—via a centralized credit-debit system—is welfare improving.

In the present model individuals with diverse tastes try to acquire and sell commodities or money by pairwise exchange. They may choose between two trading sectors characterized by different matching technologies. The *search sector* has a standard (and costless) bilateral matching technology providing random pairwise matches. Participation in the search process has positive external effects. A costly *multilateral matching technology*, provides deterministic double-coincidence matches in the *intermediated* *sector*. The latter designation reflects the interpretation of the multilateral matching technology as an atomless trade intermediary capable of providing matches compatible with a trader's preferences and production. It may be thought of as a computer which organizes exchange by processing information on the traders registered in its database. This eliminates the double coincidence of wants problem by guaranteeing exchange and consumption. This technology makes monetary transactions superfluous, but money may still be valued in the economy if the probability of a random bilateral trade makes search equally attractive for some.[1]

The model developed here is very much in the spirit of Kiyotaki and Wright, in that it studies alternative transaction mechanisms and is not a study of equilibrium price determination.[2] It also represents a natural extension of Kiyotaki and Wright, with the introduction of the multilateral matching technology marking a sharp departure. Now monetary exchange is challenged by costly deterministic matching, and both valuation of money and relative extent of the competing trade mechanisms is determined endogenously. Corner solutions exists either with no trade in the intermediated sector, or with money losing its role as a transaction facilitator (and consequently its value) when costly trading arrangements are broadly adopted. In between these two extremes there is a continuum of equilibria with concurrent search, “intermediated” trading activity and valued money. These occur when individuals similarly value the discounted utility stream provided by the different matching technologies. Characteristics of the environment such as a low degree of differentiation of commodities, a high amount of liquidity, and a high time-discount rate restrict the range of existence of monetary outcomes with costly “mediated” trade. Such equilibria are suboptimal if positive externalities are created by participation in search trade, a coordination failure due to the existence of a strategic complementarity in the individuals' choices. When matching costs are fixed at a per-capita level and the participation externality has monotonically increasing returns, a

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[1] While the focus here is the coexistence of money and an improved trade mechanism—my stylized atomless "mediator"—previous work has studied the role and scope of intermediation in bilateral non-monetary search markets. Among them, Rubinstein and Wolinsky (1987), and Cosimano (1996) examine endogenous determination and extent of intermediaries. So do Bose and Pingle (1995) in a monetary economy. In Bhattacharya and Haggerty (1989) intermediaries and producers coexist, when agents can choose to be either one, and a positive trading externality exists. Yavas (1994) shows that costly middlemen may improve welfare—if the search process is sufficiently expensive—and so does Li (1998) where middlemen allow traders to overcome information frictions.

[2] Some recent papers concerned with equilibrium price determination are Trejos and Wright (1995) and Shi (1995).

purely monetary regime proves to be superior to a mixed regime with both monetary exchange and costly trade arrangements.

The paper proceeds as follows. Section 2 presents the model, and section 3 the steady state equilibrium analysis. Welfare considerations are contained in section 4, section 5 extends the model with a more explicit matching technology, and section 6 concludes.

**2. Environment**

The population is constant and there is a unit measure of identical infinitely-lived agents indexed by $j \in [0,1]$. Time is discrete and continues forever.[3] Agents have heterogeneous preferences, constant across time, defined over a proper subset $x \in (0,1)$ of the [0,1] set of differentiated goods that can be produced in the economy. The subset $x$ is agent-specific and it does not include the agent's own production good. That is, $x$ denotes the probability that any trader j consumes the production of a randomly encounter party j'. Contingent on this event, I let $y \in [0,1]$ denote the probability that the randomly encountered party consumes the production of j. Thus $x(1-y)$ is the probability of single coincidence and $xy$ is the probability of double coincidence in a random match where someone has a good to offer. By letting $y=x$, I thus can define the measure $x$ as the proportion of people willing to consume any given good j, independent across time and matches (the special case $y=0$ is considered in section 5). Consumption of one unit of commodity j generates temporary utility payoff $u > 0$, if the commodity is in $x$, and zero otherwise, that is $u(q^j) = uqI_{\{j \in x\}}$ where, $q^j$ is the quantity of commodity j consumed. Each agent discounts the future at rate $r>0$. Since goods are differentiated in terms of the utility they provide and preferences are not defined over individual output, agents cannot consume in autarky and trade is necessary for consumption to take place.

All individuals are initially located in the search sector of the economy. An exogenously determined fraction $m \in [0,1)$ is randomly endowed with one indivisible unit of fiat money, the remaining $1-m$ are endowed with a production opportunity. Agents with money may dispose of it, and costlessly obtain a production opportunity. Since their choice depends on the strategies adopted by the others, I let $\mu_t \in [0,m]$ be

3 This is standard (see Kiyotaki and Wright 1989, for instance) and equivalent to a model with continuous time and transactions occurring at discrete points in time.

the endogenous proportion of individuals holding money in the economy at time *t*. Individual j with a production opportunity can use it once to costlessly produce one unit of indivisible output j. Besides the initial distribution, a production opportunity is obtained immediately after consumption has taken place. Given $u>0$, time discounting and costless production, production opportunities are used up as soon as they arise.

Commodities and money may be costlessly stored. The inventory technology has an upper bound set (conveniently) to one item. Since holding any inventory excludes obtaining a production opportunity, no one engages in production if carrying a good or money, and no one carries money and a good at the same time. This allows identification of traders according to their inventory. A *money trader* carries money and a *commodity trader* carries a commodity. The model is one of complete information and only individuals' trading histories are private information. This and the population assumption rule out the possibility of credit arrangements, the inventory and indivisibility restrictions are for the sake of tractability (they limit the dimensionality of the state space), while the assumptions on preferences and production technology motivate the need for trade.

There are two different matching technologies. Both match agents pairwise, one match per period. The *bilateral* matching technology matches traders according to a known random process as in Kiyotaki and Wright. By using the *multilateral* matching technology an agent incurs a cost, in utility terms, at the beginning of *t* in order to be paired with an appropriate partner during *t*. The technologies are operated in two different sectors of the economy, the search and the intermediated sector, respectively. In what follows I first introduce some notation and then describe these two technologies in more detail. At the end of each period *t*, individuals choose in which sector to trade the following period (see figure 1, more in Section 3).

[Figure 1 about here]

Let $e_t \in [0,1]$ define the probability that, in a symmetric equilibrium, an average commodity trader participates in the search sector in $t+1$. This is an endogenous variable, chosen at the end of *t* and corresponding to what I will later call the commodity trader's equilibrium *market strategy*. The equilibrium probability that an average commodity trader will participate in the intermediated sector in $t$+1 is 1- $e_t$.[4]

The bilateral matching technology is next discussed. A trader who, at the end of $t$, has decided to participate in the search sector, faces a known probability of encountering an individual in period $t$+1. This probability is defined by the continuous function $b(e_t)$: $[0,1]\rightarrow[0,b]$, $b<1$, and satisfies $b(0)=0$, $b'(e_t)\geq 0$ for $e_t \leq e'_t \leq 1$ and $b'(e_t)<0$ otherwise. Notice that while the functional form itself is *exogenous*, the equilibrium probability of a random encounter is *endogenous*, since $b(e_t)$ depends on the choice of all other commodity traders, $e_t$. This formulation of the bilateral matching technology can be interpreted as capturing a trading externality, as in Diamond (1982).[5] Since the search sector is characterized—following Jevons (1875)—by a double coincidence of wants problem, exchange and consumption may take place if both traders like what the other has. However if money is valued, single coincidence of wants *may* be sufficient for exchange and consumption to occur. I now describe the search process during any period $t$. When an agent is matched to another, with probability $b(e_{t-1})$, the individuals inspect each other's inventories and simultaneously announce whether they want to trade or not, hence trade occurs only if it is mutually agreeable. This may occur if—contingent on both holding a commodity—there is double coincidence of wants (with probability $x^2$), or if—contingent on one holding money—there is single coincidence of wants (with probability $x$). In the latter case the commodity trader announces the probability she will accept the currency offer of the money trader. No direct costs are incurred from transacting. Since both goods and money are indivisible, a successful exchange requires a one-for-one swap of items. If both parties have agreed to a transaction, exchange and consumption take place at the beginning of the following period, $t$+1, and is immediately followed by production. After a transaction has been completed the agents separate. The randomness in encounters implies that in each period an individual faces the risk to be left unmatched (with probability 1-

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4 A referee has pointed out that the model presented is observationally equivalent to an environment in which a lottery mechanism (with corner outcomes) randomly allocates agents across sectors. This is so because the trading sectors are mutually exclusive, and because stationary rational expectations equilibria with symmetric Nash strategies are considered.

5 The qualitative feature I want is commodity traders' optimal decisions influencing the likelihood of a random trade, with the latter feeding back on the former. Hence the choice for $b(e_t)$, independent of the equilibrium measure of money traders, $\mu_t$. Ruling out direct effects of $\mu_t$ is also in the spirit of the CRS meeting technology of Kiyotaki and Wright, and allows me to focus on the external effects due to adoption of different strategy profiles.

$b(e_{t-1})$), or to be unable to trade with his party (absence of mutual agreement) in which case he must wait until $t+1$ to attempt a new trade.

Traders can overcome the random matching problem by choosing to use the improved multilateral matching technology, discussed next. At the end of each period $t$ commodity traders decide whether to participate in the intermediated sector in the following period, $t+1$. In a symmetric equilibrium, the average commodity trader does so with probability $1-e_t$ (more in Section 3). Participating in the intermediated sector generates a per capita cost $\tau(e_t) = \tau + Cc(e_t)$ in utility terms, upon entrance in the sector. The function is continuous with $0 \leq \tau, C < \infty$, $c(e_t):[0,1] \to R_+$, $c(0) \leq c(1)$, $c'(e_t) \leq 0$ ($>0$) for $e_t \leq e_t''$ ($e_t > e_t''$), and $c''(e_t) > 0$. Once again, notice that the functional form of the cost is exogenous, but its equilibrium size is endogenous when $C \neq 0$.[6] These costs may be loosely interpreted as the expenses borne when resorting to the services of a trading intermediary. Once an individual has entered the intermediated sector she is matched with a partner who is holding the good she likes, provided she is not the only one to have chosen participation in that sector ($e_t \neq 1$). Costless exchange and consumption (followed by production) occur with certainty at the beginning of the following period, then the agents separate. This specification is adopted to focus on the advantages generated by a technology which, much like money, is capable of lessening the trade frictions stemming from randomness in trade. That is why all other similarities between the two competing matching technologies are preserved (one full period for trade to take place, one match per period, etc.).

### 3. Symmetric Stationary Equilibria

Consider active equilibria in which agents adopt time invariant strategies, the proportion of money traders is stationary and identical types act alike. The choice of trading sector and acceptance of money is

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Inclusion of $\mu$ would not change the quality of the results, at the expense of clarity of discussion (see Section 3).

6 While in a symmetric equilibrium the total cost is endogenous, $\tau(e_t)(1-e_t)(1-\mu_t)$, the per capita cost is only a function of the strategy $e_t$ (if $C \neq 0$). This rules out direct effects (on the multilateral matching technology) due to changes in liquidity. The convexity assumption seems to be reasonable for it has increasing returns (from participation in intermediated trade) only initially. One may interpret the subsequent decreasing returns as the consequences of an overcrowded intermediated sector.

made with the objective of maximizing the discounted expected utility from consumption.

Traders have a set of three possible choices in each period: which trading sector to choose (if the agent is holding a good), whether to accept money in exchange for her own inventory (if offered any), and whether to accept a good in exchange for her own inventory (if offered any). Consider the simplifying assumption that a commodity is not accepted if it cannot be consumed.[7] Define the strategy profile of commodity trader j as $\sigma^j$, a set of state-contingent rules specifying which trading sector will be chosen and the probability of acceptance of money. Let these two components be denoted respectively as the *market* and the *trading* strategy. Taking as given everybody else's actions, each period the average commodity trader chooses her *market* strategy $e \in [0,1]$, that is the probability—on which individuals have symmetric beliefs—that she trades in the search sector in the following period. Let $e^j \in [0,1]$ denote the best response of individual j, so that if all other commodity traders are choosing $e$ then commodity trader j chooses $e^j$. Let $\pi \in [0,1]$ define the *trading* strategy, the probability—on which individuals have symmetric beliefs—that an average commodity trader who is in the search sector accepts money. Let $\pi^j \in [0,1]$ denote the best response of individual j. Taking as given everybody else's actions, individual j chooses $\pi^j$. Finally, let $\sigma \equiv \{e, \pi\}$ and $\sigma^j \equiv \{e^j, \pi^j\}$.

Next, consider an agent holding currency. In principle he can choose the trading sector and whether to dispose of money in any period. In a stationary equilibrium, however, money traders trade only in the search sector since monetary transactions do not occur anywhere else. A commodity trader who has been matched by means of the costly technology is assured consumption. By agreeing to a money-for-goods exchange he would have to sustain at least one additional trading round before consumption can take place. Since time is discounted, accepting currency is then a dominated action, thus only commodity traders enter the intermediated sector in equilibrium.

Because of stationarity the individual's choice of disposing of money is considered only once, at the beginning of time. It determines the stationary fraction of money traders and depends on the trading and

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[7] By incorporating arbitrarily small transaction costs—as Kiyotaki and Wright do—one can rule out commodities being used as money: since goods have identical acceptability ($x$) no good provides exchange advantages over another. Jones (1977), Kiyotaki and Wright (1989) and Oh (1989) discuss the use of commodities as generally accepted media of exchange. Also, two money traders could swap inventory, an inconsequential action for the equilibrium analysis, and so is not considered.

market strategies adopted by commodity traders, $\sigma$. I consider equilibria in which currency is disposed of only if it is not valued, and hence let

$$\mu = m I_{\{\pi \neq 0\}} \tag{1}$$

denote the stationary distribution of money holdings in the economy.[8]

In equilibrium, knowledge of the trading strategy is sufficient to determine if traders initially dispose of money. From the definition of the strategies it follows that $(1-\mu)e$ and $(1-\mu)(1-e)$ are respectively the measures of commodity traders searching and in the intermediated sector, while $\mu$ are money traders. The measure of individuals in the search sector is $[(1-\mu)e+\mu]\in[0,1]$. The probabilities—conditional on the occurrence of a random match—of encountering a commodity trader, $p_c(\sigma,\mu)$, and a money trader, $p_m(\sigma,\mu)$ are

$$p_c(\sigma,\mu)=\frac{(1-\mu)e}{(1-\mu)e+\mu} I_{\{(1-\mu)e+\mu\neq 0\}} \tag{2}$$

$$p_m(\sigma,\mu)=\frac{\mu}{(1-\mu)e+\mu} I_{\{(1-\mu)e+\mu\neq 0\}}. \tag{3}$$

For example, $p_c(\sigma,\mu)=1$ and $p_m(\sigma,\mu)=0$ when $\pi=0$, $\mu=0$, and $e>0$ (money is not valued and someone trades in the intermediated sector), and $p_c(\sigma,\mu)=1-\mu$ and $p_m(\sigma,\mu)=\mu$ when $\pi>0$, $\mu=m$, and $e=1$ (money is valued and everyone is in the search sector).

Different combinations of market and trading strategies deliver different equilibria which can be completely characterized by the strategy profile of the representative commodity trader, $\sigma$, the fraction of money traders, $\mu$, and the value functions for commodity and money traders (considered below).

### 3.1. Individual's Choice

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[8] The restriction is innocuous and simplifies the analysis. Suppose an equilibrium exists where commodity traders are indifferent between accepting money or not, and money traders are also indifferent between keeping their inventory or not. This requires identical lifetime utility for money and commodity traders. Here a fraction of money holders, say $\alpha<1$, retains currency, thus $m\alpha$ money traders are left. The outcome is equivalent to one where $m'=m\alpha$ agents are endowed with money, they all keep it ($\mu=m'$), and commodity traders play $0<\pi<1$.

At the beginning of period $t$, agent j may be in three different "states" depending on her trading history. She may be holding a commodity while being located either in the search or in the intermediated sector, with expected discounted lifetime utilities respectively given by $V_{s,t}$ and $V_{i,t}$. Alternatively, she may be holding money and be located in the search sector, with expected discounted lifetime utility $V_{m,t}$. The sequence of actions during a trade round in period $t$ is as follows (see figure 1). Immediately after the beginning of $t$ a match is realized: with certainty if the individual is in the intermediated sector, and with probability b($e$) if she is in search. During $t$ matched traders must choose whether to exchange their inventory for their partner's, while unmatched individuals do nothing. Before the end of $t$, all individuals must also choose where to trade in the following period. Finally, at the beginning of $t$+1 all individuals who mutually agreed to exchange (during $t$) swap inventories, separate, and the recipients of commodities consume and produce. The ones who decided to participate in the intermediated sector enter it and suffer a utility loss $\tau(e)$. A new trade round then begins.[9]

Agents choose the strategy profile which maximizes their expected discounted lifetime utility. In a steady state where individuals act symmetrically and take everybody else's strategies as given, the value functions are given by (derivation in Appendix)

$$rV_s=b(e)p_c(\sigma,\mu)x^2u+b(e)p_m(\sigma,\mu)x\max_{\pi^j}\pi^j\{V_m-\max_{e^j}[e^jV_s+(1-e^j)(V_i-\tau(e))]\}$$

$$+\max_{e^j}(1-e^j)[V_i-\tau(e)-V_s] \qquad (4)$$

$$rV_m=b(e)p_c(\sigma,\mu)x\pi\{u+\max_{e^j}[e^jV_s+(1-e^j)(V_i-\tau(e))]-V_m\} \qquad (5)$$

$$rV_i=uI_{\{e\neq 1\}}+\max_{e^j}[e^j(V_s-V_i)-\tau(e)(1-e^j)] \qquad (6)$$

Equation (4) shows that the expected flow return to a commodity trader in the search sector, has three distinct components. With probability $b(e)p_c(\sigma,\mu)x^2$ she is in a double coincidence match with another commodity trader (both net $u$). With probability $b(e)p_m(\sigma,\mu)x$, she is in a single coincidence match with a

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9 This timing convention (attempt to get a match during the period, and exchange at the beginning of the following) is standard in the search literature, hence adopted to facilitate the comparison with similar models. It is also qualitatively inconsequential for the results (both matching and exchange could occur in the same period). The choice of sector could also be moved at the beginning of each period. The critical feature here is the presence of

money trader and must choose the probability of accepting the currency offered, $\pi^j$. By agreeing to the transaction she ends up holding currency and becomes a money trader. Since her best alternative is keeping the commodity, her flow payoff is $\{V_m - \max_{e^j} [ e^j V_s + (1- e^j)(V_i - \tau(e)) ]\}$. The third component vanishes for all market strategies except when the agent strictly prefers trading in the intermediated sector ($e^j = 0$) in which case it represents the flow payoff she derives ($V_i - \tau(e) - V_s$). Equation (5) has a similar interpretation, whereas equation (6) shows that a commodity trader who is in the intermediated sector (the fee has been assessed upon entrance, at the beginning of the period), receives utility *u with certainty* at the end of the trading round, if $e<1$ (0 otherwise). She can keep using the multilateral matching technology, paying $\tau(e)$. Going back to the search sector is accounted for by the change in her value function, $V_s - V_i$.

Consider the optimal market strategy when $\mu$ and $\sigma$ are taken as given. The choice of trading sector depends on whether switching sectors provides the commodity trader with a non-negative net payoff. Since moving to the intermediated sector is costly, her optimal market strategy is a decision rule $e^j : \tau(e) \rightarrow [0,1]$, mapping the entrance cost into a probability

$$e^j \begin{cases} = 1 \ \textit{if} \ \ \tau(e) > V_i - V_s \\ \in [0,1] \ \textit{if} \ \ \tau(e) = V_i - V_s \\ = 0 \ \textit{if} \ \ \tau(e) < V_i - V_S \end{cases} \tag{7}$$

In a similar manner, taking $\pi$, $\mu$ and $e$ as given, a commodity trader accepts money depending on whether becoming a money trader gives her an advantage, when compared to the best alternative offered by her current inventory position. Her optimal market strategy is a decision rule $\pi^j$: $V_m \rightarrow [0,1]$, mapping the expected value from holding money, relative to holding a commodity, into a probability

$$\pi^j \begin{cases} = 1 \ \textit{if} \ \ V_m > \max\{V_s, V_i - \tau(e)\} \\ \in [0,1] \ \textit{if} \ \ V_m = \max\{V_s, V_i - \tau(e)\} \\ = 0 \ \textit{if} \ \ V_m < \max\{V_s, V_i - \tau(e)\} \end{cases} \tag{8}$$

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time discounting together with trade frictions stemming from the restriction on coalition formation.

where $\max\{V_s, V_i - \tau(e)\} \equiv \max_{e^j} [e^j V_s + (1- e^j)(V_i - \tau(e))]$. In a symmetric equilibrium

$$\sigma^j = \sigma \tag{9}$$

A symmetric stationary equilibrium is defined as a set of time-invariant value functions $\{V_s, V_m, V_i\}$, strategy profile $\sigma^j$, and proportion of money traders $\mu$, such that: (i) individuals maximize their expected lifetime utilities, that is $\{V_s, V_m, V_i\}$ satisfy (4)-(6) and $\sigma^j$ satisfies (7)-(9), and (ii) given $\sigma$, and $\{V_s, V_m, V_i\}$, the stationary conditions for the distribution of types and inventory holdings are satisfied, i.e. $\mu$ satisfies (1) and $p_c(\sigma,\mu)$ and $p_m(\sigma,\mu)$ satisfy (2) and (3).

Note from (7) that it is sufficient to sign the expression $V_s - V_i + \tau(e)$ to characterize the optimal market strategy $e$ in an active equilibrium. Since in a symmetric equilibrium the value functions depend on the strategy profile adopted by the representative trader, define $\phi: \{\sigma,\mu\} \rightarrow \mathbf{R}$

$$\phi(\sigma,\mu) = V_s - V_i + \tau(e).$$

Substituting $\pi^j = \pi$, $e^j = e$ in (4)-(6), and rearranging

$$\phi(\sigma,\mu) = A[B(1+r)]^{-1} \tag{10}$$

where $A = ub(e)p_c(\sigma,\mu)x[x + a_1\pi p_m(\sigma,\mu)/p_c(\sigma,\mu)] + [1 + a_0/r - a_1 a_0/r][\tau(e)(1+r) - uI_{\{e<1\}}]$, $B = [1 + ea_0/r - ea_0 a_1/r]$, $a_0 = b(e)x\pi p_m(\sigma,\mu)$ and $a_1 = b(e)x\pi p_c(\sigma,\mu)/[r + b(e)x\pi p_c(\sigma,\mu)]$.

Note that $\phi(\sigma,\mu) \neq 0$ supports a symmetric pure market strategy, and $\phi(\sigma,\mu) = 0$ supports a symmetric mixed market strategy. Focusing on the latter, (7) implies that $e^j \in [0,1]$ is a best response if $V_s = V_i - \tau(e)$. When symmetric strategies are considered, however, the boundaries 0 and 1 cannot be part of a mixed strategy equilibrium because of the discontinuities of $\phi$.[10] If $e=1$ no deterministic matching can occur and search is preferred: in equilibrium $V_i = V_s(1+r)^{-1}$ hence $\phi > 0$ and $e^j = 1$ is the unique best response. If $e=0$ then no random matching can occur and costly matching is preferred: in equilibrium $V_s = [V_i - \tau(e)](1+r)^{-1}$ hence

[10] In equilibrium $\phi(\sigma,\mu)$ is discontinuous at $e=0,1$ because of $uI_{\{e\neq 1\}}$ and $p_m(\sigma,\mu)$ (which vanishes at $e=0$ since the latter implies $\pi=0$ and $\mu=0$). It is also discontinuous at $\pi=0$, due to (1). In particular, $\phi(\sigma,\mu)$ jumps to the positive quantity $rV_s(1+r)^{-1} + \tau(e)$ for $e=1$, and to the negative quantity $-r[V_i - \tau(e)](1+r)^{-1}$ for $e=0$.

$\phi<0$ and $e^j=0$. Consequently the symmetric equilibrium mixed market strategy is defined on the open set (0,1). Similar considerations can be made for the symmetric equilibrium mixed trading strategy $\pi$ which, as in Kiyotaki and Wright (1993), is defined on the open set (0,1).

To summarize, in a stationary symmetric equilibrium, given the others' strategies and $\mu$, both the extent of trade "intermediation" and valuation of money are endogenously determined. Outcomes are fully described by the combination of market and trading strategies. They may be *pure intermediated* ($e$=0), *pure search* ($e$=1), or *mixed search* ($e\in(0,1)$), and—depending on the trading strategy adopted—*non monetary* ($\pi$=0), *pure monetary* ($\pi$=1), or *mixed-monetary* ($\pi\in(0,1)$).

3.2. Benchmark Equilibria

Consider a benchmark formulation with fixed entrance cost and positive trading externality by letting C=0, so that $\tau(e)=\tau>0$, and $b(e)=be$. The symmetric stationary equilibrium value functions, strategies and the proportion of money traders are obtained as follows. A market strategy $e$ is conjectured. Taking it as given, I examine the possible optimal trading strategies $\pi$ (and the associated $\mu$). To confirm the existence of an equilibrium (i.e. a fixed point), I verify that the proposed $e$ is optimal (i.e., that it satisfies (7)) and derive conditions supporting its existence, for each optimal $\pi$. This includes verifying $V_s,V_m,V_i\geq 0$ and checking the sign of $\phi(\sigma, \mu)$. This process is repeated for all types of market strategies. The market and trading strategy combinations, the set of equilibria, their nomenclature, and their existence are summarized in table 1 where I retain the more general notation $\tau(e)$ and $b(e)$.[11]

[Table 1 about here]

Case I. Pure Search Equilibria ($e$=1)

When $e$=1 all commodity traders stay in the search sector, and the economy resembles the one in Kiyotaki and Wright (1993). From (10) it is immediate that this outcome is always viable since $\phi(1,\pi,\mu)>0$ $\forall\pi$.

**Proposition 1**. *There always exist equilibria with trade taking place in the search sector only. They* *can be either monetary or non-monetary.*

The three possible outcomes, labeled $S_\pi$, $\pi \in \{1, x, 0\}$, are fundamentally equivalent to the three equilibria in Kiyotaki and Wright (1993). Pure search equilibria are always possible due to the self-fulfilling nature of beliefs. If individuals believe no one trades in the intermediated sector, expected consumption from trade in that sector is zero. This makes entrance a dominated strategy even when access to an improved matching technology is totally free ($\tau=0$), a coordination failure. As already spelled out in Kiyotaki and Wright money is not valued if agents hold the common belief that no one exchanges a commodity for money ($\pi=0$).

<u>Case II. Pure Intermediated Equilibria ($e=0$)</u>

When $e=0$ then b(0)=0. If the cost of accessing the multilateral matching technology is sufficiently low then $e^j=0$, so that all individuals holding commodities avoid the search sector. Money traders are then unable to acquire commodities, thus freely dispose of money and produce. This is summarized in the following

**Proposition 2**. *Money is not valued when trade is carried out only in the intermediated sector. The condition* $\tau < u(1+r)^{-1}$ *is necessary for existence of this equilibrium, but not sufficient to guarantee money to be valueless in the economy.*

Denote the outcome as I (for intermediated) and notice that the belief $e=0$ is sufficient to support this non-monetary outcome as long as it is consistent, that is if the entrance cost is smaller than the discounted temporary utility from consumption, $u(1+r)^{-1}$. In what follows I will refer to this necessary requirement as a *feasibility condition* for intermediation to arise. The advantage to the society from using the multilateral matching technology is twofold. It eliminates the output loss due to the use of money (money drives out production opportunities) and it reduces the search costs, by speeding up the transaction process. While feasibility of $\tau$ is necessary for the existence of the pure intermediated equilibrium, it is not sufficient to guarantee money to be valueless in the economy: by Proposition 1 random search monetary equilibria are always possible. The coexistence of these two corner equilibria ($e=0,1$) explains why the possibility of resorting to a deterministic and inexpensive matching technology may deprive money of value, although not

[11] Proofs and algebra in Appendix. The examples' baseline is $b(e)=be$, $\tau=0.89$, $x=0.3$, $m=0.35$, $r=0.05$, $u=1$, $b=0.9$.

necessarily so.

Case III. Mixed Search Equilibria ($0<e<1$)

Consider $e^j=e\in(0,1)$ which, from the discussion in the previous section, is a symmetric equilibrium whenever $V_s = V_i-\tau$. I denote with

$$e^*(\pi,\mu) \equiv \{e|\ \phi(\sigma,\mu)=0,\ e^j=e\in(0,1)\}$$

the set of equilibrium mixed strategies, a function of $\pi$ and $\mu$. In order to derive sufficient and necessary conditions for the existence of mixed market strategy equilibria, the following lemmas characterize $\phi(\sigma,\mu)$ and identify the set of $\tau$'s supporting existence of a mixed market strategy equilibrium. Using $\phi(\sigma,\mu)$ and invoking symmetry of strategies define

$$\tau_\pi = \frac{u}{1+r}\{1-b(1)x^2(1-\mu)\frac{r+b(1)\pi[x(1-\mu)+\pi\mu]}{r+b(1)\pi x}\},\ \ \pi\in\{0,x,1\}$$

**Lemma 1**. $\phi(\sigma,\mu)$ *is continuous on* $e\in(0,1)$ *and* $\pi\in(0,1]$, *discontinuous at* $\pi=0$, *and* $\lim_{e\to1^-}\phi(\sigma,\mu) > \lim_{e\to0^+}\phi(\sigma,\mu)$.

**Lemma 2**. *If* $\tau_\pi < \tau < u(1+r)^{-1}$ *then* $e^*(\pi,\mu)$ *is non-empty* $\forall\ \pi\in\{0, x,1\}$.

**Lemma 3**. *Let* $e\in e^*(\pi,\mu)$, *then* $\partial\phi(\sigma,\mu)/\partial e >0\ \forall\ \pi\in\{0, x,1\}$.

Three types of symmetric stationary equilibria with coexistence of search (with or without money) and intermediated trade, labeled as $M_\pi$, $\pi\in\{0,x,1\}$, may arise. This is outlined in the following

**Proposition 3**. *There exist equilibria with* $e\in$ *(0,1) where money and costly intermediated trade may coexist. The condition*

$$\tau_\pi< \tau < u(1+r)^{-1} \qquad \pi\in\{0,x,1\} \qquad (11)$$

*is sufficient and necessary for the existence of three distinct and unique equilibria: two monetary (mixed and pure) and one non-monetary.*

Consider the non-monetary equilibrium. For commodity traders to be trading in the intermediated sector the cost $\tau$ must be feasible. Incentives must also exist for some commodity traders to be willing to trade in the search sector, hence $\tau$ cannot be too small either, $\tau>\tau_0$. This last expression requires the present value of the difference between the (certain) payoff derived from intermediated trade ($u$) and the (expected) maximum payoff attainable from search trade ($ubx^2$), to be smaller than the disutility suffered by resorting to costly matching ($\tau$).

When commodity traders accept money with probability $x$, a unique $e$ generates the mixed monetary equilibrium $M_x$ where search and intermediated trade coexist. The proportion of money traders, $\mu$, must now be taken into account in determining the conditions for the existence of equilibria. The cost of obtaining a match must exceed the present value of the (maximum) difference between the utility payoffs for the two sectors: $u$ (intermediated sector) minus $ubx^2(1-\mu)$ (search sector with full participation). Similar considerations can be made for pure monetary equilibria with intermediation, $M_1$, which are also uniquely determined.

The existence of each equilibrium is affected by trading externality, search frictions, and liquidity level. The set defined by $\left(\tau_\pi, \frac{u}{1+r}\right)$ shrinks with increasing difficulty of search. As b or $x$ fall, the set of $\tau$'s supporting a mixed market strategy equilibrium shrinks to a singleton, $u(1+r)^{-1}$. Individuals face a tradeoff between $\tau$ and the degree of differentiation of goods, $x$. If cheap intermediary services are available, some traders may search only if the bilateral matching technology promises a high likelihood of a match ($x$ large). Conversely, even a rather expensive deterministic matching technology may be attractive when tastes are highly diverse ($x$ small).[12]

This is illustrated in figure 2 which, for the baseline case, depicts the set of feasible $\{\tau, x\}$ pairs supporting the different types of interior outcomes.

[Figure 2 about here]

The horizontal line originating at point A delimits the set of feasible $\tau$, by marking the upper bound $u(1+r)^{-1}$. All parameterizations lying in the space above that line support only the corner outcome with random search, $e$=1 (monetary or not). Below the horizontal line there exists a multiplicity of equilibria,

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[12] One can verify $\tau_1 \leq \tau_x$, $\tau_0 \leq \tau_x$, and $\tau_0 \leq \tau_1$ if $x \geq (1-\mu)/(2-\mu)$. If $\pi=x$ money is least effective and $\tau_\pi$ must be larger (vs. $\pi$=1) to "penalize" more intermediated trade. Similarly, $\tau_0 \leq \tau_x$ since meeting commodity traders is less likely with money circulating. Last, $\tau_0 \leq \tau_1$ for large $x$ (money is least beneficial). In figures 2-3, $\tau_x$ always binds first.

monetary and non monetary: the two corner outcomes $e$=0,1 *always* exist, and interior outcomes $e\in(0,1)$ *may* exist. The regions (1) through (5) indicate where interior outcomes exist and, contingent on that, whether they can be monetary or not. The $\{\tau,x\}$ pairs lying in area (1) (above the curve $\tau_x$) support all interior equilibria, monetary or not, since $\tau$ satisfies (11) for all $\pi$. The opposite is true in area (4), where no equilibrium $e\in(0,1)$ exists. Partial participation in intermediated exchange (sometimes monetary, sometimes not) is an equilibrium in the remaining areas. In region (2), between the $\tau_1$ and $\tau_0$ curves, money and intermediated trade cannot coexist, and only $M_0$ exists. The opposite occurs in the area between the curves going through points A and B ($M_0$ does not exist, but $M_\pi$ does for $\pi=x$,1). Pure monetary and non-monetary interior equilibria exist in area (5), enclosed by the three curves $\tau_0$, $\tau_1$, and $\tau_x$. The role of double coincidence and costs for the existence of the different types of interior equilibria can be best understood by moving, alternatively, vertically and horizontally in the picture. Fix $\tau$. As $x$ increases the set of equilibria with some participation in intermediated exchange grows. While no such equilibria exist when the double coincidence problem is severe, the $M_0$ equilibrium first comes to exist as $x$ grows, while coexistence of money and intermediated trade ($M_1$, and $M_x$ subsequently) is also possible as $x$ heads further towards 1. Now fix $x$ and move south starting from the horizontal line originating at A. Coexistence of money and intermediated trade is possible for $\tau$ large. As $\tau$ becomes smaller, first the mixed monetary, and then the pure monetary outcomes, $M_x$ and $M_1$, cease to exist. As $\tau$ shrinks further the only interior equilibrium which still exists is non-monetary, but it disappears as $\tau$ becomes sufficiently low. That is, while money coexists with mediated trade when the latter is feasible but expensive, costly deterministic matching is strictly preferred to monetary exchange when the former is sufficiently cheap.

Figure 3 illustrates coexistence of money and intermediated trade for different money supplies.

[Figure 3 about here]

As in figure 2, the areas of existence of different equilibria are marked and enclosed between the three curves $\tau_\pi$ and the horizontal curve defining feasibility of $\tau$. High liquidity levels reduce the set of $\tau$'s supporting coexistence of intermediation: $\tau_\pi \rightarrow u(1+r)^{-1}$ as $\mu$ converges to one. An increase in money negatively affects the probability of meeting a commodity trader. At high liquidity levels this also offsets the beneficial effects of monetary trade, hence only a larger $\tau$ keeps individuals indifferent between the two trading sectors. At low liquidity levels changes in the stock of money have a different effect. When $\pi=x$ currency does not provide an advantage over barter, and an increase in $\mu$ only impairs the frequency of consumption, hence the set of feasible $\tau$'s shrinks. When $\pi=1$, a larger money supply may enhance the search process if search frictions are severe. Smaller entrance costs are thus compatible with mixed market outcomes. The role of time discounting is similar. Larger $\tau$'s are admissible when traders are more patient since, on the margin, time discounting takes a heavier toll on trades carried out in the intermediated sector. This latter feature stems from the requisite of indifference across trading sectors: commodity traders searching *must* have lower lifetime utility in equilibrium ($V_i>V_s$) and thus are less impacted by larger discounting.[13]

The comparative statics show the importance of the positive feedback from the trading externality on individuals engaged in search. When $\pi=0$, commodity traders tend to use intermediated services more when they are more impatient, $\partial e/\partial r<0$. A smaller probability of getting a random match (generated by a lower $e$) is needed to offset higher time discounting costs and keep commodity traders indifferent between the two sectors. Similarly, less individuals search as the double coincidence of wants problem becomes less severe: $\partial e/\partial x<0$ because the frequency of random encounters needs to fall to keep the indifference balance between the sectors. This also explains why $\partial e/\partial \tau<0$. Similar considerations can be made when $\pi\neq 0$. The implicit function theorem allows characterization of the solutions (see the Appendix): $\partial e/\partial \tau<0$, $\partial e/\partial r<0$, $\partial e/\partial x<0$ for any $\pi\neq 0$, and $\partial e/\partial \mu>0$ if $\pi=x$ . When $\pi=1$, $\partial e/\partial \mu$ can be either positive or negative depending on $x$ and $\mu$. If too many production opportunities are forgone ($\mu$ large), or the double coincidence of wants problems is limited ($x$ large), additional currency only weakens the random matching process, so $\partial e/\partial \mu>0$.

When one adopts the more general formulation of endogenous entrance costs ($\tau(e)$ convex) or a trading externality with some decreasing returns ($b(e)$ concave), the above results show little change. The main difference is the potential multiplicity of equilibria. When $\tau(e)$ is convex, $\tau(0)<u(1+r)^{-1}$ and $\tau_\pi < \tau(1) < u(1+r)^{-1}$ must be substituted, respectively, in Propositions 2 and 3. The inequalities are now only sufficient (not necessary) but still guarantee uniqueness (see Appendix), and the comparative statics results

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[13] Use (4), (6), $\pi=0$ and $e\in e^*(0,\mu)$ to examine $\partial V_i/\partial r$, and $\partial V_s/\partial r$. For an increase in r traders in the intermediated sector have marginal lifetime utility loss $-(u-\tau)/r^2$. Traders in search have a smaller marginal loss, $-bex^2u/r^2$, since -

depend on the size of *e* (whether it is located in the decreasing or increasing returns region of the cost function).[14] Similar considerations apply to the case of a trading externality with decreasing returns on part of its domain. Condition (11) is sufficient (but not necessary), potentially multiple mixed market strategy equilibria exist, and the set of costs supporting them is smaller (b(1)<b implies $\tau_\pi$ is larger than the benchmark case). The comparative static results depend on the size of *e* but for a different reason: high levels of participation in search trade may now be the source of negative feedback for commodity traders in search.

Furthermore, one may consider a trading externality that is not only a function of the strategy profile (hence of the mass of commodity traders in search), but of the measure of all traders searching, including the ones holding money. While changes in the amounts of liquidity—under this different modeling choice—would affect both the liquidity level and trade externality, the conditions for existence of equilibria and the comparative statics results (concerning changes in *m*) would not.[15] Finally if the utility *u* were derived at the same time $\tau(e)$ is incurred, the discounting factor r would not affect the upper bound of $\tau$ in $M_\pi$ equilibria and changes in r would not influence *e* for $\pi=\{0,x\}$. In such a case only the probability of a match vs. the cost of sure trade matters (either money is not used or traders are indifferent towards it). The equilibrium *e* would fall with higher r for $\pi=1$ and a positive trading externality. Obtaining money has a net positive payoff but the money cannot be used before one period has elapsed, so increased impatience lowers the lifetime utility of commodity traders in search; *e* must fall for the externality's feedback help maintaining indifference between the trade sectors.

## 4. Welfare Considerations

To rank the outcomes, consider the ex-ante lifetime utility of an individual defining the welfare measure

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$(u-\tau) < -bex^2u$ implies $\tau<\tau(1+r)$, at the equilibrium *e*.

[14] Consider $\pi=0$ and a larger *x* benefiting search: *e* must drop when the trading externality is positive. However if $\tau(e)$ is convex, the cost may *increase* for lower *e*. These diverse effects on matching costs—due to changes in *e*—will reflect differently on the relative value of lifetime utilities, and hence will differently affect the sign of $\partial e/\partial x$.

[15] Substitute $Q(e,\mu)=\mu+(1-\mu)e$ (the mass of search traders) instead of *e* in the trading externality. An increasing and low $\mu$ would improve even more the random matching process (vs. the original modeling choice). Hence *e* or $\tau$ would have to fall even more to restore the indifference balance. The opposite occurs for high $\mu$.

$W(k)=\mu V_m+(1-\mu)V_s$, for $e\in(0,1]$ and $k\in\{S_\pi,M_\pi\}$, and $W(I)\equiv(1+r)V_s=\dfrac{u-\tau(1+r)}{r}$ for $e$=0.[16] I first examine monetary and non-monetary outcomes separately, in order to compare welfare across different market strategies, and then compare outcomes across trading strategies (see table 2).

[Table 2 about here]

A. Non-Monetary Regimes.

Consider the benchmark model and notice that welfare is independent of the money supply when equilibria are non-monetary. For example in a pure search equilibrium $W(S_0)\equiv V_S=bx^2(u/r)$, i.e. the infinite sum of temporary utilities from consumption from the second period of life on. When a positive trading externality exists, the following is shown

**Proposition 4**. *Consider non-monetary equilibria. If the costly matching technology is sufficiently inexpensive, welfare is highest when all individuals participate in intermediated exchange. Otherwise welfare is highest when there is full participation in random search.*

Only if the alternative trading mechanism is sufficiently expensive random matching is superior to deterministic matching. In that case $W(S_0)>W(M_0)=W(I)$, and notice that welfare in the outcomes with active intermediation ($M_0$ and I) is similar due to both the indifference across trading sectors and the absence of money. The positive participation externality is instrumental since $W(S_0)<W(M_0)$ could result if large participation in search activities were to produce a negative feedback (b($e$) strictly concave, and b($e$)>b(1) on part of the domain). To illustrate the proposition consider a social planner maximizing welfare by directing traders in the sector providing the largest net payoff from trade. By construction all traders are in search before the initial period. They would consume for sure in period two if they opted for the intermediated sector in period one, while they would consume with probability $bx^2$ if they searched in period one. The planner recommends entrance in the intermediated sector (in $t$=1) if the expected discounted utility

16 This amounts to computing the expected lifetime utilities of a trader *before* the distribution of money takes place, at the beginning of $t$=1. Observe that $\mu V_m + (1-\mu)[eV_s+ (1-e)(V_i-\tau)]$ is the ex-ante lifetime utility, equivalent to W(k) for both $e$=1 and $e\in(0,1)$ since $V_i-\tau=V_s$. For $e$=0 the ex-ante lifetime utility is $W(I)=-\tau+V_i\equiv V_s(1+r)$ since *all* traders enter the intermediated sector in period one (suffering $-\tau$), and start consuming *only* in the second period of life. Their lifetime utility is $-\tau$ plus the infinite discounted sum of net utility $u-\tau$ from period 2 on.

derived from search trade (in $t$=2) is less that the discounted payoff derived from deterministic matching (in $t$=2) net of the disutility generated by the entrance cost (incurred at $t$=1), that is only if $\tau < u(1+r)^{-1}(1-bx^2)=\tau_0$ (see Appendix).

This is best illustrated by a numerical example. The baseline $\tau$=0.89 is feasible ($u(1+r)^{-1}$=0.952) and satisfies inequality (11) ($\tau_0$=0.875), hence non-monetary equilibria with either full or partial participation in costly intermediation can both exist ($e$=0.808 supports $M_0$). Since non-monetary search equilibria are always possible, I evaluate welfare at each of the three equilibrium market strategies: $W(S_0)$=1.62, and $W(M_0)$=W(I)=1.31. If $\tau<\tau_0$ then $M_0$ does not exist, and two corner outcomes are left to compare. If, for instance, $\tau=\tau_0$-0.01 then W(I)=1.83>$W(S_0)$=1.62.

It is immediate to see that the proposition is robust to alternative assumptions. In particular if the entrance cost occurs at the time of consumption, and not the period before, would only modify the bound on $\tau$ to $u(1 - bx^2)$.

B. Monetary Regimes

Consider monetary outcomes and observe that equilibria where $\pi=x$ are welfare dominated by equilibria where $\pi$=1. This is due to the displacement of production created by money. The equilibrium $\pi$=1 is superior because consumption is more frequent in the face of an identical loss of production, and one may interpret $\pi=x$ as a coordination failure. For this reason in what follows I focus only on pure monetary equilibria with $\pi$=1, for the benchmark formulation.

**Proposition 5**. *Consider equilibria with valued money. Suppose the equilibrium with partial participation in costly trade intermediation exists. Then the latter outcome is welfare dominated by the equilibrium where all traders participate in random matching*.

When fiat money is valued, a benevolent planner would not allocate individuals across the two sectors because of two distinct disadvantages. In the absence of full participation, the average payoff from search trade would be negatively affected because of the positive participation externality. A further loss is caused by the costly matching process. The optimal action is to direct all traders to the search sector where both commodities and monetary assets are traded. There appears to be a strategic complementarity in the choice of market strategies since full participation is critical to achieve the best outcome for the average trader.

When monetary exchanges are an integral component of the payment system, using up resources to partially overcome search frictions may lead to an inferior outcome. The introduction of a costly trading technology innovation—allowing individuals to avoid currency transactions—appears to be socially undesirable unless there is a generalized adoption of the new method of transacting. This result seems to retain some of the flavor of Hart (1975), although the present framework is non-competitive.

One can interpret the occurrence of $e \in (0,1)$ as a coordination failure. Usage of money negatively affects welfare because of the crowding-out of production and an additional cost is due to the use of costly matching. Since random trade benefits from participation, the overall efficiency of the exchange process can be improved by reaching a corner solution, a result that conforms to the analysis contained in Diamond (1982). A role for a superior authority is foreseen in addressing this coordination failure. Note the importance of the nature and the form of the trading externality, since the result need not hold when the externality does not have constant and increasing returns from participation. This is easily verified from the proof of the proposition (by substituting b(1) for b and b($e$) for b$e$). An illustration is contained in figure 4 where the interior equilibria $M_1$ are superior, when they exist (this depending on the quantity of money).

[Figure 4 about here]

The figure is drawn for the baseline parameters with b($e$) strictly concave, $b(e)=b-(d-e)^2$ and d=0.5. Two equilibrium interior $e$'s, and two corner solutions exist. At the corner $e$=0 welfare W(I) is constant across money supplies. Due to the “crowding out” effect of money, welfare at the other corner, $W(S_1)$, is hump-shaped and greater than W(I) only on a subset of $\mu$ values. Because two interior roots exist, two welfare measures are reported for $M_1$, $W(M_1\text{-L})$ and $W(M_1\text{-H})$ (respectively, the low and high root). While the interior equilibrium is the superior one, in this example the one with the largest participation in random trade is the best overall, $W(M_1\text{-L})<W(M_1\text{-H})$. It can be verified that, for each $\mu$, the largest $e$ generates a substantially larger (smaller) probability of meeting a commodity (money) trader, but only a marginally smaller b($e$). The latter is a negative external effect due to the concavity of b($e$) which, however, is more than compensated by an increased frequency of consumption matches.

Finally, observe that the proposition *does not* imply that whenever there is a monetary equilibrium welfare is lowest for *any* participation in the costly intermediation technology. Furthermore, the ranking of welfare across participation depends on both quantity of money and costs, since the *equilibrium e* is a function of the underlying parameters. In particular, interior outcomes (monetary or not) do not exist on the entire parameter space. For instance, low $\tau$ may rule out $M_1$ (if (11) is violated), and so may large $\mu$'s since $\tau_1$ increases in $\mu$ (see figure 4).

C. Comparison Across Regimes

In the light of the above considerations, I now consider comparison of welfare across stationary economies with different trading regimes. Of particular interest is considering whether money generally retains its welfare improving role for *any* degrees of market integration. Townsend (1983) for instance—in ranking welfare across stationary economies with different economic environments and exogenously imposed financial regimes—finds that autarky is welfare-dominated by a monetary regime, which in turn is inferior to a non-monetary credit-debit regime. In accordance with the above, I examine three comparable stationary economies where the economic environment is ex-ante identical (unlike Townsend's) but ex-post heterogeneous since different financial regimes may arise as an equilibrium outcome. I consider, respectively, the non-monetary random search regime ($S_0$), the monetary regime with random search only ($S_1$), and the regime with non-monetary trade and costly matching only (I).

**Proposition 6**. *Consider an economy with very diverse tastes, and an inexpensive costly matching technology. Welfare is largest when all traders avoid random matching*.

One may interpret the proposition above as saying that monetary regimes are not the optimal choice for economies where an efficient—although costly—trading technology is available and where the degree of specialization is high (very diverse tastes). Money acquires value because of its ability to *partially* defeat trade frictions deriving from imperfectly functioning spot markets. But an economy with pervasive trade frictions may have an incentive to *fully* defeat such frictions, even if some resources need to be used up in the process. The amount of per-capita resources cannot be too large, though: from the proof of the proposition

$$\tau<u(1+r)^{-1}\{1-b(1-\mu)x[\mu+x(1-\mu)\} \qquad (12)$$

which has an intuitive explanation. Since accomplishing trade is difficult (low $x$), money is beneficial. Non-monetary outcomes with random search (Townsend's autarky) are thus eliminated from the set of optimal candidates. Non-monetary outcomes with less than full participation in costly matching trade are also eliminated (since $\tau<\tau_0$ for small $x$). Widespread acceptance of money is also a better outcome than partial acceptance ($\pi$<1), hence the latter is not a good candidate either. Additionally, a monetary regime with no intermediation is better than one with partial participation in intermediated trade ($W(S_1)>W(M_1)$ in Proposition 5), thus leaving $S_1$ (Townsend's monetary regime) and I (Townsend's credit-debit regime) to be compared. Now consider a benevolent planner. He would eliminate money and have all individuals use deterministic matching only when the net present payoff stream it guarantees, $-\tau + (u-\tau)/r$, exceeds the discounted sum of the expected payoff in the money-only regime, $\{\mu[b(1-\mu)xu]+(1-\mu)[b(1-\mu)x^2u]\}/r$. The difference between these two payoff streams determines a "least efficiency" level which the multilateral matching technology must guarantee, i.e. $\tau$'s upper bound in (12). Below that level monetary exchange is preferable.[17] This is illustrated in figure 5 in which W(I)* is a globally superior outcome when drawn for $\tau$ equal to the minimum upper bound in (12) (achieved at $\mu=(1-2x)/(2-2x)$) minus 0.01.

[Figure 5 about here]

Note also that when $M_1$ exists it is superior to I, and—in line with proposition 4—$W(M_1)$ converges to W(I) as $\mu\to 0$. This because W(k) puts weight $\mu>0$ on $V_m$, and $V_m>V_s=-\tau+V_i=W(I)$. It must here emphasized that the initial stock of money is not constant across monetary/non-monetary equilibria, and thus affects the stock of output. This "crowding out" feature is taken into account in Proposition 6 since it directly affects the trade-off between money and the costly matching technology.[18] This is best understood by observing that the set of admissible $\tau$'s increases not only as $x$ falls but also as the initial money stock grows, causing a displacement of production. From (12), if $\mu$ is close to one, the upper bound for $\tau$ is close to $u(1+r)^{-1}$.

Finally, it is easy to confirm that mixed monetary strategies will still generate a dominated outcome in the general case where $b(e)$ is concave or $\tau(e)$ is convex. Because of the non-linearities engendered, however, it is difficult to rank the multiple equilibria across the degree of endogenous participation, $e$, for $\pi=0,1$. For $\tau(e)$ convex proposition 5 is unchanged, and proposition 4 still holds if $\tau(0)<u(1+r)^{-1}(1-bx^2)$. Proposition 6 need not hold since an $M_1$ equilibrium may still exist despite (11) being violated (as noted in

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[17] The difference in payoffs must be positive (i.e. $\tau\geq 0$), assured by r small. Namely, $(1+r)^{-1}$ must exceed the sum of the weights on the utility for money traders, $\mu b(1-\mu)x$, and commodity traders, $(1-\mu)b(1-\mu)x^2$.

[18] Relaxing the storage assumption eliminates the "crowding out" but requires solution to a complex stationary distribution of multiple inventories of objects. See for instance Camera and Corbae (forthcoming).

section 3.2). A concave b($e$) implies that, when money is not valued, full participation in costly trade is the best outcome if $\tau<u(1+r)^{-1}[1-b(1)x^2]$. However, it is easily seen from table 2 that if money is valued and the supply of money is sufficiently small, partial participation in costly trade is superior to the corner outcome with random search (as an illustration, compare W($M_1$) in figure 4 to W(I)* in figure 5). Proposition 6 can thus be amended in the more general case where the trading externality does not exhibit monotonically increasing returns. When mediation costs are small the full participation in costly matching is still the superior outcome. However, a monetary economy with a moderate money supply and *some* degree of participation in costly trade can be the "second best".

## 5. An Explicit Intermediation Technology

So far the multilateral matching technology has been interpreted as a "black box", for tractability. I now sketch a model with a more complete structure and a more explicit intermediation technology capable of endogenizing $\tau$ and its relationship to the degree of market participation $e$, and where double coincidence matches can be attained by an intermediary which arises endogenously.

To simplify the analysis I consider the case where there is never double coincidence so that exchange requires either money or intermediation ($y$=0). This requires only a slight modification to the original environment. Now the initial population is divided in N types $i\in\{1, 2,\ldots,N\}$ each in equal proportion $x$=1/N. An individual of type $i$ consumes only good $i$, preferences are as in section 2, and normalize $u$=1. Type $i$ can produce one divisible unit of good $i$+1 (modulo N) contingent on previous consumption, suffering fixed disutility $c\geq0$. Goods cannot be stored. A proportion $m$ of each type is initially endowed with one unit of money (which they can dispose of), I call them *buyers* and their lifetime utility is $V_m$. The remaining proportion 1-$m$ of each type (the *sellers*) is initially endowed with a production opportunity, I call them *sellers* and their lifetime utility is $V_s$. There is one intermediation technology that can be owned by one (atomless) individual. At the beginning of life and before the initial distribution of endowments, anyone can choose to own it and to become the intermediary. Since the intermediary is of measure zero, the stationary distribution of money or production opportunities is unaffected by this initial choice. In choosing between trade or ownership of the technology, an individual compares the respective ex-ante lifetime utilities (before the initial distribution, that is). The intermediary is not part of the initial distribution of endowments, but his technology allows costless transformation of any good in his consumption good.

Furthermore, he can make his position known to everybody, and can keep record of transactions occurring only *within* (but not across) each period. One way to describe the intermediation technology in $t$ follows. Each participant in the intermediated sector is identified according to her type, her unit production collected, an account opened in her name and credited for the production. If the individual is not capable of producing she is turned away. A fraction $\tau(e)\in[0,1]$ of her good is kept by the intermediary and transformed into personal consumption (during the period). The fraction is a function of the participation rate, provides the intermediary with utility $\tau(e)$, and represents the per-capita payment for the trading services. Having produced, the participant is entitled to $1-\gamma(e)$ units of her consumption good. Before the end of $t$ all producers receive $1-\gamma(e)$ units of their consumption, and this balances their accounts. At the beginning of $t+1$ traders and intermediary consume, the accounts are reset, and a new trading round begins.

Consider stationary strategies which are symmetric across the N types. A seller's market strategy $e\in[0,1]$ now represents the equilibrium fraction of sellers of each type $i$ participating in search ($1-e$ have chosen intermediated trade). Money traders cannot participate in intermediation since, not having consumed in the past, they cannot currently produce.[19] As before the probability of a random match is $b(e)$, but now $x$ is the probability of single coincidence, while 0 is the probability of double coincidence. Because of the latter (and costly production) random exchange requires money, with buyers making a take-it-or-leave-it offer to sellers (one unit of good in exchange for one money). In equilibrium there is a proportion $x(1-\mu)$ of sellers of type $i$, $x(1-\mu)e$ are in the search sector, and $x(1-\mu)(1-e)$ are in the intermediated sector. There are also $x\mu$ buyers of type $i$ searching. Because of symmetric strategies and uniform distribution of types, an *equal* measure $x(1-\mu)(1-e)$ of sellers of *each type* participates in intermediation, so that an average seller entering the intermediated sector has the certainty of consumption as long as $\tau(e)<1$ (the participation fee is not too high) and $e<1$ (participation in intermediation is the equilibrium strategy). Since the fraction of good $i$ which is *not* consumed by the intermediary is $x(1-\mu)(1-e)(1-\tau(e))$, in equilibrium each seller $i$ who has entered receives $1-\gamma(e)=1-\tau(e)$ consumption.

I now set $c=0$ (as in the previous sections) and note that the value functions are very similar to (4)-(6)

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[19] They could be let free to enter the intermediated sector, and maybe the intermediary or some participants would accept money in exchange for commodities. I suspect this could be an equilibrium if the distribution of types is non-uniform or asymmetric strategies across types cause a non-uniform distribution of intermediated goods.

since in a symmetric monetary equilibrium where $\sigma'=\sigma=\{e,\pi\}$

$$rV_s=b(e)p_m(\sigma,\mu)x\pi\{V_m-[eV_s+(1-e)V_i]\}+(1-e)(V_i-V_s)$$

$$rV_m=b(e)p_c(\sigma,\mu)x\pi\{[eV_s+(1-e)V_i]+1-V_m\}$$

$$rV_i=[1-\tau(e)]I_{\{e\neq 1\}}+e(V_s-V_i).$$

I now discuss the equilibrium strategies, considering the trading strategy $\pi$ first. It is clear that a non-monetary, and a pure monetary equilibrium always exist, so I can focus on $\pi=1$ when $e>0$.[20]. Next, taking as given $\sigma$ consider the choice of becoming the intermediary, where his ex-ante lifetime utility is

$$V=\sum_{t=2}^{\infty}\frac{(1-\mu)(1-e)\tau(e)}{(1+r)^{t-1}}\equiv\frac{(1-\mu)(1-e)\tau(e)}{r}$$

at the beginning of the initial period.[21] Since the ex-ante lifetime utility of a trader is $(1-\mu)V_s+\mu V_m$ when $e\in(0,1)$, then

$$V=(1-\mu)V_s+\mu V_m \tag{13}$$

must hold in order for any individual to be indifferent between the two economic activities. When $e\in(0,1)$ equation (13) determines the endogenous $\tau(e)$ as a function of $\sigma$ and the parameters. At the corner $e=1$ (13) must hold as a strict inequality ("less than" sign), V=0, and $\tau(e)$ is indeterminate. At the corner $e=0$ (13) must be replaced by $V=V_i$. Now consider the market strategy $e$. Because of the timing of intermediation costs (7) is slightly different. Taking as given $\sigma$ and $\tau(e)$, a seller's best response is a symmetric equilibrium if

$$e\begin{cases}=0 & \text{if } V_i>V_s\\ \in(0,1) & \text{if } V_i=V_s\\ =1 & \text{if } V_i<V_s\end{cases} \tag{14}$$

[20] A mixed monetary equilibrium may exist only if $c>0$, for $e\in(0,1]$. A seller sets $\pi\in(0,1)$ if $V_m-c-\max\{V_s,V_i\}=0$ which, due to the absence of barter matches, implies $\max\{V_s,V_i\}=0$ (see the value functions). A buyer buys if $\max\{V_s,V_i\}+1-V_m>0$, and this implies $V_m>0$. When $c=0$ conjecturing $\pi\in(0,1)$ implies $\max\{V_s,V_i\}=0=V_m$, a contradiction.

[21] Recall that agents choose their market strategies before the beginning of t=1, thus the intermediary can operate from t=1, and initially consumes at the beginning of t=2.

where the mixed strategy does not include the corners 0 and 1 because in equilibrium $V_s=V_i(1+r)^{-1}$ if *e*=0 and $V_i=V_s(1+r)^{-1}$ if *e*=1. Because equilibria are now characterized by both (14) and (13), it is clear why this framework is capable of endogenizing both the intermediation cost, $\tau(e)$, and its relationship to the market participation rate *e*.

It is easy to see that the existence results are robust to the new specification (see the Appendix). Both corner outcomes exist, the monetary equilibrium *e*=1 always, and the non-monetary intermediated equilibrium whenever *e*=0 and $\tau(e)=1/2$. It is also easy to show that a unique monetary equilibrium with intermediation exists for positive small $\mu$, and such that the intermediation cost is bounded away from $\tau(0)$. Because of the trading externality, the monetary outcome with some intermediation is still dominated by the monetary equilibrium with only random matching, as in the previous section. However, the non-monetary equilibrium with only costly matching is welfare superior to monetary search for any money supply. It is easy to see that in the best possible case (*x*, b close to one) the monetary outcome guarantees one unit to only half of the population ($\mu=1/2$), still worse than getting half a good forever ($\tau=1/2$ when *e*=0).

This framework could be easily amended to study other issues. For instance one could allow for competing intermediation technologies to characterize the endogenous extent of intermediaries and the resulting distribution of prices. One could also consider how different participation across types (say, $e_i \neq e_i$ for type $i \neq j$) or endogenous choice of types would affect availability and efficiency of intermediation as compared to a monetary regime. Finally, one could think of shocks to production in an economy where intermediaries can keep records of past transactions, offer credit, and issue private money, in order to study its coexistence with fiat money.

## 6. Conclusions

The paper has examined the robustness of monetary equilibria in a prototypical absence-of-double-coincidence economy. Individuals can do away with the trade frictions due to random matching by resorting to a costly trade mechanism not requiring usage of media of exchange. A standard bilateral matching technology characterized by a participation externality is available in the *search sector*. Costly and deterministic matching is available in the *intermediated sector*. Agents maximize their lifetime expected utility by choosing a strategy profile specifying not only whether currency is to be accepted, but also whether (and how often) to resort to costly trading arrangements.

This model is a natural extension of Kiyotaki and Wright (1993), with different trading regimes arising depending on individuals' beliefs and optimizing behavior. Monetary and non-monetary symmetric stationary equilibria are both possible, with either exclusive or simultaneous utilization of each of the two separate matching technologies. In each case the extent of deterministic matching is endogenously determined, its occurrence depending on the size of its cost relative to the probability of a random trade. In particular, trading regimes where valued money coexists with "mediated" exchange are also possible, and there may be multiple equilibria. Factors influencing the existence of different types of equilibria include not only the degree of search friction and amount of liquidity, but also the extent and form of the trade externality.

Equilibrium valuation of money fades away at one corner, when only "mediated" costly exchange of commodities takes place. When one considers Kiyotaki and Wright (1993) as the opposite corner solution, where only random matching trades occur, it is seen that money is quite robust to introduction of a costly and improved trading mechanism. In fact, a whole continuum of monetary equilibria exists in between, ranging from very small degrees of market "integration" (the end close to Kiyotaki and Wright's monetary outcome) to very large degrees of market "integration" (the end close to the opposite corner solution, with widespread deterministic matching). This result, I suspect, would not be affected by the inclusion of endogenous price formation in the model, via bargaining over divisible output.

Because of the dominance of (either of the) corner solutions, the model suggests the existence of a strategic complementarity in the choice of the trading technology, similar to Diamond (1982). The occurrence of dominated outcomes—which may be interpreted as coordination failures—could perhaps be ascribed to the absence of a coordination technology, and may justify the need for a superior authority (a government?) in coordinating trade patterns. In particular, coexistence of money with costly trade arrangements delivers a suboptimal outcome when costs are fixed and a positive trading externality exists. A monetary regime a' la Kiyotaki and Wright would be preferred only when the alternative matching mechanism is too expensive, otherwise avoiding monetary transactions would be best.

The simple framework presented has allowed me to focus on the implications that a costly trading technology innovation has on the equilibrium valuation of money. Future research should be directed at addressing the purpose of intermediaries as a holders of inventories (either real or nominal) in a monetary economy, and better motivate their endogenous role, possibly by considering an intermediated sector where multiple matchmakers may compete.

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## APPENDIX

In what follows I let $p_c \equiv p_c(\sigma,\mu)$, $p_m \equiv p_m(\sigma,\mu)$ and $\phi \equiv \phi(e,\pi,\mu)$ $\forall e,\pi,\mu$, when no confusion arises.

Value functions

The Bellman equation for a commodity trader in the search sector, a money trader, and a commodity trader operating in the intermediated sector are respectively given by

$$V_{s,t} = (1+r)^{-1}\{b(e_{t-1})p_{c,t}x^2\{u + \max_{e^j} [e_t^j V_{s,t+1} + (1- e_t^j)(V_{i,t+1} - \tau(e_t))]\}$$

$$+ b(e_{t-1})p_{c,t}(1-x^2) \max_{e^j} [e_t^j V_{s,t+1} + (1- e_t^j)(V_{i,t+1} - \tau(e_t))]$$

$$+ b(e_{t-1})p_{m,t}x \max_{e^j} \{\pi_t^j V_{m,t+1}+(1-\pi_t^j) \mathit{Max}_{e^j} [e_t^j V_{s,t+1}+(1- e_t^j)(V_{i,t+1}-\tau(e_t))]\}$$

$$+b(e_{t-1})p_{m,t}(1-x) \max_{e^j} [e_t^j V_{s,t+1}+(1-e_t^j)(V_{i,t+1}-\tau(e_t))]$$

$$+[1-b(e_{t-1})] \max_{e^j} [e_t^j V_{s,t+1}+(1-e_t^j)(V_{i,t+1}-\tau(e_t))]\}$$

$$V_{m,t} = (1+r)^{-1}\{b(e_{t-1})p_{c,t}x\pi_t[u + \max_{e^j} [e_t^j V_{s,t+1} + (1- e_t^j)(V_{i,t+1} - \tau(e_t))]]$$

$$+ b(e_{t-1})p_{c,t}(1-x\pi_t)V_{m,t+1} + b(e_t)p_{m,t}V_{m,t+1} + [1-b(e_t)]V_{m,t+1}\}$$

$$V_{i,t}=(1+r)^{-1}\{uI_{\{e_{t-1}<1\}} + \max_{e^j} [e_t^j V_{s,t+1}+(1-e_t^j)(V_{i,t+1}-\tau(e_t))]\}$$

which in the steady state are seen to generate (4)-(6) after eliminating the time subscript, multiplying both sides by (1+r), factoring the common terms on the right hand side, and rearranging.

Proof of Proposition 1

To demonstrate $e$=1 is an equilibrium it is sufficient to show that $\phi(1,\pi,\mu)>0$ $\forall$ $\pi$, $\mu$. When $e$=1, $A=ub(1-\mu)x[x+a_1\mu\pi/(1-\mu)]+\tau(1+r)[a_0(1-a_1)/r+1]$, $B=1+a_0(1-a_1)/r$, $a_0=b\mu x\pi>0$, and $a_1=bx\pi(1-\mu)/[r+bx\pi(1-\mu)]<1$. Since $1-a_1>0$ then $A,B>0$ and $\phi(1,\pi,\mu)>0$ $\forall\pi$, so that $e^j=1$ by (7) and (9) is satisfied. To show that both $\pi=0$ and $\pi\neq0$ are possible equilibria let $e$=1. Then $p_c=1-\mu$ and $p_m=\mu$ and from (4)-(6) $V_S>0$ if

$$1\text{-}\mu\text{-}\mu \max_{\pi^j} \pi^j b(1\text{-}\mu)(x\text{-}\pi)(r+bx\pi)^{-1}>0 \quad \text{(A.1)}$$

Then $\pi^j=\pi=0$ is an equilibrium if $V_m\text{-}V_s < 0$, in which case $V_m = 0$, and individuals endowed with money dispose of it ($\mu = 0$). Evaluating (A.1) at $\mu=\pi=0$ verifies $V_s > 0$. When $\pi^j=\pi=1$, $\mu=m$ and one can verify that $V_m > V_s >0$. For $\pi^j=\pi=x$, $V_m= V_s>0$ and $\mu=m$. ■

Proof of Proposition 2

It is sufficient to show that if $e$=0, an agent holding money would dispose of it. First observe that if $e$=0, b(0)=0, and, from (10), $a_0=a_1=0$, $A=\tau(1+r)\text{-}u$, and $B$=1. Then $\phi(0,\pi,\mu)<0$ if $\tau<u(1+r)^{-1}$ so that $e^j=0$ is a best response. Now conjecture $e$=0 and $\tau<u(1+r)^{-1}$. From (4) and (6) $V_s=(V_i\text{-}\tau)/(1+r)>0$, since $V_i>\tau$, so that $e$=0 is an equilibrium. Now assume $\pi,\mu\neq0$, so that $p_c(0,\pi,\mu)=0$ and $p_m(0,\pi,\mu)=1$, and recall that $V_s,V_i>0$. From (5), $V_m=0$, so that $\pi^j=0$—from (8)—a contradiction. Now consider $\pi=\mu=0$, so that $p_c(0,0,\mu)=p_m(0,0,\mu)=0$. Since $V_s>V_m=0$ and $\pi^j=0$, holding currency is not optimal. ■

Proof of Lemma 1

Consider a symmetric equilibrium where $e^j=e\in(0,1)$ then rearrange (10) as

$$\phi=\{ubex^2p_c[r+be\pi(xp_c+\pi p_m)]+[\tau(1+r)\text{-}u](r+be\pi x)\}/\{[r+be\pi x(p_c+ep_m)](1+r)\} \quad \text{(A.2)}$$

a composition of continuous functions for $e\in(0,1)$ and $\pi\in(0,1]$. Because of (1), $\phi$ has a discontinuity at $\pi=0$, with $p_c=1$ and $p_m=0$, and $\phi(e,0,\mu) = \tau\text{-}u(1\text{-}bex^2)(1+r)^{-1}$ for $e\in(0,1)$. The deleted limits of $p_m$ and $p_c$ as $e$ approaches 0 and 1 are different for $\pi=0$ and $\pi\neq0$. When $\pi=0$ as $e\to0^+$ then $p_c\to1$ and $p_m\to0$, whereas $p_c\to1$, and $p_m\to0$ as $e\to1^-$. Hence $\lim_{e\to0^+}\phi=\tau\text{-}u/(1+r)$, $\lim_{e\to1^-}\phi=\tau\text{-}u(1\text{-}bx^2)/(1+r)$, and $\lim_{e\to1^-}\phi > \lim_{e\to0^+}\phi$. When $\pi\neq0$, as $e\to0^+$ then $p_c\to0$, and $p_m\to1$, while as $e\to1^-$ then $p_c\to1\text{-}\mu$, and $p_m\to\mu$. As $e\to0^+$ then $p_c\to0$, $p_m\to1$,while as $e\to1^-$ then $p_c\to1\text{-}\mu$, and $p_m\to\mu$. From (A.2) $\lim_{e\to0^+}\phi=\tau\text{-}u(1+r)^{-1}$, $\lim_{e\to1^-}\phi= \tau - u(1+r)^{-1} + ubx^2(1\text{-}\mu)\{r+b\pi[x(1\text{-}\mu) +\pi\mu]\}\cdot[(r+b\pi x)(1+r)]^{-1}$, and it is easily seen that $\lim_{e\to1^-}\phi> \lim_{e\to0^+}\phi$. ■

Proof of Lemma 2

Let $e^j=e\in(0,1)$ be an equilibrium for any $\pi$. Recall that b(1)=b, use Lemma 1, and consider $\tau\in[\tau_\pi,u(1+r)^{-1}]$. Consider the boundaries first. If $\tau=u(1+r)^{-1}$ then $V_i-\tau=0<V_s$ $\forall e\in(0,1)$ hence $\phi>0$ and $e^j=1$. If $\tau=\tau_\pi$ then $V_i-\tau>V_s>0$ $\forall e\in(0,1)$ hence $\phi<0$ and $e^j=0$. Consequently $e\notin(0,1)$ when $\tau=\tau_\pi$ or $\tau=u(1+r)^{-1}$.

Now consider the open set $(\tau_\pi, u(1+r)^{-1})$. If $\tau<u(1+r)^{-1}$ then $\lim_{e\to 0^+}\phi<0$ $\forall$ $\pi\in\{0,x,1\}$. If $\tau>\tau_\pi$ $\forall\pi$, then the limits of $\phi(e,0,\mu)$, $\phi(e,x,\mu)$, and $\phi(e,1,\mu)$ as $e\to 1^-$ are positive. By the Bolzano intermediate value theorem $\tau_\pi<\tau<u(1+r)^{-1}$ is sufficient and necessary for the existence of at least one $e\in(0,1)$ such that $\phi=0$, $\forall$ $\pi\in\{0,x,1\}$. Thus $e^*(\pi,\mu)$ is non-empty, and it contains at least one element $\forall\pi$. ■

Proof of Lemma 3

When $\phi=0$ then $V_i-\tau$ is unaffected by changes in $e$. Let $e\in e^*(\pi,\mu)$ and use (4)-(5). If $\pi=\{0,x\}$ then $\partial V_s/\partial e>0$ since $p_m=0$ and both $p_c$ and b$e$ are increasing in $e$. If $\pi=1$

$$rV_s=bep_cx^2u[r+bex+p_mbe(1-x)] \tag{A.3}$$

and $\partial V_s/\partial e=(ep_c+bep'_c)x^2u[r+bex+p_mbe(1-x)]+bep_cx^2u[bx+p'_mbe(1-x)+p_mb(1-x)]>0$, since $p'_m<0$ but $p'_mbe(1-x)+p_mb(1-x)>0$, where $p'_m$ and $p'_c$ are partial derivatives with respect to $e$. Consequently $\partial V_S/\partial e>0$ $\forall\pi\in\{0,x,1\}$.■

Proof of Proposition 3

Let $\tau_\pi<\tau<u(1+r)^{-1}$. Conjecture $e\in e^*(\pi,\mu)$, then $\phi=0$ and using (4)-(6)

$$rV_s=ubex^2p_c+bep_mx\max_{\pi^j}\pi^jubexp_c(\pi-x)(r+bex\pi)^{-1} \tag{A.4}$$

then $\tau<u(1+r)^{-1}$ guarantees $V_i-\tau\equiv V_s>0$ in a symmetric equilibrium.

i) Let $\pi=0$. Then $\mu=0$, $p_c=1$ and $p_m=0$. From (8) $\pi^j=0$ if $V_s>V_m$, that is if the last term on the right hand side of (A.4) is negative. This is always the case since $\pi=0=\mu$ and $V_s>V_m=0$. Because of Lemma 2 there is at least one $e^j=e\in(0,1)$ which satisfies (7), since $\tau_0<\tau<u(1+r)^{-1}$ is assumed. Using (A.2), it is easily seen

that $e$ is unique, since $\phi(e,0,\mu)=0$ when $e=e^*(0,\mu)\equiv\{(bx^2)^{-1}[1-\tau(1+r)/u]\}$. Since $sup(e)=1$ (let $\tau=\tau_0$), and $inf(e)=0$ (let $\tau=u(1+r)^{-1}$), then (11) is sufficient and necessary for existence of a unique symmetric equilibrium $e^j=e\in(0,1)$.

ii) Consider $\pi\in(0,1)$. Then $\mu=m$ and $V_s=V_m$ only if the last term on the right hand side of (A.4) is zero, which occurs for $\pi=x$, in which case $\pi^j=\pi$ satisfies (8). Because of Lemma 2 there is at least one $e^j=e\in(0,1)$ which satisfies (7), since $\tau_x< \tau <u(1+r)^{-1}$ is assumed. From (10), an equilibrium $e$ is a real root of the quadratic expression

$$e^2ubx^2(1-\mu)+[(1-\mu)e+\mu][\tau(1+r)-u] \tag{A.5}$$

which is convex in $e$ if $\tau<u(1+r)^{-1}$, is increasing in $\tau$, and non-positive if $\tau=\tau_x$, $\forall e\in(0,1)$. It is seen that (11) is sufficient and necessary for existence of a unique symmetric equilibrium $e^j=e\in(0,1)$; convexity of (A.5) implies the equilibrium is unique and decreasing in $\tau$, $sup(e)=1$ (let $\tau=\tau_x$) and $inf(e)=0$ (let $\tau=u(1+r)^{-1}$). Also, (11) guarantees $\phi(0+\varepsilon,x,\mu)<0$ and $\phi(1-\varepsilon,x,\mu)>0$, for $\varepsilon>0$ small, then (A.5) intersects the horizontal axis in just one point on (0,1), the other intersection being on the negative axis. Thus only one root of (A.5), the largest, is acceptable:

$$e^*(x,\mu) = \{-(1-\mu)[\tau(1+r)-u]+\sqrt{(1-\mu)^2[\tau(1+r)-u]^2-4ubx^2(1-\mu)\mu[\tau(1+r)-u]}\}[2ubx^2(1-\mu)]^{-1}.$$

iii) Finally consider $\pi=1$, $\mu=m$. From the last term on the right hand side of (A.4) $V_m-V_s>0$ always holds, hence $\pi^j=1$ from (8). Because of Lemma 2 there is at least one $e^j=e\in(0,1)$ which satisfies (7), since $\tau_1<\tau<u(1+r)^{-1}$ is assumed. Since this $e$ implies $\phi(e,1,\mu)=0$, and this amounts to finding the solutions to $A=0$ in (10), $e$ has to be one of the roots of

$$ube^2x^2(1-\mu)\{r[(1-\mu)e+\mu]+be^2x(1-\mu)+be\mu\}+[\tau(1+r)-u](r+bex)[(1-\mu)e+\mu]^2 \tag{A.6}$$

I now show that the solutions is unique, $sup(e^*(1))=1$ and $inf(e^*(1))=0$. When $\tau=u(1+r)^{-1}$ then $e=0$ is the unique real root of (A.6) and when $\tau=\tau_1$, $e=1$ is the only real root. Since (A.6) is increasing in $\tau$, the expression is negative for $\tau=\tau_1$, $\forall\, e\in(0,1)$. Consequently (11) is necessary and sufficient for the existence of a unique symmetric equilibrium $e^j=e\in(0,1)$. This is shown by letting $\tau=\tau_1$, so that (A.6) becomes

$$e^2\{r[(1-\mu)e+\mu]+be[ex(1-\mu)+\mu]\}-\{r+b[x(1-\mu)+\mu]\}[(1-\mu)e+\mu]^2\frac{r+bex}{r+bx} \tag{A.7}$$

Since $ex(1-\mu)+\mu < (1-\mu)e+\mu$, then a maximum for (A.7) is

$$e^2(r+be)-\{r+b[x(1-\mu)+\mu]\}[(1-\mu)e+\mu]\frac{r+bex}{r+bx} \qquad \text{(A.8)}$$

whose second partial derivative with respect to $e$ is $(r+3be)(r+bx)-\{r+b[x(1-\mu)+\mu]\}bx(1-\mu)$. Suppose $e\leq x$; since $e<(1-\mu)e+\mu$, $e(r+be)<r+bex$, and $\{r+b[x(1-\mu)+\mu]\}/(r+bx)>1$, then (A.8) is negative. Suppose $e>x$; the second derivative of (A.8) with respect to $e$ is positive, so the function is strictly convex. Since (A.7) vanishes at $e$=1, it lays below the horizontal axis and so is negative for $e\in(0,1)$ and $\phi(e,1,\mu)<0$ if $\tau<\tau_1$.

Now assume (11) holds, rewrite (A.6) as $f(e)+[\tau(1+r)-u]g(e)$ where $f(e)$ is a polynomial of fourth degree in $e$ and $g(e)$ is a polynomial of third degree in $e$. The second derivative of (A.6) with respect to $e$ is quadratic in $e$, so there are at most two inflection points on (0,1) if the second derivative of (A.6) is zero and the third does not vanish. If no inflection point exists (A.6) is convex on (0,1) since the first derivative of (A.6) is non-positive at $e$=0. Therefore (A.6) vanishes at only one point $e\in(0,1)$. Suppose instead that inflection points do exist for $e\in(0,1)$. That is, suppose $f''(e)+[\tau(1+r)-u]g''(e)=0$ and $f'''(e)+[\tau(1+r)-u]g'''(e)\neq0$. Since $f''(e)$ is quadratic and $f'''(e)>0$ for $e>0$, and $g''(e)$ is linear and increasing in $e$, then $f''(e)+[\tau(1+r)-u]g''(e)=0$ is satisfied at most by one $e\in(0,1)$. That is (A.6) has at most one inflection point on $e\in(0,1)$. Since (A.6) is negative at $e$=0 and positive at $e$=1 (because of (11)), it must be convex for $e\leq e_L$, $e_L\in(0,1)$ and concave after that. Even in this case, (A.6) crosses the horizontal axis just once. Consequently the solution is unique.

Therefore for $\pi\in\{0,x,1\}$ there is a unique $e^*(\pi,\mu)$ whenever $\tau_\pi<\tau<u(1+r)^{-1}$. ■

Characterization of the mixed market strategies $e$.

i) Let $\pi$=0. From the proof of Proposition 3 recall that $e=\{(bx^2)^{-1}[1-\tau(1+r)/u]\}$ is the unique equilibrium. It is immediate that $e$ is decreasing in $\tau$, r, and $x$.

Now let $\pi\neq0$ and $\partial e/\partial j = -(\partial\phi/\partial j)/(\partial\phi/\partial e)$, $j\in\{r,x,\pi,\mu,\tau)$.

ii) Let $\pi=x$, then $V_s$ is unaffected by $\tau$, it is increasing in $e$ (by Lemma 3), and $x$. Consider an increase in $\mu$. If $e\in(0,1)$ the partial derivative of $rV_s$ with respect to $\mu$ is, from (4)-(5), $-xube^2x/\{r^2[(1-\mu)e+\mu]^2\}<0$ since $V_m-V_s= ubexp_c(e,\pi)(\pi-x)(r+bex\pi)^{-1}=0$. $V_i$ is decreasing in $\tau$ and unaffected by $e$, $x$ and $\mu$, from (6).

Then $\partial\phi/\partial e>0$, $\partial\phi/\partial x>0$, $\partial\phi/\partial\mu<0$, and $\partial\phi/\partial\tau>0$ when evaluated at the equilibrium $e$ and $\pi=x$. Consequently $\partial e/\partial x<0$, $\partial e/\partial\tau<0$, and $\partial e/\partial\mu>0$. Finally, by taking the partial derivative of (A.5) with respect to r, $\tau[e(1-\mu)+\mu]>0$ so that $\partial\phi/\partial r>0$ and $\partial e/\partial r<0$.

iii) Now consider $\pi=1$. The change in $V_s$ and $V_i$ for changes in $\tau$, $e$ and $x$ is similar to when $\pi=x$. The partial derivative of (A.6) with respect to r is increasing. Therefore $\partial\phi/\partial e>0$, $\partial\phi/\partial x>0$, $\partial\phi/\partial r<0$, $\partial\phi/\partial\tau>0$, $\partial e/\partial x<0$, $\partial e/\partial\tau<0$, and $\partial e/\partial r<0$. Since $V_m-V_s\neq 0$, if both $x$ and $\mu$ are sufficiently large, commodity traders would suffer from a further increase in liquidity. Using (A.3) when $e\in e^*(\pi,\mu)$, $V_s$ is seen to be non-decreasing in $\mu$ only if

$$-p'_c[r+bex+p_m be(1-x)]\leq p_c p'_m be(1-x) \tag{A.9}$$

where $p'_c$ and $p'_m$ denote partial derivatives with respect to $\mu$. (A.9) is continuous on $\mu\in(0,1)$ and for $x$ large it is violated (consider $x\approx 1$). As $\mu\to 1$ then $p'_c\to -e$, $p_c\to 0$, $p'_m\to e$ and $p_m\to 1$. As $\mu\to 0$ then $p'_c\to 0$, $p_c\to 1$, $p'_m\to 1$ and $p_m\to 0$. Therefore (A.9) is violated also as $\mu\to 1$. By continuity, there exists at least one $\mu'\in(0,1)$ such that for $\mu\geq\mu'$ $\partial V_s/\partial\mu\leq 0$, so that $\partial\phi(e,1,\mu)/\partial\mu<0$ and $\partial e/\partial\mu>0$. Conversely for sufficiently small $\mu$, $\partial V_s/\partial\mu>0$ so that $\partial e/\partial\mu<0$. ■

Solutions $e\in(0,1)$ when $\tau(e)$ is convex

Since $\tau(e)$ is convex and $\tau(1)>\tau(0)$ (see section 2), Lemma 2 guarantees that $\tau_\pi<\tau(1)<u(1+r)^{-1}$ is sufficient for $e^*(\pi,\mu)$ to be non-empty, but is not necessary because $\tau(e)$ can be potentially as small as zero. Because of a convex $\tau(e)$ finding the set $e^*(\pi,\mu)$ implies finding the roots on (0,1) of some convex function. This—in turn—implies a potential multiplicity of equilibria, and a unique solution if $\tau_\pi<\tau(1)<u(1+r)^{-1}$. If $\pi=0$ then $e^*(0,\mu)$ contains the roots of the polynomial $\tau(e)(1+r)-u(1-ebx^2)$, when $\pi=x$ $e^*(x,\mu)$ contains the roots of $e^2ubx^2(1-\mu)+[e(1-\mu)+\mu][\tau(e)(1+r)-u]$, and when $\pi=1$ $e^*(1,\mu)$ contains the roots of $ube^2x^2(1-\mu)\{r[(1-\mu)e+\mu]+be^2x(1-\mu)+be\mu\}+[\tau(e)(1+r)-u](r+bex)[(1-\mu)e+\mu]^2$.

Proof of Proposition 4

Consider $\pi=\mu=0$ and $\tau(e)=\tau$. Comparing $S_0$ to $M_0$, from table 2 welfare is largest if $e=1$, since $b(e)=be<b$ for $e<1$. $W(I)=W(M_0)$ whenever $M_0$ exists, since $W(I)\equiv-\tau+V_i=V_s\equiv W(M_0)$ for $e\in(0,1)$. Equilibrium I dominates $S_0$ if $\tau<u(1+r)^{-1}(1-bx^2)=\tau_0$. Since (11) is necessary for existence of $M_0$, then $\tau_0<\tau<u/(1+r)$ is sufficient (and necessary) to support both pure and mixed market strategies and to guarantee that $S_0$ welfare-dominates both $M_0$ and I. If $\tau<\tau_0$ (11) is violated, $M_0$ is not supported, and $W(I)>W(S_0)$.■

Proof of Proposition 5

Consider $\pi=1$ and let $\tau_1<\tau<u(1+r)^{-1}$. From table 2, $W(S_1)>W(M_1)$ if, after some algebra, $(r+bex)[(1-\mu)x+\mu][\mu+e(1-\mu)](e+\mu) > bexe^2(1-x)(1-\mu)\mu$. Consider a minimum for the left hand side (letting r=0) so that $[(1-\mu)x+\mu][\mu+e(1-\mu)](e+\mu) > e^2(1-x)(1-\mu)\mu$, which is always satisfied since $e^2(1-\mu)\mu + [\bullet] >e^2(1-x)(1-\mu)\mu$ where $[\bullet]$ is a positive term. ■

Proof of Proposition 6

Let $\tau$ be feasible. Recall that pure monetary equilibria are superior to mixed monetary equilibria, $S_x$ and $M_x$, and that $W(S_1)>W(M_1)$ if $M_1$ exists. If $x<x_L=(1-\mu)[\mu+x(1-\mu)]$ then $W(S_1)>W(S_0)$. Let $x<x_L$. Then $W(I)>W(S_1)$ if $\tau < u(1+r)^{-1}\{1-b(1-\mu)x\ [\mu+x(1-\mu)]\}$, which satisfies $\tau<\tau_1< u(1+r)^{-1}$ and $\tau>0$. Since $\tau<\tau_1$ and $\tau_1<\tau_0$ (because $x<x_L$) then both mixed market strategy equilibria $M_0$ and $M_1$ are *not* supported.■

Existence of equilibria for the endogenous cost formulation.

Case $e=1$. This is always a monetary equilibrium since under the conjecture $e=1$ then $V_s>V_i=0$ and $V=0<(1-\mu)V_s+\mu V_m$. Case $e=0$. This non-monetary intermediated equilibrium exists for $\tau(0)=1/2$. When $e=0$ then $\mu=0$, and $V_i=[1-\tau(e)]/r>V_s=V_i/(1+r)>V_m=0$, and $V=V_i$ is uniquely solved by $\tau(0)=1/2$.

Case $e\in(0,1)$. Assume $b(e)=be$. In this monetary equilibrium with search and intermediation

$V_m = \frac{b(e)p_c x}{r + b(e)p_c x}$. Two conditions must be met (i) the equilibrium $e$ must be a best response, i.e. $V_s = V_i$, and (ii) the per capita cost must solve (13). Thus we have two equations in the two unknowns $e$ and $\tau(e)$. Let's consider (13) under the conjecture $V_s = V_i$ (to be verified below), that is $V = \frac{(1-\mu)[1-\tau(e)]}{r} + \mu V_m$ which implies

$$\tau(e) = \frac{1}{2-e}\left(1 + \frac{r\mu}{1-\mu} V_m\right) \tag{A.10}$$

a non-linear expression in $e$. It's easy to see that there is a continuum of pairs $\{\tau(e), e\}$ such that the cost of intermediation is increasing in $e$, and $\tau(e) \in (1/2,1)$ for $0 < e < e(\mu) < 1$. Both sides of the expression are continuous on $e \in (0,1)$. As $e \to 0$ then $V_m \to 0$ and (A.10) implies $\tau(e) \to 1/2$. As $e \to 1$ then the left hand side disappears, $b(e) \to b$, and $p_c = 1-\mu$, and (A.10) implies $\tau(e) \to 1 + \frac{rb\mu x}{r + b(1-\mu)x} > 1$. Furthermore since $V_m$ is monotonically increasing in $e$ then the $\tau(e)$ satisfying (A.10) is monotonically increasing in $e$. Consequently there exist an $e(\mu) < 1$ such that (A.10) is satisfied for all $e \in (0, e(\mu))$, and $\tau(e) \in (1/2,1)$. Observe also that as $\mu \to 0$ then $\lim_{e \to 1} \tau(e) \to 1$, hence $e(\mu) \to 1$.

Now we verify the existence of an optimal $e$ such that $V_i = V_s$ (as conjectured). This requires

$$\tau(e) = 1 - rV_s. \tag{A.11}$$

Since $V_s$ is monotonically increasing in $e$, then the right had side of (A.11) is decreasing in $e$. Observe also that as $\mu \to 0$ then $V_s \to 0$ $\forall e > 0$, so that a maximum for the right hand side of (A.11) is 1. By continuity, for $\mu$ sufficiently small there exists a unique $e^* \in (0,1)$ such that both (A.10) and $V_i = V_s$ are satisfied by a unique feasible $\tau(e) \in (1/2,1)$. Note, however, that the equilibrium $\tau(e)$ is bounded away from 1/2. This since $\tau(e) \to 1/2$ only as $e \to 0$ (from (A.10)), but this is in contradiction with (A.11), which requires $\tau(e)$ to converge to 1, since $V_s, V_m \to 0$ as $e \to 0$.■

Table 1. The Set of Possible Equilibria, Existence and Lifetime Utilities

| Eq'm: $\{e,\pi,\mu\}$ | Conditions for existence | Lifetime utilities |
|---|---|---|
| I: $\{0,0,0\}$ | $\tau(0) < u(1+r)^{-1}$ | $V_i = (u - \tau(0))/r$ |
| $S_0$: $\{1,0,0\}$ | always possible | $V_s = b(1)x^2(u/r)$ |
| $S_x$ : $\{1,x,m\}$ | always possible | $V_s = V_m = b(1)(1-\mu)x^2(u/r)$ |
| $S_1$ :$\{1,1,m\}$ | always possible | $V_s = b(1)x^2[1-\mu + \mu b(1)(1-\mu)(1-x)(r+bx)^{-1}](u/r)$ |
| | | $V_m = b(1)(1-\mu)x\,[1 - b(1)(1-\mu)(1-x)(r+bx)^{-1}](u/r)$ |
| $M_0$: $\{e\in e^*(0),0,0\}$ | $\tau_0 < \tau(1) < u(1+r)^{-1}$ | $V_s = V_i - \tau(e) = b(e)x^2(u/r)$ |
| $M_x$:$\{e\in e^*(x),x,m\}$ | $\tau_x < \tau(1) < u(1+r)^{-1}$ | $V_s = V_m = V_i - \tau(e) = b(e)x^2\,p_C(e,x)(u/r)$ |
| $M_1$:$\{e\in e^*(1),1,m\}$ | $\tau_1 < \tau(1) < u(1+r)^{-1}$ | $V_s = V_i - \tau(e) = b(e)x\{p_c(e,1)x$ |
| | | $+p_m(e,1)b(e)xp_C(e,1)(1-x)[r+b(e)x]^{-1}\}(u/r)$ |
| | | $V_m = b(e)p_c(e,1)x\{1- b(e)xp_C(e,1)(1-x)$ |
| | | $\cdot[r+b(e)x]^{-1}\}(u/r)$ |

Table 2. The Set of Possible Equilibria and Welfare

| Eq'm: $\{e,\pi,\mu\}$ | Welfare |
|---|---|
| I: $\{0,0,0\}$ | $[u-\tau(0)(1+r)]/r$ |
| $S_0$: $\{1,0,0\}$ | $b(1)x^2(u/r)$ |
| $S_x$ : $\{1,x,m\}$ | $b(1)(1-\mu)x^2(u/r)$ |
| $S_1$ :$\{1,1,m\}$ | $b(1)(1-\mu)x\,(u/r)[\mu+x(1-\mu)]$ |
| $M_0$: $\{e\in e^*(0),0,0\}$ | $b(e)x^2(u/r)$ |
| $M_x$:$\{e\in e^*(x),x,m\}$ | $b(e)p_c(e,x)x^2(u/r)$ |
| $M_1$:$\{e\in e^*(1),1,m\}$ | $b(e)p_c(e,1)x(u/r)\{\mu+x(1-\mu) + b(e)x(1-x)(r+b(e)x)^{-1}[(1-\mu)p_m(e,1)-\mu p_c(e,1)]\}$ |

Figure 1. Timing of events

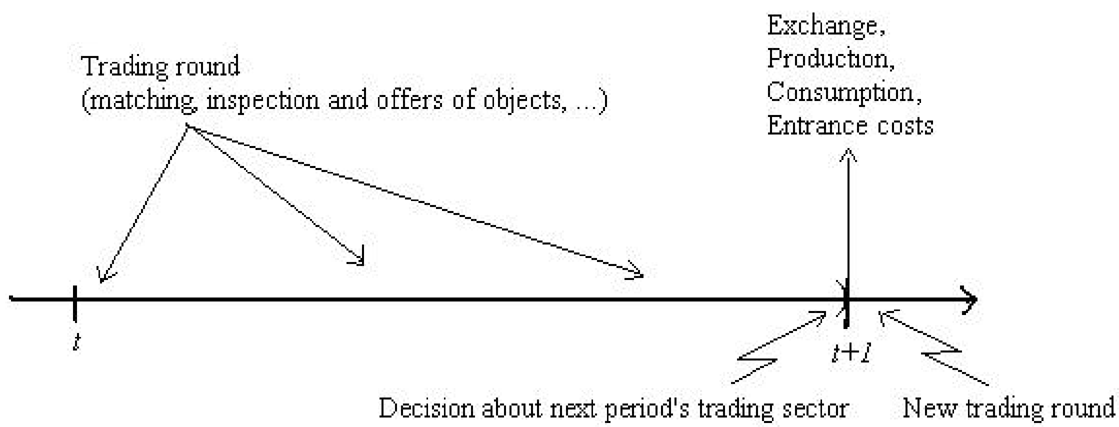


Figure 2. Existence of equilibria on the τ-*x* space

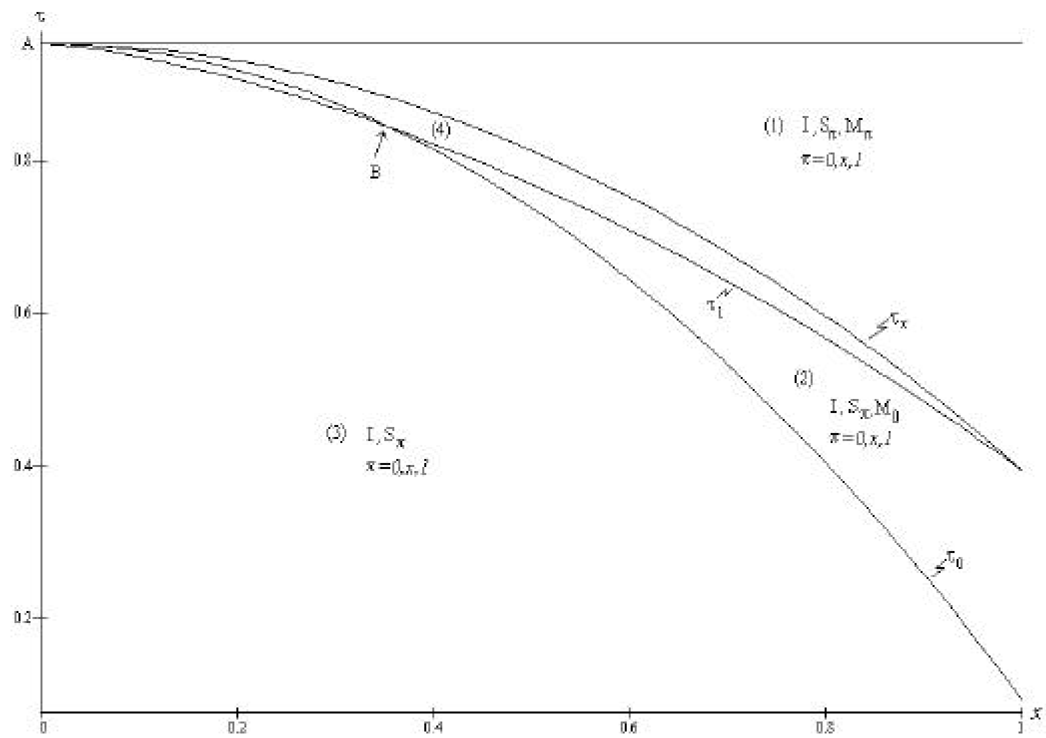

Figure 3. Existence of equilibria on the τ-μ space

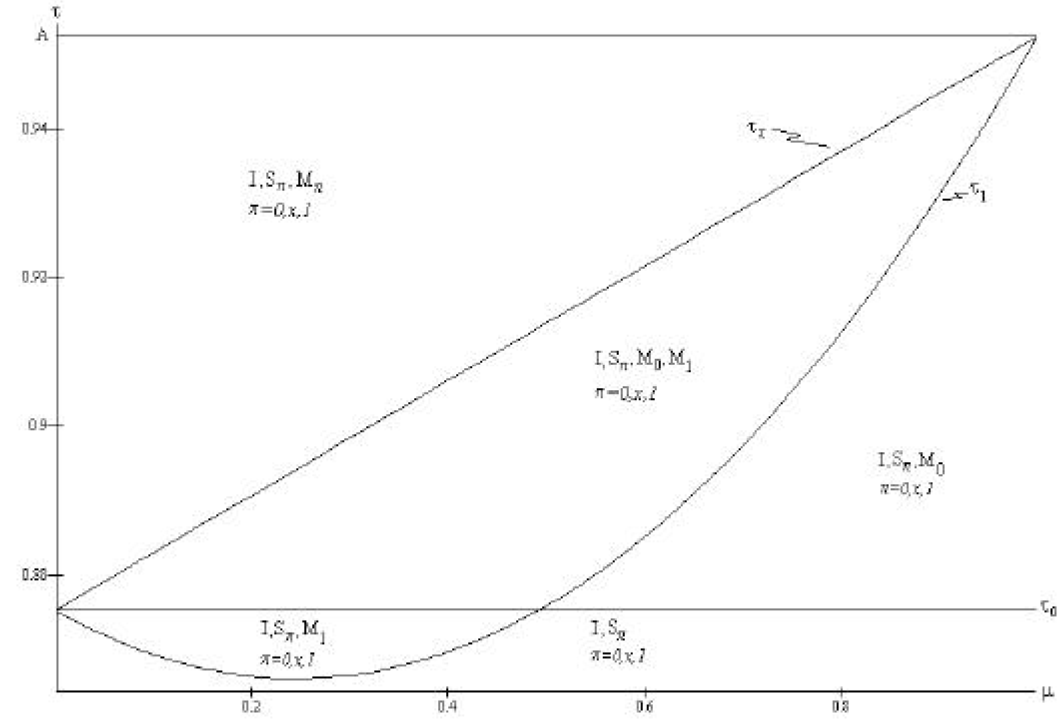


Figure 4 - Welfare for b(e) concave

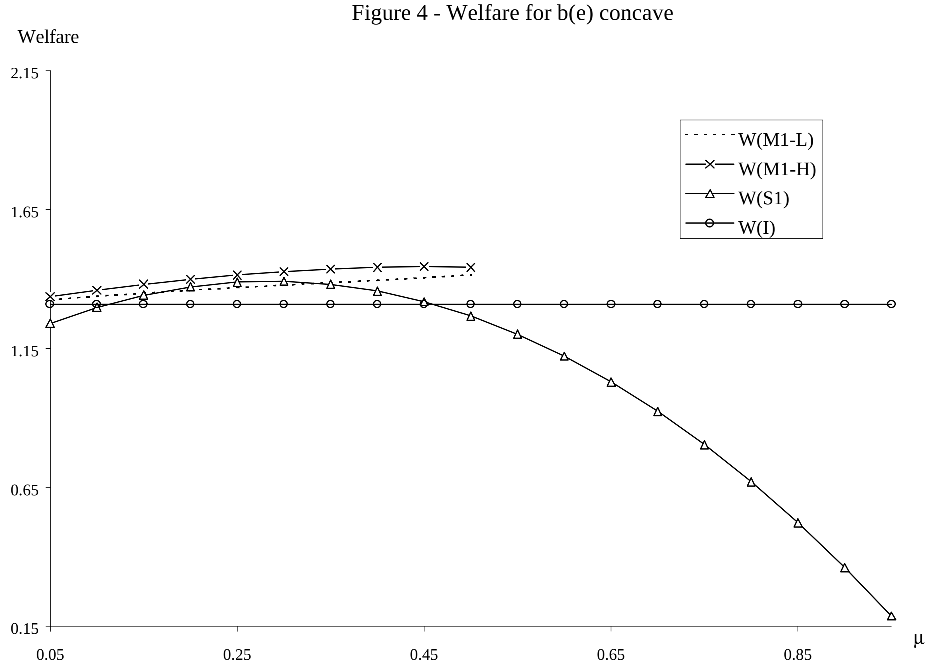

Figure 5 - Welfare for b($e$) linear