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Inside and Outside Money as Alternative Media of Exchange

We study a random-matching model of money in which a subset of people, called bankers, have known histories and the rest, called nonbankers, have unknown histories. Earlier, we showed that if there are no outside assets, then an optimal arrangement has bankers issuing objects, banknotes, that are used in trades involving nonbankers. Here, the same model is used to compare such exclusive use of inside money to the exclusive use of outside money. We show that the set of implementable outcomes using outside money is a strict subset of the set using inside money.

IN AN EARLIER PAPER, Cavalcanti and Wallace (1999), we generalized a random-matching model of money by weakening the assumption that individual trading histories are private information. We assumed that a subset of people, called bankers, have known histories and that the rest, called nonbankers, have unknown histories. Under the assumption that there are no outside assets, we showed that an optimal arrangement has bankers issuing a form of inside money, banknotes, that function as a medium of exchange and that, in particular, are used in trades involving nonbankers. Here we use the same model to compare the functioning of inside money and outside money. We show that the set of implementable outcomes using outside money is a strict subset of the set using inside money. We also present a limited result concerning the sense in which the use of inside money is better.

Loosely speaking, those results arise for the following reason. While outside and inside money function in the same way in trades between nonbankers, they function in different ways in trades between nonbankers and bankers. If only outside money is used, then the purchasing capability of a banker depends on the banker's previous trades; if inside money is used, that purchasing capability need not be dependent on previous trades because the banker may be able to issue additional inside money at any time.

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Our assumed environment is a fairly standard random-matching model, except that (i) each person has a technology for creating durable and intrinsically useless objects that potentially serve as inside money and (ii) some people, the bankers, have known histories. Our approach to comparing the functioning of inside and outside money in that environment is in the spirit of mechanism design. For our environment, there is a large class of implementable allocations, allocations that are subgame perfect equilibria for some game. In general, such allocations make use of both inside money and outside money, an intrinsically useless object that individuals cannot create. For our comparison between the functioning of inside and outside money, we focus on two subsets of implementable allocations. One subset ignores the outside money. That subset is implementable because people are willing to ignore and discard outside money which is an intrinsically useless object. The other subset ignores inside money. That subset is also implementable, and for essentially the same reason. Our main result is that the subset that uses inside money and ignores outside money is larger than the subset that uses outside money and ignores inside money.

In section 1, we describe the environment in detail. In section 2, we describe two sets of symmetric and stationary allocations. One set uses outside money exclusively, while the other uses inside money exclusively. The distinction between using inside money and using outside money appears in the form of different laws of motion for money holdings—in a way that is consistent with our loose explanation given above. In section 3, the implementable allocations are described. They are shown to be allocations that satisfy some participation constraints. Those constraints, which arise solely from the assumption that no one can commit to future actions, are very different for bankers and nonbankers. Bankers can be punished in the future for current actions because their actions become part of a public record, while nonbankers cannot be punished because their actions do not become part of a public record. In section 4, we present the result that the set of implementable allocations using outside money is a strict subset of that using inside money. Then, in section 5, we present a limited result showing that there are good allocations that are implementable using inside money, but are not implementable using outside money. We end, in the conclusion, by commenting on the special assumptions we make and by saying why we think our model is an attractive model of inside money.

1. THE ENVIRONMENT

Time is discrete and the horizon is infinite. There are S distinct, divisible, and perishable types of goods at each date and there is a $[0, 1]$ continuum of each of S specialization types of people, where $S > 2$: a person whose specialization type is s consumes only good s and produces only good $s + 1$ (modulo S), for $s = 1, 2, \dots, S$. Each person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. Utility in a period is given by $u(x) - y$, where x is the amount consumed and y is the amount produced. The function u is defined on $[0, \infty)$, is increasing, twice differentiable, and satisfies $u(0) = 0$, $u'' < 0$, and $u'(0) = \infty$. Moreover, there exists $\tilde{x} > 0$ such that $u(\tilde{x}) = \tilde{x}$.

In each period, people are randomly matched in pairs. There are two kinds of meetings: single-coincidence meetings, those between a type s person (the producer) and a type $s + 1$ person (the consumer) for some s ; and no-coincidence meetings, those in which neither person produces what the other consumes. (Because the number of types, S , exceeds two, there are no double-coincidence meetings.) We assume that people cannot commit to future actions, so that those who produce have to get a future reward for doing so.¹

The society is able to keep a public record of the actions of a fraction B of each type of person, where $B \in (0, 1)$. It has no public record for any one else. As we will see, a person whose history is known, a banker, can be induced to produce without receiving something tangible in exchange, because the person can be rewarded in the future for actions they take currently; in contrast, a person whose history is not known, a non-banker, must receive something tangible in order to produce.

We also assume that each person has a technology that permits the person to create indivisible and perfectly durable objects, called notes. The notes issued by a single person are uniform, but are distinguishable from those issued by any one else, so that counterfeiting is not a problem. We also assume that any person is able to destroy or freely dispose of notes that the person has acquired. Outside money also consists of indivisible and perfectly durable objects. To keep the model simple, we assume that each person can carry from one date to the next at most one unit of notes issued by others or one unit of outside money.

Finally, we assume that people in a meeting know each other's specialization type, asset holdings, and identity in the sense of banker or nonbanker.

2. STATIONARY AND SYMMETRIC ALLOCATIONS

An allocation describes what happens in different kinds of meetings. Here we define two sets of stationary and symmetric allocations, inside-money allocations and outside-money allocations. The respective implementable allocations are subsets of those.

One symmetry we impose is across specialization types. Another is that all notes issued by bankers are treated identically and that all notes issued by nonbankers are ignored, are worthless. For our purposes, these restrictions on notes are innocuous; if anything, they make it harder for us to prove that the set of implementable inside-money allocations is larger than the set of implementable outside-money allocations. In any case, from now on notes refer to those issued by bankers.

We impose stationarity in several senses. At the start of a date, prior to being randomly matched, each person has a *state*. The state for a nonbanker is money holdings, outside money for an outside-money allocation, notes (banknotes) for an inside-money allocation, and in either case a member of the set $\{0, 1\}$. Despite the fact that bankers have known histories, we assume that there are also only two possible states for a banker, states that without loss of generality can also be taken to be the members of the

1. The above specification is essentially that in Shi (1995) and Trejos and Wright (1995).

set $\{0, 1\}$. For an outside-money allocation, the state for a banker is, like that of a nonbanker, holdings of outside money. For an inside-money allocation, the state for a banker has a different interpretation; it says something about the history of the banker. (Note holding is not a state variable for a banker because all banknotes are treated identically and a banker can always print a new one.) In particular, implicit in an inside-money allocation is a function that maps possible histories into the set of two states.

The above symmetries and stationarities permit us to describe inside- and outside-money allocations using the same notation. Almost all the notation we need pertains to what happens in single-coincidence meetings. We allow what happens to depend on the identity (banker or nonbanker) and state (0 or 1) of the producer and the consumer. Thus, we let production (and consumption) in a single-coincidence meeting be denoted $y_{ij}^{kl} \in R_+$ where the superscripts denote identity— $k, l \in \{b \text{ (banker)}, n \text{ (nonbanker)}\}$ and k , the first superscript, is the identity of the producer and l , the second, is that of the consumer—and the subscripts denote states— i , the first subscript, is that of the producer and j , the second, is that of the consumer. We also need to describe state-transitions in such meetings. We let $p_{ij}^{kl} \in \{0, 1\}$ denote the transition for the producer and let $q_{ij}^{kl} \in \{0, 1\}$ denote that for the consumer, where the superscripts and subscripts have the same meaning as for production. Here 0 means keep the current state and 1 means switch to the other state.² We also need some notation to describe the possibility that a banker gives a gift of money, either outside money or a note, in a no-coincidence meeting with a nonbanker. (Although nonbankers never give gifts, a general notation for such gifts is helpful.) We let $r_{ij}^{kl} \in \{0, 1\}$ describe whether a person with identity k in state i switches states in a no-coincidence meeting with a person with identity l in state j (again, 0 means keep the current state and 1 means switch states). Finally, we need notation for the distribution of bankers and nonbankers across states. We let x_i^k with $k \in \{b, n\}$ and $i \in \{0, 1\}$ denote the fraction of each production-consumption specialization type who have identity k (b for banker, n for nonbanker) and who are in state i . Because each person must be in one of the states, these fractions satisfy

$$\sum_i x_i^b = B \text{ and } \sum_i x_i^n = 1 - B. \quad (1)$$

As part of stationarity, we require that the fraction of bankers in each state and the fraction of nonbankers in each state are constant. We express the conditions that the x_i^k are constant by equating the inflow into state 1 to the outflow from state 1 for both nonbankers and bankers. In expressing these and other conditions, we anticipate one of the participation constraints; namely, that a nonbanker does not freely dispose of money. We also anticipate the stationarity requirement that no one freely disposes of outside money. For nonbankers, the inflow-equal-outflow condition is, therefore,

2. Were we only describing the use of outside money, we could get by with a single money transfer variable that describes whether or not the trading partners exchange money holdings.

$$x_0^n \sum_j x_j^b [p_{0j}^{nb} + q_{j0}^{bn} + (S-2)r_{0j}^{nb}] = x_1^n \sum_j x_j^b q_{j1}^{bn}. \quad (2)$$

For bankers, the inflow-equal-outflow condition is

$$x_0^b \sum_j x_j^n [p_{0j}^{bn} + q_{j0}^{nb} + (S-2)r_{0j}^{bn}] = x_1^b \sum_j x_j^n [p_{j1}^{bn} + q_{j1}^{nb} + (S-2)r_{j1}^{bn}]. \quad (3)$$

The right-hand side of (2) is relatively simple because we have built into it that non-bankers give up money at most when they are consumers.

Finally, there are restrictions implied by the preservation of outside-money holdings in all meetings and by the preservation of note holdings in meetings between non-bankers. We present these without qualifications and invoke them as needed in the definitions that follow.

$$p_{ii}^{kl} = q_{ii}^{kl} = r_{ii}^{kl} = 0 \quad (4)$$

and

$$p_{ij}^{kl} = q_{ij}^{kl} \text{ and } r_{ij}^{kl} = r_{ji}^{lk}. \quad (5)$$

The first, (4), says that if both people in a meeting have the same state, then neither can switch to a different state; the second, (5), says that one person in a meeting switches to a different state if and only if the other does.

We can now define (stationary and symmetric) inside- and outside-money allocations.

DEFINITION 1: *An inside-money allocation is $(x_i^k, y_{ij}^{kl}, p_{ij}^{kl}, q_{ij}^{kl}, r_{ij}^{kl})$ for $i \in \{0, 1\}$ and $k \in \{b, n\}$ that satisfies (1)–(3), and (4) and (5) for $k = l = n$.*

Notice that (4) and (5) must hold only when two nonbankers meet. Only in those meetings are initial note holdings necessarily preserved. In contrast, (4) and (5) hold for all meetings in an outside-money allocation.

DEFINITION 2: *An outside-money allocation is $(x_i^k, y_{ij}^{kl}, p_{ij}^{kl}, q_{ij}^{kl}, r_{ij}^{kl})$ for $i \in \{0, 1\}$ and $k \in \{b, n\}$ that satisfies (1)–(5).³*

The fact that the second definition is more restrictive is responsible for the strict subset result.⁴

Before proceeding, we give a simple example of each kind of allocation. We start with an example of an outside-money allocation.

EXAMPLE 1: *In single-coincidence meetings between bankers, the amount y is produced and consumed and no money changes hands. In all other single-coincidence*

3. In fact, one of (2) and (3) is redundant.

4. In an outside-money allocation, the constant amount of outside money per specialization type is $x_1^b + x_1^n \in [0, 1]$. Definition 2 permits the planner to choose that amount.

meetings, the amount y is exchanged for a unit of outside money if and only if that is consistent with preservation of money holdings in the meeting and the unit upper bound on holdings. In all other meetings nothing happens. (In terms of our notation, the nonzero output and state-transition variables are $y_{ij}^{bb} = y_{01}^{bn} = y_{01}^{nb} = y_{01}^{nn} = y$; $p_{01}^{nb} = p_{01}^{bn} = q_{01}^{bn} = p_{01}^{nn} = q_{01}^{nn} = 1$.)

For this example, it follows from (2) that the inflow into nonbankers' holdings of money per type is $S^{-2}x_0^n x_1^b$ and that the outflow is $S^{-2}x_1^n x_0^b$. Also, it is obvious that equality between the two implies equality between inflows and outflows for bankers. Therefore, the specification in Example 1 and x_i^k that satisfy $x_0^n x_1^b = x_1^n x_0^b$ and (1) constitute an example of an outside-money allocation.⁵ (Of course, such an allocation may or may not satisfy participation constraints, which will be described in the next section.)

The example of an inside-money allocation we give is one in which nothing depends on the state of bankers. The crucial features of it are that a banker issues a note if and only if the banker meets a nonbank producer without a note, and that the banker acquires and destroys (redeems) a note if and only if the banker meets a nonbank consumer with a note.

EXAMPLE 2: *In single-coincidence meetings between bankers, the amount y is produced and consumed. In all single-coincidence meetings between nonbankers, the amount y is exchanged for a note if and only if that is consistent with preservation of note holdings in the meeting and the unit upper bound on holdings. A nonbanker produces y for a banker and receives a note if and only if the nonbanker does not have a note. A banker produces y for a nonbanker and receives (and destroys) a note if and only if the nonbanker has a note. In all other meetings nothing happens. (In terms of our notation, the non zero output and state-transition variables are $y_{ij}^{bb} = y_{i1}^{bn} = y_{0i}^{nb} = y_{01}^{nn} = y$; $p_{0i}^{nb} = q_{i1}^{bn} = p_{01}^{nn} = q_{01}^{nn} = 1$.)*

In Example 2, no action depends on banker histories. Therefore, any x_i^b that satisfy (1) are acceptable. As for the x_i^n , according to (2) the inflow of notes into the hands of nonbankers per type is $S^{-2}Bx_0^n$ and the outflow is $S^{-2}Bx_1^n$. Therefore, $x_0^n = x_1^n = (1 - B)/2$.

As may be obvious, there does not exist an outside-money allocation that duplicates the trades in Example 2. In fact, later on, we use Example 2, with a restriction on y to insure implementability, to show that there are implementable inside-money allocations that cannot be duplicated by outside-money allocations.

3. IMPLEMENTABLE ALLOCATIONS

We use a weak notion of implementability. For us, an *implementable allocation* is one that is the outcome of a subgame perfect equilibrium for some game.⁶ We now

5. Example 1 has no gifts from bankers to nonbankers. In what follows, it is important to remember that this is just an example; the absence of such gifts is not a general feature of outside-money allocations. Indeed, the possibility of such gifts is what makes it difficult to prove that any outside-money allocation can be dominated in welfare terms by an inside-money allocation.

6. This notion is weak in that we do not require that the allocation be the outcome of all equilibria of the game. For a discussion of various notions of implementability, see Kreps (1990, p. 702).

show that the set of such allocations coincide with the set of allocations that satisfy some participation constraints, provided we assume that each person has the option to behave autarkically in any meeting and that nonbankers have the option to dispose of money.

It is convenient to express the participation constraints in terms of discounted expected utilities. We let v_i^k denote the expected discounted utility for a person with identity k who begins a period in state i . The stationarity implies that v_i^k can be expressed implicitly in terms of an allocation by

$$S(1 - \beta)v_i^k = \sum_{l,j} x_j^{i'} \{u(y_{ji}^{lk}) - y_{ij}^{kl} + \beta[p_{ij}^{kl} + q_{ji}^{lk} + (S - 2)r_{ij}^{kl}](v_{i'}^k - v_i^k)\}, \quad (6)$$

where $i' \neq i$. Notice that for a given allocation, (6) consists of two pairs of equations, each pair being two simultaneous linear equations in v_0^k and v_1^k . Those equations have a unique solution. (Among the ways to establish that is by a trivial contraction mapping argument.)

The first constraint is that

$$v_i^k \geq 0. \quad (7)$$

The second is a restriction on production by bankers,

$$-y_{ij}^{bl} + \beta[p_{ij}^{bl}v_{i'}^b + (1 - p_{ij}^{bl})v_i^b] \geq 0, \quad (8)$$

where $i' \neq i$.

For nonbankers, there are three constraints. One expresses the requirement that nonbankers not dispose of money. The other two require that nonbankers have non-negative gains from trade when they consume and when they produce. They are, respectively,

$$v_1^n \geq v_0^n, \quad (9)$$

$$u(y_{ji}^{ln}) + \beta[q_{ji}^{ln}v_{i'}^n + (1 - q_{ji}^{ln})v_i^n] \geq \beta v_i^n \quad (10)$$

and

$$-y_{ij}^{nl} + \beta[p_{ij}^{nl}v_{i'}^n + (1 - p_{ij}^{nl})v_i^n] \geq \beta v_i^n, \quad (11)$$

where $i' \neq i$.

We now show, in a very standard way, the equivalence between the set of implementable allocations and those that satisfy (6)–(11).

LEMMA 1: *An allocation, either an inside-money one (see Definition 1) or an outside-money one (see Definition 2), is implementable if and only if there exist v_i^k such that (6)–(11) hold.*

PROOF. First, suppose that an allocation satisfies (6)–(11). We can associate with any allocation, whether an inside-money allocation or an outside-money allocation, the following game. In meetings, people respond simultaneously by saying agree or disagree. If both say agree, then the relevant part of the allocation is carried out in that meeting. If at least one says disagree, then that meeting is autarkic: there is no production and any nonbanker keeps the initial state. Moreover, if a banker says disagree (defects), then that becomes part of the public record that becomes known to everyone at the beginning of the next date. Here are the proposed supporting equilibrium strategies. If there has not been a publicly recorded defection, then everyone plays agree. If there has been a recorded defection, then potential producers in single-coincidence meetings play disagree and others play anything. Consider the post-defection actions. Given that other producers will not produce in any future meeting, it is a dominant strategy of a producer, banker or nonbanker, to disagree and, thereby, not produce. It is dominant in the narrow sense that it is best no matter what the trading partner plays. It follows that prior to a defection a banker views the consequence of defecting to be permanent autarky; by definition of the above game, playing disagree gives rise to autarky in the meeting, and the above strategies imply permanent autarky thereafter. Therefore, a banker is willing to play agree provided the current action gives a net payoff that is at least as good as permanent autarky, a condition that is implied by (8). As regards nonbankers, it is obvious that they are willing to play agree and not dispose of money if (9)–(11) hold.

Now suppose that an allocation is implementable. Because never producing is an option, (7) and (8) must hold. And because autarky in a meeting and disposal of money is always possible without further consequences for a nonbanker, (9)–(11) must hold. \square

Notice that the difference between (8), on one hand, and (10) and (11), on the other hand, is the payoff from defecting. For bankers, the payoff from defecting is permanent autarky; for nonbankers, the payoff is autarky in the meeting only. One consequence of (11) is that a necessary condition for positive nonbanker production is that the nonbanker switches states. In other words, nonbankers do not engage in gift giving. That is not true of bankers. Inequality (8) does not imply that a banker must switch states in order to produce. Notice also that the only banker activity that is restricted is banker production. That is because banker defection leads, by definition, to autarky in the meeting and to subsequent permanent autarky. It follows that a banker may be tempted to defect only when the banker is asked to experience current disutility, namely, when asked to produce. In particular, a banker is never tempted to defect by issuing a note when the allocation says that a note should not be issued—as is always the case in what we are calling outside-money allocations and as may be the case for inside-money allocations. Finally, notice that inside- and outside-money allocations are not distinguished by different participation constraints. Therefore, as asserted above, what distinguishes the two kinds of allocations are the different restrictions on state transitions given in Definitions 1 and 2.

4. THE STRICT SUBSET RESULT

We begin by showing that there are implementable inside-money allocations that are not implementable outside-money allocations. We accomplish that by showing that there are implementable Example 2 allocations that are not implementable outside-money allocations. We first describe the set of implementable Example 2 allocations.

LEMMA 2: *Let $\rho \equiv S(1 - \beta)/\beta$ and let y'' be the unique positive solution for y of $u(y)/y = 1 + 2\rho/(1 + B)$. An Example 2 allocation is implementable if (and only if) $y \in [0, y'']$.*

The proof appears in Cavalcanti and Wallace (1999, Lemma 3) and will not be repeated here. It amounts to verifying that any such allocation satisfies (6)–(11). We now show that there is no equivalent outside-money allocation.

PROPOSITION 1: *There exist implementable inside-money allocations that are not outside-money allocations.*

PROOF. The proof is by contradiction. As noted above, in any Example 2 allocation, $x_i^n = (1 - B)/2$. Therefore, to duplicate the volume of trade among nonbankers for any implementable Example 2 allocation (see Lemma 2), the outside-money allocation must have the same nonbanker distribution of outside money. Now consider single-coincidence meetings in which the banker is the consumer and the nonbank producer does not have money. In the Example 2 allocation, the volume of trade per specialization type in such meetings is $S^{-2}B(1 - B)/2$. To duplicate that volume in the outside-money allocation, we must have $x_1^b = B$ and hence $x_0^b = 0$. Therefore, the right side of (2) is $q_{11}^{bn}B(1 - B)/2$. This is 0 for the outside-money allocation because, by (4), q_{11}^{bn} is 0 for an outside-money allocation. But, then, by (2), there is no inflow into holdings of outside money by nonbankers, which implies that they never produce for bankers. \square

Next we prove the set inclusion.

PROPOSITION 2: *Any implementable outside-money allocation is an implementable inside-money allocation.*

PROOF. This is immediate from the definitions because the conditions that an implementable inside-money allocation must satisfy are weaker than those that must hold for an implementable outside-money allocation. (To elaborate, given an implementable outside-money allocation, the duplicating inside-money allocation is constructed as follows. At the initial date, prior to meetings, divide the set of bankers for the inside-money allocation into two sets, one of measure x_1^b and the other of measure x_0^b , where these are the measures with and without money in the outside-money allocation. Now let the former set be designated inside-money state 1 and the latter inside-money state 0. Then choose all other components of the inside-money allocation to be those of the outside-money allocation.) \square

Propositions 1 and 2 give us the strict subset result.

5. A SUPERIORITY-OF-INSIDE-MONEY RESULT

We have said almost nothing about the allocations that are implementable inside-money allocations, but are not implementable outside-money allocations. One suspects that some of those are good allocations. The strongest such result would be the following. Given any implementable outside-money allocation, there is a Pareto-superior implementable inside-money allocation, where we distinguish among bankers and nonbankers with and without outside money. Weaker results would distinguish among fewer than those four types by averaging over types with weights given by population fractions. We have been able to produce only a result of the latter sort, using a representative-agent welfare criterion, and only for a region of the parameter space.

The representative-agent criterion is $v = \sum_{i,k} x_i^k v_i^k$. By (6) and (2)–(3), it follows that

$$S(1 - \beta)v = \sum_{i,j,k,l} x_i^k x_j^l z(y_{ij}^{kl}), \quad (12)$$

where $z(y) \equiv u(y) - y$. Notice that y_{ij}^{kl} appears in v only by way of the function z and that no state-transition variables appear in v . That makes it easy to produce an argument. Our result is that in a region of the parameter space, one we describe explicitly, there is an implementable inside-money allocation that produces a larger v than any outside-money allocation. The inside-money allocation is like the Example 2 allocation except that bankers also produce y for nonbankers without money and $y = y^* \equiv \arg \max z(y)$.

EXAMPLE 3: *Let the nonzero components of the output and state-transition variables be given by $y_{ij}^{bb} = y_{ij}^{bn} = y_{0i}^{nn} = y_{01}^{nn} = y^*$; $p_{0i}^{nb} = q_{i1}^{bn} = p_{01}^{nn} = q_{01}^{nn} = 1$.*

It follows, just as for the Example 2 allocation, that nothing depends on the state of a banker and that (2) implies $x_i^n = (1 - B)/2$. We first give a sufficient condition for implementability of this allocation.

LEMMA 3: *Let $\rho \equiv S(1 - \beta)/\beta$. If $z(y^*) \geq y^*$ and $\rho \leq \min\{B, (1 - 3B)/2\}$, then the example 3 allocation is implementable.*

The proof is given in the appendix. Example 3 has bankers produce for nonbankers twice as frequently as nonbankers produce for bankers. Discounting aside, the first hypothesis deals with this by making the reward for consuming y^* twice the disutility of producing y^* . In addition, Example 3 gives a nonbanker consumption in a meeting with a banker whether or not the nonbanker has money.⁷ Therefore, if B is too large,

7. Notice that the Example 3 allocation would not be implementable if nonbankers could hide money, because according to it they get the same consumption in a meeting with a banker whether or not they have money and because they prefer not to give up their money.

then a nonbanker has little incentive to produce to acquire money. The second hypothesis, which requires that $B < 1/3$, helps produce satisfaction of the nonbanker production participation constraint, (11).

We now prove that no outside-money allocation gives as large a value of v as does the Example 3 allocation.

PROPOSITION 3: *No outside-money allocation attains as high a v as that implied by the Example 3 inside-money allocation.*

The proof, which appears in the appendix, shows that there is no stationary distribution of outside money that achieves as high trading frequencies as those of Example 3. That suffices because the production levels in Example 3 are best according to v .

Even granting the representative-agent criterion, Proposition 3 is, of course, only relevant for the region of the parameter space for which Example 3 is implementable (see Lemma 3). In our attempts to get a more general result, we have continually run into difficulties related to the fact that an arbitrary outside-money allocation can contain gifts from bankers to nonbankers. That possibility accounts for the inclusion of a gift in Example 3. And that, in turn, makes Example 3 implementable only in a region of the parameter space.

We do not know whether a general result concerning superiority of inside-money allocations is possible. In particular, we have not produced a counterexample, an implementable outside-money allocation that cannot be dominated by some implementable inside-money allocation.

6. CONCLUSION

The idea that imperfect knowledge of histories is necessary for essentiality of money goes back at least to Ostroy (1973). [See Kocherlakota (1998) for a discussion and for related results.] Our model makes a distinction between inside and outside money. As regards outside money, the model provides an example in which only some knowledge of histories implies inessentiality of outside money. Kocherlakota and Wallace (1998) provide a closely related model in which there is a common lag in updating *every* person's history. Because of that symmetry, the flavor of their result is that there has to be almost complete knowledge—that is, a sufficiently short lag—in order that outside money be inessential. Here, at least for comparisons across steady states, it is sufficient for inessentiality of outside money that there be a positive measure of people with known histories. However, inside money is essential provided there is a positive measure of people with unknown histories.

Our results concerning the inessentiality of outside money are obtained in a special environment and for a limited set of allocations for that environment. One special assumption is that money is indivisible and that people can hold only either 0 or 1 unit of it. Related to that assumption is the very limited dependence on banker history allowed in our specification of allocations. However, the logic of our arguments seems not to depend on those assumptions or other features of the model. In particular, we expect to be able to establish the set inclusion result (Proposition 2) for general indi-

vidual holdings of money and for a general specification of dependence on banker histories because any such enrichment of the state space will be common to both inside- and outside-money allocations. We suspect, though, that enriching the state space will make it more difficult to show that there are inside-money allocations that are not outside money allocations (Proposition 1) because general individual holdings of money will enlarge the set of allocations that are implementable using outside money. Nevertheless, the result should still hold because, no matter what dependence on banker histories is allowed, a nonbanker produces only in exchange for something tangible.

In our proof of Lemma 1, we support the threat of permanent autarky against a defecting banker by global autarky. Although defection and, hence, global autarky do not occur in equilibrium, the threat of punishing defection by global autarky is from some points of view not credible. Although we could support the threat with only punishment of the defector, that seems to require that people play weakly dominated strategies in some meetings because nonbankers cannot be punished for trading with a defecting bank. In any case, we should emphasize that our set inclusion result survives so long as the threat against a defecting bank is at least as severe for inside-money allocations as for outside-money allocations. Punishing with permanent autarky is one example that satisfies that condition.

The assumptions we have made dictate the form taken by inside money in the model. Given those assumptions, it seemed natural to us to label the inside money “banknotes,” but in some respects that terminology may be misleading. In an inside-money allocation, nonbankers who produce for bankers receive a security in exchange. The fact that they produce to get the security makes the security resemble a form of trade credit. The security resembles historically observed banknotes because it is a bearer security with no explicit maturity that gets passed around among nonbankers and, as in some actual banking systems, is redeemed by all bankers.⁸ Notice, also, that aside from the indivisibility and zero-one set of individual holdings, the model’s banknotes resemble electronic money in the form of stored value cards provided that nonissuers can transfer value between cards—the analogue of two nonbankers transferring banknotes between them in the model.

The assumptions also account for the somewhat unrealistic feature built into allocations that a banker’s note issue does not depend on the banker’s previous note issue. As we noted in Cavalcanti and Wallace (1999), the known banker’s history includes the banker’s cumulated note issues and redemptions. In our model, actions could be made dependent on such state variables. We did not assume such dependence for two reasons: it would make the presentation more complicated and it would almost certainly not be desirable in our setting. From an *ex ante* point of view, it is desirable that a potential consumer in a single-coincidence meeting consume no matter how many times the person consumed in the past and analogously for a potential producer. Of course, it is well known how to amend the model to make history dependence desir-

8. Nothing in the model prevents some asymmetrical treatment of banknotes. For example, one could imagine dividing the set of bankers into positive measure subsets and having each bank redeem notes issued only by members of its subset. In fact, it is only because each bank has measure zero that we cannot have a bank redeem only its own notes. Whether any such asymmetric redemption scheme is desirable in another matter. We suspect that it is not.

able: introduce private information for bankers along the lines of what Green (1987) and successors have done. In fact, a small step in that direction was taken in Cavalcanti and Wallace (1999). There, however, we did not compare the use of inside and outside money.

Despite all its extreme assumptions, we think that our model represents an attractive starting point for more elaborate models of inside money. The main ingredients of the model seem entirely plausible. The issuers of inside money are those about whom much is known. The nonissuers are those about whom little is known. Moreover, because little is known about nonissuers and for other reasons, including absence-of-double-coincidence difficulties, trade among nonissuers is enhanced by the presence of a tangible medium of exchange. Finally, our model is also attractive from the viewpoint that the proof of the pudding is in the eating. The roles of inside money and outside money as alternative media of exchange have been debated at least since the nineteenth century controversies associated with the so-called banking and currency schools. Our model seems to be the first that produces results about the allocations that are implementable using each kind of medium of exchange. Because the model implies that more allocations are implementable using inside money than using outside money, it does not support the currency school view.

A. APPENDIX

A.1 Proof of Lemma 3

(Let $\rho \equiv S(1 - \beta)/\beta$. If $z(y^*) \geq y^*$ and $\rho \leq \min\{B, (1 - 3B)/2\}$; then the Example 3 allocation is implementable.)

Let \hat{v}_i^k denote expected discounted utility implied by the Example 3 allocation. The allocation satisfies the banker participation constraint, (8), if $\beta \hat{v}_i^b - y^* \geq 0$. From (6) and the description of Example 3,

$$\rho \beta \hat{v}_i^b = \left(B + \frac{1 - B}{2} \right) z(y^*) - \frac{1 - B}{2} y^* \geq B y^*, \quad (13)$$

where the inequality uses the first hypothesis. It follows that

$$\beta \hat{v}_i^b - y^* \geq \left(\frac{B}{\rho} - 1 \right) y^* \geq 0, \quad (14)$$

where the first inequality follows from (13) and the second from the second hypothesis. (Large B helps satisfy (8) because in meetings between bankers, bankers produce and consume with the same frequency.)

From (6) and the definition of Example 3, we have

$$(\rho + 1 + B) \beta \Delta = \frac{1 - B}{2} [u(y^*) + y^*] + B y^*, \quad (15)$$

where $\Delta \equiv \hat{v}_1^n - \hat{v}_0^n$. For satisfaction of (11), we require $\beta\Delta - y^* \geq 0$. Using (15), we get

$$\begin{aligned}\beta\Delta - y^* &= \frac{\frac{1-B}{2}[u(y^*) + y^*] + By^*}{\rho + 1 + B} - y^* \\ &\geq \frac{\frac{1-B}{2}3y^* + By^*}{\rho + 1 + B} - y^* \\ &= \left[\frac{3(1-B)/2 + By^*}{\rho + 1 + B} - 1 \right] y^* \geq 0,\end{aligned}\tag{16}$$

where the first inequality uses the first hypothesis and the second uses the part of the second hypothesis which says that $\rho \leq (1 - 3B)/2$. As for the other two nonbanker participation constraints, (9) and (10), the reader can easily deduce from (15) that they are satisfied. \square

A.2 Proof of Proposition 3

(No outside-money allocation attains as high a v as that implied by the Example 3 inside-money allocation.)

Let v_{out} denote the v implied by an outside-money allocation. We proceed by obtaining an upper bound on v_{out} . By (12), the best production level is y^* . Hence, we set production at y^* , except when it must be zero (when a nonbank producer has money or meets someone without money). Therefore,

$$S(1 - \beta)v_{out} \leq [B + x_0^n(x_1^b + x_1^n)]z(y^*),\tag{17}$$

where the first term arises from meetings in which the producer is a banker and the second from meetings in which the producer is a nonbanker. Denoting the v implied by Example 3 by \hat{v} , it follows that

$$S(1 - \beta)\hat{v} \leq \left[B + \frac{(1-B)(1+B)}{4} \right] z(y^*),\tag{18}$$

where, again, the first term arises from meetings in which the producer is a banker and the second from meetings in which the producer is a nonbanker, and where we use the fact that $x_1^n = (1 - B)/2$ for Example 3.

There are two cases to consider. One is where money is not used in trades between bankers and nonbankers in the outside-money allocation. Then it follows that nonbankers only produce for other nonbankers and the best money distribution is $x_0^n = x_1^n = (1 - B)/2$. The result in this case is that v_{out} satisfies $S(1 - \beta)v_{out} \leq [B + (1 - B)(1 - B)/4]z(y^*)$, which implies $v_{out} < \hat{v}$.

The other case is where money is used in trades between bankers and nonbankers. Because (4) holds for all k and l for an outside-money allocation, (2) becomes

$$x_0^n x_1^b M = x_1^n x_0^b q_{01}^{bn}, \quad (19)$$

where $M \equiv p_{01}^{bn} + q_{10}^{bn} + (S - 2)r_{01}^{bn}$. The assumption that outside money is used in trades between bankers and nonbankers implies that $q_{01}^{bn} = 1$ and $M \geq 1$. Then, (19) and (1) imply that x_1^b can be expressed in terms of x_0^n as

$$x_1^b \left(M + \frac{1 - B - x_0^n}{x_0^n} \right) = \frac{(1 - B - x_0^n)B}{x_0^n}. \quad (20)$$

Therefore, letting $\theta \equiv 1 - B$, we have

$$\begin{aligned} x_0^n (x_1^b + x_1^n) &= x_0^n \left[\frac{(\theta - x_0^n)B}{Mx_0^n + \theta - x_0^n} + \theta - x_0^n \right] \\ &\leq x_0^n \left[\frac{(\theta - x_0^n)B}{\theta} + \theta - x_0^n \right] = \frac{x_0^n (\theta - x_0^n)}{\theta}. \end{aligned} \quad (21)$$

It follows that $x_0^n (x_1^b + x_1^n)$ is maximized at $x_0^n = \theta/2$, which implies $x_0^n (x_1^b + x_1^n) = (1 - B)(1 - B)/4$. By (17) and (18), we again get $v_{out} < \hat{v}$. \square

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