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Economic Theory

Decentralized credit and monetary exchange without public record keeping*

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Summary. We relax a standard assumption on the matching technology in a search model of money. In particular, agents may remain in a long-term partnership as long as it is in their self-interest. With this simple modification, it is possible to support self-enforcing, intertemporal trade which resembles credit without a public record keeping device. We examine conditions for co-existence of currency and credit and the welfare gains/losses associated with the introduction of money.

Keywords and Phrases: Search frictions, Partnerships, Gift-giving.

JEL Classification Numbers: E0.

1 Introduction

In an important paper, Kocherlakota (1998) established the inessentiality of money in the presence of memory (defined to be a perfect public record keeping device). In particular, he showed that any incentive feasible allocation that could be implemented with money could be implemented with memory. In fact, since there is randomness in the matching technology in standard search models of money, there are allocations that can be implemented with memory that cannot be implemented with money (e.g. a match where there is a coincidence of wants but the potential

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buyer has no money since he was lucky enough to have purchased in his last match). While there will be no public record keeping device in this paper, we will show how long-term partnerships enable agents to exploit match-specific information partitions to their advantage.

Credit arrangements are precluded in standard monetary search models by a combination of assumptions on technologies (more specifically, matching, commitment, and record keeping technologies). In this paper we wish to maintain the decentralized nature of trading relationships in standard search models (so that all trade is bilateral, there is no centralized (third party) commitment or record-keeping technologies), but we relax an assumption on the matching technology so that two agents who were randomly matched may establish a long-term partnership if it is in their interest. In this case a match can generate a valuable information partition (their history of trades) that respects the other decentralized assumptions of prototypical search models and upon which Pareto improving strategies by the two agents (like credit) can be conditioned.

Specifically, we extend the standard indivisible goods search model of Kiyotaki and Wright (1993) by allowing matched pairs of agents to stay together, as long as it is mutually beneficial.[1] During any match, there are random arrivals of nonstorable production opportunities. In a double coincidence match, an agent will sometimes be *able* to produce when there is no possibility of *quid-pro-quo* trade. Whether the agent is *willing* to produce (or give a gift or extend credit) is a central concern of this paper. The partnership dissolves after production through an exogenous technology shock that makes the production site at which both agents were matched permanently inoperable. Hence, as long as there are outside options (other possible valuable partnerships) the steady-state equilibrium still involves search.

A number of issues in the organization of exchange arise in our model. Can efficient bilateral "credit" relationships be self-enforcing? Can such "credit" partnerships co-exist with valued fiat money? In particular, does monetary exchange weaken incentives of such partnerships? Does monetary exchange even lead to an improvement in ex-ante welfare? Since matched agents' relationships is a dynamic game, there are, in the usual sense, many possible equilibria. We examine the possibility of multiple, steady state monetary equilibria determined by agents' beliefs about the future "credit worthiness" of their trading partner.

Several papers take a somewhat different approach than ours to relaxing the assumptions on matching, information, and commitment made in earlier search models. One closely related paper is that of Kranton (1996), who examines how the presence of an outside option of anonymous monetary exchange can cause reciprocal exchange to disintegrate.[2] In the first part of her analysis, households are restricted exogenously to be in credit partnerships or the money market; in the second part where households can choose their participation, eventually everyone in the population will engage in the same mode of exchange depending on the

[1] A previous version of this paper considered divisible goods, but this paper makes similar points in a much simpler environment.

[2] Diamond (1990) examines pairwise credit equilibria in a search model without money. While not a search model, Aiyagari and Williamson (2000) also examine incentive effects due to the lack of commitment associated with monetary exchange on credit arrangements.

initial size of the money market. In our paper, credit and monetary exchange can co-exist in the limit. Kocherlakota and Wallace (1998) introduce an incomplete public record keeping technology of exchange histories which enables agents to punish deviations from gift exchange. Unlike that paper, however, our framework does not rely on a public record keeping technology but shows how a decentralized search environment with enduring bilateral matches admits the possibility of match specific histories and punishment strategies.[3] Jin and Temzelides (2000) consider credit histories which are localized instead of economy-wide. Shi (1996) includes a commitment technology; collateral can be used to tie borrowers and lenders together.

The paper has two main sections. In Section 2, we study a simple economy without fiat money and establish existence of double coincidence equilibrium long-term credit partnerships. Without fiat money, however, there are many single coincidence matches where there are potential gains from trade which go unexploited. For this reason, in Section 3, we add fiat money to the environment. We show how the presence of currency weakens incentives to form credit partnerships. In particular, we establish there always exists a region of the parameter space where the incentive to extend credit when one has money is weakened. We also provide examples where money and credit partnerships co-exist and show the possibility that certain equilibria can be dominated ex-ante by equilibria without money.

2 A search economy without money

2.1 Environment

The economy consists of a continuum $[0, 1]$ of infinitely lived agents, each of whom specializes in the production of a distinct, nonstorable, indivisible good. Time progresses continuously and is infinite. At each date t,an agent is either matched with a partner at a production location or searching for a potential partner.

2.1.1 Preferences, production, and endowments

We assume that there exists an exogenous parameter that captures the extent to which real commodities and tastes are differentiated. In particular, x equals the proportion of commodities that can be consumed by any given agent, and x also equals the proportion of agents that can consume any given good. Thus for a pair of agents i and j selected at random in our economy, the probability that i wants to consume the good that j produces is x. Furthermore, we assume that conditional on the event that i wants to consume the good that j produces, the probability that j wants to consume the good that i produces is y.[4] Consuming one of his consumption goods yields agent i utility $u > 0$. An agent's rate of time preference is given by $\rho > 0$.

[3] Another difference is that we do not pose the problem in a mechanism design framework but look for non-cooperative, perfect equilibria.

[4] This specification is from Rupert, Schindler, and Wright (2001). A similar version that endogenizes specialization can be found in Camera, Reed, and Waller (forthcoming).

Each agent has the capability to produce one unit of a nonstorable good at a production location. Agents are associated with their production good - agent i produces good i from which he derives no utility. Agent specific production opportunities arrive at each location randomly; the arrival times are exponentially distributed with parameter α. The arrival time processes are independent across locations and individuals. These opportunities cannot be stored. The disutility cost of production is $c \in (0, u)$. Conditional on production, the probability of an exogenous breakdown of the technology (such that no further production can take place at that site) is $\delta/2\alpha$ is independent across locations and time.[5] This exogenous breakdown ensures that there is a steady stream of new participants in the search pool. Since an agent derives no utility from production of his own good, autarky yields a payoff of zero.

2.1.2 Matching

At each date an agent is either searching for a new trading partner or matched at a production location. Agents waiting for a trading partner are matched in pairs through a random process. When two agents meet, only their prior trading histories are private information. Unlike prior monetary search models, we allow matches to continue if both agents choose to stay together at their production location.

We normalize the arrival rate of matches for households in the search pool to one. Given preferences and technologies, the probability of a single-coincidence match at any given time is $x(1-y)$ and the probability of a double-coincidence match is xy.

2.1.3 Actions and the order of play

An agent who is unmatched has two choices: engage in search or drop out of the economy. An agent who drops out of the economy receives the autarkic payoff 0.

The play of agents who have been matched unfolds as follows.

- When agents i and j meet, each simultaneously decides whether to stay together at their given production location or resume search. Let $a_s^{ij} \in \{0, 1\}$ denote agent i's decision to stay ($a_s^{ij} = 1$) in a match with j or separate (exit the match) and re-enter the search pool ($a_s^{ij} = 0$).
- An agent who receives a production opportunity chooses whether or not to produce. Let $a_p^{ij} \in \{0, 1\}$ denote agent i's decision to produce ($a_p^{ij} = 1$) in a match with j or not produce ($a_p^{ij} = 0$).[6] Without loss of generality, if production takes place, we assume exchange and consumption takes place immediately.
- Immediately following the production decision, each agent decides whether to stay together or to search.

[5] Such shocks are uninsurable since no one else in the economy knows what happens at a production site. Alternatively, one can interpret these breakdowns as death by one of the partners (with subsequent birth away from the production location by an identical household).

[6] Given the assumptions on the production arrival process, two agents at a production location will not simultaneously receive production opportunities.

2.2 Equilibrium

Characterizing an equilibrium where agents form partnerships and extend credit to one another is simple in this environment. It is easy to show that since production is costly, single-coincidence matches have no value since there is no way to undertake $quid-pro-quo$ exchange. On the other hand, when two agents who have a double coincidence meet, they can play a repeated game in which one agent who has a production opportunity is willing to extend "credit" to his partner for fear of losing future opportunities to consume.

We define a *credit strategy* as one in which agents in a partnership produce (issue credit) unless there is an instance of previous default in their trading history. In particular, agent $i's$ *credit strategy* (denoted σ_C^i) specifies that he: (i) chooses to search rather than remain in autarky, (ii) chooses to continue to search upon a single-coincidence match, (iii) chooses to form a credit partnership upon a double coincidence match, (iv) chooses to expend effort upon arrival of a production opportunity and stays in the double coincidence match provided there is no previous instance of non-effort at a production opportunity, and (v) chooses not to produce and separates if there is a previous instance of non-effort at a production opportunity in the double coincidence match.

More formally, in the match between i and j, let $a(\tau)$ denote their action profile at the τ^{th} countable event. For instance, $a(0) = (a_s^{ij}(0), a_s^{ji}(0))$ denotes the stay or search actions at the event associated with their first meeting, while if τ is an event where agent i receives a production opportunity then $a(\tau) = (a_p^{ij}(\tau) = 1, a_p^{ji}(\tau) = 0)$ denotes a gift by agent i to agent j. A *history* in the repeated game between agents i and j is denoted $h(\tau+1) = (a(0), a(1), \ldots, a(\tau))$. Let $A^i(h(\tau+1))$ denote player i's feasible actions at event $\tau+1$ when the history is $(h(\tau+1)$. Let H^τ denote the set of all possible histories at event τ. Then a *pure strategy* for agent i is a set of maps (ζ^i) where $\zeta^i : H \rightarrow A^i$. Finally, let σ denote the set of probability distributions over the pure strategies. If we let s^{ij} denote the probability that agent i stays in the match with agent j and p^{ij} denote the probability that agent i produces when he receives a production opportunity in the match with agent j, then the last two nodes of σ_C^i can be written: (iv) $(p^{ij}(\tau) = 1, s^{ij}(\tau) = 1$ provided $h(\tau')$ contains no $a_p^k(\tau') = 0, \forall \tau' < \tau, k \in \{ij, ji\}$; (v) and $(p^{ij}(\tau) = 0, s^{ij}(\tau) = 0$ if $h(\tau')$ contains $a_p^k(\tau') = 0, \forall \tau' < \tau, k \in \{ij, ji\})$.

Let $\Gamma(t)$ denote the proportion of agents in search and $\Phi(t)$ denote the proportion of agents in double-coincidence partnerships. Then

$$\Gamma(t) + \Phi(t) = 1. \tag{2.1}$$

If we let $s(t) = s^{ij}(t)$ denote the probability that a partnership continues (i.e. a hazard rate), then the law of motion for the proportion of agents in search is given by:

$$\dot{\Gamma} = \delta\Phi - xys\Gamma. \tag{2.2}$$

That is, inflows occur through a breakdown of the production technology (probability $\delta/2\alpha$) when either one of the partners receives a production opportunity (which arrives at rate 2α) in a double-coincidence partnership (of which there is proportion Φ). Outflows occur through matches of those in the search pool (of which there is proportion Γ) of pairs who choose to form a partnership (since $s = 1$) in the case of a double coincidence (xy).

Definition 2.1. *A symmetric, steady state credit equilibrium without money is a strategy profile Σ_C and a stationary distribution $\{\Gamma, \Phi\}$ induced by Σ_C, of agents in double-coincidence partnerships such that for each agent i, σ_C^i is a Nash Equilibrium in every history and Φ satisfies (2.1) and (2.2) with $\dot{\Gamma} = 0$.*

We next show how to verify that such an equilibrium exists in certain regions of the parameter space. On the equilibrium path, two agents playing σ_C always produce at every opportunity and never reach the punishment phase. To show this, let V and W be the lifetime utilities achieved by an agent matched and in search who plays σ_C, given that all other agents play Σ_C. Lifetime utilities must satisfy:

$$\begin{aligned} \rho V &= \alpha\,(u - c) + \delta\,(W - V) \\ \rho W &= xy\,(V - W)\,. \end{aligned}$$

These imply:

$$0 < V - W = \frac{\alpha\,(u - c)}{\rho + \delta + xy}. \tag{2.3}$$

Now suppose that i entertains the possibility of a one-shot deviation from $s^{ij} = 1$ in his match with j. In that case he solves

$$\max_{s^{ij}} s^{ij} V + (1 - s^{ij}) W$$

But since $V > W$, he chooses $s^{ij} = 1$, establishing it is a best response to form a partnership (condition (iii)). It is also clear that $W > 0$, establishing that it is a best response to continue searching if only one agent likes the other's good (conditions (i) and (ii)) since there is no possibility of trade in single coincidence matches without money.

To check that i has no incentive to "default" (condition (iv)), we examine a one-shot deviation from expending effort at the arrival of a production opportunity. If i deviates, according to Σ_C he is punished with search so that for effort to be a best response it must be the case that

$$-c + \frac{\delta}{2\alpha} W + \left(1 - \frac{\delta}{2\alpha}\right) V \geq W$$

or using (2.3)

$$\frac{\alpha\left(1 - \frac{\delta}{2\alpha}\right)(u - c)}{\rho + \delta + xy} \geq c. \tag{2.4}$$

Thus, the "no default" $p^{ij} = 1$ condition is less likely to bind the higher is utility (u) and the arrival rate of production opportunities (α) and more likely to bind the higher is the rate of time preference (ρ)[7], the probability of exogenous breakdown of the partnership (δ), the cost of expending effort (c), and the lower are search frictions (xy, since your outside options are easier to find).

Finally, given that all agents are playing Σ_C, in a history where one agent has defaulted, the punisher has no incentive to deviate from search (which yields $W > 0$) since his partner will not produce at subsequent opportunities according to Σ_C thereby yielding 0 if he stays in the partnership (condition (v)).

The final detail to complete our analysis of credit equilibria without money is to determine the proportions of agents searching and matched. Since $\Gamma + \Phi = 1$,the steady state proportion of agents in search is given by:

$$\Gamma = \frac{1}{1 + \frac{xy}{\delta}}.$$

Notice that the proportion of agents in search is increasing in the separation arrival rate (δ) and decreasing in the matching probability (xy).

We summarize these results in the following proposition.

Proposition 2.2. *Provided* $\frac{\alpha\left(1-\frac{\delta}{2\alpha}\right)(u-c)}{\rho+\delta+xy} \geq c$*, a symmetric, steady-state credit equilibrium without money exists.*

3 A search economy with money

It is clear in the above analysis that while credit partnerships improve upon autarky, there are many single-coincidence matches that go unexploited. This suggests that fiat money could enhance the efficiency of the search economy. The interesting issue is to what extent money affects credit partnerships by weakening incentives for partners with money to produce.

3.1 Environment

Population, preferences, and productive technologies remain as in the previous section. What is new is that initially, a fraction $M \in (0, 1)$ of the agents are endowed with one unit of indivisible fiat money. Only one unit of money can be stored. Agents can receive production opportunities while they hold money (i.e. money does not displace production) and can trade using money, goods, or both. We assume that money can be freely discarded.

Let $\Gamma_m(t)$ be the fraction of the entire population who are searching and have money $m \in \{0, 1\}$ at time t. Potential trading partners are drawn from the pool of all other searchers. The arrival rate of partners with money is $\gamma_1(t)$, where $\gamma_1(t) = \Gamma_1(t)/(\Gamma_0(t) + \Gamma_1(t))$ is the probability that a randomly selected searcher holds money. Thus, the arrival rate of single-coincidence partners who have money is $x(1 - y)\gamma_1$ and the arrival rate of double-coincidence partners who have money is $xy\gamma_1$.

[7] Consistent with the standard folk theorem result.

Four kinds of matches are technically possible: single-coincidence (which we denote SC), double-coincidence with neither agent holding money (DC0), double-coincidence with one agent holding money (DC1), and double-coincidence with both agents holding money (DC2).We denote the proportions of the entire population who are: (1) holding money inventory $m \in \{0,1\}$ while searching by $\Gamma_m(t)$; (2) in SC matches by $\Theta(t)$; and (3) in DC0, DC1, and DC2 matches by $\Phi_0(t)$, $\Phi_1(t)$, and $\Phi_2(t)$. Thus,

$$\Gamma_0 + \Gamma_1 + \Phi_0 + \Phi_1 + \Phi_2 + \Theta = 1. \tag{3.1}$$

Furthermore,

$$\Gamma_1 + \Phi_2 + \frac{1}{2}(\Phi_1 + \Theta) = M. \tag{3.2}$$

3.2 *Equilibrium*

There are many different types of equilibria that can arise in this economy. In this section, we focus on an equilibrium that resembles the equilibrium studied in Section 2.2 in that credit strategies are a best response in all double coincidence matches, but monetary exchange takes place in single coincidence matches. We believe this is the most interesting since it maximizes production and exchange (i.e. at all production opportunities in all single and double coincidence matches). In the conclusion, we discuss the consequences of other types of exchange strategies, which may call for the exchange of currency even within double coincidence matches.[8]

We construct a *symmetric, steady-state credit equilibrium with money* by (1) conjecturing a credit strategy in double coincidence matches and a quid-pro-quo strategy in single coincidence matches; (2) finding the implied steady-state proportions of agents in each type of relationship; (3) calculating steady-state utilities implied by the conjectured strategies; and (4) verifying that the strategies are individually rational after every history given the steady state payoffs.[9]

3.2.1 Strategies

As in Section 2.2, we define a *credit strategy with money* as one in which agents in a given double coincidence match produce (issue credit) unless there is an instance of previous default in their trading history, regardless of his partner's money holdings. In particular, agent $i's$ credit strategy with money specifies that he: (i) chooses

[8] It is worth keeping in mind that credit equilibria always depend on agents' beliefs that others are creditworthy (i.e. that $p^{ij}(\tau) = 1$ at the relevant nodes). When there is a credit equilibrium there is generally a quid-pro-quo equlibrium that looks like the monetary equilibrium in previous search models, except that agents in DC1 matches choose to trade repeatedly. We discuss this further in the Section 4.

[9] It is possible to formally state a definition of equilibrium in much the same way as in Section 2.2 with the modification that a strategy is a mapping from the space of all possible histories and all possible money holdings to an action. Rather than clutter the body of the paper with more notation, the reader should see the earlier version of this paper.

to search rather than remain in autarky, (ii) chooses to enter a single-coincidence match as either a buyer or a seller, (iii) chooses to expend effort (give up money) as a seller (buyer) in single coincidence matches, (iv) chooses to form a credit partnership upon a double coincidence match irrespective of money holdings, (v) chooses to expend effort upon arrival of a production opportunity without exchange of money and stay in the double coincidence match provided there is no previous instance of non-effort at a production opportunity, and (vi) chooses not to produce and begins to search if there is a previous instance of non-effort at a production opportunity in the match.

3.2.2 Population proportions

Let s_z, s_0, s_1, and s_2 denote the probability that two agents choose to form a single coincidence or double coincidence match where the match has $0, 1$, or 2 units of money (e.g. $s_z = s_z^{ij}(0)s_z^{ij}(0)$ where $s_z^{ij}(0)$ is the probability that agent i chooses to enter the single coincidence match upon first meeting agent j). The laws of motion for partnerships and the proportion of agents actively engaged in search are given by

$$\dot{\Gamma}_0 = \frac{\alpha}{2}\Theta + \delta\Phi_0 + \frac{\delta}{2}\Phi_1 - \left[x(1-y)s_z\gamma_1 + xys_0\gamma_0 + xys_1\gamma_1\right]\Gamma_0 \quad (3.3)$$

$$\dot{\Gamma}_1 = \frac{\alpha}{2}\Theta + \frac{\delta}{2}\Phi_1 + \delta\Phi_2 - \left[x(1-y)s_z\gamma_0 + xys_1\gamma_0 + xys_2\gamma_1\right]\Gamma_1 \quad (3.4)$$

$$\dot{\Phi}_0 = xys_0\gamma_0\Gamma_0 - \delta\Phi_0 \quad (3.5)$$

$$\dot{\Phi}_1 = xys_1\left(\gamma_0\Gamma_1 + \gamma_1\Gamma_0\right) - \delta\Phi_1 \quad (3.6)$$

$$\dot{\Phi}_2 = xys_2\gamma_1\Gamma_1 - \delta\Phi_2 \quad (3.7)$$

$$\dot{\Theta} = x(1-y)s_z\left(\gamma_0\Gamma_1 + \gamma_1\Gamma_0\right) - \alpha\Theta. \quad (3.8)$$

The stock of searchers Γ_m with money m is augmented by the completion of single-coincidence trade (Θ term) and breakups of DC matches ($\Phi_{m+m'}$ terms). The stock is depleted by the start of SC and DC matches (Γ_m term). The stock of agents in DC or SC relationships ($\Phi_{m+m'}$ or Θ terms) is augmented by new matches which stay together and is depleted by exogenous breakups. Each breakup resturns one trader to Γ_0 and one trader to Γ_1.[10]

3.2.3 Steady state lifetime utilities

We label the value functions of agents in search $W(m)$, in single-coincidence matches $Z(m)$, and in double-coincidence partnerships $V(m, m')$ where m' is the money holdings of one's partner. Under the conjectured equilibrium the value

[10] The differential equations preserve the total population and fraction of agents with money, but do not determine them: $\dot{\Gamma}_0 + \dot{\Gamma}_1 + \dot{\Phi}_0 + \dot{\Phi}_1 + \dot{\Phi}_2 + \dot{\Theta} = 0$ and $\dot{\Gamma}_1 + \dot{\Phi}_2 + \frac{1}{2}\dot{\Phi}_1 + \frac{1}{2}\dot{\Theta} = 0$. Thus, they contain two redundancies (i.e. they are not independent equations). The two accounting identities (3.1) and (3.2) provide the necessary normalizations. Equations (3.1)-(3.8) have two solutions, but only one is strictly positive.

functions in search are

$$\rho W(0) = x(1-y)s_z\gamma_1 [Z(0) - W(0)] + xys_0\gamma_0 [V(0,0) - W(0)] + xys_1\gamma_1 [V(0,1) - W(0)] \quad (3.9)$$

and

$$\rho W(1) = x(1-y)s_z\gamma_0 [Z(1) - W(1)] + xys_1\gamma_0 [V(1,0) - W(1)] + xys_2\gamma_1 [V(1,1) - W(1)] \quad (3.10)$$

where s_z, s_0, s_1, s_2 are all 1. The value functions for a seller and a buyer in a single coincidence match are given by

$$\rho Z(0) = \alpha [-c + W(1) - Z(0)] \quad (3.11)$$

$$\rho Z(1) = \alpha [u + W(0) - Z(1)] . \quad (3.12)$$

Finally, the value functions in double-coincidence partnerships are

$$\rho V(m, m') = \alpha(u - c) + \delta [W(m) - V(m, m')] . \quad (3.13)$$

3.2.4 Incentive conditions

The credit strategy conjectured in part (1) is individually rational provided the following incentive feasibility constraints are satisfied. First, for any monetary equilibrium, we must verify that money is not freely disposed.[11]

$$W(1) \geq W(0),\ Z(1) \geq Z(0),\ \text{and } V(1, m') \geq V(0, m'). \quad (3.14)$$

Second, we must verify that agents choose to search, as well as enter single and double coincidence matches (nodes (i), (ii), and (iv)). These are given by

$$Z(m) \geq W(m) \geq 0,\ m \in \{0, 1\} \quad (3.15)$$

$$V(m, m') \geq W(m),\ (m, m') \in \{0, 1\}. \quad (3.16)$$

Third, we must verify that buyers and sellers find it is rational to exchange money for goods in single coincidence matches at node (iii)

$$-c + W(1) \geq Z(0) \text{ and } u + W(0) \geq Z(1). \quad (3.17)$$

Fourth, we must verify that when it is a partner's turn to produce in a double coincidence partnership, he does so as in (v) given the punishment strategy in (vi)[12]

$$-c + \frac{\delta}{2\alpha} W(m) + \left(1 - \frac{\delta}{2\alpha}\right) V(m, m') \geq W(m), (m, m') \in \{0, 1\}. \quad (3.18)$$

[11] Notice that some of these conditions follow trivially. For instance, from (3.13) we have

$$\rho(V(1, m') - V(0, m')) = \delta \left[W(1) - V(1, m') - \left(W(0) - W(0, m')\right)\right]$$
$$\iff (\rho + \delta)(V(1, m') - V(0, m')) = \delta [W(1) - W(0)]$$

so that if $W(1) \geq W(0)$, then $V(1, m') \geq V(0, m')$.

[12] It is simple to see that since the credit strategy calls for no transfer of money, a receiver of the good is happy to accept it.

Finally, we must verify that the punishment strategy in (vi) is individually rational, given beliefs that others are playing the credit strategy. In particular, given that all agents are playing the credit strategy, in a history where one agent has defaulted, if the punisher chooses to search he receives $W(m)$ while if he remains with his partner he receives 0 since according to the credit strategy his partner will not produce at subsequent opportunities. But this condition is simply $W(m) \geq 0$,which has already been stated in (3.15).

3.2.5 Existence and characterization

The introduction of money affects the outside options of agents in credit partnerships. In particular, when it is the turn for an agent with a unit of money to expend costly effort, his outside option in the (off-the-equilibrium path) event of "default" is augmented since he can search for a single coincidence match where he is a buyer. The next proposition provides a simple way to illustrate these effects on incentives by considering what happens if a measure zero set of agents have money. It shows that the incentive constraint for an agent with money in a double coincidence partnership is more likely to bind than in the economy of Section 2.2.

Proposition 3.1. *In a region of the parameter space where* $\frac{\alpha\left(1-\frac{\delta}{2\alpha}\right)(u-c)}{\rho+\delta+xy} > c$ *and a neighborhood of* $M = 0$*, the incentives to produce in a credit relationship are weakened if one partner has money.*

Proof. See Appendix.

We illustrate how money affects existence of a credit equilibrium in Figure 1. The parameters used to generate the figure are $U = 2$, $c = 1$, $\rho = 0.05$, $x = 1$, $y = 0.1$, $\alpha = 1$, and $\delta = 0.5$. For these parameter values, the conditions for a credit equilibrium without money (2.4) are satisfied. Figure 1 plots four of the most important incentive constraints as the quantity of money M is varied in $(0, 1)$: that a seller produces in a single coincidence match (3.17) which we term IC-SC; that a household with or without money holdings in a credit partnership, which we term IC-DCM and IC-DC0 respectively, with a production opportunity expends effort (3.18); and that a seller enters a single coincidence match (3.15) which we term Z0-W0. In the figure, an incentive constraint is violated when its value falls below 0.[13] The incentive constraint that a seller in a single coincidence match produces is satisfied for all values of M and has the standard hump-shape in search models. Furthermore, the incentive constraint that agents in a double coincidence match where neither partner has money is always satisfied. The incentive problems arise in credit partnerships where at least one agent has money and entry by sellers into single coincidence matches. In particular, for values of $M \in [0.17, 0.61]$ a partner with money in a double coincidence match chooses not to produce since his outside option is so high. In this region a full credit equilibrium with money fails to exist. When money becomes less valuable (i.e. $M > 0.61$), the constraint fails to bind anymore. When $M \geq 0.85$, a potential seller no longer chooses to enter a single

[13] For example, we express IC-DCM as $\left(1 - \frac{\delta}{2\alpha}\right)[V(1, m') - W(1)] - c$.

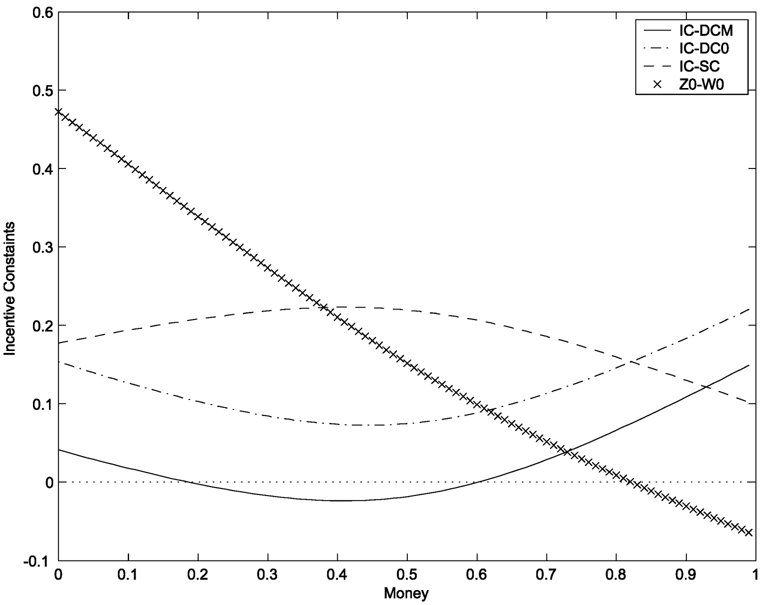

Figure 1. Credit and money incentives

coincidence match since money is less valuable when he himself becomes a buyer searching for the few remaining sellers; instead he would like to enter a credit partnership. Thus, again, a full credit equilibrium fails to exist in this region. It is very interesting that there's actually a role for money to complement credit in the sense that for $M \in (0.61, 0.85)$, a full credit equilibrium exists while in $[0.17, 0.61]$ it does not because credit breaks down.

Table 1 illustrates both existence and the value of ex-ante welfare in our framework as we vary δ, M, ρ, and y.[14] Besides the full credit equilibrium with money (called "C-M" in the table) and the credit equilibrium without money (called "C only"), we construct (by endowing agents with appropriate beliefs) two other possible equilibria: one where no one enters double coincidence matches (this yields a monetary equilibrium where exchange takes place in single coincidence matches only, called "SC only") as well as an equilibrium where no one enters a double coincidence match if one of the partners has money (called "DC0-M").[15] The first four rows of the table provide parameter values and the next four rows provide the ex-ante welfare figures when an equilibrium exists and notes the incentive condition which is violated when an equilibrium does not exist (DNE).

[14] By "ex-ante welfare" we mean $\Gamma_0 W(0) + \Gamma_1 W(1) + \Phi_0 V(0,0) + \Phi_1 \left[\frac{1}{2}V(0,1) + \frac{1}{2}V(1,0)\right] + \Phi_2 V(1,1) + \Theta \left[\frac{1}{2}Z(0) + \frac{1}{2}Z(1)\right]$.

[15] By "endowing agents with the appropriate beliefs" to construct a given equilibrium, we simply mean the same as what is done in an autarkic equilibrium; if an agent believes that no one will accept his money, then it is a best response not to accept money. Similarly, if an agent believes that his partner will not produce for him in a double coincidence match, then it is a best response not to enter it.

Table 1

δ	0.5	0.5	0.4	0.5	0.55	0.3	0.2
M	0.5	0.5	0.5	0.5	0.5	0.85	0.5
ρ	0.005	0.05	0.1	0.1	0.05	0.05	0.05
y	0.1	0.1	0.2	0.2	0.2	0.45	0.1
C only	33.3	3.33	3.33	DNE^{b}	DNE^{a,b}	12.00	6.67
C-M	51.5	DNE^{a}	DNE^{a,c}	DNE^{a,b,c}	DNE^{a,b}	DNE^{c}	7.44
DC0-M	41.4	4.08	2.47	2.31	DNE^{b}	DNE^{c}	5.11
SC only	31.0	3.10	1.43	1.43	2.85	0.98	3.10
QPQ1-M	39.3	3.89	2.27	2.15	4.26	DNE^{d}	4.67
C-QPQ1-M	48.5	4.82	3.21	DNE^{e}	DNE^{e}	DNE^{f}	DNE^{g}

Parameters: $U = 2, c = 1, \alpha = 1, x = 1$.

[a] means $\left(1 - \frac{\delta}{2\alpha}\right)[V(1,m') - W(1)] - c < 0$

[b] means $\left(1 - \frac{\delta}{2\alpha}\right)[V(0,m') - W(0)] - c < 0$

[c] means $Z(0) - W(0) < 0$

[d] means $Z^Q(0) - W^Q(0) < 0$

[e] means $\left(1-\frac{\delta}{2\alpha}\right)\left[V^Q(1,1)-W^Q(1)\right] -c < 0$ and

$\left(1-\frac{\delta}{2\alpha}\right)\left[V^Q(0,0)-W^Q(0)\right] -c < 0$

[f] means $Z^Q(1) - W^Q(1) < 0$ and $V^Q(1,0) - W^Q(1) < 0$

[g] means $V^Q\,(1,0) - V^Q\,(0,0) < 0$

The first column of Table 1 is a parameterization where all four possible equilibria suggested above exist. Notice that the "C-M" equilibrium yields higher ex-ante welfare than the "C only" equilibrium, which illustrates why money may be introduced into such an economy (i.e. so that single coincidence matches don't go unexploited). The second column is a parameterization where the C-M equilibrium fails to exist (i.e. $M = 0.5$ as in Fig. 1) since the incentive condition (3.18) is violated. Nevertheless, a "DC0-M" equilibrium exists and welfare dominates the "C only" equilibrium. The third column is interesting since it illustrates a principle close to Hart's (1975) result that the introduction of a market (i.e. money) in a world of incomplete markets (i.e. a search model) need not yield a welfare improvement. While Hart's result was due to general equilibrium price effects and our paper has "fixed prices", the effect of credit strategies (s_z, s_0, s_1, s_2) on population proportions is endogenous and generates the welfare result. In particular, $\Gamma = 0.80, \Phi = 0.20$ in the environment without money while in the economy with money $\Gamma_0 = 0.38, \Gamma_1 = 0.42, \Phi_0 = 0.04, \Theta = 0.16$, which has implications for valuations as well. Alternatively, one could view this as an example of a coordination failure since no agent with money wants to freely dispose of their money, yet the presence of money means that ex-ante welfare is lower than in its absence. The fourth column illustrates the effect of an increase in the exogenous breakup rate, which causes the incentive constraint in the "C only" environment to be violated even though it is not violated in the "DC0-M" equilibrium. This is because a breakup in the "DC0-M" equilibrium results in a longer average time for a partner without money to consume since she may pass through a single coincidence match and hence the incentive to deviate from the partnership is lower relative to the "C only" equilibrium. The fifth column simply illustrates that there are parameter values where we cannot support any reciprocal exchange, while there exists a "SC only" equilibrium where all exchange between agents with a single coincidence of wants involves monetary transfers. The sixth column illustrates that with sufficiently high stocks of money, the value of earning money in a single coincidence match is sufficiently small to cause the seller to search for more valuable double coincidence matches, thereby violating incentive constraint (3.15) and leading to non-existence of "C-M" and "DC0-M" equilibria despite the fact that a monetary equilibrium without credit exists. Again, this provides an example of Hart's result.

4 Directions for future research

In this paper, we chose to focus on a very particular set of strategies; credit (or reciprocal exchange) takes place in double coincidence matches while monetary exchange (which is a form of quid-pro-quo) takes place in single coincidence matches. We believe focusing on credit equilibria of this variety is the most relevant since, when it exists, it maximizes production and exchange (i.e. at all production opportunities in all single and double coincidence matches). It is clear that there are many other possible strategies. While a full characterization of all equilibria is left for future research, in an earlier version of this paper we considered quid-pro-quo strategies in double coincidence matches where one of the partners has money.[16] In particular, a quid-pro-quo strategy in a DC1 match would basically replicate a sequence of single coincidence matches without having to leave the partnership (thereby saving on search costs). Such strategies could even be used off-the-equilibrium path in the punishment phase.[17]

More specifically, the value functions associated with quid-pro-quo strategies are given by[18]

$$\rho V^Q(0,1) = \alpha\left[-c + V^Q(1,0) - V^Q(0,1)\right] + \frac{\delta}{2}\left[W^Q(1) - V^Q(1,0)\right]$$
$$\rho V^Q(1,0) = \alpha\left[u + V^Q(0,1) - V^Q(1,0)\right] + \frac{\delta}{2}\left[W^Q(0) - V^Q(0,1)\right]$$

and the relevant incentive condition is

$$-c + \frac{\delta}{2\alpha}W^Q(1) + \left(1 - \frac{\delta}{2\alpha}\right)V^Q(1,0) \geq W^Q(0).$$

[16] We focused on all possible credit and *quid-pro-quo* equilibria (which amounted to 24 possible equilibria). We were able to show that all but 7 of these equilibria could be ruled out.

[17] While such strategies may be renegotiation proof, since they result in weaker punishment, they may not exist in regions of the parameter space where credit strategies do.

[18] To see this, consider an agent in DC1 not holding money. Nothing happens until he can produce and his production opportunities arrive at rate α. After production, the match terminates with probability $\frac{\delta}{2\alpha}$. in the event of termination, his net gain is $W^Q(1) - V^Q(0,1)$, where we have indexed the value functions by superscript Q to make clear that these are associated with a different strategy (i.e. even though the form of the value functions in search, single coincidence matches, and even DC0 and DC2 are the same, since all depend on each other, a different strategy used in DC1 will change the other values). If the match continues, which occurs with probability $\left(1 - \frac{\delta}{2\alpha}\right)$, his net gain is $V^Q(1,0) - V^Q(0,1)$. Hence we have:
$\rho V^Q(0,1) = \alpha\left[-c + \frac{\delta}{2\alpha}\left[W^Q(1) - V^Q(0,1)\right] + \left(1 - \frac{\delta}{2\alpha}\right)\left[V^Q(1,0) - V^Q(0,1)\right]\right]$ which simplifies to that in the text.

While such strategies lead to less trade than the credit strategies considered in the previous sections, quid-pro-quo exchange does not have the same commitment problem and hence may exist for parameterizations where certain types of credit strategies do not (e.g. the second through fifth columns of Table 1). Furthermore, there are minor modifications to the laws of motion to take into account that DC1 breakups, which are conditional on production,[19] occur less frequently with quid-pro-quo strategies. In particular, the terms preceding Φ_1 in equations (3.3), (3.4), and (3.6) are all cut in half.[20]

There are some interesting equilibria that we can construct (again by endowing agents with the appropriate beliefs) using quid-pro-quo strategies. For instance, in the SC equilibrium constructed above, quid-pro-quo exchange was taking place only in single coincidence matches. However, such potentially welfare improving exchange could have taken place in DC1. In the ninth row of Table 1 we construct just such an equilibrium (called "$QPQ1 - M$"). It is also clear that we can endow agents with beliefs such that households play quid-pro-quo strategies in DC1 while they play credit strategies in DC0 and DC2 and trade with money in SC matches; we call such an equilibrium "$C - QPQ1 - M$". The last row of Table 1 lists the parameter values under which there exists a $C - QPQ1 - M$ equilibrium; the incentive conditions that are violated in regions of non-existence and ex-ante welfare in regions of existence. The final column of Table 1 provides an interesting example where the free disposal condition for money is violated in DC1 credit partnerships is violated. That is, while disposing of money implies that she may not consume next in and out of the relationship, an agent can turn the inefficient QPQ trading strategy in DC1 into an efficient credit strategy in DC0. Figure 2 plots ex-ante welfare, where an equilibrium exists, under the parameterization of Figure 1.

With the introduction of quid-pro-quo strategies, it is possible to construct rational expectations equilibria where the economy fluctuates between a high-level equilibrium with abundant credit and a low-level equilibrium where credit dries up. In particular, a sunspot variable can coordinate beliefs about which strategy (credit vs. quid-pro-quo) used in DC1 matches. Let Σ denote a transition matrix defined on a sunspot state where beliefs are consistent with credit strategies in all partnerships (call it o for optimism) and on a sunspot state where beliefs are consistent with quid-pro-quo strategies in DC1 partnerships (call it p for pessimism). In regions of the parameter space where there are $C - M$ and $C - QPQ1 - M$ steady state equilibria, Σ is a 2×2 identity matrix (i.e. $\Sigma_{oo} = \Sigma_{pp} = 1$ where $\Sigma_{ii'}$ denotes the probability of the sunspot variable moving from state i to state i'. If the incentive conditions hold with strict inequality in those regions of the parameter space, then by the upper hemi-continuity of the mixed strategy decision rules considered in this paper, for $\Sigma_{ii'}$ sufficiently close to 1, for $i = o, p$, there will exist a stationary

[19] With quid-pro-quo strategies, production occurs only on arrival of an opportunity to someone not holding money in DC1.

[20] That is, the terms $\frac{\delta}{2}\Phi_1$ in equations (3.3) and (3.4) are now $\frac{\delta}{4}\Phi_1$ and the term $\delta\Phi_1$ in (3.6) is now $\frac{\delta}{2}\Phi_1$.

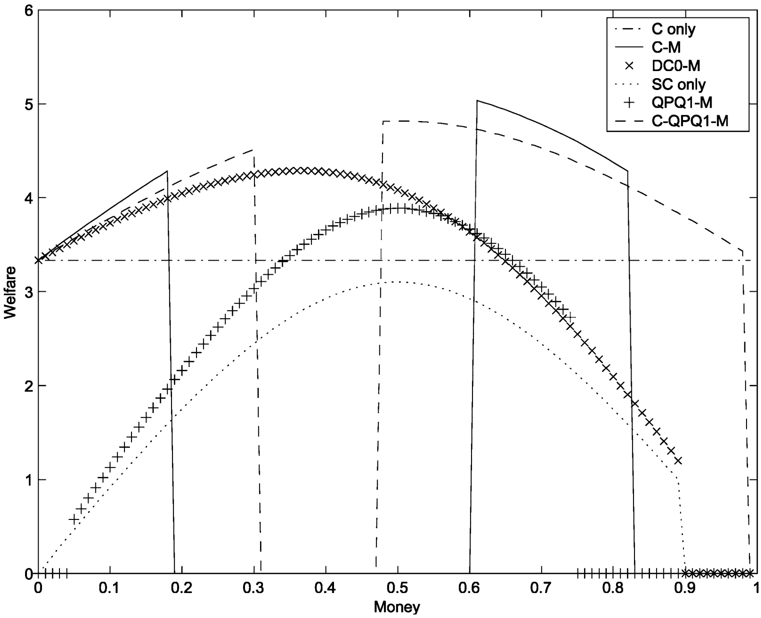

Figure 2. Welfare across different exchange strategies

sunspot equilibrium.[21] In this way, we can generate rational expectations business cycles in a search framework like that of Diamond and Fudenberg (1989). Unlike their paper, where the ultimate source of the fluctuations arise out of a trading externality and there is no money or credit, our fluctuations in output arise out of commitment problems associated with intertemporal trade.

An interesting extension of this framework with enduring relationships would be to consider an environment where preferences are such that all bilateral matches are single coincidence, but enduring partnerships can be multilateral so that trading networks form through time. This is left for future research.

5 Appendix

Proof of Proposition 3.1. What is to be shown is that incentives to produce in a credit relationship ($\left(1-\frac{\delta}{2\alpha}\right)[V(m,m')-W(m)] \geq c$) are weakened (i.e.$V(1,0)-W(1) < V-W$) if one partner has money. The structure of the proof is to evaluate $V(1,0)-W(1)$ at $M=0$ in the measure zero event where a household has a unit of money (in which case the laws of motion are trivial) and to evaluate $V-W$ at parameters such that the incentive constraint (2.3) holds with strict inequality. Then,

[21] Notice that at the moment of the switch of beliefs, the populations proportions are identical since the strategies do not differ in their stay or search decision. Since the value functions are bounded in $[0, \frac{u}{\rho}]$ and population proportions are bounded in $[0,1]$, then we can always find a sufficiently small transition probability that the incentive conditions continue to be satisfied in a stochastic steady state. For a similar construction of a sunspot equilibrium with endogenous state variable, see Cooper and Ejarque (1995).

since all value functions and laws of motion are continuous in a neighborhood of $M = 0$, the result follows.

Recall we established in (2.3) of Section 2 that $V - W = \frac{\alpha(u-c)}{\rho+\delta+xy}$. Now we derive $V(1,0) - W(1)$. If $M = 0$, the value function with no money in search (3.9) is

$$\rho W(0) = xy\left[V(0,0) - W(0)\right] \tag{5.1}$$

which is identical to the credit equilibrium in Section 2.2 since the population proportions are identical (i.e. $\gamma_0 = \Gamma(0)/(\Gamma(0)+\Gamma(1)) = 1$and $\gamma_1 = 0$). Next we consider the value to an agent with a unit of money (a measure zero event) given by

$$\rho W(1) = x(1-y)\left[Z(1) - W(1)\right] + xy\left[V(1,0) - W(1)\right] \tag{5.2}$$

which differs from $W(0)$ really only due to single value of money vs. search. The value functions in single coincidence and double coincidence matches are identical to those ((3.11),(3.12),(3.13)) in the previous section.

Subtracting (5.2) from (3.13) with $(m, m') = (1, 0)$ yields

$$\begin{aligned}\rho\left[V(1,0) - W(1)\right] = &\left\{\alpha(u-c) + \delta\left[W(1) - V(1,0)\right]\right\}\\ &- \left\{x(1-y)\left[Z(1) - W(1)\right] + xy\left[V(1,0) - W(1)\right]\right\}\end{aligned}$$

or

$$V(1,0) - W(1) = \frac{\left\{\alpha(u-c) - x(1-y)\left[Z(1) - W(1)\right]\right\}}{(\rho+\delta+xy)}.$$

Since $V - W = \frac{\alpha(u-c)}{\rho+\delta+xy}$,then

$$V(1,0) - W(1) < V - W \Leftrightarrow Z(1) > W(1)$$

which states that the condition for money to weaken incentives in a credit partnership amounts to the condition on the value of money in single coincidence matches.

To determine whether $Z(1) > W(1)$, substitute (3.12) and (3.13) with (m, m') $= (1, 0)$ into (5.2) and (3.13) with $(m, m') = (0, 0)$ into (5.1)to yield 2 equations in 2 unknowns:

$$(\rho + xy)\, W(0) = \frac{xy}{(\rho+\delta)}\left[\alpha(u-c) + \delta W(0)\right] \tag{5.3}$$

$$(\rho + x)W(1) = \frac{\alpha x(1-y)}{(\alpha+\rho)}\left[u + W(0)\right] + \frac{xy}{(\rho+\delta)}\left[\alpha(u-c) + \delta W(1)\right]. \tag{5.4}$$

Solving (5.3):

$$W(0) = \frac{xy\alpha(u-c)}{\rho\left[\rho+\delta+xy\right]} \tag{5.5}$$

into (5.4) yields

$$\left(\frac{\rho(\rho+\delta)+x\left[\rho+\delta(1-y)\right]}{(\rho+\delta)}\right)W(1)=\frac{\alpha x(1-y)}{(\alpha+\rho)}\left[u+\frac{xy\alpha(u-c)}{\rho\left[\rho+\delta+xy\right]}\right]+\frac{xy\alpha(u-c)}{(\rho+\delta)} \tag{5.6}$$

or

$$W(1)=\frac{\left[\frac{\alpha x(\rho+\delta)(1-y)}{(\alpha+\rho)}\left[u+\frac{xy\alpha(u-c)}{\rho[\rho+\delta+xy]}\right]+xy\alpha(u-c)\right]}{\rho(\rho+\delta)+x\left[\rho+\delta(1-y)\right]} \tag{5.7}$$

Hence to establish $Z(1)>W(1)$, we can use (3.12) and (5.7) to yield

$$\begin{aligned}Z(1)&=\frac{\alpha}{\alpha+\rho}\left[u+\frac{xy\alpha(u-c)}{\rho\left[\rho+\delta+xy\right]}\right]\\&>\frac{\left[\frac{\alpha x(\rho+\delta)(1-y)}{(\alpha+\rho)}\left[u+\frac{xy\alpha(u-c)}{\rho[\rho+\delta+xy]}\right]+xy\alpha(u-c)\right]}{\rho(\rho+\delta)+x\left[\rho+\delta(1-y)\right]}=W(1)\end{aligned}$$

or letting

$$A\equiv\frac{\alpha}{\alpha+\rho}\left[u+\frac{xy\alpha(u-c)}{\rho\left[\rho+\delta+xy\right]}\right]$$

we have

$$\begin{aligned}A\quad&>\quad\frac{\left[Ax\left(\rho+\delta\right)\left(1-y\right)+xy\alpha(u-c)\right]}{\rho(\rho+\delta)+x\left[\rho+\delta(1-y)\right]}\\&\iff(\rho+\delta)u>-xyc\end{aligned}$$

which always holds.

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