ELSEVIER Journal of Monetary Economics 47 (2001) 377–415

Journal of MONETARY ECONOMICS

www.elsevier.nl/locate/econbase

Dirty money☆

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Received 19 April 1998; received in revised form 9 May 2000; accepted 8 June 2000

Abstract

An inter-governmental body is encouraging the replacement of currency with the objective of discouraging illegal economic activities. This policy is analyzed in a search-theoretic model where individuals choose legal or illegal production, settle trades via monetary or costly intermediated exchange, and where the government imperfectly monitors monetary transactions. Stationary monetary equilibria with both legal and illegal productions exist, in which case the over-provision of currency may increment the extent of illegal production. This result holds also in the presence of intermediated exchange of legal goods. Equilibria with differing transaction patterns and degrees of illicit activities coexist. © 2001 Elsevier Science B.V. All rights reserved.

JEL classification: C78; D83; E40

Keywords: Search; Money; Illegal activities; Underground economy

☆ The paper's basic idea originated from a conversation with Beth Ingram, whom I thank. It is my pleasure to acknowledge helpful comments from both Steve Williamson and an anonymous referee, which have helped to substantially improve the paper from an earlier version. I also thank Merwan Engineer, Chris Waller, Randy Wright, seminar participants at Iowa State University, the ESE 1998 conference at Penn State University, the SED 1998 meetings in Oxford, and the Econometric Society summer 1999 meetings. Filip Vesely and Johannes Winkler have provided research assistance.

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place in a similar environment, such a model naturally lends itself to the investigation of how government policy and different transaction technologies may affect the extent of illegal activities. The model is kept simple to allow a clear exposition of the mechanics of possible links between currency and underground activities. While both money and transaction patterns are not taken as primitives, the existence of production-specific costs, externalities, and trade frictions are. These features motivate the need for trade and reflect some of the main characteristics, which generate economic incentives for illicit activities. In particular, I assume that there is a cost advantage in the production of illegal output, which is subject to a participation externality. Another important primitive is a standard restriction on coalition formation, and specialization in consumption and production. Bilateral transactions are costless, suffer from absence of double coincidence, and larger trade groups are infinitely costly to form. This creates scope for currency to be valued *in equilibrium* for its role as a medium of exchange.

Stationary economies with different initial amounts of currency are examined. I prove that there are monetary equilibria where legal and illegal activities coexist. In particular, when cash transactions involve illegal commodities, non-monetary legal trades may also take place. I provide conditions under which a larger initial money stock is associated with a greater pervasiveness of illegitimate activities. The intuition for the result rests in currency's capacity to lessen trade frictions by affecting the ability to buy and sell. Illicit sales guarantee higher payoff, but *must* rely on availability of currency, and are hampered by government enforcement, more of which increases the return from the legal alternative. Legal sellers may also carry out cashless trades. Consequently a greater money stock may increase the returns to illicit production, thus providing incentives to engage in them. Finally, I show that equilibria with no illicit production and differing degrees of trade intermediation may coexist. In this case, subsidization of the alternative transaction system may be preferred to increased enforcement of monetary transactions or reductions of the currency stock, and lead to a more efficient outcome. The analysis thus provides support for the proponents of the view that the social costs of a country's payment system can be reduced by promoting a shift to electronic payments.[4]

2. Environment

The model is built around a standard absence of double coincidence environment (see Kiyotaki and Wright, 1989). Time is discrete and continues forever. There is a constant population composed of a continuum of infinitely lived

[4] Humphrey et al. (1996) estimate the cost to society of making payments as laying between 2% and 3% of GNP annually. Humphrey and Berger (1990) suggest that debit cards and Automated Clearing House payments are the least expensive relative to all other methods, except cash.

individuals of measure one, and there is a continuum of non-storable goods that can be produced at each date, also of measure one. Individuals and goods are uniformly distributed among N different sets denoted by $i \in \{1, 2, \ldots, N\}$, each of equal measure $x = 1/N$. Where no confusion arises I will refer to any individual (or good) belonging to set i as individual (or good) i. Furthermore, each good can be one of two different types, *legal* or *illegal* (more below).

Individuals are ex-ante identical in the sense that they are all specialized in both consumption and production. Consumption of one unit of any commodity j by an individual i generates temporary utility $u > 0$ if $j = i$ and 0 otherwise. Every individual i discounts future utility at rate $r > 0$. The initial endowments are as follows. An exogenously determined proportion $m \in (0, 1)$ of individuals of each type is initially randomly endowed with one unit of indivisible fiat money, where indivisibility is adopted for tractability. Every individual i has also the ability to costlessly produce up to one unit of a perishable good $i + 1$ (modulo N). Production, however, requires the possession of a *production technology*. Also, I let the holding of money or of a production technology be costless, and mutually exclusive, and I impose a unit storage upper bound. Money can be freely discarded at the beginning of life, so I let $\mu \in [0, m]$ denote the *endogenous* proportion of individuals i, holding one unit of money in the steady state. In a symmetric equilibrium, the one studied, μ also represents the equilibrium proportion of the population holding money. The remaining proportion, $1 - \mu$, of individuals i is free to acquire a production technology. I will call the agents holding money *buyers*, and the others *sellers*.

Sellers, are further partitioned into *legal* and *illegal*, according to the production technology owned. At the beginning of each date two types of production technologies, legal and illegal, are available for acquisition by individual i to manufacture good $i + 1$. Production technologies can be freely discarded but can only be acquired at the beginning of a period. I let $k \in \{g, b\}$ denote the type (g is legal, b is illegal). Acquisition of the legal production technology generates disutility $c \in (0, u)$, while acquisition of the illegal production technology generates disutility $c_b(\lambda): [0, 1] \to [0, c]$, a continuously differentiable function of λ, the stationary *endogenous* proportion of sellers who own a legal production technology.

It is assumed that illegal production is subject to a negative participation externality, in that $c_b(\lambda)$ is a decreasing and convex function of λ, while satisfying $c_b(0) = c > c_b(1) = 0$. That is to say, illegal production has a cost advantage over the legal alternative, $c - c_b(\lambda)$, which varies endogenously with the relative extent of participation in the two activities. It shrinks as the proportion of illicit sellers, $1 - \lambda$, rises (i.e. as λ falls).[5] This specification seems to be a reasonable

[5] The assumption of cost differentials between legal and illegal is standard. For instance, in Grossman and Shapiro's (1988) study of the trade in counterfeit products, counterfeiters have a comparative advantage in the production of low quality goods, but suffer a risk of confiscation of illegitimate product by a government.

first-order approximation, and also serves several important purposes. First, the mounting cost of taking on illicit activities as more do so, is a simple device to model increased deterrence efforts, whereby the government more vigorously monitors and punishes attempted acquisitions of illegal technologies. Second, because of their distinctive "nonconformist" or "socially deviant" nature, the negative correlation between participation and return associated to illegal activities reflects the sociological concept of negative frequency dependence of those payoffs derived by expropriative or deviant strategies. This notion, encountered in sociological models of criminal or illegal behavior, is reflected in the observation that participants in criminal activities suffer from a negative feedback effect due, for example, to increased competition for markets and customers, or disputes among the individuals involved (see Cohen and Machalek (1988), or Vila and Cohen (1993) for a discussion). Third, it rules out the outcome where monitoring occurs but production is entirely illicit, which is a feature difficult to reconcile with the designation "deviant" given to illegal activities.

Since own production does not provide utility, exchange is required for consumption to take place. The economy, however, suffers from an extreme restriction on coalition formation in that only bilateral and random matches are allowed. Because of preferences and technologies, traders in each match experience an absence of double coincidence of wants. The type of partner (inventory and preferences) and the actions taken in the match are observed, trading histories are private information, so that the possibility of writing contractual arrangements for intertemporal trade is neglected, and trade needs to be facilitated either by an intermediary, or by a medium of exchange.

2.1. Matching technologies

At the beginning of each date, t, an individual i can choose to access either one of two different *matching technologies*, each providing one match in a period. The *random matching* technology provides "standard" random matches, as in Kiyotaki and Wright (1989), and is available to all individuals. The *costly* matching technology provides deterministic matches, as in Camera (2000), but is *only* available to legal sellers. The technologies are operated in two different sectors of the economy, the *search* and the *intermediated* sector, respectively.

A legal seller i who accessed the intermediated sector in t produces and delivers her good to a fictitious intermediary, and is guaranteed one unit of good i at the beginning of the following period, at which point she also suffers a loss $\phi \geqslant 0$, interpreted as a service fee. This overcomes trade frictions due to random encounters, specialized consumption and production, and is intended to approximate the existence of a trading scheme which, compared to random search, is better organized and does not require the physical transfer of a medium of exchange.[6] Clearly, in a symmetric equilibrium this type of exchange requires that some sellers (of each type i) participate in deterministic matching.

I let $e \in [0, 1]$ denote the endogenous stationary proportion of legal sellers (of every set i) who have chosen deterministic matching in a symmetric equilibrium (more in Section 3). Traders who have chosen the search sector at t are randomly matched in pairs during the period. The probability of meeting agents with whom trade is possible is a function of the strategies adopted, preferences, and the quantity of money. Since goods are not storable, only matches between i and $i-1$ or $i+1$ may result in a transaction. In particular, a seller i meets a buyer $i+1$ with probability μx, a buyer i meets a legal seller $i-1$ with probability $(1-\mu)\lambda(1-e)x$, and a buyer i meets an illegal seller $i-1$ with probability $(1-\mu)(1-\lambda)x$. In all of these instances sellers produce, and exchange takes place at the beginning of the following period involving a swap of output for one unit of money.

2.2. Externalities and government monitoring

It is assumed that consumption of illegally produced goods reduces the lifetime utility of others, an external cost denoted as $\xi(v)$, a function of the equilibrium aggregate consumption of illegally produced goods, v, where $\xi(0) = 0$.[7] To study how government punishment may serve as an exogenous discipline on illegal production activities, I assume the existence of an atomless government, modeled as such since it allows simplification of the steady state distribution of objects, and it makes the relationship between policy and equilibrium strategies more clear-cut.[8] The government possesses a technology

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[6] Imagine that seller i provides her good $i-1$ to a dealer, who in return delivers some fraction of good i he received from a seller $i+1$, and so on. The dealer has preferences defined over all commodities, the ability to keep record of intra-period transactions, and ϕ is a function of the bid–ask spread he charges. A more explicit modeling of trade intermediaries would not add additional insight at the expense of more complexity.

[7] Negative externalities (from consumption or production) appear to motivate government intervention and regulation. For instance UN Secretary Kofi Annan has stated that "Illicit drugs destroy innumerable individual lives and undermine our societies." The executive director of the UN International Drug Control Programme has suggested that "The illicit production of drugs diverts human and natural resources from more productive activities and weakens the foundation for long-term economic growth" (United Nations, 1997).

[8] The main results are invariant to modeling the government as a subset of agents following exogenous trading strategies (see Camera, 1998). This modeling choice is adopted by Aiyagari and Wallace (1997), where the study of policy rests on the assumption that government agents are essentially "vending machines" who store goods. Other examples are Li (1995), Green and Weber (1996), and Soller Curtis and Waller (2000) where government agents are automata confiscating currency, or Aiyagari et al. (1996) and Li and Wright (1998) where government agents follow prescribed rules for acceptance of different objects.

enabling costless and perfect monitoring of intermediated transactions, an assumption meant to capture the difficulty that illegal activities have in taking advantage of well organized markets. Only costly and imperfect monitoring of monetary transactions is possible, with the government suffering a loss $z(\gamma) \geqslant 0$ to monitor cash transactions with probability $\gamma \in [0, 1)$. If a transaction involving an illegally produced commodity is discovered, a penalty is imposed before the trade is executed.[9] Since all transactions involve equal amounts of indivisible goods, it seems natural to assume that both the probability of observing an illegal transaction and the penalty levied are constant (i.e. independent of the size of the transaction). I make the penalty correspond to the seizure of entire output (the optimal amount), and let the amount confiscated be used as government consumption, providing resources to support monitoring.

3. Symmetric stationary equilibria

I consider stationary rational expectations equilibria. I confine attention to outcomes where individuals adopt symmetric Nash strategies taking both distribution of objects and strategies of others as given. In equilibrium, production and trading decisions are individually optimal, given the correctly perceived strategies and distribution. They are also time-invariant, and identical for individuals of the same type. Furthermore, individual actions are based on the correct evaluation of the gains from trade in each possible match. I also adopt the simplifying assumption that agents can hold at most one unit of money, and I only consider cases wherein individuals adopt pure strategies in the acceptance of money.[10]

The outline of events is as follows. At date t individuals may be buyers or sellers. Sellers may choose between legal and illegal production, the second of which is cheaper. Trade may take place using money, but only legal sellers may trade using the costly intermediation technology. Exchange and consumption occur at the beginning of $t+1$. A new production technology may then be acquired by all who have nothing stored. Two features of the structure of the exchange process differentiate the illegal sellers' set of trade opportunities from the ones available to all others. The first feature is their inability to use trade intermediation. The second is the probability of output confiscation due to

[9] Alternatively, the government could randomly inspect all sellers, unconditional on transactions. Transactions monitoring, however, allows me to juxtapose the incentives generated by the two different trading technologies, monetary or non-monetary: only by using cash in a transaction, can enforcement be sometimes avoided.

[10] Generalizing the model to encompass mixed strategies would not add new insight at the cost of added complexity for the analysis.

government monitoring. Thus, while all trade activities are subject to frictions, illegal activities face additional obstacles.

Finally, I briefly discuss production and pricing in equilibrium. Since exchange requires ownership of either money or a good, individuals with no inventory must acquire a production technology before a match is realized. If a buyer is met and a single coincidence exists, then production immediately takes place. The quantity produced is a function of the bargaining protocol adopted. Buyers make take-it-or-leave-it offers to sellers. Since production is costless and with unit capacity, in equilibrium, buyers will ask for one unit and the nominal price in all monetary transactions is one. In this way, the potentially complex issue of the distribution of prices and inventories can be avoided.[11] A non-monetary stationary equilibrium always exists, in which case no seller accepts currency. In what follows, I focus on the stationary choice of production technology and trading sector for a representative individual i who has no inventory, and then discuss the stationary distribution of production technologies across individuals.

3.1. Value functions

At date t an individual may be in two different states depending on her past actions. She may be a seller (legal or illegal), or she may be a buyer (with money). As a legal seller, she chooses a matching technology for the period, i.e. the trading sector. As an illegal seller or a buyer, she searches. Sellers in the search sector who have met a buyer, must choose whether to accept money, which they always do in a monetary equilibrium. Buyers who have met sellers decide whether to accept the good offered, which they do contingent on single coincidence. Sellers, who have entered the intermediated sector, are deterministically matched. At the beginning of the following period, all who have been successfully matched exchange inventories and consume, and sellers who traded in the intermediated market must also pay the service fee. Immediately after exchange, all individuals without inventory choose a production technology. Sellers who did not exchange have the option of disposing of their original technology and acquire another one. Because technologies are costly, they are never discarded in equilibrium (but this may occur out of equilibrium).

The choice of relevant actions is described by the strategies of an individual, a set of rules specifying which actions to take contingent on her state (i.e. her inventory). In a stationary equilibrium I let $s \in [0, 1]$ define the probability that

[11] Shi (1995) and Trejos and Wright (1995) study equilibrium price formation with divisible goods and indivisible money. Green and Zhou (1998) and Zhou (1999) have considered the opposite case (divisible money and indivisible goods). Camera and Corbae (1999) and Taber and Wallace (1999) have studied environments with divisible goods and partially divisible money. Molico (1996) has studied numerical economies with divisible goods and money.

an agent without inventory chooses to acquire a legal production technology. I let $e \in [0, 1]$ define the probability that an individual owning a legal production technology chooses costly trade. I denote the stationary strategy profile as $\sigma = \{s, e\}$, on which individuals have symmetric beliefs, while $\sigma' = \{s', e'\}$ is an individual's best response when she takes as given the strategies of others.

Let V_g, V_b, and V_m denote the stationary value functions of respectively, a legal seller, an illegal seller, and a buyer at any date. Each individual chooses her strategy profile to maximize her lifetime utility while taking distribution of objects and everybody else's strategies as a given. I let

$$\Pi \equiv \max_{s' \in [0,1]} \{s'[V_g - c] + (1 - s')[V_b - c_b(\lambda)]\}$$

denote the continuation value for someone without inventory (e.g. a buyer who has just bought). This is the best alternative between the two production technologies. In an equilibrium with existence of legal sellers, legal technologies must be at least as preferred as illegal ones, and are not discarded. This implies that $\max\{\Pi, V_g\} = V_g$. Similarly, in an equilibrium where illegal production is held $\max\{\Pi, V_b\} = V_b$.

It follows that, in an equilibrium where production technologies are not discarded, the value functions must satisfy the standard flow-return version of the Bellman equation:

$$rV_g = \max_{e' \in [0,1]} \{e'[-\phi + (\Pi + u - V_g)1_{\{e \neq 0\}}] + (1 - e')\mu x(V_m - V_g)\}, \tag{1}$$

$$rV_b = \mu x(1 - \gamma)(V_m - V_b) - \mu x\gamma(\Pi - c_b), \tag{2}$$

$$rV_m = (1 - \mu)x[\lambda(1 - e) + (1 - \lambda)(1 - \gamma)](\Pi + u - V_m). \tag{3}$$

Expression (1) shows that the return to a seller owning a legal production technology is comprised of two components. First, she can choose to trade in the intermediated sector (term multiplying e'), in which case she pays the cost ϕ. If there is positive participation (the indicator function $1_{\{e \neq 0\}}$) she consumes with certainty receiving utility, u. Her net continuation payoff is the value of her best production alternative, Π, minus her return from search, V_g.[12] Clearly, if legal production is at least preferred to illegal, then $\Pi - V_g = -c$. Second, the legal seller can search (term multiplying $1 - e'$), in which case she meets an

[12] Clearly one must verify that in equilibrium she does not discard her unused technology in favor of another. In general the value from search holding technology k is $\max\{\Pi, V_k\}$, and an unused technology is discarded if $\Pi > V_k$.

appropriate buyer at rate μx. By selling she becomes a buyer with a gain from trade given by her net continuation payoff, $V_m - V_g$. Expression (2) is comparable to (1). The main difference is the absence of the choice of trading sector, and the $1-\gamma$ multiplicative term denoting those instances in which monetary transactions do not get monitored. When a transaction does get monitored confiscation of output occurs, and the individual suffers a net loss represented by the value of acquiring a new production technology, Π, minus her return from search, V_b. Clearly, if illegal production is at least preferred to illegal, then $\Pi - V_b = -c_b(\lambda)$. Expression (3) is similarly interpreted.[13]

3.2. Distribution of money and goods

Consider the stationary distribution of money. Because of free disposal, equilibrium valuation of money requires ex-ante buyers to be at least indifferent to the acquisition of *some* production technology. That is, money holders compare the two equilibrium lifetime utilities V_m and Π. Money is discarded if $V_m < \Pi$, which implies $V_m = 0$ in equilibrium, and it is evident that there cannot exist a monetary equilibrium where money holders are indifferent between holding money and not holding, $V_m = \Pi$.[14] In this case, $V_m = 0$, too. Thus, no one discards money, in equilibrium, if it is believed to have value, and $V_m > \Pi > 0$ is necessary. For a given equilibrium σ, I define the proportion of buyers in the economy as

$$\mu = \begin{cases} m & \text{if } V_m > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Next, I consider the stationary distribution of production technologies across all $1-\mu$ sellers, characterized by λ. Only traders exiting a match with no inventory choose a production technology because in equilibrium unused production technologies are not discarded. In a symmetric stationary equilibrium s denotes the probability that a legal production technology is chosen. While this choice affects the proportion of legal sellers, λ is not necessarily equal to s when some illegal trades do not go through ($\gamma > 0$), and some intermediated trades occur ($e > 0$). Consider the proportion of legal sellers. At each date t, *outflows* are due to legal sellers (i) who have traded in the search sector and have

[13] Because $\xi(v)$ represents an external cost, inclusion of it in the value functions does not change the analysis. It must be included, however, in the definition of societal welfare. This is done in Section 5.

[14] Suppose money is believed to have value and $V_m = \Pi$, thus someone endowed with it may discard it. In equilibrium sellers do not dispose of unused technology k, $\Pi < V_k$. But $V_m = \Pi < V_k$ implies that the payoff from selling for money, $V_m - V_k$, is negative, contradicting the conjecture of valued money.

become buyers, and (ii) who have traded in the intermediated sector and have subsequently chosen illegal production. *Inflows* are due to traders who were not legal sellers, were without inventory, and have chosen to acquire a legal production technology. That is to say, buyers who have bought, and illegal sellers who had their inventory confiscated. Thus, in the steady state λ must satisfy

$$\mu x(1-\mu)[\lambda(1-e)+(1-\lambda)(1-\gamma)]s + (1-\mu)(1-\lambda)\mu x\gamma s - (1-\mu)\lambda[(1-e)\mu x + e(1-s)] = 0. \tag{5}$$

The first term shows the rate at which buyers become legal sellers. After a single coincidence match, they always trade with a legal seller, but they trade with probability $1-\gamma$ with an illegal seller. The second term shows the rate at which illegal sellers matched to a buyer, are subject to monitoring and confiscation, and acquire a legal production technology. The last term shows the sum of the rates at which those legal sellers who traded in search and become buyers, and those who trade in the intermediated sector and become illegal. Using (5), the stationary equilibrium proportion of legal sellers must satisfy

$$\lambda = \lambda(\sigma) \equiv \frac{s\mu x}{\mu x + e(1-s)(1-\mu x)}. \tag{6}$$

I note that the function $\lambda(\sigma): [0,1]\times[0,1] \to [0,1]$ is continuously differentiable, monotonically increasing in s and μ, convex in s, concave in μ, decreasing in e, $\lambda(1,e)=1$, $\lambda(0,e)=0$, $\lambda(s,0)=s$, and $\lambda(s,1)<1$.

Finally, recall that in a symmetric stationary equilibrium the choice of sector by a representative legal seller is denoted by e, and that legal sellers who access the intermediated sector exit it following exchange. Hence, e defines the stationary proportion of legal sellers in the intermediated sector, while $1-e$ the fraction of legal sellers in the search sector.

3.3. Individual best responses

An agent chooses σ' taking as given both σ and the distribution of money and goods. Using Π, define

$$\Delta(\sigma) = V_b - c_b(\lambda) - V_g + c \tag{7}$$

as a measure of the net relative value of the two production technologies in equilibrium. Using (1) let

$$K(\sigma) = \mu x(V_m - V_g) - [-\phi + (u-c)1_{\{e\neq 0\}}] \tag{8}$$

measure the value of search relative to costly trade in equilibria with some legal sellers.[15]

A trader without inventory chooses a production technology, and contingent on being a legal seller chooses a trading sector. She does so taking value functions, strategies of all others, and distribution of objects as given. The two choices, s' and e', are intertwined, but for simplicity I discuss them separately. Consider the choice of trading sector, conditional on holding a legal production technology, and given σ. She chooses deterministic matching only if she obtains a larger payoff than random matching. When the two alternatives generate the same payoffs, she will randomize, and she will avoid deterministic matching otherwise. Her equilibrium choice must satisfy the best response correspondence

$$e' \begin{cases} = 0 & \text{if } K(\sigma) > 0, \\ \in [0,1] & \text{if } K(\sigma) = 0, \\ = 1 & \text{if } K(\sigma) < 0. \end{cases} \tag{9}$$

Now consider the choice of production technology. She chooses the one which provides her with the highest return, net of costs. In particular, she will become a legal seller only if that guarantees her a higher lifetime utility (net of cost). When the two alternatives generate identical lifetime utilities she will randomize, and choose illegal production otherwise. That is, in equilibrium

$$s' \begin{cases} = 0 & \text{if } \Delta(\sigma) > 0, \\ \in [0,1] & \text{if } \Delta(\sigma) = 0, \\ = 1 & \text{if } \Delta(\sigma) < 0. \end{cases} \tag{10}$$

In a symmetric equilibrium the elements of σ' must both be a fixed point of the respective best response correspondences,

$$\sigma' = \sigma. \tag{11}$$

Since individuals do not produce if their expected lifetime utility is negative, an equilibrium is active (i.e. with production) only if the continuation value is positive

$$\Pi > 0, \tag{12}$$

otherwise no production technology is acquired. In a monetary equilibrium money must be valued, $V_m > 0$. This requires that buyers buy in every single coincidence match

$$\Pi + u > V_m, \tag{13}$$

[15] If $\lambda = 0$ and a seller holds a legal technology, (8) is still valid since $\lambda = 0$ implies $e = 0$, $V_b \leqslant V_g$, hence $\Pi = V_g - c$.

but it does not imply that *all* sellers must accept money, although some have to. Since I consider only pure strategies in the acceptance of money, in a monetary equilibrium *some* seller k must be strictly better off from accepting money, that is

$$V_m > \min\{V_g, V_b\}. \tag{14}$$

3.4. Definition of equilibrium

A *symmetric stationary equilibrium* is a set of non-negative value functions $V = \{V_g, V_b, V_m\}$, a strategy profile σ, a proportion of legal sellers λ, and a proportion of buyers μ such that: (i) individuals maximize their expected lifetime utilities and act symmetrically, i.e. for a given proportion of buyers and distribution of sellers $\{\mu, \lambda\}$, V and σ must satisfy (1)–(3) and (9)–(12) and also (13) and (14) if the equilibrium is monetary; (ii) given value functions and symmetric strategies $\{V, \sigma\}$, the distribution of buyers and sellers, $\{\mu, \lambda\}$, satisfies the stationarity conditions (4)–(6).

To summarize, in a symmetric stationary equilibrium the strategy profile σ, the fraction of legal sellers λ, and the proportion of buyers μ, are endogenously determined taking as given the initial stock of currency m. Outcomes are fully described by the combinations of μ, λ, and the elements of σ.

4. A benchmark

A benchmark model is now provided, wherein I consider the linear cost function $c_b(\lambda) = c(1 - \lambda)$ and absence of monitoring costs, $z(\gamma) = 0\,\forall\gamma$. I let $s^* \in (0, 1)$ and $e^* \in (0, 1)$ denote a mixed strategy. I refer to those outcomes, in which legal and illegal sellers coexist, as *interior*, $s = s^*$, in which case $\lambda = \lambda^* \in (0, 1)$. The opposite case is represented by *corner* solutions, where only legal or illegal production technologies are chosen, $s \in \{0, 1\}$ and $\lambda \in \{0, 1\}$. A non-monetary equilibrium always exists, since money is an intrinsically useless and inconvertible object. Furthermore, because of the absence of double coincidence, illegal sellers must always trade via monetary exchange. They also have a lower lifetime utility than their legal counterparts, due to monitoring. This is summarized next. All proofs are in Appendix A.

Lemma 1. *Money is valued in equilibria where illegal sellers exist,* $\lambda < 1$, *but it need not be valued if* $\lambda = 1$. *Also,* $V_b \leqslant V_g$ *for* $\gamma \geqslant 0$, *and if money buys legal goods it also buys illegally produced goods.*

Since legal sellers may resort to deterministic matching, they need not engage in monetary exchange. Thus, it seems natural to partition the set of possible active equilibria in two subsets. First, there is a subset of equilibria where the costly matching technology is never used, $e = 0$, in which case *all* exchanges are monetary. I refer to equilibria in this subset as *search* equilibria. Second, there is a subset of equilibria wherein legal transactions may be intermediated, $e \in (0, 1]$. In this case money may or may not be used for the purchase of legal commodities. Equilibria where $e = e^*$ are referred to as *intermediated*, and I call outcomes where $e = 1$ *deterministic*.

I discuss existence for these subsets of equilibria separately, adopting the following procedure. Given a candidate strategy e, I conjecture an equilibrium strategy s. The resulting strategy profile implies a unique distribution of goods and money (λ and μ) via (4) and (6), used to determine the value functions. I use the functions $\Delta(\sigma)$ and $K(\sigma)$ to provide conditions under which σ is a fixed point of the best response correspondences (9) and (10). I do so by defining regions of the parameter space, which support the proposed σ, thus verifying the existence of an equilibrium. This procedure is repeated for each possible strategy s. I then consider another candidate e until all nine possible equilibrium combinations of e and s are exhausted. To make the discussion clearer, I present the results in terms of λ instead of s. Since there is a one-to-one map between the two, information on s can be easily recovered using (6), or by looking at the proofs. I start by considering the existence of *search* equilibria.

4.1. Search equilibria: $e = 0$

In these equilibria all transactions are monetary, since $e = 0$.[16] Three search equilibria may exist, but do not coexist, and in all cases $\lambda = s$. There is an interior outcome where legal and illegal productions coexist, $\lambda = \lambda^*$, and corner outcomes with only legal sellers, $\lambda = 1$, or only illegal sellers, $\lambda = 0$. Sufficient existence conditions depend on extent of monitoring, and cost of the legal production technology.

Proposition 1. *Let*

$$C_0 \equiv \frac{(1-\mu)\mu x^2 \gamma}{(r+x)[r+\mu x(1-\gamma)] + (1-\mu)\mu x^2 \gamma}.$$

Three search equilibria may exist:

(i) *If $\gamma = 0$ then $\lambda = 0$ is the only search equilibrium.*

(ii) *If $\gamma \neq 0$ then*

(a) *If $c/u \leqslant C_0$ then $\lambda = 1$ is the only search equilibrium.*

(b) *If $c/u > C_0$ then $\exists$ a unique $\lambda^* \in (0, 1)$ such that $\lambda = \lambda^*$ is the only search equilibrium.*

[16] Since $e = 0$ is sufficient for $e' = 0$ (see (8)), no legal producer would use intermediation, even if available.

Because $\lambda = s$ in equilibrium, I can discuss the distribution of production with the help of Fig. 1, depicting the individual best response s'. Consider $\gamma > 0$. If the cost of acquisition of a legal production technology is below uC_0, then $\lambda = 1$ since legal production is preferred. This corresponds to the dark line drawn at $s' = 1$ and its broken continuation, in which case $s = 1$ is the only fixed point. If the cost c exceeds that lower bound, however, preferences over legal production are a function of the distribution implied by the strategies of others. In particular, $\lambda = 0$ if $s > s^*$, while legal production is strictly preferred otherwise $\lambda = s' = 1$ if $s < s^*$. These components of her best response correspond to the two dark horizontal lines, and reflect the fact that to the left (right) of s^* the cost $c_b(\lambda)$ is too high (low) due to large (small) participation in illegal production. Coexistence of legal and illegal sellers, $\lambda = \lambda^*$, occurs when individuals randomize, whereby the dark line going through s^* denotes the mixed best response. The only fixed point is s^*, in which case the disparity in valuations is identical to their cost differential, $V_g - V_b = c - c_b(\lambda)$. As legal production technologies become cheaper ($c/u \to C_0$) they become more attractive and λ^* grows, corresponding to the movement of the dark vertical line toward 1. This increases the gap between the two technologies' costs, and restores indifference. The opposite occurs when c rises, in which case λ^* falls towards the lower bound s_L. As monitoring efforts fall, illegal production becomes even more attractive and this lower bound shrinks to zero. When γ vanishes $s' = 0$ is the best response (dotted line and dark continuation on horizontal axis) and, irrespective of c/u, $\lambda = 0$ is the unique equilibrium.

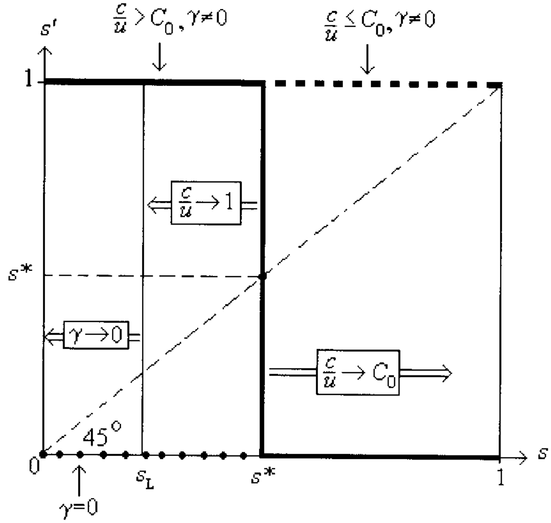

Fig. 1. Best response s' when $e = 0$.

The figure also illustrates the participation externality's role in ruling out outcomes where monitoring occurs but production is entirely illicit, and in rendering the proportion of legal sellers stable to small perturbations. Since the frequency of transactions depends on the amount of currency, exchange is difficult when liquidity is limited, and there is an incentive to choose (less costly) illegal production. As more select it, however, the cost advantage shrinks until it disappears. In the case least benign to illicit production ($\lambda = 0$), *any* probability of being caught makes illegal production the poorest choice. This relegates the degenerate outcome $\lambda = 0$ *only* to the case where monitoring is absent. Moreover, given a belief λ^*, a perturbation, which exogenously changes the distribution of sellers, will also alter the structure of incentives. For instance, if the proportion of illegal producers were to increase, $\lambda < \lambda^*$, the participation externality would raise the cost of illicit production. This, in turn, would provide incentives to choose legal production ($s = 1 > s^*$ in the figure) until λ falls back to λ^*, restoring indifference.[17]

The following corollary, draws attention to the role of monitoring and money stock in supporting the different types of equilibria.

Corollary to Proposition 1. *Let a search monetary equilibrium exist and* $\gamma \neq 0$. *Then,* (i) λ^* *is always an equilibrium for some* μ; (ii) *there exists a minimum positive level of government monitoring,* γ_0, *such that if* $\gamma < \gamma_0$ *then* $\lambda = \lambda^*$; (iii) *if* $\gamma \geqslant \gamma_0$ *then* $\lambda = 1$, *only if* μ *is bounded away from zero and one*; (iv) λ^* *is decreasing in* c, $\lambda^* \to 1$ *as* $c/u \to C_0$, $\lambda^* \to s_L$ *as* $\mu \to 1$, *and* $\lambda^* \to 0$ *as* $\mu \to 0$; *and* (v) *if* $\lambda = \lambda^*$ *is an equilibrium, then* λ^* *is monotonically increasing in* γ *if* γ *is sufficiently small.*

One interesting result is legal and illegal sellers *always* coexist for some currency level (μ close to 0 or 1), and there is a minimum level of monitoring below which participation in illicit activities always occurs, irrespective of the currency supply. Fig. 2 (drawn for the baseline $u = 1, r = 0.05, x = 0.5, c = 0.15$, and $\phi = 0.8$) illustrates it. When $e = 0$, the u-shaped curve (going through E, C, D, and F) represents the locus of points $c/u - C_0 = 0$, which separates the interior equilibria $\lambda = \lambda^*$, from the corner $\lambda = 1$. Since C_0 is increasing in γ (always) and in μ (for μ sufficiently small), all points inside the curve support $\lambda = 1$ as the only search equilibrium (areas 5–8). In the region outside, $\lambda = \lambda^*$ is the unique equilibrium with the exception of the case $\gamma = 0$ (where $\lambda = 0$). The negative link between monitoring and returns to illicit production explains why,

[17] These results are not robust to the cost assumption. For instance, a fixed $c_b < c$ supports $s = e = 0$ for some $\gamma > 0$. As long as the cost advantage can compensate the expected loss of confiscation, agents will choose to produce illicit goods even with some positive (but not too large) rate of monitoring.

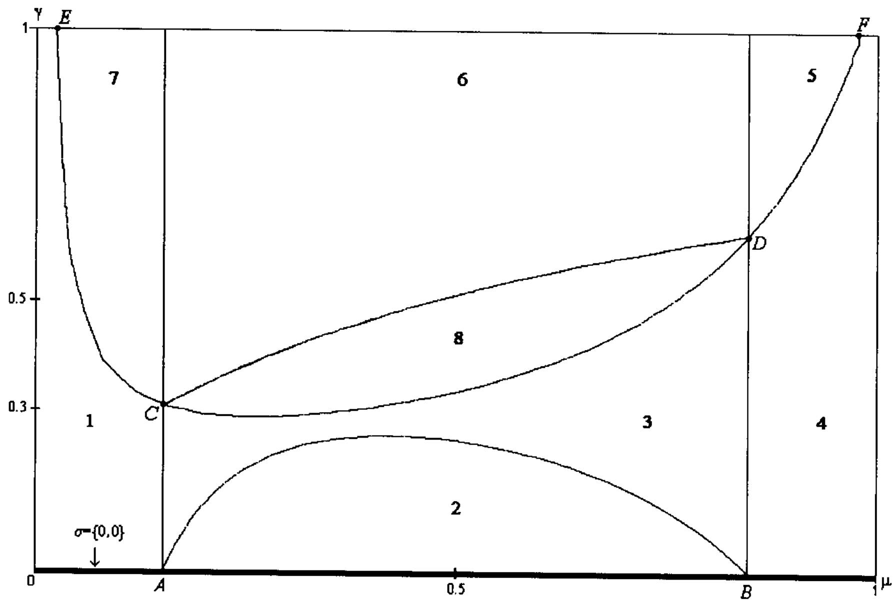

Fig. 2. Equilibria. $e = \lambda = 0$ if $\gamma = 0$; $e = 0$ and $\lambda = 1$ in 5–8; $e =$ and $\lambda = \lambda^*$ in 1–4; $e = \lambda = 1$ everywhere; $e = 1$ and $\lambda = \lambda^*$ in 2 (multiple equilibria may exist in 1 and 4 if γ is small); $e = e_1$ and $\lambda = 1$ in 6; $e = e_s$ and $\lambda = \lambda^*$ on part of 2, 3, and 8.

for some currency stocks, λ is increasing in γ and there exists a threshold γ_0 below which illegal production takes place. Note that $\lambda = 0$ for $\gamma = 0$, but V_b falls with increased monitoring, hence individuals will choose legal production until the marginal agent is indifferent ($\lambda > 0$ above the horizontal axis, when $e = 0$). Since more monitoring obstacles illicit transactions, and since $c_b(\lambda)$ falls in λ, a smaller proportion of illicit sellers is needed to support the mixed strategy as γ grows. Fig. 3 illustrates this: when $\gamma = 0.2$, λ is uniformly lower than its value when $\gamma = 0.3$. Thus λ and γ move in the same direction until γ reaches an upper bound beyond which illegal transactions are just too difficult to accomplish, and so disappear. This bound is a function of both availability and valuation of currency. More currency improves the likelihood of meeting a buyer, but it also makes consumption matches less frequent, which decreases the value of money (compare the set of μ supporting $\lambda = 1$ in Figs. 2 and 3, for $\gamma = 0.3$).[18] Fig. 3 illustrates another important feature: λ reaches a peak at

[18] This implies V_m concave in μ, u-shaped C_0, and explains why $s = 1$ only for $0 \ll \mu \ll 1$. If $\mu \approx 1 (\approx 0)$ it is very difficult to buy (sell). This sharply lowers money's value and makes low-cost illicit production more appealing.

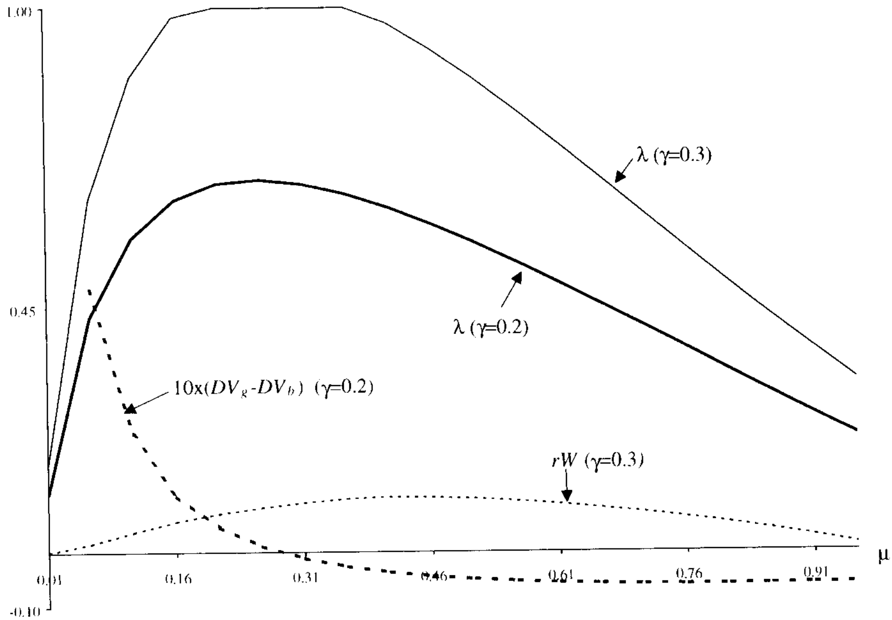

Fig. 3. λ and the money supply for $e = 0$.

interior values of the money supply. I next provide a sufficient condition for this to occur.

Proposition 2. *Let $\lambda = \lambda^*$ be a search equilibrium. If $c/u < x/(r + x)$, then there exists a quantity of money bounded away from zero and one that maximizes λ.*

A policy implication is that restricting the currency supply may serve as an endogenous discipline on illegal production, as the FATF and some economists seem to suggest. The intuition is quite simple. Recall that in an equilibrium with coexisting production technologies, the disparity in their valuation and cost must be identical, that is $V_g - V_b = c - c_b(\lambda)$. If the returns from legal production fall (grow), relative to its illicit alternative, then the cost disparity must also fall (grow), which occurs if λ drops. Since variations in the initial stock of currency initially increase the gap $V_g - V_b$, but affect it differently for larger amounts of currency, the response of λ will be hump-shaped. Fig. 3 illustrates this feature for $\gamma = 0.2$, by plotting $\mathrm{D}V_g - \mathrm{D}V_b$, i.e. the change in $V_g - V_b$ over μ ($\mathrm{D}V_k = V_k(\mu') - V_k(\mu)$, where $\mu' > \mu$). When μ is small the disparity in the valuation of production technologies grows (at increasingly smaller rates) and this is matched by the increase in λ. As μ keeps growing, however, V_g rises at a smaller rate than V_b (eventually both drop), the gap between the two returns shrinks, and λ falls to restore indifference.[19] The growth in disparity between sellers' lifetime utilities caused by a larger money stock, when μ is small, can be explained by the effect that currency has on both the ability to buy and sell, and also on the frequency of confiscation. Added currency improves transaction rates for all sellers (but diminishes it for buyers), generating larger returns to production. However, this creates a greater confiscation frequency, which results in increased expected losses for illegal sellers.[20] Thus, even if the effect on lifetime utilities is positive, it is less pronounced for illegal sellers, and λ must increase. As μ and λ grow, the bite of increased confiscation is lessened (since $c_b(\lambda)$ shrinks), while the surplus from illicit transactions remains the highest. This changes the direction of adjustment of $V_g - V_b$, so that λ starts to fall. This response is reinforced as μ keeps rising because the (beneficial) effect of added currency fades away, until it is reversed (a common result in this type of models). This explains the sufficient condition of the proposition. When trade frictions are limited (r small and x large), the exchange process benefits the least from added currency hence there exists an interior μ that maximizes λ.

4.2. *Deterministic equilibria*: $e = 1$

In these equilibria legal trade is entirely intermediated, and if currency exchange takes place, it fosters only illicit transactions. Because of the costly nature of intermediation, a necessary condition for $e = 1$ is that deterministic matching be *feasible*. That is, the net lifetime utility of a legal seller, $V_g - c$, must be positive (Eq. (12) is satisfied). This amounts to the pair of inequalities

$$\phi < \phi_{\mathrm{H}} \equiv u - c(1 + r) \quad \text{and} \quad \frac{c}{u} < \frac{1}{1 + r}, \tag{15}$$

since $\phi \geqslant 0$, by assumption. This means that the discounted value of the net payoff from a (future) intermediated transaction, $(u - \phi)/(1 + r)$, must be larger than the cost c of becoming a legal seller today.

Obviously, because intermediated transactions are perfectly monitored, sole production of illegal output is never an equilibrium. Two deterministic equilibria may co-exist when intermediation is feasible: one corner non-monetary $\lambda = 1$, and one interior monetary $\lambda = \lambda^*$. Below I provide sufficient conditions

[19] This result is not robust to the assumption of constant c_b. It can be shown that $V_g - V_b$ grows with μ, for μ small. Thus, if an interior equilibrium existed and $V_g - V_b$ were positively affected by λ, then λ would initially fall.

[20] The value to search increases by $x(V_m - V_g)$ for a legal seller, and by $x(1-\gamma)(V_m - V_b) - x\gamma c_b(\lambda)$, for an illegal seller. The difference between these two (first minus second) is positive and increasing in λ, in an interior equilibrium.

for their existence and I underscore the possibility of multiplicity of interior solutions.

Proposition 3. *Let*

$$C_1 \equiv \frac{\mu x^2(1-\mu)(1-\gamma)^2}{r[r+x(1-\gamma)]+\mu x\gamma+\mu x^2(1-\mu)(1-\gamma)^2},$$

$$\phi_{L1} \equiv (u-c)\left[1-\frac{\mu x^2(1-\gamma)^2(1-\mu)}{r+x(1-\gamma)}\right]+\frac{c\mu x\gamma}{r+x(1-\gamma)},$$

$$\phi_{H1} \equiv (u-c)\left[1-\frac{\mu x^2(1-\mu)(1-\lambda)(1-\gamma)}{r+\mu x+x(1-\mu)(1-\lambda)(1-\gamma)}\right],$$

$$\bar{C} \equiv C_1|_{\gamma=0} \quad \textit{and} \quad \varepsilon > 0 \textit{ small}.$$

(i) *If* (15) *does not hold, then no deterministic equilibria exist.*
(ii) *If* (15) *holds, then*
  (a) $\lambda = 1$ *is the only non-monetary deterministic equilibrium and always exists.*
  (b) *If* $c/u < C_1$, $\phi_{L1} < \phi < \min\{\phi_H, \phi_{H1}\}$, *and* $\gamma \in [0, \varepsilon]$, *then* $\exists$ *a unique* $\lambda^* \in (0,1)$ *such that* $\lambda = \lambda^*$ *is the only monetary deterministic equilibrium.*

Corollary. *If* $\lambda = \lambda^*$ *is a deterministic equilibrium and* $c/u > \bar{C}$, *then* λ^* *is a set with two elements.*

One interesting result is that even if legal producers choose intermediated trade, illegal production will not necessarily disappear, in which case monetary transactions are *entirely* illicit. This is due to two elements: (i) the returns to search relative to deterministic trade for a legal seller (determining e), and (ii) the returns to illicit relative to licit production (determining λ). Clearly, if intermediation is feasible there is always a non-monetary equilibrium with legal production ($e = \lambda = 1$) because illegal transactions will not be undertaken if money is believed valueless. Not surprisingly, extensive monitoring of monetary trades is sufficient to produce this result, and only when monitoring is limited, $\lambda = \lambda^*$ may exist. The latter also requires deterministic matching to be sufficiently, but not excessively, expensive. Illegal sellers should not switch to legal production, being attracted by cheap intermediation, whereas legal sellers must prefer intermediation to search, so ϕ must be bounded above. It also requires limited legal production costs, $c/u < C_1$, otherwise the too expensive legal technology would be avoided.

Another feature worthy of note is the potential multiplicity of equilibria with coexistence of legal and illegal sellers, which results from the existence of a strategic complementarity in illicit production. Since $e = 1$, only illicit producers transact with money; therefore, the existence of a larger segment of illegal sellers generates better trade opportunities in the search sector. All else equal, this makes illicit endeavors more profitable. The existence of a participation externality, however, implies that smaller involvement in illegal activities decreases their cost. These two contrasting effects, strategic complementarity and participation externality, explain why two participation levels, a high and low λ^*, may coexist. In Fig. 2 (where $\phi < \phi_H = 0.8425$) a unique interior deterministic equilibrium exists in region 2, below the hump-shaped curve going through A and B (defining the implicit function $\phi - \phi_{L1} = 0$).[21] In that area ϕ is sufficiently large, so illegal sellers do not prefer to switch technology to access intermediation. This lower bound is ϕ_{L1}, and is influenced by the frequency of monetary exchange, hence the currency stock, monitoring, and fraction of illicit sellers. It is changes in μ and γ which, via their effect on ϕ_{L1}, define the boundary of region 2. On that region ϕ is also below a certain upper bound, ϕ_{H1}, which guarantees legal sellers' strict preference for intermediation. This constraint is lessened as both γ and λ increase, since the existence of more legal sellers lowers the frequency of monetary exchange and the returns to illicit production. In region 2 there are $\lambda < 1$ such that this constraint is never binding.

4.3. *Intermediated equilibria: $e = e$*

Two possible intermediated monetary equilibria, where $e = e^* \in (0, 1)$, may exist (but do not coexist) in the presence of government monitoring: one corner and one interior.

Proposition 4. *Let*

$$C_e \equiv \frac{(1-e)(1-\mu)\mu x^2\gamma}{[r + \mu x + x(1-\mu)(1-e)][r + \mu x(1-\gamma)] + (1-e)(1-\mu)\mu x^2\gamma},$$

$$\phi_L \equiv (u - c)\left[1 - \frac{\mu x^2(1-\mu)}{r + x}\right],$$

$$e_s \equiv \frac{\mu x[e_1 - \gamma(1-s)]}{s\mu x + (\gamma - e_1)(1-s)(1-\mu x)},$$

$$e_1 \equiv 1 - \frac{(u - c - \phi)(r + \mu x)}{x(1-\mu)[\phi - (u-c)(1-\mu x)]}.$$

[21] Consider region 2. Since ϕ_{L1} is convex in γ, then $\phi > \phi_{L1}$. It can be shown that $\phi < \phi_{H1}$ for some $\lambda < 1$ (not necessarily for all λ, since γ is not restricted to a neighborhood of zero, as in Proposition 3). Thus $\phi_{L1} < \phi_{H1}$ for some $\lambda < 1$, in region 2. For the baseline parameters $\phi < \phi_H$, while c/uC_1 if $\mu \in (A, B)$, i.e. the curve defined by $c/u - C_1 = 0$ lies above region 2. Therefore $\phi_{L1} < \min\{\phi_H, \phi_{H1}\}$ for some $\lambda < 1$, hence interior deterministic equilibria exist. It can be shown that for $\gamma = 0$ and $r = 0.25$ two equilibrium λ^* exist in a left neighborhood of that μ satisfying $c/u = \bar{C}$.

(i) *If* (15) *does not hold or* $\gamma = 0$, *then no intermediated equilibria exist.*
(ii) *If* (15) *holds*, $\gamma > 0$, *and* $\phi > \phi_L$, *then*
   (a) *If* $c/u \leqslant C_{e_1}$, $e^* = e_1$ *and* $\lambda = 1$ *is the only intermediated equilibrium.*
   (b) *If* $c/u \in (C_{e_1}, \bar{C})$ *and* $\gamma = e_1$, *then* $\exists$ *a unique* $s^* \in (0,1)$ *and a corresponding* $\lambda^* \in (0,1)$ *such that* $e^* = e_{s*}$ *and* $\lambda = \lambda^*$ *is the only intermediated equilibrium.*

Corollary. *Search, intermediated and deterministic equilibria may coexist. In particular,* (i) *search equilibria exist for any* ϕ, *and if* $\gamma > 0$ *there are regions of the parameter space such that* (ii) *search, intermediated and deterministic equilibria with* $\lambda = 1$ *coexist, and* (iii) *intermediated and search equilibria with* $\lambda = \lambda^*$ *coexist.*

Intermediated equilibria require indifference between the two productions *and* the two matching technologies, i.e. between sustaining a cost to receive a deterministic match or undertake random monetary trades for free. To rule out $e = 0, 1$, ϕ must be feasible but not too low, hence the lower bound ϕ_L. Another necessary ingredient is the discipline imposed by government monitoring. Obviously $\gamma = 0$ cannot be an intermediated equilibrium because of illicit production's cost advantages. Thus the existence of equilibria depends on γ and c. Since the strategies e^* and s^* are interdependent, however, pinning down the set of γ which supports an interior intermediated equilibrium, is a difficult endeavor. In particular, cash purchases are more difficult as monitoring grows, and since in this equilibrium the gross payoff to a buyer, $V_g + u - c$, is constant, the value of money tends to fall when γ rises. Because of the existence of a strategic complementarity in trading with money, V_m tends to fall also with more extensive intermediation. Hence, when intermediation is substantial, $e \approx 1$, the returns to monetary trade tend to fall with γ, and vice versa. In equilibrium, however, money's value *must* be independent of γ, since monetary trade gains must equal the constant gains from deterministic trade ($V_g = 1$, in Fig. 4). Indifference is maintained by appropriate adjustments of the distribution of sellers (see Fig. 4). When γ falls cash trades must be penalized by less legal sellers and more intermediation (λ drops and e rises). Thus, not only should monitoring be not excessive (otherwise legal sellers would avoid monetary trade altogether), but also, to support an intermediated equilibrium, its level must vary with the extent of monetary and legal trade. This also suggests that e^* should respond differently to increases in the initial money supply, positively if μ is small, and negatively if otherwise (in Fig. 4, e^* is a hump-shaped function of μ). Indifference between trading sectors requires changes in μ exerting a positive effect on monetary transactions to be counterbalanced by a smaller fraction of legal sellers undertaking monetary trade.

In Fig. 2, intermediated equilibria may exist if μ is bounded away from the corners: $\lambda = 1$ in region 6 (above the curve going through CD, defining the

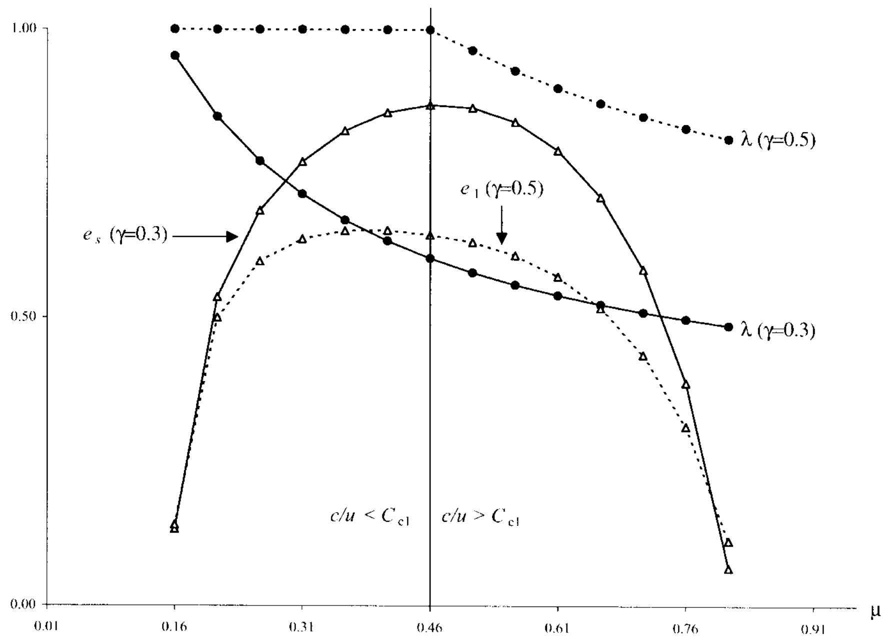

Fig. 4. λ and the money supply for $e = e$.

implicit function $c/u - C_{e_1} = 0$), while $\lambda = \lambda^*$ on some part of the region below (2, 3, and 8).[22] The role of government monitoring is also evident since $\lambda = 1$ is an intermediated equilibrium only if γ is sufficiently large. This is best illustrated in Fig. 4 where e_s and λ, are traced across μ for $\gamma = 0.3$ and 0.5. When $\gamma = 0.5$, $\lambda = 1$ if $\mu \leqslant 0.46$ and $\lambda < 1$ otherwise, since $c/u < C_{e_1}$ only to the left of the vertical line. Figs. 2 and 4 provide also a good illustration of coexistence of equilibria. For instance, search, deterministic and intermediated outcomes with $\lambda = 1$ coexist in region 6, where $c/u < C_{e_1} < C_0$, while monetary equilibria with $\lambda \leqslant 1$ coexist for $\mu = 0.31$ and $\gamma = 0.3$. Fig. 4 also shows an ever declining λ^*, in an intermediated equilibrium, a feature summarized in the following.

Proposition 5. *Let $\lambda = \lambda^*$ be an intermediated equilibrium. Then λ is strictly decreasing in μ, and is maximized by a positive quantity of money bounded away from one.*

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[22] C_{e_1} is increasing in γ. In the picture $\phi > \phi_L$ and $c/u < \bar{C}$ if $\mu(A, B)$ (ϕ_L and are independent of γ). These are the same bounds on money needed for existence of the equilibrium $e = 1$ and $\lambda = \lambda^*$ when $\gamma = 0$, since $\phi_L = \phi_{L1}$ for $\gamma = 0$.

The monotonic decline of λ is rooted in the constant return from intermediation. For equilibrium trading sector indifference, changes in the money stock must be compensated by changes in the sellers' distribution. Changes in μ, however, affect V_b and thus alter the *relative* value of licit production. When currency is scarce, an increase in it boosts the returns to illicit production so that λ must fall to restore indifference. As μ keeps growing e^* falls triggering greater participation in monetary trade. This improves illegal sellers' ability to transact and furthers participation in illicit activities.

5. Policy and welfare

Since consumption of illegally produced commodities generates a negative externality, the government would like to curtail it. In the presence of costly monitoring, however, the existence of equilibria depends on the government's ability to finance it. As indicated earlier, confiscation provides an amount of resources proportional to the frequency of illicit transactions, denoted by $Z \equiv (1-\mu)(1-\lambda)\mu x\gamma$, so that $z(\gamma) \leqslant Z$ must hold. While there are obviously reasonable functional forms for $z(\gamma)$, which can support interior equilibria, monitoring would be infeasible in monetary equilibria where no confiscation takes place ($\lambda = 1$, $e < 1$).[23] For this reason, I amend the model by letting the government levy (and commit to) a lump-sum tax τ each time a legal production technology is acquired. If I let c_g denote the cost of a legal production technology, then $c \equiv c_g + \tau$ still represents the cost of becoming a legal seller. Taxation generates $T \equiv T(\lambda, e)$ revenue, $G \equiv Z + T$ denotes government resources at each date, and I also assume that the government is able to use its resources to subsidize deterministic matching. I do so by letting $\beta \in [0,1]$ denote the share of government resources devoted to monitoring, and $\rho \leqslant \Phi$ denote the subsidy to a seller who has chosen intermediation, where Φ is now the per capita intermediation cost so that $\phi \equiv \Phi - \rho$ still is the per capita deterministic matching fee. Given τ, feasibility of monitoring and subsidy requires

$$z(\gamma) \leqslant \beta G \quad \text{and} \quad \rho(1-\mu)\lambda e \leqslant (1-\beta)G. \tag{16}$$

I will call any tax, monitoring, and subsidy plan which satisfies (15) and (16) a *feasible government plan*. In this way the previous discussion of existence of equilibria is essentially unaffected, and I can address the important question of how the government could most efficiently eradicate illicit production by allocating resources between monitoring and subsidies to deterministic matching.

One important implication of the coexistence of equilibria is the need for an integrated approach to combating illicit production. Individuals' inability to

[23] E.g. $z(\gamma)$ increasing, $0 = z(0) < z(1) = 1$, and $z'(\gamma) < \partial Z/\partial\gamma$ is a good candidate since $Z = 0$ at $\gamma = 0$, and $Z < 1\, \forall \lambda, \gamma$.

coordinate actions and select equilibria implies that isolated control of currency, monitoring, or subsidies is not sufficient to eradicate illicit production. For instance, when legal transactions are only monetary, enforcement alone cannot eliminate illegal production if the value of its alternative is sufficiently small (for small money supplies, for example). Also, a mere reduction in intermediation costs will not necessarily help, due to the coexistence of search with deterministic equilibria. This is summarized next.

Proposition 6. *Government control of monitoring is necessary, but not sufficient to guarantee a monetary outcome with only legal sellers. Additionally, subsidies to intermediation, or changes in the currency stock, are by themselves not sufficient to guarantee an outcome with only legal sellers.*

This result, however, does not imply that subsidization of a non-cash-based (and cheaply monitored) payment system is outright ineffective. Rather, the model suggests that sufficiently inexpensive trade intermediation might help in driving out illicit activities if individuals elect to conduct cash-less transactions. The coexistence of multiple equilibria, however, implies that beliefs over transaction patterns and strategies of others have implications for the expected disparity in valuation of alternative productions, and hence for the outcome. Assuming that the government could influence beliefs and "select" equilibria, what equilibrium should be chosen? In particular, what is the relative efficiency of those outcomes where only legal sellers exist? This is an interesting question since different such equilibria coexist on some region of the parameter space. And if money can be a substitute for intermediated trade, which trading institution is most efficient at eliminating illicit activities? To provide an answer, I denote efficiency of an outcome by the societal welfare achieved, defining the latter as the equally weighted lifetime utility of all but illegal producers, inclusive of externalities:

$$W \equiv \mu V_m + (1-\mu)\lambda V_g - \xi(v), \tag{17}$$

where $v \equiv (1-\mu)\mu x(1-\lambda)(1-\gamma)$ is the endogenous measure of illegally produced goods consumed at any date. Now suppose there is some feasible government plan, which supports all three outcomes with only legal sellers ($e = 0, 1, e_1$). Which one is the most efficient?

Proposition 7. *Suppose that, given τ, there are subsidy and monitoring schemes capable of supporting search, intermediated and deterministic equilibria with $\lambda = 1$. Then the deterministic outcome is welfare superior to the search equilibrium if $\phi \leqslant (u-c)[1-\frac{1}{4}x]$, while the intermediated equilibrium is always welfare dominated.*

Suppose that, given τ, government resources can be used to eradicate illicit production whether in the presence of non-monetary intermediation, monetary trades, or a mix of both. In comparing any two trading arrangements, a planner would choose the one generating the largest positive net payoff. Clearly, a cash-less economy would be ideal whenever, on the margin, the per capita cost of accomplishing a cash-less transaction does not exceed the per capita gain engendered by the avoidance of monetary (and random) transactions. When this holds for $\mu = \frac{1}{2}$ (generating the highest monetary transactions frequency, $\frac{1}{4}x$) it will hold for all money supplies, hence the sufficiency of the requirement. The inferiority of intermediated outcomes is a consequence of the existence of a strategic complementarity in monetary exchange. Recall that in this equilibrium sellers are indifferent between the two competing technologies, money and intermediation, but a larger volume of monetary transactions would take place if everyone participated in search, a positive trade externality. When taxes are allowed to vary across outcomes, however, equilibria with coexistence of money and intermediation need not be inefficient. In fact, given $\lambda = 1$ and τ, as e drops so does the overall frequency of transactions, and hence government revenue (T being the smallest when $e = 0$). It follows that, compared to search, an intermediated equilibrium generates a larger tax revenue. Intermediated outcomes, however, require more monitoring than search, and possibly also subsidies.[24] If the resulting larger expenditure is more than compensated by the increase in revenue, there would be space for a reduction in per capita taxes (hence c) and a potential increase in welfare (respect to search). More generally, when μ, τ, γ, and ρ are optimally chosen to support $\lambda = 1$ and maximize W (for some e), welfare comparisons yield ambiguous results, except when matching fees are so low that intermediation is strictly preferred.[25]

As a final observation, restrict attention to equilibria where trade is entirely monetary ($e = 0$). Suppose that there is a feasible government plan capable to discipline the economy into licit production by choosing some currency stock other than $\frac{1}{2}$. Would the reduction in the frequency of trade (entailed by choosing $\mu \neq \frac{1}{2}$) be unambiguously optimal? The answer is negative, and is best illustrated by rewriting welfare (in a search equilibrium) as

$$rW = \mu(1-\mu)x[\lambda + (1-\lambda)(1-\gamma)](u-c) - v(V_m - V_g) - r\xi(v), \tag{18}$$

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[24] Since $C_{e_1} < C_0$ and C_{e_1} increases in γ, γ must exceed the minimum value necessary to support $e = 0$ (which solves $c/u = C_0$) if $e = e_1$. Since monitoring costs increase in γ, a search equilibrium requires less government spending.

[25] To compare optimal plans one can solve a constrained programming problem for each equilibrium e to maximize W by choice of τ, β, γ, and m, subject to the constraints due to feasibility of monitoring, subsidies, and existence of equilibria. E.g., when $e = 0$ solve $\max_{m,\gamma,\tau} W$ subject to $c/u = C_0$, $z(\gamma) = T$, and $\mu = m$. When $e = e_1$ one more choice variable is added (β), and the constraints are $c/u = C_{e_1}$, $z(\gamma) = \beta T$, $\rho(1-\mu)e_1 = (1-\beta)T$, $\mu x(V_m - V_g) - (u - c - \phi) = 0$, and $\mu = m$.

where the second term, $v(V_m - V_g)$, represents the loss due to the existence of illegal sellers (whose presence detracts from societal welfare). Suppose that λ decreases in μ around $\frac{1}{2}$. The government could decrease the frequency of illicit transactions, v, by reducing the cash stock. This would not necessarily improve welfare, due to two counterproductive effects. Reducing μ impairs the ability to sell and depresses licit activities ($\mu(1-\mu)$ drops). Second, because of its role as a transaction facilitator, money's value would increase making the presence of illicit sellers an even greater penalizing factor ($V_m - V_g$ rises). It is the size of these effects and the form of consumption externality, which ultimately determine if a reduction in the currency stock may lead to welfare gains. The implication is that policies calling for a reduction of the currency stock to curtail illicit activities are not necessarily the best course of action. Fig. 3 shows such an instance since welfare is not maximized at any money stock supporting $\lambda = 1$.[26]

6. Concluding remarks

I have developed a simple general equilibrium model capable of characterizing the links between availability of currency and extent of illegal economic activities. This was motivated by the desire of providing a rationale for policy recommendations explicitly directed at encouraging the replacement of cash transfers to combat illicit activities. In the search-theoretic model proposed, individuals can engage in illegal and legal production and perform monetary or intermediated cash-less transactions. The government monitors imperfectly (perfectly) the monetary (intermediated) exchange process, and confiscates illicitly produced goods because their consumption generates negative externalities.

I have shown that different stationary equilibria coexist contingent on the size of the returns to legal production, the severity of impediments from illicit output trades, and cash availability. Monetary outcomes with only legal production exist if monitoring is sufficiently extensive, and the money supply is moderate. Transactions may be entirely cash-less when intermediation is sufficiently inexpensive. When enforcement is not too extensive, however, there are monetary equilibria where licit and illicit productions coexist. Individuals' incentives to engage in either of the two activities is susceptible to changes in the initial stock of currency. In particular, there are regions of the parameter space over which an over-provision of currency may induce larger participation in illegal production. This is more likely in environments where legal trade frictions are not too

[26] Monitoring costs are convex in the frequency of monetary transactions, $z(\gamma) = \gamma[(1-\mu)\mu x]^2$, $\tau = 0.05$, and $c_g = 0.1$. $\gamma = 0.3$ satisfies $z(\gamma) \leqslant G$ for all μ, $T = (1-\mu)\lambda s + \mu x[(1-\mu)(1-e)\lambda + (1-\mu)(1-\lambda)(1-\gamma)]s + (1-\mu)(1-\lambda)\mu x\gamma s$, and $\xi(v) = 0.5v$.

pervasive, in which case increased liquidity may negatively affect the gap existing between returns to legal and illegal production.

If the government's objective is a reduction of illicit activities, these results provide a rationale for policy recommendations directed at limiting the amount of cash in circulation. Nonetheless, this study indicates that as long as money's role as a transaction facilitator is critical for trade activity, using currency as an exogenous discipline on illicit undertakings may have unintended consequences. There are several reasons for this. First, the observed coexistence of equilibria with different transaction patterns and extent of illicit production seems to suggest the need for an integrated approach to combating illicit production. In particular, I have shown that mere implementation of arbitrary reductions in currency does not necessarily rid the economy of all illicit activities. Furthermore, the welfare analysis suggests that subsidization of alternative payment systems may be a more efficient way to curtail illicit activities than increased enforcement of monetary transactions. Finally, even when one restricts attention to equilibria where trade is entirely monetary, using money as a disciplining tool may have drawbacks, because stripping the economy of (some of) its currency may depress *all* trading activities and decrease welfare. While this study can serve as a first step in the direction of providing theoretical support to the common belief of an existing link between a country's illicit economic activities and the organization of its payment systems, a more sophisticated formulation of the environment may provide additional insight (consideration of fiscal policy, for example).

Appendix A

Proof of Lemma 1. Suppose $\lambda \in [0, 1)$ and $V_m = 0$. By (4), $\mu = 0$ hence $V_b = 0$. But then $s = \lambda = 1$, a contradiction. If $\lambda = 1$ both monetary and non-monetary equilibria are admissible depending on ϕ and e. From (4), if $\phi = u - c$ then $e = 0$, so that exchange requires valued money. By inspection of (1) and (2), $V_b < V_g$ if $\gamma > 0$ and $V_b = V_g$ if $\gamma = 0 \, \forall \lambda \in [0, 1]$. Hence if currency is accepted by legal sellers, $V_m > V_g$, it is also accepted by illegal sellers. The reverse is not necessarily true, since legal sellers have the option to access intermediated trade. □

Proof of Proposition 1. Suppose $e = 0$ is a monetary equilibrium for some s. Then $K(\sigma) > 0$, hence $e = 0$ is a fixed point of (9). In all the following proofs let $a(\lambda) = \lambda(1 - e) + (1 - \lambda)(1 - \gamma)$.

Case $s = 0$: (6) implies $\lambda = 0$, $c_b(0) = c$, and $\Delta(\sigma) < 0 \, \forall \gamma > 0$ since $V_g > V_b$, hence $s = 0$ is not an equilibrium. However $\Delta(\sigma) = 0$ if $\gamma = 0$ since $V_g = V_b$. Thus $s' \in [0, 1]$, and $s = 0$ is a fixed point. Since $\Pi = V_g - c = V_b - c_b(0)$ no free disposal of legal production technologies occurs. The equilibrium is monetary since $V_m > V_b > 0$.

Case $s = 1$: (6) implies $\lambda = 1$ and $c_b(1) = 0$. To verify that $s = 1$ is an equilibrium, it must be a fixed point of the best response correspondence (10). This requires $\Delta(\sigma) < 0$ for $\sigma = \{1, 0\}$. If $\gamma = 0$ then $s = 1$ is not a fixed point of (10), thus consider $\gamma > 0$. Under the conjectured σ, $\Pi = V_g - c > V_b - c_b(1) = V_b$, thus sellers holding an illegal production technology (out of equilibrium) dispose of it to become legal sellers. Using (1)–(3), it is easy to show that $V_m - V_g > 0 > V_b - V_g$, hence if the equilibrium exists it is monetary. Next, $\Delta(\sigma) < 0$ if $c/u \leqslant C_0$. Here

$$C_0 \equiv \frac{(1-\mu)\mu x^2 \gamma}{(r+x)[r+\mu x(1-\gamma)] + (1-\mu)\mu x^2 \gamma} \in (0, 1)$$

is strictly positive in γ and μ, increasing in γ, $1 > \lim_{\gamma \to 1} C_0 > \lim_{\gamma \to 0} C_0 = 0$. For all $\gamma > 0$, it can be shown that C_0 is concave in μ, increasing in μ for $\mu \leqslant \bar{\mu} < 1$ (decreasing otherwise), and $C_0 \to 0$ as $\mu \to 0, 1$. Since $c/u < 1$ by assumption, then $\forall \gamma, \mu > 0$ there exists a non-empty subset $(0, C_0 u] \subset (0, u)$. If $c/u \in (0, C_0)$ then $\Delta(\sigma) < 0$, so that $s = 1$ is the only fixed point of (10). If $c/u = C_0$ then $\Delta(\sigma) = 0$, hence $s' \in [0, 1]$ is the best response, whose only fixed point is $s = 1$. Thus if $c/u \in (0, C_0]$, $s = 1$ is the unique search equilibrium.

Case $s = s^*$: (6) implies $\lambda = s$, and $0 < c_b(\lambda) < c$. If $\gamma = 0$ then $\Delta(\sigma) \geqslant 0$ because $V_b = V_g$ and $c_b(\lambda) \leqslant c$ for all λ. Hence s^* cannot be an equilibrium. Thus consider $\gamma \neq 0$. Under the conjectured σ, $\Pi = V_g - c = V_b - c_b(\lambda) < V_b < V_g$, thus $\Pi - V_b = c_b(\lambda)$. Using (1)–(3) one can easily show that $V_m - V_b > V_m - V_g > 0$, hence money is valued. Using (6), (1)–(3), and after a few manipulations, it can be shown that $\Delta(\sigma) \propto L(s) - R(s)$, where

$$L(s) = c[\mu x(\lambda - \gamma) + \lambda r] \quad \text{and} \quad R(s) = \gamma \mu x \frac{(u-c)(1-\mu)xa(\lambda)}{r + \mu x + (1-\mu)xa(\lambda)}.$$

Note that $L'(s) > 0$ and $R'(s) > 0$, $L''(s) = 0$, and $R''(s) < 0 \, \forall s$ (since λ is monotonically increasing in s), hence $\Delta(\sigma)$ is strictly convex in s. Additionally, $\lim_{s \to 1} R(s) > 0 > \lim_{s \to 0} L(s)$, hence $\lim_{s \to 0} \Delta(\sigma) < 0$. It can be shown that $\lim_{s \to 1} \Delta(\sigma) \geqslant 0$ whenever $c/u \geqslant C_0$. Note that $[uC_0, u) \subset (0, u)$ is non-empty. If $c/u \in (C_0, 1)$, by the intermediate value theorem it follows that there exists an s^* such that $\Delta(\sigma) = 0$ for $s = s^*$, $\Delta(\sigma) < 0$ for $s < s^*$, and $\Delta(\sigma) > 0$ for $s > s^*$. Since $\Delta(\sigma)$ is strictly convex in s, then s^* is the unique fixed point of (10), and $\Delta(\sigma)$ is increasing in s around s^*. Also, $\lim_{\gamma \to 0} C_0 = 0$, then an interior equilibrium exists for any c, and clearly $s^* \to 0$. If $c/u = C_0$ then $s' = 1$ is the best response, hence s^* is not an equilibrium. Hence if $c/u \in (C_0, 1)$, $\sigma = \{s^*, 0\}$ is the unique search equilibrium, and $\lambda = \lambda^* = s^*$. □

Proof of Corollary to Proposition 1. Let $\gamma > e = 0$. Recall that $\lambda = s$. Suppose $c/u < C_0$ for some μ, in which case $s = 1$. Since C_0 is increasing in γ, concave in

μ, and $\lim_{\gamma\to 0} C_0 = 0$, then there exists a $\gamma_0 > 0$ such that $c/u - C_0 = 0$ when $\gamma = \gamma_0$ and $\mu = \bar{\mu}$ (the μ which maximizes C_0). It follows that $\lim_{s\to 1} \Delta(\sigma) > 0$ for all $\gamma < \gamma_0$, and since $\lim_{s\to 0} \Delta(\sigma) < 0$, then there exists an $s^* \in (0,1)$ such that $\Delta(\sigma) = 0$ when $s = s^*\ \forall \mu \neq 0$. If $\gamma \geqslant \gamma_0$ then $c/u \leqslant C_0$ at $\mu = \bar{\mu}$. Since C_0 is concave in μ, and $C_0 \to 0$ as $\mu \to 0, 1$, then there exist two values of μ in the unit interval, $\mu_L(\gamma) < \mu_H(\gamma)$, such that $c/u > C_0$ for $\mu \notin [\mu_L(\gamma), \mu_H(\gamma)]$. Hence, when $\gamma \geqslant \gamma_0$, $s = s^*$ is an equilibrium only if $\mu < \mu_L(\gamma)$, or $\mu > \mu_H(\gamma)$.

I now show that when s^* is an equilibrium, it is decreasing in c and γ. From the previous proof note that $\Delta(\sigma)$ is increasing in s, and

$$\frac{\partial \Delta(\sigma)}{\partial c} \propto \lambda - \frac{\gamma \mu x}{r + \mu x + (1-\mu) x a(\lambda)}.$$

In equilibrium $L(s) = R(s)$, and since $R(s) > 0$ then $L(s)$ must be positive, which is satisfied if $\lambda > \mu x \gamma/(r + \mu x)$. This last inequality implies that $\partial \Delta(\sigma)/\partial c > 0$, in equilibrium. Thus, by the implicit function theorem, s^* is decreasing in c. Its lower bound is $s_L = \mu x \gamma/(r + \mu x) < 1$ and solves $\lim_{c\to u} \Delta(\sigma) = 0$, and since $\lim_{c\to uC_0} \Delta(\sigma) = 0$, $s^* < 1$. It is easy to see that since in equilibrium $R(s) - L(s) = 0$, as $\mu \to 1$ $s^* \to s_L$ and as $\mu \to 0$ $s^* \to 0$.

Finally I show a sufficient (but not necessary) condition for s^* to be monotonically increasing in γ. Suppose that, $c/u > C_0$ hence $s = s^*$ is a search equilibrium. One can show that

$$\frac{\partial \Delta(\sigma)}{\partial \gamma} \propto -c + \frac{(u-c)(1-\mu)x}{r + \mu x + (1-\mu) x a(\lambda)} \times \left\{ -1 + \gamma(1-\lambda)\left[1 + \frac{r + \mu x}{r + \mu x + (1-\mu) x a(\lambda)}\right]\right\}.$$

As $\gamma \to 0$ the term in curly brackets reaches -1. Continuity implies $\partial \Delta(\sigma)/\partial \gamma < 0\ \forall \gamma \leqslant \gamma(\mu)$, where $\gamma(\mu)$ is the unique positive value of γ which makes the term in curly brackets vanish. Since $\Delta(\sigma)$ increases in s, the implicit function theorem implies that, when $s = s^*$, s^* must be monotonically increasing in γ for $\gamma < \gamma(\mu)$. □

Proof of Proposition 2. Let $\gamma > e = 0$. Consider $\gamma \leqslant \gamma_0$, which, by the previous corollary, implies $s = s^*\ \forall \mu \in (0,1)$. I show that $ds^*/d\mu \geqslant 0$ for some $\mu \leqslant \hat{\mu}$ and $ds^*/d\mu < 0$ otherwise. Recall that $\Delta(\sigma) \propto L(s) - R(s)$, and $\lambda = s^*$ in equilibrium.

In equilibrium $\lim_{\mu\to 0} s^* = 0$, so that $\lim_{\mu\to 0} a(\lambda) = 1 - \gamma$, and it follows that $\lim_{\mu\to 0} \partial \Delta(\sigma)/\partial \mu|_{s=s^*} < 0$. In equilibrium, as $\mu \to 1$: $s \to s_L = \gamma x/(r + x)$ and $a(\lambda) \to 1 - \gamma + \gamma x^2/(r + x)$,

$$\partial L(s)/\partial \mu \to -\frac{r c \gamma x}{r + x}, \quad \partial R(s)/\partial \mu \to -\gamma x(u-c) a(\lambda)/(r + x).$$

Thus, one can verify that $\lim_{\mu\to 1} \partial\Delta(\sigma)/\partial\mu|_{s=s^*} > 0$ if $x/r > c/(u-c)a(\lambda)$. Since $a(\lambda) \leqslant 1$ then $c/u < x/(r+x)$ is a sufficient condition because $x/(r+x) > C_0$, thus it is consistent with the requirement for existence of $s = s^*$.

It can be shown that $\partial^2 R(s)/\partial\mu^2 < 0$ and $\partial^2 L(s)/\partial\mu^2 = 0$. Consequently

$$\left.\frac{\partial^2\Delta(\sigma)}{\partial\mu^2}\right|_{s=s^*} > 0 \quad \forall\lambda,\mu.$$

Using the intermediate value theorem, there is a unique $\hat{\mu}\in(0,1)$ such that $\partial\Delta(\sigma)/\partial\mu|_{s=s^*} \leqslant 0$ if $\mu \leqslant \hat{\mu}$, and positive otherwise. Since

$$ds^*/d\mu = -\left.\frac{\partial\Delta(\sigma)}{\partial\mu}\Big/\frac{\partial\Delta(\sigma)}{\partial s}\right|_{s=s^*} \quad \text{and} \quad \frac{\partial\Delta(\sigma)}{\partial s} > 0 \quad \forall s,$$

then $ds^*/d\mu \geqslant 0$ for $\mu \leqslant \hat{\mu}$ (negative otherwise). From the previous corollary, $\gamma > \gamma_0$ supports a search equilibrium with $s = s^*$ for $\mu < \mu_L(\gamma)$ or $\mu > \mu_H(\gamma)$, and $s = 1$ otherwise. Thus any $\mu\in[\mu_L, \mu_H]$ maximizes λ. Since in equilibrium $\lim_{\mu\to 0}\partial\Delta(\sigma)/\partial\mu < 0$, $\partial\Delta(\sigma)/\partial s > 0$, and $\partial^2\Delta(\sigma)/\partial\mu^2 > 0$, then s^* must be increasing for $\mu < \mu_L(\gamma)$ and decreasing for $\mu > \mu_H(\gamma)$. □

Proof of Proposition 3. (Conjecture and equilibrium where $e = 1$, in which case $K(\sigma) < 0$ for $e' = 1$ to be a fixed point of (9)).

Case $s = 0$: Suppose money is valued. Since $\lambda = 0$, a seller holding a legal production technology (out of equilibrium) prefers search since $K(\sigma) \equiv \mu x(V_m - V_g) + \phi > 0$; thus $e' = 0$, which provides a contradiction.

Case $s = 1$: $\lambda = 1$, $V_m = \mu = 0$, and $V_b = (V_g - c)/(1+r)$, i.e. illegal sellers switch production. Thus $\Delta(\sigma) < 0$, and $s = 1$. Since $\sigma = \{1,1\}$ and $\mu = 0$ imply $u - c - \phi > \mu x(V_m - V_g) = 0$, then $e = 1$ is a fixed point of (9).

Case $s = s^*$: (6) implies $\lambda\in(0,1)$. In a monetary equilibrium $\Delta(\sigma) = 0$ whenever $\Pi = V_g - c = V_b - c_b(\lambda)$, which implies no disposal of production technologies. $V_m > 0$ whenever ϕ is feasible, since $V_g - c > 0$ (hence $V_g + u - c > 0$). In equilibrium $V_g = (u-c-\phi)/r$, and $K(\sigma) < 0$ if $(u-c-\phi)/r > [\mu x/(r+\mu x)]V_m$, satisfied whenever $\phi < \phi_{H1}$, where

$$\phi_{H1} \equiv (u-c)\left[1 - \frac{\mu x^2(1-\mu)a(\lambda)}{r + \mu x + x(1-\mu)a(\lambda)}\right] < 1$$

and increasing in γ and s. It can be shown that $\Delta(\sigma)$ is strictly decreasing in γ, and when $e = 1$ is an equilibrium, then $\lim_{s\to 0}\Delta(\sigma) < 0$, so s^* is bounded away from one or zero. Additionally, since $V_m \to 0$ as $\lambda \to 1$, and $V_g - c > 0$, it is easy to verify that $\lim_{s\to 1}\Delta(\sigma) < 0$. Because $\Delta(\sigma)$ is highly nonlinear in s, I consider $\Delta(\sigma)$ as a function of λ, and find conditions such that $\Delta(\sigma) = 0$ for some $\lambda\in(0,1)$. Since λ maps into a unique s (from (6)), I then find the equilibrium s^*.

It is easily shown that $\Delta(\sigma)$ is strictly concave in λ, and $\lim_{\lambda\to 0}\Delta(\sigma)>0$ iff $\phi>\phi_{L1}$, where

$$\phi_{L1}\equiv(u-c)\left[1-\frac{\mu x^2(1-\gamma)^2(1-\mu)}{r+x(1-\gamma)}\right]+\frac{c\mu x\gamma}{r+x(1-\gamma)},$$

increasing in γ. Since ϕ must be feasible, the interval (ϕ_{L1},ϕ_H) must be non-empty: $\phi_{L1}<\phi_H$ when

$$\frac{c}{u}<C_1\equiv\frac{\mu x^2(1-\mu)(1-\gamma)^2}{r[r+x(1-\gamma)]+\mu x\gamma+\mu x^2(1-\mu)(1-\gamma)^2},\tag{A.1}$$

C_1 decreasing in γ, and $C_1<(1+r)^{-1}$. Consequently if $c/u<C_1$, and $\phi\in(\phi_{L1},\phi_H)$, then the intermediate value theorem assures the existence of a $\lambda^*\in(0,1)$ such that $\Delta(\sigma)=0$ for $\lambda=\lambda^*$. Moreover, since $\Delta(\sigma)$ is strictly concave in λ, and $\lim_{\lambda\to 0}\Delta(\sigma)>0$, then λ^* is unique, because $\Delta(\sigma)$ vanishes only at one $\lambda\in(0,1)$. Since (6) implies λ^* is a one to one map from $s\in[0,1]$ onto $[0,1]$, then there exists a unique s^*. Furthermore, $C_1\to 0$ as $\gamma\to 1$, so C_1 is strictly decreasing in γ, and $c/u<C_1$ for some $\gamma\neq 0$ if

$$c/u<C_1|_{\gamma=0}\equiv\bar{C}\equiv\frac{\mu x^2(1-\mu)}{r(r+x)+\mu x^2(1-\mu)},$$

satisfied only for μ bounded away from one and zero, and when $\phi_{L1}<\phi_H$ holds and a unique s^* satisfies $\Delta(\sigma)=0$ for *some* $\gamma>0$.

Finally, $(\phi_{L1},\phi_H)\cap(\phi_{L1},\phi_{H1})$ must be non-empty so that, given $s=s^*$ there are ϕ such that $e'=1$ is a best response. I now provide sufficient conditions for $\phi_{L1}<\phi_{H1}$, in which case $\phi\in(\phi_{L1},\min\{\phi_{H1},\phi_H\})$ supports $\sigma=\{s^*,1\}$. It is evident that $\phi_{L1}<\phi_{H1}$ if $\gamma=0$ ($\phi_{H1}-\phi_{L1}\to 0$ as $s^*\to 0$). Since ϕ_{H1} and ϕ_{L1} are increasing in γ, as $\gamma\to 1$ the difference $\phi_{H1}-\phi_{L1}$ can be positive or negative. Suppose $\phi_{H1}-\phi_{L1}=0$ for $\gamma=\gamma(s^*)\in(0,1)$. Then $\phi_{L1}<\phi_{H1}$ for $\gamma<\gamma(s^*)$, and for γ around zero $\phi_{L1}<\min\{\phi_{H1},\phi_H\}$. Now let (A.1) hold, and $\phi_{L1}<\phi<\min\{\phi_{H1},\phi_H\}$. Then $s=s^*$ and $e=1$ satisfy $\Delta(\sigma)=0$ and $K(\sigma)<0$.

When $s=s^*$ illegal sellers accept money but legal sellers may or may not, out of equilibrium. They will do so if $\phi>(u-c)[1-x(1-\mu)(1-\lambda)(1-\gamma)]$, unlikely to hold for large μ, since it would clash with (15). If $V_m\leqslant V_g$, legal sellers do not accept money, but illegal still do since it is easy to verify that $V_b<V_m\forall\gamma\geqslant 0$. □

Proof of Corollary to Proposition 3. Recall that $\Delta(\sigma)$ is strictly concave in λ. If $c/u<\bar{C}$, then $\lim_{\lambda\to 0}\Delta(\sigma)<0$. Since $C_1<\bar{C}$ and $\lim_{\lambda\to 1}\Delta(\sigma)<0$, then if $\sigma=\{s^*,1\}$ is an equilibrium for $c/u>\bar{C}$, $\Delta(\sigma)$ vanishes in two distinct points $s^*\in(0,1)$. □

Proof of Proposition 4. Suppose $e = e^*$ is an equilibrium. This requires $K(\sigma) = 0$ and feasibility of deterministic matching, (15).

Case $s = 0$: This cannot be an equilibrium for any γ (see similar case in the Proof of Proposition 3).

Case $s = 1$: (6) implies $\lambda = 1$ and $c_b(1) = 0$. If $\gamma = 0$ then $V_g = V_b$, thus $\Delta(\sigma) > 0$, hence $s' = 1$ is not a best response. Thus consider $\gamma \neq 0$. Under the conjecture $s = 1$, $\Pi = V_g - c > V_b$ (which I will verify) so sellers with an illegal production technology (out of equilibrium) will dispose of it. Under the proposed σ, $V_m - V_g > 0$, and $\Delta(\sigma) \leqslant 0$, if

$$\frac{c}{u} \leqslant C_e$$

$$\equiv \frac{(1-e)(1-\mu)\mu x^2\gamma}{[r + \mu x + x(1-\mu)(1-e)][r + \mu x(1-\gamma)] + (1-e)(1-\mu)\mu x^2\gamma},$$

in which case $s = 1$. Note that $0 \leqslant C_e < C_0 < 1 \forall e \in (0,1)$. For all $\gamma, \mu > 0$, $C_e > 0$, increasing in γ, $0 = \lim_{\gamma\to 0} C_e < \lim_{\gamma \to 1} C_e < 1/(1+r)$, C_e is concave in μ, increasing in μ for $\mu \leqslant \bar{\bar{\mu}} \in (0,1)$ (decreasing otherwise), and $C_e \to 0$ as μ approaches 0 or 1.

Given $\sigma = \{1, e^*\}$ and $\gamma > 0$, determine e^*. $K(\sigma) = 0$ whenever $\mu x(V_m - V_g) = u - c - \phi$, uniquely satisfied by

$$e = e_1 \equiv 1 - \frac{(u - c - \phi)(r + \mu x)}{x(1-\mu)[\phi - (u-c)(1-\mu x)]};$$

$e_1 < 1$ if $\phi > (u-c)(1-\mu x)$; $e_1 > 0$ if

$$\phi > \phi_L \equiv (u-c)\left[1 - \frac{\mu x^2(1-\mu)}{r + x}\right]. \tag{A.2}$$

Since $\phi_L > (u-c)(1-\mu x)$ then $\phi > \phi_L$ is sufficient for $e_1 \in (0,1)$. Since ϕ must be feasible then $\phi \in (\phi_L, \phi_H)$, a non-empty interval whenever $c/u < \bar{C}$. Thus, given $s = 1$ and $\gamma \neq 0$, if $c/u < \bar{C}$ and $\phi \in (\phi_L, \phi_H)$, then $e' = e_1$. It is easy to confirm that $C_e < C_0 < \bar{C} < (1+r)^{-1}$. Given $e = e_1$ and any $\gamma, \mu > 0$ there exists a non-empty subset $(0, uC_e] \subset (0, u)$ on which c can take values. If $c/u \in (0, C_{e_1})$ then $\Delta(\sigma) < 0$ so $s = 1$ and $e = e_1$ are optimal. If $c/u = C_{e_1}$ then $\Delta(\sigma) = 0$, thus $s' \in [0,1]$ and $s = 1$ is the only fixed point. Thus if $c/u \in (0, C_{e_1}]$ and $\phi \in (\phi_L, \phi_H)$, $\sigma = \{1, e_1\}$ is the unique intermediated equilibrium.

Case $s = s^*$: If $\gamma = 0$ then $\Delta(\sigma) > 0$ and s^* cannot be an equilibrium. Thus consider $\gamma > 0$. Under the conjectured σ, $\Pi = V_g - c = V_b - c_b(\lambda) < V_b < V_g$. It is easy to show that $V_m - V_g > 0$ and that $\Delta(\sigma) \propto L(s) - R(s)$, where $L(s) = c[\mu x(\lambda - \gamma) + \lambda r]$, and

$$R(s) = \gamma \mu x \frac{(u-c)(1-\mu)xa(\lambda)}{r + \mu x + (1-\mu)xa(\lambda)}.$$

Since λ is monotonically increasing and convex in s, then $L'(s) > 0$ and $L''(s) > 0$, $R'(s) < 0$ if $\gamma < e$, while $R'(s) > 0$ and $R''(s) < 0$ if $\gamma > e$. Additionally, $\lim_{s\to 0} R(s) > 0 > \lim_{s\to 0} L(s)$, hence $\lim_{s\to 0} \Delta(\sigma) < 0$, while it can be shown that $\lim_{s\to 1} \Delta(\sigma) \geqslant 0$ whenever $c/u \geqslant C_e$.

Given the conjectured σ and $\gamma > 0$, $K(\sigma) = 0$ whenever

$$a(\lambda) = \frac{(u-c-\phi)(r+\mu x)}{x(1-\mu)[\phi-(u-c)(1-\mu x)]} \equiv 1 - e_1. \tag{A.3}$$

The right hand side of (A.3) is between zero and one whenever $\phi \in (\phi_L, \phi_H)$, which requires $c/u < \bar{C}$. The unique solution to (A.3) is

$$e^* = e_s \equiv \frac{\mu x[e_1 - \gamma(1-s)]}{s\mu x + (\gamma - e_1)(1-s)(1-\mu x)},$$

e_s strictly decreasing in γ, increasing in e_1, increasing in s if $\gamma > e_1$ (decreasing otherwise), and $\lim_{s\to 1} e_s = e_1$. The numerator and denominator of e_s can both be negative, depending on e_1 and s, thus there are different combinations of the parameters and s such that $e_s \in (0,1)$. I derive a sufficient (but not necessary) condition for $e_s \in (0,1)$ given that $\phi \in (\phi_L, \phi_H)$ and $c/u < \bar{C}$. First, $e_s \geqslant 0$ if the numerator is non-negative, i.e. $e_1 \geqslant \gamma(1-s)$, and if the denominator is positive, i.e.

$$\mu > \frac{1}{x}\frac{(1-s)(e_1-\gamma)}{s+(1-s)(e_1-\gamma)}.$$

Second, $e_s \leqslant 1$ if

$$\mu \geqslant \frac{1}{x}\frac{(1-s)(e_1-\gamma)}{s(1-e_1)}.$$

Both the last two inequalities are not binding as long as $e_1 \leqslant \gamma$. The first inequality does not bind because in such a case its numerator would be negative and the denominator positive (since $\gamma < 1$), and the second inequality for a similar reason. Thus if $\gamma(1-s) \leqslant e_1 \leqslant \gamma$ then $e_s \in (0,1)$. Given $\phi \in (\phi_L, \phi_H)$, $c/u < \bar{C}$, and $s = s^*$, then there exists a non-empty set $(e_1, e_1(1-s))$ on which γ can take values such that $e = e_s$ is the unique equilibrium.

Since $C_e < \bar{C} < (1+r)^{-1}$, and $e_s \to e_1$ as $s \to 1$, whenever $e = e_s$ and $\gamma, \mu > 0$ there exists a non-empty subset $[uC_{e_1}, u\bar{C}) \subset (0,u)$ on which c can take values, and which guarantees non-emptiness of (ϕ_L, ϕ_H). If $c/u \in (C_{e_1}, \bar{C})$, the intermediate value theorem assures existence of an s^* which satisfies $\Delta(\sigma) = 0$, $\Delta(\sigma) < 0$ for $s < s^*$, and $\Delta(\sigma) > 0$ for $s > s^*$. Since $\Delta(\sigma)$ is strictly increasing in s if $\gamma < e$, and $\Delta(\sigma)$ is strictly convex in s if $\gamma > e$, then s^* is the unique fixed point of (10), and

$\Delta(\sigma)$ is strictly increasing in s around s^*. Since $\lim_{\gamma \to 0} C_e = 0$, then $s^* \to 0$, which exists for any $c/u < \bar{C}$. If $c/u = C_{e_1}$, then $s' = 1$ hence s^* is not an equilibrium. Thus if $c/u \in (C_{e_1}, \bar{C})$ then $\sigma = \{s^*, e_s\}$ is the unique intermediated equilibrium. □

Proof of Corollary to Proposition 4. Coexistence of $e = 0, 1, e_1$ for $\lambda = 1$. It can be verified that $C_{e_1} < C_0 < \bar{C} < (1+r)^{-1}$, If $c/u < \bar{C}$, (ϕ_L, ϕ_H) is non-empty (Proof of Proposition 4), hence there exists a feasible ϕ. Let $c/u < C_{e_1}$, $\phi \in (\phi_L, \phi_H)$, $\gamma > 0$, and verify that the sufficient conditions for the existence of $\sigma = \{1, 0\}$, $\sigma = \{1, 1\}$ and $\sigma = \{1, e_1\}$ are all satisfied: (i) $\sigma = \{1, 0\}$ is an equilibrium since $c/u < C_{e_1} < C_0$ (Proof of Proposition 1), (ii) $s = e = 1$ can always be an equilibrium since ϕ is feasible (Proof of Proposition 3), and (iii) $\sigma = \{1, e_1\}$ is an equilibrium since $c/u < C_{e_1}$ and $\phi \in (\phi_L, \phi_H)$ is feasible. Search equilibria always exists since $e' = 0$ is the best response to $e = 0$ (see Proposition 1). Therefore $\sigma \equiv \{s, 0\}$ coexists with all other equilibria for any arbitrarily small ϕ.

Coexistence of $e = 0$, e^* for $\lambda = \lambda^*$. Let (15) hold, $\bar{C} > c/u > C_0$, $\gamma = e_1$, and $\phi > \phi_L$. Then $\sigma = \{s^*, 0\}$ coexists with $\sigma = \{s^*, e^*\}$ since $C_0 > C_{e_1}$, so that $c/u \in (C_{e_1}, C_0)$ holds. □

Proof of Proposition 5. Let $\gamma, \mu > 0$, and suppose an intermediated equilibrium exists, i.e. $\sigma = \sigma^* \equiv \{s^*, e^*\}$. Since $K(\sigma) = 0$, V_g in equilibrium is a constant, $V_g = (u - c - \phi)/r$. Since V_m is a function of V_g, then V_m is also a constant. Rearranging (1)–(3) in the conjectured equilibrium $\Delta(\sigma) = 0$ whenever

$$\lambda^* = \gamma \frac{rV_g + c\mu x}{rc + c\mu x},$$

where $\lambda^* > \gamma$ since $V_g > c$, from (15). Clearly $\partial\lambda^*/\partial\mu < 0$ whenever σ^* is an equilibrium, since

$$\left.\frac{\partial V_g}{\partial \mu}\right|_{\sigma^*} = 0.$$

Notice that this does not imply that s^* is strictly decreasing in μ, however, since the marginal change in e^* must also be considered.[27] Using λ^*,

$$s^* = \frac{\gamma[u - \phi - c(1 - \mu x)][\mu x + e(1 - \mu x)]}{c\mu x(r + \mu x) + \gamma e(1 - \mu x)[u - \phi - c(1 - \mu x)]},$$

[27] In the baseline case s increases around $\mu = 0.16$ (where e is increasing rapidly) while λ is always decreasing.

$s^* > 0$ for $\phi < \phi_H$ ($u - \phi - c(1 - \mu x) > 0$ in that case), and $s^* < 1$ for

$$\gamma < \frac{r + \mu x}{u - \phi - c(1 - \mu x)},$$

hence $s^* \to 1$ as

$$\mu \to \hat{\hat{\mu}} \equiv \frac{\gamma(u - c - \phi) - r}{x(1 - c\gamma)}.$$

$\phi > \phi_L$ is necessary for the equilibrium, an inequality equivalent to μ being in a proper subset of (0, 1), call it M. If $\sigma = \sigma^*$ exists on M, and $\hat{\hat{\mu}} \in M$, then $s = 1$ for $\mu \leqslant \hat{\hat{\mu}}$ and $s < 1$ for $\mu > \hat{\hat{\mu}}$. If $\hat{\hat{\mu}}$ is (weakly) smaller than the lower bound of M, then $s < 1\ \forall \mu$ which support σ^*. $\sigma = \sigma^*$ is not an equilibrium if $\hat{\hat{\mu}}$ is (weakly) larger than the upper bound of M (neither to the left of $\hat{\hat{\mu}}$, since $s = 1$, nor to the right); since this upper bound must be strictly less than one, $\hat{\hat{\mu}}$ must be bounded away from one if σ^* is an equilibrium. □

Proof of Proposition 6. Recall that $\sigma \equiv \{s^*, 0\}$ coexists with other equilibria $\forall \gamma > 0$, $\phi \geqslant 0$, and some μ. Hence control over γ is not sufficient to guarantee $\lambda = 1$, but is necessary since $e \neq 1$ and $\lambda = 1$ requires $\gamma > 0$ (Propositions 1 and 4). Control of μ is not sufficient since $\sigma \equiv \{s^*, 0\}$ exists for γ sufficiently low (Corollary to Proposition 1). Control over ϕ is not sufficient for $\sigma \equiv \{1, 1\}$ because of its coexistence with $\sigma \equiv \{s^*, 0\}$, for some μ and γ. □

Proof of Proposition 7. Consider $s = \lambda = 1$, hence $T = (1 - \mu)[\mu x(1 - e) + e]\tau$, while $Z = v = 0$ for all e. Assume that $z'(\gamma) > 0$ and, given τ, a feasible government plan. Compare welfare in the three outcomes, search, intermediated, and deterministic, respectively denoted by $W(0)$, $W(e_1)$, and $W(1)$. $e = 0$ is an equilibrium when $c/u \leqslant C_0$ and $\gamma > 0$. Note that $rV_g \equiv \mu x(V_m - V_g)$ while $rW = r\mu(V_m - V_g) + rV_g$, thus $rW = \mu(1 - \mu)x(u - c)$. The "optimal" amount of money is $\mu = 1/2$, since it maximizes the frequency of exchange. $e = 1$ is always an equilibrium, in which case $\mu = 0$, $T = \tau$, and $rW = u - c - \phi$. $e = e_1$ is an equilibrium when $c/u \leqslant C_{e_1}$, $\gamma > 0$. Since $e = e_1$ iff $rV_g \equiv \mu x(V_m - V_g) = u - c - \phi$, then

$$rW = \frac{(u - c)(1 - \mu)(1 - e)\mu x}{1 - (1 - \mu)ex/(r + x)} \equiv (u - c - \phi)\frac{r + x}{x}.$$

Given c, $rW(e_1) < rW(0)$ since

$$\frac{(1 - e)}{1 - (1 - \mu)ex/(r + x)} < 1.$$

Therefore $e = e_1$ is always dominated. Recall that $\mu = 1/2$ maximizes $rW(0)$, thus a sufficient condition for $rW(1) \geqslant rW(0)$ is $\phi \leqslant (u-c)[1-\mu(1-\mu)x]|_{\mu=0.5}$. □

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