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Economic Theory

Exposita Notes

Distributional aspects of the divisibility of money: an example*

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Summary. I highlight the importance of the distributional aspects of money's divisibility by comparing a search-theoretic model with random transfers of indivisible money balances, to one with deterministic transfers of partially divisible balances. Randomization allows price flexibility, as if money were fully divisible. Partial divisibility does not, but allows money redistributions. An example of the relevance of such 'extensive margin' aspects of divisibility is provided.

Keywords and Phrases: Divisibility, Lotteries, Money distribution, Search.

JEL Classification Numbers: D30, D83, E40.

1 Introduction

There has been recent interest in randomized monetary trades (lotteries) within the context of matching models where individual money balances are indivisible (Berentsen, Molico, and Wright, 2002). This is partly due to difficulties encountered when working with divisible money, as this creates endogenous heterogeneity in nominal wealth and market prices, that can substantially lessen analytical tractability (e.g. Green and Zhou, 2002).

The use of lotteries on transfers of indivisible money balances can capture *some* aspects of the notion of divisibility of money, while preserving tractability. Indeed, recent work suggests similitudes between models with divisible-money or lotteries

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on indivisible money.[1] This note clarifies what aspects of money divisibility lotteries can and cannot capture.

Lotteries on indivisible money balances convexify the space of feasible nominal price offers. This price flexibility captures an 'intensive margin' aspect of money divisibility. Efficient trades can be sustained, when money has a great value, *as if* balances were divisible. The reason is buyers can spend less than their holdings *on average*, and so in equilibrium they do not overconsume. The problem with this is buyers spend none or all of their money, but never portions of it. Price changes thus *cannot* have redistributive consequences, as it would happen if balances were divisible. This 'extensive margin' aspect of money divisibility may be significant for allocative efficiency.

To make this point I contrast the allocation achieved in an indivisible-money divisible-goods matching model with lotteries, to an allocation achieved when there are no lotteries (so the intensive margin aspect of divisibility is removed) but fractions of money balances can be spent (so the extensive margin aspect can be captured). For a given money supply, I show how in equilibrium, randomized exchange would occur on regions of the parameter space where buyers would also choose to spend fractions of their holdings. I then show that for some parameters there exists an allocation with partially divisible balances and no lotteries that is superior, in terms of ex-ante welfare, to the allocation achieved via lotteries on indivisible balances. The reason is the large distributional effects present when agents can spend portions of their balances.

2 Environment

The environment is as in Camera and Corbae (1999). Time is continuous, there is a continuum of perishable goods, and a continuum of infinitely lived agents of unit mass. Agents are equally distributed across J different types, indexed by j, with $x = 1/J$. Agents specialize in production and consumption. An agent of type j consumes only good j, and can only produce good $j + 1$. Production of q goods generates disutility $-q$. Consumption of q desired goods generates period utility $u(q)$, with $u'(q) > 0$, $u''(q) < 0$ and $u'(0) = \infty$. I let $u(q) = \frac{q^{1-\gamma}}{1-\gamma}$ so that $u(q) - q$ is maximized at $q = q^* = 1$. The instantaneous discount rate is $r > 0$. Agents meet bilaterally and at random via a Poisson process with arrival rate $\alpha > 0$. Thus, contingent on a meeting, there is probability x of single coincidence. Trade must be monetized due to limited commitment, enforcement and unobservability of trading histories.

Agents' money balances are bounded. An individual can hold up to a nominal quantity of money $0 < \bar{M} < \infty$. Initially, a quantity of money M is randomly distributed, where $0 \leq M \leq \bar{M}$. Individual balances $\bar{M}$ can be divided in $N \in \mathbb{N}$ countable parts, and I call the smallest part a 'token', having nominal value $\bar{M}/N$.

[1] Berentsen and Rocheteau (2001), focus only on intensive margin effects of divisibility, comparing lotteries on indivisible money to divisible-money models with degenerate distributions. Ravikumar and Wallace (2001) suggest their use of lotteries helps deliver results that "stand a good chance" to hold in a divisible-money version of their model.

Given M and $\bar{M}$, divisibility increases with N and the nominal value of a token falls. Nominal money balances are indivisible when $N = 1$, and partially divisible otherwise. I normalize $\bar{M} = 1$, one monetary unit, as in Kiyotaki and Wright (1993).[2]

3 Symmetric stationary monetary equilibria

I study monetary equilibria where distributions are stationary, and symmetric time-invariant Nash strategies are adopted. Let b denote an agent's money balances, defined on the set

$$B \equiv \left\{0, \frac{1}{N}, \frac{2}{N}, .., \frac{N-1}{N}, 1\right\}.$$

Because of symmetry, let m_b denote the probability of a randomly encountered agent having b money, where $\{m_b\}_{b \in B}$ is a probability measure over B such that $E(\{m_b\}) = M$ (the population is a set of positive Lebesgue measure). Precisely,

$$M = \sum\nolimits_{b \in B} b m_b \quad \text{and} \quad \sum\nolimits_{b \in B} m_b = 1$$

so that M corresponds to average balances, and NM is the mass of tokens in the economy.

An agent's strategy depends on his state, b, and on the aggregate state, summarized by the distribution of money $\{m_b\}$. Exchange must be quid-pro-quo since barter is unfeasible, there is no credit, and goods are non-storable. It follows that an agent in a single coincidence match can generally be a buyer or a seller, unless $b = 1$ (he can only buy) or $b = 0$ (he can only sell). The terms of trade are endogenously formed, based on take-it-or-leave-it offers from buyers to sellers. Consequently, in equilibrium the seller accepts offers that leave him zero surplus. I let V_b denote the stationary expected lifetime utility of someone with b money.

3.1 Indivisible balances and lotteries

Let $N = 1$. Hence, agents can have either 1 or 0, and the money distribution has mass on two points, $m_1 = M$ and $m_0 = 1 - M$. A buyer can offer to spend 1 or 0 money, for any given amount of goods, so nominal prices are not flexible. To introduce a notion of price flexibility I consider lotteries on money transfers (see Berentsen , Molico, and Wright, 2002).

A buyer makes a take-it-or-leave-it offer $\{q, \tau\}$ to a seller, asking for q goods and offering to transfer his entire balances with probability τ (commitment is assumed). The equilibrium value functions must satisfy the standard functional equations

$$\rho V_0 = m_1 \left[\tau \left(V_1 - V_0\right) - q\right] \tag{1}$$
$$\rho V_1 = \left(1 - m_1\right) \left[u\left(q\right) - \tau \left(V_1 - V_0\right)\right] \tag{2}$$

[2] Suppose $\bar{M} = \$1$ and $M = \$0.50$. If $N = 1$ a dollar is indivisible: half of the agents must have a token, and half have none. When the dollar is made divisible in two, $N = 2$, the nominal value of a token is $\$0.50$. Each agent can hold two tokens of smaller denomination but the quantity of money does not change. For example, we can give $\$1$ and $\$0.50$ respectively to 25% and 50% of the population.

where $\rho = \frac{r}{\alpha x}$ captures the extent of trading difficulties. The expressions in (1)–(2) tell us that the instantaneous return to an agent is proportional to the surplus from trading; $u(q) - \tau(V_1 - V_0)$ to a buyer, and $\tau(V_1 - V_0) - q$ to a seller. Hence, a monetary equilibrium requires $V_1 > V_0 \geq 0$.

The buyer faces a constrained maximization problem. Having all the bargaining power, the buyer chooses q to maximize the surplus $u(q) - q$, selecting τ to make the seller indifferent, $\tau(V_1 - V_0) - q = 0$. That is

$$\max_{q,\tau} [u(q) - \tau(V_1 - V_0)] \quad \text{s.t.} \quad \tau = \frac{q}{V_1 - V_0} \text{ and } \tau \leq 1$$

This is $\max_{q,\lambda} \left\{ u(q) - q + \lambda(1 - \frac{q}{V_1 - V_0}) \right\}$, where $\lambda \geq 0$ is the Lagrange multiplier on $\tau \leq 1$. The key first order condition is

$$u'(q) = 1 + \frac{\lambda}{V_1 - V_0}.$$

The equilibrium pair $\{q, \tau\}$ must satisfy $q = \tau(V_1 - V_0)$. Two cases may arise, depending on whether the constraint $\tau \leq 1$ is binding, or not. If $\tau = 1$ then $\lambda > 0$, hence $q < q^*$; if $\tau \leq 1$ then $\lambda = 0$, hence $q = q^*$. A unique monetary equilibrium exists.

Lemma 1. *A large γ and small ρ support the use of lotteries. Precisely,*

$$\{q, \tau\} = \begin{cases} \{q^*, \hat{\tau}\} & \text{if } m_1 \leq m(\rho, \gamma) \\ \{\hat{q}, 1\} & \text{if } m_1 > m(\rho, \gamma) \end{cases}$$

where $\hat{q}, m(\rho, \gamma) \in (0, 1)$, and $\hat{\tau} \in (0, 1]$ falls as γ grows or ρ falls.

Proof. To find the equilibrium $\{q, \tau\}$ note that $q = \tau(V_1 - V_0)$ and (1)–(2) imply

$$V_0 = 0 \text{ and } V_1 = \frac{1 - m_1}{\rho + \tau(1 - m_1)} u(q).$$

In equilibrium $q = \tau V_1$. Substituting for V_1 and $u(q) = \frac{q^{1-\gamma}}{1-\gamma}$, we have $q = q(\tau)$ where

$$q(\tau) = \left[\frac{\tau(1 - m_1)}{\rho + \tau(1 - m_1)} \cdot \frac{1}{1 - \gamma} \right]^{\frac{1}{\gamma}}.$$

Suppose $q = q^* = 1$. There is a unique τ, call it $\hat{\tau}$, such that $q(\hat{\tau}) = 1$, with $\hat{\tau} = \frac{\rho(1-\gamma)}{\gamma(1-m_1)}$. Also, $\hat{\tau} < 1$ if $m_1 < m(\rho, \gamma) = 1 - \frac{\rho(1-\gamma)}{\gamma}$, $\hat{\tau} = 1$ if $m_1 = m(\rho, \gamma)$, and $m(\rho, \gamma) \geq 0$ if $\rho \leq \frac{\gamma}{1-\gamma}$. If $m_1 > m(\rho, \gamma)$, no $\tau \leq 1$ is consistent with $q = q^*$. Thus, suppose $\tau = 1$. Then there is a unique q, call it $\hat{q}$, such that $\hat{q} = q(1)$ with $\hat{q} = \left(\frac{A}{1-\gamma} \right)^{\frac{1}{\gamma}}$, and $A = \frac{1-m_1}{\rho+1-m_1}$. It is easy to verify that $\hat{q} < q^* = 1$ when $m_1 > m(\rho, \gamma)$. Notice that $\lim_{\rho \to 0} m(\rho, \gamma) = \lim_{\gamma \to 1} m(\rho, \gamma) = 1$. □

A high curvature of the utility function, γ large, and low trading frictions, small ρ, give buyers an incentive to spread consumption over time. Buyers would like to

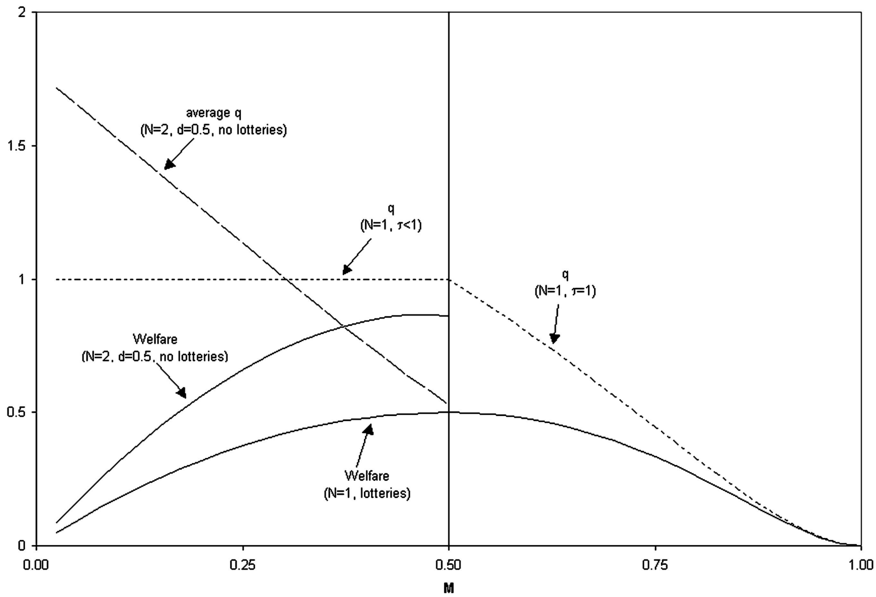

Figure 1

spend only some of their balances today, but since money is indivisible the best they can do is to randomize on money transfers. This convexifies the money offer set, allowing buyers to 'spend' *any* amount between 0 or 1, on average. As the curvature of the utility function or trading frictions fall, there is an incentive to spend even less, and the equilibrium probability τ falls.[3]

When $\tau < 1$ trades are efficient. Money has a value greater than the surplus-maximizing quantity, $V_1 \geq q^*$. Thus, buyers can ask for q^* and spend less than a unit of money, on average. When $\tau = 1$ trades are inefficient as buyers are 'cash constrained.' Since $V_1 < q^*$ buyers would spend more than their money unit if they could. In both cases, the equilibrium price is unique, $\frac{\tau}{q} = \frac{1}{V_1}$. Since V_1 falls in M, prices rise in M; q falls *only if* $\tau = 1$ (see Fig. 1, for $M > 0.5$).[4]

3.2 Partially divisible balances without lotteries

Now I rule out lotteries and let $2 \leq N < \infty$ thus the quantity of money M is distributed via tokens that are N times smaller than before. Buyers can now spend *some* portions of their unit balances, hence nominal price offers, for any given q, are not fully flexible.

[3] Interestingly, prices go to zero as $\rho \to 0$, since $\lim_{\rho\to 0} V_1 = \infty$, because $\lim_{\rho\to 0} \hat{\tau} = 0$ while $q = q^*$, a constant. Essentially as $\rho \to 0$ sellers produce but never get paid. Initial sellers will never consume and will aways produce q^*, while initial buyers will never produce and will only consume. This does not happen if $\tau = 1$, since $\lim_{\rho\to 0} V_1 = \lim_{\rho\to 0} \hat{q} = \left(\frac{1}{1-\gamma}\right)^{\frac{1}{\gamma}}$.

[4] Absent lotteries $q = V_1 < q^*$. Thus, lotteries improve the allocation along the 'intensive margin.'

Since the model is that of Camera and Corbae (1999), who normalize $\bar{M} = N$, I omit unnecessary detail, and refer to their study for proofs of claims made in this section. To start consider price formation. Because balances can be heterogeneous, I refer to agents with large (small) balances as being 'rich' ('poor'). A buyer with b money can make a take-it-or leave-it offer to a seller with $s \in B$ money as follows. The buyer requests $q_{s,b}(d)$ goods in exchange for d money, where feasibility implies $0 \leq d \leq \min\{b, 1-s\}$. The seller can accept or reject. The optimal offer pair $\{d, q_{s,b}(b)\}$ leaves the seller indifferent to the trade.

Here, many patters of trade are possible. To study equilibria where buyers spend small amounts of money, I focus on the case where the optimal money transfer is the smallest possible, $d = 1/N$ or one token. Due to take-it-or-leave it offers the quantity traded is independent of the buyer's wealth. In a trade with seller $s < 1$, the equilibrium nominal price is $\frac{1}{Nq_s}$, where $q_s = V_{s+1/N} - V_s$.

The stationary equilibrium value function is $V_0 = 0$, and for $b > 0$

$$\rho V_b = \sum_{s \in B \setminus \{1\}} m_s \left[u(q_s) - (V_b - V_{b-1/N})\right].$$

The value to having b money is a function of the frequency of matching with sellers s, m_s, and the trade surplus expected, $u(q_s) - (V_b - V_{b-1/N})$. One can show that $0 \leq V_b < \infty$ and $\{V_{b+1/N} - V_b\}$ is a decreasing sequence. Thus, the wealthy value money less than the poor; this has two consequences. There is equilibrium price dispersion. The price is $q_s^{-1} = (V_{s+1/N} - V_s)^{-1}$, the inverse of the seller's valuation for the money offered. Hence, trades with rich sellers occur at a higher price than with poor sellers. Second, trades are generally inefficient, $q_s \neq q^*$ (see Fig. 1). This hinges on the non-convexity of the set of money offers, but also on the heterogeneity in money valuations.[5] Here

$$V_b = \frac{1 - A^{bN}}{1 - A} V_{1/N} \quad \text{for } b \neq 0$$

$$V_{1/N} = \left[\frac{A}{(1-m_1)(1-\gamma)} \sum_{s=0}^{(N-1)/N} m_s A^{sN(1-\gamma)}\right]^{\frac{1}{\gamma}}$$

so that to find prices we have to characterize $\{m_b\}$. One can prove that the stationary distribution of money is unique and censored-geometric; specifically, $\{m_b\}$ satisfies

$$m_b = m_0 \left(\frac{1-m_0}{1-m_1}\right)^{bN} \quad \text{for } b \in B \setminus \{0, 1\}$$

$$\frac{m_0\left[(1-m_0)(1+N) - NM\right]^{N+1}}{1 - m_0(1+NM)} = N^N (1-M)^N$$

$$\text{and } m_1 = \frac{1 - m_0(1+MN)}{(1-m_0)(1+N) - MN}.$$

[5] The surplus in a match (s, b) is $u(q_{s,b}(d)) - q_{s,b}(d) + V_{s+d} - V_s - (V_b - V_{b-d})$. Since $V_{s+d} - V_s \neq V_b - V_{b-d}$ in general, and money holdings are observable, the buyer would not necessarily offer d in order to get $q_{s,b}(d) = q^*$.

The divisibility of balances affects their distribution. For instance, if $M = 1/2$ then $m_b = (N+1)^{-1}\ \forall b$.[6] What's more, divisibility allows a beneficial redistribution of money from rich to poor, an 'extensive margin' effect. Now that $N > 1$, agents can spend fractions of their balances, and a class of 'moderately wealthy' agents arises, since $(m_0 + m_1)|_{N>1} < (m_0 + m_1)|_{N=1} = 1$. This raises the volume of trade, by lowering the fraction of agents who cannot buy or sell. It also improves trade efficiency, by reducing dispersion in valuations and mass of agents with extreme valuations (who produce either too little or too much). Hence, the redistribution allowed by partial divisibility raises the value of money, prices, and welfare (see Fig. 4 and 5 in Camera and Corbae, 1999).

Lemma 2. *A large γ and small ρ support the equilibrium where $d = 1/N$. Thus the equilibria (i) with lotteries and $N = 1$, and (ii) without lotteries, $N > 1$, and $d = 1/N$, generally coexist.*

Proof. Use Lemma 1, and the proof of Proposition 2 in Camera and Corbae (1999). □

Once again, buyers make small expenditures if $1/\gamma$ and ρ are low.[7] As the curvature of preferences rises (γ rises), buyers are less willing to spend lots if the price is low, as the marginal utility from consumption diminishes rapidly. As trade frictions drop (ρ falls) future consumption is discounted less, thus buyers are less likely to postpone a trade to search for a better price. Also, the value a token approaches a constant, independent of nominal wealth. Thus, buyers have less incentives to postpone trades also because price dispersion is low.

3.3 Divisibility and distributional effects

It is now clear that buyers want to make small monetary trades if they want to preserve their nominal wealth for future consumption, but also want to consume as frequently as possible, buying a little something even if prices are steep. Spending fractions of balances or randomizing on the transfer of indivisible balances both allow a reduction in *average* expenditure. The problem with indivisibility is that trading does not allow beneficial redistributions of the supply of money M. To show it, I consider the most limited form of divisibility, $N = 2$, when narrow monetary redistributions can occur. However, I give an example where an allocation achieved in this case dominates, in ex-ante welfare terms, the allocation under lotteries on indivisible balances.

Proposition. *For $M \leq 0.5$, there exists an equilibrium allocation with partially divisible money balances ($N = 2$) and no lotteries that yields higher ex-ante*

[6] One can verify that the expression that solves for m_0 is an identity when $m_0 = (1+N)^{-1}$ and $M = 1/2$.

[7] Since $\{V_{b+d} - V_b\}$ is a decreasing sequence, ρ small and γ large satisfy $u(q_s) > V_{1/N}$ for $s = (N-1)/N$, and $V_{(N-1)/N} + u(q_s) - V_1 > \max_{\hat{d}\leq 1}\left\{V_{1-\hat{d}} + u(q_s(\hat{d})) - V_1\right\}$ for $s = 0$. The poorest buyer spends all he has even if the price is high. The richest buyer spends the least he can, even if the price is low.

welfare than the best allocation attainable with indivisible money balances ($N = 1$) and lotteries.

Proof. Consider the equilibrium $d = \frac{1}{N}$ when $N = 2$ without lotteries, versus $N = 1$ with lotteries. Let $W(N)$ denote ex-ante welfare. I focus on γ large and ρ small, as they are necessary to induce small monetary trades (Lemmas 1 and 2). I also choose $M = 0.5$ as, under lotteries and $N = 1$, this quantity of money implies efficient trades and maximum number of matches.[8] This is the best that lotteries can do, as $W(1)$ achieves a maximum at $M = 0.5$, as I show below.

I use the standard notion of ex-ante welfare $W = \sum_{b\in B} m_b V_b$. Consider $N = 1$ with lotteries:

$$W(1) = \begin{cases} \dfrac{m_1(1-m_1)\gamma}{\rho(1-\gamma)} & \text{if } m_1 \leq m(\rho,\gamma) \\ m_1\left(\dfrac{A}{1-\gamma}\right)^{\frac{1}{\gamma}} & \text{if} \end{cases}$$

The most efficient outcome, $W(1) = \frac{\gamma}{4\rho(1-\gamma)}$, occurs when $m_1 = 0.5 < m(\rho,\gamma)$ ($m(\rho,\gamma) > 0.5$ if γ and ρ are sufficiently small). In this case the number of trade matches is maximized, $m_1(1-m_1) = 1/4$ when $M = 0.5$ and $q = q^*$ so trade surplus is maximized (see Fig. 1).

Now consider $N = 2$ without lotteries. In this case $m_b = \frac{1}{N+1}$ $\forall b$ hence $W(2) = \sum_{b\in B} m_b V_b > \frac{N}{N+1} V_{1/N}$ since V_b is increasing in b. Let $\gamma = \rho = 0.5$. It is easy to verify that $d = 0.5$ is an equilibrium, in which case $\frac{N}{N+1} V_{1/N} \approx 0.67$. Furthermore, if $\rho \leq 0.5$ then $m(\rho,\gamma) \geq 0.5$, hence $W(1) = 0.5$. Thus, $W(2) > W(1)$ around $\gamma = \rho = 0.5$. □

For $\gamma = \rho = 0.5$, Figure 1 illustrates the two economies: (i) $N = 2$ without lotteries, and (ii) $N = 1$ with lotteries. Since when $N = 2$ multiple equilibria may be possible, I focus on the one where $d = 0.5$, i.e. every 'rich' buyer spends a fraction of his money balances.[9] The illustration indicates this strategy is an equilibrium if $M \leq 0.5$, beyond which the value of money drops so that rich buyers may desire to spend all their balances. This makes sense, since in the economy where $N = 1$, the value of money (hence q) also falls for $M > 0.5$.

Thus, consider $M \leq 0.5$. In economy (i) buyers carry out small expenditures but average consumption is inefficient, either too high (if money is scarce), or too low (if money is plentiful). While consumption is efficient in economy (ii), ex-ante welfare is greater in economy (i) because money is more widely distributed so more agents can consume, relative to economy (ii). This beneficial extensive margin effect hinges on the buyers' ability to spend only half of their unit balances. The redistribution of money it generates, relative to economy (ii), is so beneficial

[8] When $N > 1$ and no lotteries are allowed, the number of trade matches is also maximized, as the distribution of money is uniform: $\sum_{b\in B\setminus\{0\}} m_b \sum_{s\in B\setminus\{1\}} m_s = \left(\frac{N}{N+1}\right)^2$.

[9] When $N = 2$, it can be proved that $d = 1/N$ only if $M < \hat{M}$, where $\hat{M} \geq 0.5$. A different trade pattern arises for $M > \hat{M}$ (see Camera and Corbae, 1999). When $N = 1$, $\tau \leq 1$ only if $M \leq 0.5$.

that it overtakes the trading inefficiencies due to rigidities in monetary offers (spend $d = 0.5$ or $d = 1$).[10]

4 Final remarks

This study has provided intuition on the importance of distributional aspects of asset divisibility, in search-theoretic models of money. When individual money balances are divisible, changes in market prices affect the distribution of money, trade opportunities, hence allocative efficiency. Thus, care must be taken in 'approximating' divisible-money models via models of randomized exchange of indivisible money balances. The latter approach captures the intensive margin aspect but ignores the extensive margin aspect of money divisibility.

I postulate that *if* abstracting from perfect divisibility of money is needed to construct an economy with analytically tractable money distributions, it may be reasonable to consider a model with randomized monetary trades where agents can *also* spend portions of their balances. Preliminary work (Berentsen, Camera, and Waller, 2004) indicates this modeling avenue has the potential to capture both the intensive and extensive margin aspects of a fully-divisible money model.

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[10] This is not a general result. For M small, q is constant in economy (ii), but not in economy (i). As $\rho \to 0$ then V_1 diverges in economy (ii) but converges to $(\frac{1}{1-\gamma})^{\frac{1}{\gamma}}$ in economy (i). Thus $W(1) > W(2)$ for ρ small enough.