Designing a Trading Post*

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November 5, 2004

Abstract

We apply mechanism design to a spatial environment with directed bilateral and multilateral matching. We show that if households are patient and have sufficiently heterogeneous tastes, and the costs of operating multilateral exchanges are sufficiently low, then the solution to the design problem can be implemented as a monetary trading post economy with nonlinear prices. In particular, if households are sufficiently risk averse, we show that the constrained optimum may not be implementable with linear prices.

1 Introduction

Studying efficient ways to organize exchange is one of the objectives of monetary theory. Shapley and Shubik [7] were among the first to pose a static model that organized multilateral exchange at trading posts between agents who have endowments of goods which may differ from what they prefer to consume. Recently, Howitt [3] studies a dynamic environment with some of the explicit spatial, information, and enforcement frictions employed in search models of money (e.g. Kiyotaki and Wright [4]), but which allows directed matching to trading posts.

In this environment, Howitt studies whether monetary exchange at trading posts dominates barter. Trading posts, which are restricted to exchange in only two goods, mitigate search costs between households but are themselves costly to operate. There are n nonstorable goods in his economy and $n(n-1)$ household types (i.e. each household is endowed with one of the n goods but desires one of the other $(n-1)$ goods). In any active symmetric equilibrium, the minimum number of barter shops that must open is $n(n-1)/2$ while only n monetary shops are necessary. Since there is a fixed cost κ to running a shop, it is then clear that monetary exchange economizes on such costs. On the other hand, for monetary exchange to take place in this spatial setting, consumption this period must be

*We wish to thank Subir Bose, V.V. Chari, Peter Howitt, Narayana Kocherlakota, Max Stinchcombe, Tom Wiseman, and especially Neil Wallace and Randy Wright for helpful comments on this paper.

paid for with money earned last period (which is not the case for barter). Howitt shows (Proposition 3) that if households are sufficiently impatient, a monetary equilibrium which is robust to deviations to barter exchange fails to exist.

In this paper, we adopt a mechanism design approach to address the question of the dominance of monetary exchange. Unlike Howitt, directed bilateral matching as in Corbae, Temzelides, and Wright [1] is not ruled out by our environment. We focus on a region of the parameter space where the mechanism would direct households to trading posts rather than bilaterally match them. Of course the existence of such a solution to the mechanism design problem depends on the value that households receive in bilateral matches.

We characterize the set of incentive feasible allocations and define an optimum problem that maximizes the ex-ante welfare of households subject to incentive feasibility in much the same way as Kocherlakota and Wallace [6]. We implement the constrained efficient allocation as a trading post economy. This is important since we do not know whether prior specifications of trading post economies actually solve an optimum problem. For instance, shops in Howitt's paper post linear prices, but in general an allocation rule may call for a nonlinear schedule. We show in fact that there are regions of the parameter space where linear price functionals at trading posts do not implement the constrained optimum.

We proceed as follows. Section 2 describes the environment. Section 3 defines the mechanism design problem. In section 3.1 we solve a relaxed version of the mechanism design problem (that of maximizing ex-ante household welfare subject to aggregate resource feasibility and participation by those agents running the trading post). In section 3.2, we provide conditions on parameters such that the solution to this relaxed problem also satisfies the general mechanism design problem. Section 4 shows how the allocation of the mechanism design problem can be implemented via nonlinear prices at a trading post. In many ways, our decentralization resembles the Lucas, Cass, Yaari model with an endogenous cash-in-advance constraint studied by Townsend [8] in the first *Models of Monetary Economies.* Section 5 briefly discusses robustness of the results.

2 Environment

Time is discrete and the horizon is infinite. The economy is composed of a unit measure of households indexed by $h \in [0,1]$ and a countably infinite number of brokers indexed by $b \in \mathbb{N}$.[1] A household is composed of two members: a shopper denoted s and a worker denoted w.

There are a finite number n of divisible, nonstorable goods. Besides nonstorable goods, there is another storable and divisible good that yields no utility to households or brokers denoted 0. We denote the set of goods types by $G = \{0, 1, ..., n\}$.Good 0 is in fixed supply and distributed uniformly, in per capita amount $\overline{m}$, at the beginning of time to households only.

[1] Websters defines a broker as "one who acts as an intermediary".

Each household specializes in the production of one good, but has preferences defined over one of the other $n-1$ goods. Household type is indexed by (i,j) where i is the household's production type and $j \neq i$ is the household's consumption type. Specifically, the type space is given by $\Theta = \{(i,j) \in \{1,\ldots,n\} \times \{1,\ldots,n\} : i \neq j\}$. Thus, there are $n(n-1)$ types of agents and we assume each type is of equal measure. For pairs of households, this specification admits both a double coincidence of wants (between type (i,j) and (j,i) households) as well as a single coincidence of wants (between type (i,j) and (j,k) households where $i,j \neq k$).

There are $n(n+1)/2$ types of brokers. Of these types, $n(n-1)/2$ specialize in pairs of Θ, whom we call barter brokers, while n specialize in $\{0,i\}$ pairs, whom we call monetary brokers. Brokers cannot produce goods, but can exchange and consume the two types of goods that they specialize in. There are a countably infinite number of each type of broker. Denoting the type of broker b by θ^b, without loss of generality, we assume broker $b \in \{1,...,n\}$ is a monetary type, broker $b \in \{n+1,...,n(n+1)/2\}$ is a barter type, and this pattern repeats itself. [2].

Households and brokers reside at locations denoted $\lambda \in [0,1]$. Locations do not explicitly enter household or broker preferences nor production technologies. Without loss of generality we let broker b permanently reside at $1/b$. Each period households are relocated in $[0,1]\backslash\{\frac{1}{b}, b \in \mathbb{N}\}$ according to an i.i.d random process (which implies a household's current residence is independent of its past residence as well as its tastes and money holdings).[3] While brokers cannot direct themselves to a location other than their residence, all households simultaneously direct each of their members to one location each period (of course a household could direct one or both of its members to stay home).

After households direct their members to locations, matches between household members and/or brokers can form. A match at a household residence can only be between a member of that household and a member of another household who has directed itself to that location. If more than two households direct their members to a household residence, no match can be formed at that location. A match at a broker residence can only be between the broker and a household member who has directed itself to that location. Each period, a household member can at most be part of one match, but a broker can participate in multiple matches. Production and consumption can only take place in a match.[4] As in Townsend [9] (p. 963), agents can also make oral announcements (i.e. messages) in their respective match at their given location. Members of the same household can communicate when they are at the same location, but not if they are in separate matches at different locations.

It is costly for a broker to open a trading post. That is, only if the broker

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[2] In particular, broker b is of type $\{i,j\}$ when $\mathrm{mod}_{n(n+1)/2}\,(b) = \left(in - \frac{i(i-1)}{2}\right) + j - i$, for $i < j$.

[3] The existence of such an iid process is assured by the Kolmogorov Extension Theorem.

[4] For example, the worker cannot produce at a barter post and bring a good home for the shopper to consume and a shopper cannot wait for the worker of the household to arrive back with money to go shopping.

incurs disutility $-\kappa$ can he form multiple matches at his residence in that period. Given the fixed disutility cost, operating a trading post is characterized by increasing returns. Brokers have linear preferences over their specialized types of goods. Specifically, if an (i, j) broker b opens a post, he receives momentary utility given by $c_i^b + c_j^b - \kappa$, where $c_i^b \geq 0$ and $c_j^b \geq 0$ are his consumption of good i and j (and of course, if i or j is good 0, it yields no utility).

It takes a fixed amount of effort for the worker of a household to produce one unit of goods. In particular, if a type i worker takes effort $\ell = 1$ she produces one unit of the type i good and if she chooses $\ell = 0$, she produces zero units of the good. Only the shopper can consume and only the worker can produce. Preferences of a type (i, j) household h are given by the utility function $u(c_j^{hs}) - e\ell_i^{hw}$ where $c_j^{hs} \in \mathbb{R}_+$ is the household consumption of a good of variety j, $\ell_i^{hw} \in \{0, 1\}$ is the household's effort, and $e > 0$ is the disutility of effort. We assume that u is a strictly increasing, concave function. Households and brokers discount the future at a common rate $\beta < 1$.

Any agent can keep records about actions and messages from only a countable number of agents with whom he is matched in any given period. This restriction on record keeping does not affect households since they participate in countable number of matches in their life. However, the restriction does constrain the written records of a broker since he may receive an uncountable number of announcements and observe an uncountable number of actions of households with whom he is matched. Of course, the restriction does not rule out record keeping of broker aggregate consumption and changes in money holdings in all periods.

At the beginning of any period, all agents can send and receive announcements across locations (Townsend [9] (p. 965) defines this cross location message sending as telecommunication). This cross location communication can include announcements about recorded actions. After these announcements, there is no cross location communication though within location communication is possible as described above.

Household locations, types, and money holdings are unobservable. Households are also not recognizable on the basis of other characteristics (e.g. households do not have their names written on their forehead). Furthermore, actions (production, exchange, and consumption) for any type of agent in one location are unobservable at another location. However, while oral announcements are not "observable" outside of a match, actions at any location are observable by those agents at that location. One implication of our observability and communication assumptions is that a broker's actions in period t is known in period $t + 1$ by all agents in the economy.

Finally, we assume that all exchange must be voluntary (i.e. there is no technology to enforce exchange) and that there is free entry by brokers.

The timing of events in any period is given by the following stages: (i) In any of a countably infinite number of rounds within this stage, brokers choose whether to open a trading post (and incur $-\kappa$) and agents make announcements (which can include broker rules for allocating the two goods they specialize in at their residence); (ii) Household location shocks are drawn; (iii) Household members direct their search to a residence; (iv) After having formed matches in the previous stage, brokers choose whether to revise their rules for allocating goods; (v) production, exchange, and consumption decisions take place at a given residence; and (vi) Workers and shoppers return to their residence with any money accumulated at stage (v).

3 Mechanism Design Problem

A mechanism is a mapping from messages to an allocation both of which are consistent with the above environment.[5] An allocation for any given period is a specification of the location to which each member of the household is directed and what trades they should make (which determines household production, consumption, and money holdings) as well as which types of trading posts should be open and what trades should occur at those posts (which determines broker consumption). One implication of our environment (that recordkeeping is limited to information from a countable number of agents in any period) is that except for a set of measure zero, households are anonymous in the sense that there is no record on their past actions and announcements. For this reason, in what follows we will only focus on mechanisms which treat households as anonymous.

The mechanism makes proposals which prescribe actions to households and brokers. Proposals are accepted or rejected. In the case of rejection, the household or broker must stay at its residence and is precluded from trade for that period. In the case of household proposals, the mechanism may direct the two members to the same location or different locations. The mechanism proposes two four-tuples, one to the worker and one to the shopper. Since a worker can only produce and a shopper can only consume, it is without loss of generality to consider proposals to each which specify exchange of one good and/or money (i.e. either giving what they produce or getting what they consume). A four-tuple to one of the members of the household is

$$(\lambda, (v, q), d) \in T = [0,1] \times G \times \mathbb{R}^2$$

where λ denotes the location where the household's member is directed, q is the amount of good v the household's member receives (if $q > 0$) or gives up (if $q < 0$), and d is the amount of money to be received (if $d > 0$) or given up (if $d < 0$). Given the locational aspects of our environment, these two proposals can be made independent or can be conditioned on one another. Let $\iota = 0$ if the proposals are independent and $\iota = 1$ if they are conditioned on one another. Therefore, a pure proposal to household h, γ_H^h, is a nine-tuple

$$\gamma_H^h = \left(\iota^h, \left(\lambda^{hw}, \left(v^{hw}, -\ell^{hw}\right), d^{hw}\right), \left(\lambda^{hs}, \left(v^{hs}, c^{hs}\right), d^{hs}\right)\right) \in \Gamma_H = \{0,1\} \times T \times T$$

[5] Throughout we assume the mechanism has commitment.

where Γ_H is the set of all possible pure proposals for a household. Clearly, if the household sends a message about taste $\widetilde{\theta}^h = (i, j)$, then $v^{hw} = i$ and $v^{hs} = j$.

In order to be consistent with the spatial, communication, and enforcement assumptions, the household can reject one of the two five-tuples if they are directed to different locations, that is $\lambda^{hw} \neq \lambda^{hs} \Rightarrow \iota^h = 0$. In other words, if the two members of the household are sent to different locations, they cannot be forced to jointly comply with both parts of the proposal. For instance, the mechanism may direct an (i, j) type household to two monetary trading posts; in that case, the proposal is

$$\left\{0, \left(\frac{1}{b^{hw}}, (i, -\ell^{hw}), d^{hw}\right), \left(\frac{1}{b^{hs}}, (j, c^{hs}), -d^{hs}\right)\right\}$$

where b^{hs} and b^{hw} denote the brokers operating the monetary trading posts to which the shopper and worker are directed. If, for example, the household had a lot of accumulated money balances, it might choose to reject the first five-tuple of the proposal but accept the second five-tuple (i.e. consume but not produce). In contrast, if the mechanism directs both members of household h of type (i, j) to an $\{i, j\}$ barter post, the proposal is

$$\left\{1, \left(\frac{1}{b^h}, (i, -\ell^{hw}), 0\right), \left(\frac{1}{b^h}, (j, c^{hs}), 0\right)\right\}$$

where b^h denotes the broker operating the barter post where both members have been sent.[6] In this case, since both members are at the same location, it's possible for the mechanism to force compliance of both parts of the proposal (however not necessary).

There are some obvious implications of our environment for how the mechanism directs households. If the mechanism directs a household to a barter trading post, it would direct both members of the household to the same broker.[7] The mechanism can also direct a household to another household's residence. Given the timing and informational restrictions in the environment, the mechanism must direct households without knowledge of their characteristics (i.e. independent of their messages). Given this, there is no gain to directing households to particular residences except for the fact that the mechanism can set the probability of not meeting another household to zero. In this case, directed matching amounts to random matching. Furthermore, it is clear that proposals which direct agents to household residences must be mixed since the match

[6] The mechanism would not direct the household to give up or get money since this would violate the restriction that only 2 goods can be exchanged at a trading post.

[7] To see this point, notice that if the worker is being directed to a broker different from the shopper, then whatever he receives in exchange for his production cannot be consumed (since consumption can only take place at the location of the household's shopper). Moreover, since households are anonymous, future allocation of goods cannot depend on whether the worker produces. Therefore, if the mechanism proposes that the worker produce at a barter post where his shopper is not directed, since this action only generates a static production cost and no consumption gain for the household, the worker will simply reject this proposal given that there is no impact on future payoffs of the household whatsoever.

realization will entail either no coincidence, a single coincidence, or a double coincidence of wants.

In the case of broker pure proposals, the mechanism proposes to broker b, the three-tuple $\gamma_B^b = \left(c^b, \widehat{c}^b, d^b\right) \in \Gamma_B = \mathbb{R}_+^2 \times \mathbb{R}$ where c^b is consumption of the first type of good broker b is specialized in, $\widehat{c}^b$is consumption of the second type of good broker b is specialized in, and d^b denotes the change in the broker' money holding (where we use the convention that a $\varnothing$ proposal to broker b implies he does not open). Obviously, a barter broker has $d^b = 0$ and a monetary broker has $\widehat{c}^b = 0$.[8] A broker who has positive money holdings can receive a proposal that directs him to open in order to diminish his money holdings and consume (accomplished by directing workers to his post).

Before messages are sent in a given period, the mechanism assigns numbers $h \in [0,1]$ to households independent of their past messages and current characteristics (due to anonymity). Clearly the number a household is assigned is independent across periods.

An allocation in a given period is a member of $\Delta\left(\times_{h\in[0,1]}\Gamma_H^h \times_{b\in\mathbb{N}} \Gamma_B^b\right)$ where Γ_H^h is the set of all proposals associated with household h and Γ_B^b is the set of all proposals associated with broker b. That is, an allocation is a probability distribution over the set of all possible combinations of proposals to all households and brokers. Note that a member of $\times_{h\in[0,1]}\Gamma_H^h \times_{b\in\mathbb{N}} \Gamma_B^b$ is a function from $[0,1] \times \mathbb{N}$ to $\Gamma_H \cup \Gamma_B$ which assigns proposals to households and brokers. Therefore, an allocation allows for randomization which will have to respect the restrictions of our environment over proposal assignments.

In any period t, households can send a message about their current characteristics as well as the history of past characteristics, proposals, and actions. That is, in period t household h sends message $\widetilde{\eta}_t^h \in \mathcal{H}_t^h$.[9] In this case, the general message space in period t is $\times_{h\in[0,1]}\mathcal{H}_t^h$. The mechanism's period t allocation rule, $\mathcal{M}_t$, is a function from the general message space in period t, $\times_{h\in[0,1]}\mathcal{H}_t^h$, to the set of allocations, $\Delta\left(\times_{h\in[0,1]}\Gamma_H^h \times_{b\in\mathbb{N}} \Gamma_B^b\right)$

$$\mathcal{M}_t : \left(\times_{h\in[0,1]}\mathcal{H}_t^h\right) \to \Delta\left(\times_{h\in[0,1]}\Gamma_H^h \times_{b\in\mathbb{N}} \Gamma_B^b\right)$$

which maximizes household utility subject to truthful implementability as well as other restrictions implied by the environment.[10]

Let the period t conditional distribution over proposals for household h (broker b) be denoted as $\mathcal{M}_t^h$ ($\mathcal{M}_t^b$). If in period t the other households are sending the message $\times_{h'\in[0,1]\backslash h}\eta_t^{h'}$, then by sending $\widetilde{\eta}_t^h \in \mathcal{H}_t^h$, household h will be offered the mixed proposal $\mathcal{M}_t^h\left(\widetilde{\eta}_t^h \times_{h'\in[0,1]\backslash h} \eta_t^h\right)$. Now for any realization γ_H^h in the support of $\mathcal{M}_t^h\left(\widetilde{\eta}_t^h \times_{h'\in[0,1]\backslash h} \eta_t^h\right)$ from this mixed proposal, we can define an

[8]For example, if broker b with type $\theta^b = \{i, 0\}$ accepts proposal $(c^b, 0, d^b)$, he opens a monetary trading post and consumes c^b units of good i, accumulating or decumulating money d^b.

[9]Notice that it is possible for both members of the household to send different messages; just include both in $\widetilde{\eta}_t^h$ and $\widetilde{\omega}_t^h \subset \widetilde{\eta}_t^h$.

[10]If the mapping between locations directs the household to another household location, the mapping must be measure preserving. If it is not, then resource feasibility may not hold.

indicator function which accounts for acceptance or rejection of γ_H^h. In particular, $1_t^{hs} = 1$ if household h accepts the shopper's part of the proposal and $1_t^{hs} = 0$ if it rejects in period t. Similarly, $1_t^{hw} = 1$ if the household accepts the worker's part of the proposal and $1_t^{hw} = 0$ if it rejects in period t.

For the aggregate pure proposal $\gamma = \times_{h\in[0,1]}\gamma_H^h \times_{b\in\mathbb{N}} \gamma_B^b$, let

$$\left(\iota_\gamma^h, \left(\lambda_\gamma^{hw}, \left(v_\gamma^{hw}, -\ell_\gamma^{hw}\right), d_\gamma^{hw}\right), \left(\lambda_\gamma^{hs}, \left(v_\gamma^{hs}, c_\gamma^{hs}\right), d_\gamma^{hs}\right)\right)$$

denote the elements of corresponding proposal γ_H^h for household h and $(c_\gamma^b, \widehat{c}_\gamma^b, d_\gamma^b)$ denote the elements of corresponding proposal γ_B^b for the broker b. Furthermore let η_t^{-h} denote $\left(\times_{h'\in[0,1]\setminus h}\eta_t^{h'}\right)$. In this case, we can define the value function for household h with current characteristics $\left(\lambda_t^h, \theta_t^h, m_t^h\right)$ as

$$\begin{aligned} & V_t\left(\lambda_t^h, \theta_t^h, m_t^h; \eta_t^{-h}\right) \qquad (1) \\ = \quad & \max_{\widetilde{\eta}_t^h \in \mathcal{H}_t^h} E_{\mathcal{M}_t\left(\widetilde{\eta}_t^h \times \eta_t^{-h}\right)}\left[\max_{1_t^{hs}, 1_t^{hw}}\left[\begin{array}{c} u\left(1_t^{hs}\cdot c_\gamma^{hs}\right) - e\cdot 1_t^{hw}\cdot \ell_\gamma^{hw} \\ +\beta\left[V_{t+1}\left(\lambda_{t+1}^h, \theta_{t+1}^h, m_t^h + 1_t^{hw}\cdot d_\gamma^{hw} - 1_t^{hs}\cdot d_\gamma^{hs}, \eta_{t+1}^{-h}\right)\right]\end{array}\right]\right]\end{aligned}$$

where if $\iota_\gamma^h = 1$, then $1_t^{hs} = 1_t^{hw}$.

Note that anonymity implies that a household can always reject a proposal in one period and receive the same proposal as those who didn't reject the previous proposal conditional on sending the same message. A rejection, however, may affect the household's money holdings and hence its choice of future messages.

Then the full programming problem can be written as

$$\max \int_0^1 V_0\left(\lambda_t^h, \theta_t^h, \overline{m}; \eta_t^{-h}\right) dh \qquad (2)$$

subject to the following feasibility constraints:

$$E_{\left\{\times_{h\in[0,1]}\eta_t^h\right\}}\left[\sum_{s\geq t}\beta^s\left(c_{\gamma_s}^b + \widehat{c}_{\gamma_s}^b - \kappa\cdot 1_{\left\{(c_\gamma^b, \widehat{c}_\gamma^b, d_\gamma^b)\neq\varnothing\right\}}\right)\right] \geq 0, \quad \forall t, b \qquad (3)$$

which is the broker participation constraint where $1_{\left\{(c_\gamma^b, \widehat{c}_\gamma^b, d_\gamma^b)\neq\varnothing\right\}}$ denotes an indicator function whether broker b opens his trading post in period s;

$$\overset{*}{1}{}_t^{hs} = \overset{*}{1}{}_t^{hw} = 1, \quad \forall t, h \qquad (4)$$

which is the household participation constraint and $\overset{*}{x}$ denotes an optimal action;

$$\overset{*}{\widetilde{\eta}}{}_t^h = \eta_t^h, \quad \forall t, h \qquad (5)$$

which is the incentive compatibility constraint;

$$C_\gamma^{v,\lambda} = \int_{\left\{h\,:\,\lambda_\gamma^{hw}=\lambda \text{ and } v_\gamma^{hw}=v\right\}} \ell_\gamma^{hw} - \int_{\left\{h\,:\,\lambda_\gamma^{hs}=\lambda \text{ and } v_\gamma^{hs}=v\right\}} c_\gamma^{hs} \geq 0 \qquad (6)$$

which is resource feasibility for nonstorable good v at location λ (where $C_{\gamma}^{v,\lambda}$ denotes the remaining amount of good v at location λ that can be consumed by a broker or disposed of freely);[11]

$$\begin{aligned} D_{\gamma}^{\lambda} &= \int_{\left\{h:\lambda_{\gamma}^{hw}=\lambda\right\}} d_{\gamma}^{hs} + \int_{\left\{h:\lambda_{\gamma}^{hs}=\lambda\right\}} d_{\gamma}^{hw} \geq 0, \quad \forall\lambda \in [0,1] \setminus \left\{\frac{1}{b} : b \in \mathbb{N}\right\} \quad (7) \\ D_{\gamma}^{\lambda} &\geq d_{\gamma}^{b}, \quad \forall\lambda \in \left\{\frac{1}{b} : b \in \mathbb{N}\right\} \\ m_{t+1}^{h} &= m_{t}^{h} + d_{\gamma}^{hw} - d_{\gamma}^{hs} \geq 0 \\ m_{t+1}^{b} &= m_{t}^{b} + d_{\gamma}^{b} \geq 0 \end{aligned}$$

which is resource feasibility of the storable good at household and broker residencies, respectively;

$$\lambda_{\gamma}^{hw} = \frac{1}{b} \Longrightarrow v_{\gamma}^{hw} \in \theta^{b}, \quad \lambda_{\gamma}^{hs} = \frac{1}{b} \Longrightarrow v_{\gamma}^{hs} \in \theta^{b} \tag{8}$$

(where θ^{b} is broker b's type) which is a consistency restriction that states if a given good is to be produced or consumed at a broker location, it must also be exchanged there;

$$\#\left\{h : \lambda_{\gamma}^{hw} = \lambda \text{ } or \text{ } \lambda_{\gamma}^{hs} = \lambda\right\} \setminus r(\lambda) \leq 1, \quad \forall\lambda \in [0,1] \setminus \left\{\frac{1}{b} : b \in \mathbb{N}\right\} \tag{9}$$

which is the restriction on matching at a household residence (where $r(\lambda)$ denotes the household which resides at location λ);

$$\begin{aligned} \lambda_{\mathcal{M}_t\left(\times_{h\in[0,1]}\eta_t^h\right)}^{hw} &= \lambda_{\mathcal{M}_t\left(\times_{h'\in[0,1]\setminus h}\eta_t'^{h'}\times\eta_t^h\right)}^{hw}, \quad \forall t, h \quad (10) \\ \lambda_{\mathcal{M}_t\left(\times_{h\in[0,1]}\eta_t^h\right)}^{hs} &= \lambda_{\mathcal{M}_t\left(\times_{h'\in[0,1]\setminus h}\eta_t'^{h'}\times\eta_t^h\right)}^{hs}, \quad \forall t, h \end{aligned}$$

which is an informational constraint on the mechanism implied by the environment that requires the location to which any given household is sent must be independent of the other households' messages $\times_{h'\in[0,1]\setminus h}\eta_t'^{h'}$ (in other words, if where to send households depended on other households' messages, then households could communicate within a period via the mechanism, which is ruled out in the environment); and

$$\lambda_{\gamma}^{hw} \neq \lambda_{\gamma}^{hs} \Longrightarrow \iota_{\gamma}^{h} = 0 \tag{11}$$

which is the constraint that the members of a given household cannot communicate across different locations (which also rules out brokers communicating across different locations within a period).

[11]Note that in the case of a household residence (i.e. $\lambda \neq \frac{1}{b}$), the integral denotes summation.

3.1 Relaxed Programming Problem

In this section, we solve a programming problem where certain incentive and resource conditions are relaxed. First, while information cannot be transferred across locations (i.e. (10) and (11) are imposed), incentive compatibility (5) is not imposed. Second, while brokers must satisfy individual rationality (3), the household individual rationality constraint (4) is *only* used to rule out directing workers of (i, j) households to $\{i, k\}$ barter posts $k \neq j$. Third, we impose *only* economywide resource feasibility, not location specific feasibility (6). Thus, we neglect some constraints that must be satisfied in the full mechanism design problem of maximizing households' utility.[12] This gives us an upper bound for household utility which is actually possible to attain in certain regions of the parameter space. That is, under certain conditions, the neglected constraints do not bind in the full mechanism design problem considered in subsection 3.2.

We make the following parametric assumptions on u, n, e, and κ.

Assumption 1. (a) $n \geq 3$; (b) $u'(1) > e$; and (c) $u'(1 - n\kappa)\left[1 - \frac{\kappa n^2}{n-1}\right] > e$.

Condition (a) is used to ensure that if all goods are traded in the economy, then the cost associated with monetary posts ($n\kappa$) is less than or equal to that of barter posts ($\frac{n(n-1)\kappa}{2}$). Condition (b) ensures that all workers produce (instead of running lotteries as in Hansen [2]). Condition (c) will be used to rule out any bilateral matching.

Our strategy for solving this problem is to maximize worker production of each of the goods in the economy, distribute just enough goods to brokers in order to cover the fixed costs of running trading posts, and then distribute the remaining goods to shoppers in order to maximize their utility.

Recall that the worker of an (i, j) household can be directed to a monetary trading post, a barter trading post, or a household's location (including remaining at one's own location.) If the worker is directed to a trading post, clearly good i should be one of the two types of good being traded at that post (i.e. (8) must be satisfied on the production side). Therefore the only monetary trading posts to which the worker of an (i, j) household can be directed are $\{i, 0\}$ monetary posts. It is also important to realize the worker cannot be directed to an $\{i, k\}$ barter trading post where $k \neq j$. This is a consequence of the following facts: (i) the shopper of the (i, j) household cannot consume at an $\{i, k\}$ barter post (i.e. (8) must be satisfied on the consumption side); (ii) by rejecting the proposal that directs the worker to an $\{i, k\}$ barter post, the household avoids $-e$ disutility associated with production; and (iii) due to anonymity, this cannot have any effect on the household's current utility (due to (11)) or future utility (since the household can always pretend it has sent the worker to the barter post and there is no way to verify it). In other words, if the mechanism directs the worker of an (i, j) household to an $\{i, k\}$ barter post, it would necessarily violate (4). Thus, the worker of an (i, j) household can only be directed potentially to $\{i, 0\}$ or $\{i, j\}$ trading posts.

[12] Restrictions (8) and (10) are still imposed as well.

If the worker is directed to a household's location, then since the location to which she is directed is independent of the other households' messages (constraint (10)) and location shocks are iid, the probability of meeting a household who wants good i is at most $\frac{1}{n}$. To see this point, notice that $n-1$ type households like good i (all $\{k,i\}$ types for $k \neq i$) and since the worker is matched randomly with at most one of the $n(n-1)$ types of households (due to (9)), if the match occurs at all, an upper bound on the probability that she will produce is $\frac{1}{n(n-1)} \times (n-1) = \frac{1}{n}$.

Production can take place at a monetary post, a barter post, and/or at household residences. In any given period t, the mechanism can direct $\mu_t^{(i,j)}$ fraction of all (i,j) households to an $\{0,i\}$ monetary post, $\phi_t^{(i,j)}$ fraction of them to an $\{i,j\}$ barter post, $\chi_t^{(i,j)}$ fraction of them to bilateral matches, and $\left(1-\mu_t^{(i,j)}-\phi_t^{(i,j)}-\chi_t^{(i,j)}\right)$ fraction of them to do nothing. Thus, an upper bound for production of good i by the workers of (i,j) households is given by

$$\frac{1}{n(n-1)}\left[\mu_t^{(i,j)}+\phi_t^{(i,j)}+\frac{1}{n}\times\chi_t^{(i,j)}\right]$$

since all of the workers who are directed to posts can at most produce 1 unit of good i and those who are directed to bilateral matches can find someone with a coincidence of wants with at most probability $\frac{1}{n}$. Then total goods production in period t, will be equal to

$$TP_t=\sum_i\sum_{j\neq i}\left(\frac{1}{n(n-1)}\left[\mu_t^{(i,j)}+\phi_t^{(i,j)}+\frac{1}{n}\times\chi_t^{(i,j)}\right]\right) \tag{12}$$

In terms of disutility, producing this many goods generates a total cost to households of eTP_t.

Since a broker must be compensated for the cost of operating a post (κ), a relaxed version of (6) implies total goods available for consumption by households is given by

$$TC_t=TP_t-\sum_i\left[C_t^{\{0,i\}}+\sum_{j>i}C_t^{\{i,j\}}\right] \tag{13}$$

where $C_t^{\{0,i\}}$ is the time t consumption of a broker who operates an $\{0,i\}$ monetary post and $C_t^{\{i,j\}}$ is the time t consumption of the broker who operates the barter shop $\{i,j\}$.[13]

The participation constraints for a broker operating a monetary or barter post respectively given in (3) can be stated as:

$$\kappa\sum_t\beta^t\times 1_{\{\mu_t^i>0\}}\leq\sum_t\beta^t C_t^{\{0,i\}} \tag{14}$$

[13]The reason why the second summation is over $j>i$ is because there is no difference between $\{i,j\}$ and $\{j,i\}$ shops, so we use this counting convention to avoid double counting.

$$\kappa \sum_t \beta^t \times 1_{\left\{\phi_t^{(i,j)}>0 \text{ or } \phi_t^{(j,i)}>0\right\}} \leq \sum_t \beta^t C_t^{\{i,j\}} \tag{15}$$

where $1_{\left\{\mu_t^i>0\right\}}$is an indicator function assigning 1, iff at least for one j,we have $\mu_t^{(i,j)} > 0$ (that is at least one monetary $\{0, i\}$ post should be open so that type i workers can be directed to that post) and $1_{\left\{\phi_t^{(i,j)}>0 \text{ or } \phi_t^{(j,i)}>0\right\}}$ is an indicator function assigning 1, iff $\phi_t^{(i,j)} > 0$ or $\phi_t^{(j,i)} > 0$ (that is at least one barter $\{i, j\}$ post should be open so that type i workers from the (i, j) households, or type j workers from the (j, i) households can be directed to that post).

Since $u(\cdot)$ is a concave function, an upper bound for economy-wide household utility given in (2) can be obtained by assuming all households consume an equal share of total production net of broker consumption (i.e. TC_t). Thus, we write the relaxed programing problem as:

$$V_0^R = \max_{\left\{\mu_t^{(i,j)},\phi_t^{(i,j)},\chi_t^{(i,j)}\right\}_{t,i,j}} \sum_{t=0}^{\infty} \beta^t \left\{u\left(TC_t\right) - eTP_t\right\} \tag{16}$$

subject to (14) and (15), where TP_t and TC_t are defined in (12) and (13).

In the following lemma, we show that the solution to the relaxed problem has all households directed to money shops.

Lemma 1 (Relaxed Program) *Given Assumption 1, the solution for the Relaxed Program (16) is given by:*

$$\mu_t^{(i,j)} = 1 \textit{ and } \phi_t^{(i,j)} = \chi_t^{(i,j)} = 0 \quad \forall i \neq j, t$$

$$C_t^{\{0,i\}} = \kappa \textit{ and } C_t^{\{i,j\}} = 0 \quad \forall i \neq j, t$$

While proofs appear in the appendix, the key elements of this one are as follows. First, Assumption 1.b implies it is always optimal to produce (a degenerate lottery) since $u'(TC_t) > u'(1) > e$. Second, if the fixed cost of operating a monetary trading post $\{0, i\}$ or a barter trading post $\{i, j\}$ is already incurred, then increasing returns implies it is optimal to direct the workers of (i, j) households to those posts, instead of bilateral matches (which at the margin increases each household's expected production from at most $1/n$ to 1). Third, since at most two types of workers can be directed to a barter trading while $(n-1)$ types of workers can be directed to a monetary trading post, then Assumption 1.a implies that replacing barter posts with monetary posts will weakly increase the objective function. Finally, it must be optimal to open a post. By the third point, we need only consider adding a monetary post. If we direct $(n-1)$ types of workers (each of which consists of a measure $1/(n(n-1))$ of workers) to the monetary post instead of bilateral exchange, production increases from $1/n$ to 1. The contribution in production of this change is

$$(n-1) \times \frac{1}{n(n-1)} \times \left(1 - \frac{1}{n}\right) = \frac{n-1}{n^2}.$$

Since we have to account for the costs of opening the post, this change yields a $\frac{n-1}{n^2} - \kappa$ increase in consumption. Assumption 1.c. guarantees that the utility benefits of such a move are positive.

3.2 When is relaxed enough?

In this section we show that under the following assumption, a solution to the relaxed problem satisfies all the constraints of the complete problem (2)-(11).

Assumption 2. $\beta \geq \frac{e}{u(1-n\kappa)}$.

An allocation that solves the relaxed problem directs the shopper from a household which sends a message that includes $\widetilde{m} = \overline{m}$ and $\widetilde{\theta} = (i, j)$ to an $\{0, j\}$ post where he exchanges his $\overline{m}$ units of money for $1 - n\kappa$ units of consumption good j and the worker to a $\{0, i\}$ post where she exchanges 1 unit of good i for $\overline{m}$ units of money. In particular, each period along the relaxed problem equilibrium path the mechanism makes the proposal

$$\left\{0, \left(\frac{1}{b^{\{i,0\}}}, (i, -1), \overline{m}\right), \left(\frac{1}{b^{\{j,0\}}}, (j, 1 - n\kappa), -\overline{m}\right)\right\} \tag{17}$$

to any household that claims to be an (i, j) type with $\overline{m}$ units of money and the household accepts. The mechanism sends proposal $(1, (\widetilde{\lambda}, (0, 0), 0), (\widetilde{\lambda}, (0, 0), 0))$ for any other message that includes $\widetilde{m} \neq \overline{m}$.

The mechanism's proposal for broker b, for $b \in \{1, .., n\}$ is given by

$$(\kappa, 0, 0) \tag{18}$$

and for $b > n$ is given by $\varnothing$. That is, in every period, for $b \in \{1, .., n\}$, broker b will open a monetary shop $\{b, 0\}$, trade good b with money, consume κ units of good b without accumulating any money, and the rest of the brokers will do nothing. Therefore, we have $b^{\{i,0\}} = i$, in (17).

By definition, consistency conditions (8) to (11) hold. Since no household is being directed to a household residence, and brokers $b > n$ are not open, we need only verify the resource feasibility constraint (6) for brokers, $1, 2, \ldots, n$. But given the allocation from the relaxed problem directs the workers of $(n - 1)$ types (each producing 1 unit of say good i) to the post and the shoppers of $(n - 1)$ types (each consuming $(1 - n\kappa)$ units of good i) to the post with the broker receiving $C_{\gamma}^{i,\lambda} = \kappa$, resource feasibility is trivially satisfied.

Since brokers with $b > n$ neither open a trading post nor consume, their participation constraint (3) automatically holds. Moreover, for those brokers who open a trading post, $b \in \{1, \ldots, n\}$, every period they consume exactly κ units of the good, thereby satisfying the participation constraint (3).

Finally, we need to verify that (4) and (5) also hold. For an (i, j) household h located at λ_t^h with m_t^h units of money, given anonymity it is a weakly dominant strategy to send the message $\left(\lambda_t^h, (i, j), \overline{m}\right)$ about its characteristics, and receive the proposal (17). By sending any other message, the household receives an allocation which it could have received by sending the $\left(\lambda_t^h, (i,j), \overline{m}\right)$ message. To see this, first note that (17) is independent of one's message about λ_t^h so truthful reporting about location is trivially satisfied. Second, note that if the household lies about its tastes, it receives a proposal which yields either no money to the worker (since it cannot produce the good at that post) and/or no utility to the shopper, both results it can achieve by choosing $1_t^{hw} = 0$ and/or $1_t^{hw} = 0$. Third, if the household sends any message about m_t^h other than $\overline{m}$, it receives no goods or money, which it can again achieve by choosing $1_t^{hw} = 0$ and/or $1_t^{hw} = 0$. While the first two parts of this argument confirm that households will send truthful messages about their location and tastes, it does not establish that households will not lie about their money holdings (in fact it confirms that they will lie if $m_t^h \neq \overline{m}$). All we need to establish for incentive compatibility (5) then is whether households will ever deviate from $m_t^h = \overline{m}$. We use (4) and induction to verify that they don't.

Along the equilibrium path, any household h that starts period t in state $(\lambda_t^h, \theta_t^h, m_t^h = \overline{m})$, reports truthfully, and accepts the mechanism's proposed path receives lifetime utility equal to:

$$V_t\left(\lambda_t^h, \theta_t^h, \overline{m}; \eta_t^{-h}\right) = \frac{1}{1-\beta}\left[u\left(1-n\kappa\right) - e\right] \tag{19}$$

To establish under what conditions (4) holds, a household can always reject a proposal in period t (i.e. choose $1_t^{hs} \neq 1$ and/or $1_t^{hw} \neq 1$) and still receive a proposal like (17) conditional on sending a message that includes. $\widetilde{m}_{t+1} = \overline{m}$ in period $t+1$ from the mechanism due to anonymity. Given the indicator function for acceptance or rejection of proposals, we can consider any general deviation from the stated allocation as

$$V_t^D\left(\lambda_t^h, \theta_t^h, \overline{m}; \eta_t^{-h}\right) = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\left[1_\tau^{hs} \times u\left(1-n\kappa\right) - 1_\tau^{hw} \times e\right] \tag{20}$$

subject to

$$\overline{m} + \sum_{s=t}^{\tau-1} \overline{m} \cdot \left(1_s^{hw} - 1_s^{hs}\right) \geq \overline{m} \cdot 1_\tau^{hs} \quad \forall \tau. \tag{21}$$

The endogenous cash-in-advance constraint (21) follows since under the stated allocation rule, the locational restrictions in our environment implies that in order to consume in period t (i.e. $1_t^{hs} = 1$) the household should have at least $\overline{m}$ units of money.

Lemma 2 (Participation) *Provided Assumptions 1 and 2 hold, for any possible deviation which satisfies (21), the individual participation constraint (4) is satisfied (i.e.* $V_t \geq V_t^D$*).*

The necessity of Assumption 2 can be understood by the following one-shot deviation; the worker chooses $1_t^{hw} = 0$ and the shopper chooses $1_{t+1}^{hs} = 0$, while they accept all other proposals. In this case, the worker avoids e disutility today but does not receive utility from consumption tomorrow. The deviation is beneficial if

$$e \geq \beta u(1 - n\kappa)$$

which is just Assumption 2.

Proposition 3 (Relaxed⟹Complete) *Provided Assumptions 1 and 2 hold, the solution for the relaxed problem (16), which is given in (17) and (18) satisfies all the constraints of the complete problem (3)-(11) and hence solves (2)*

Proof. Given lemma 2 (Participation) and the fact that all households begin with $m_0^h = \overline{m}$, (5) holds. The rest of the constraints are automatically satisfied as shown above. ■

We show in Figure 1 that there exists a non-empty region of the parameter space such that Assumptions 1 and 2 hold. Specifically, the figure depicts the

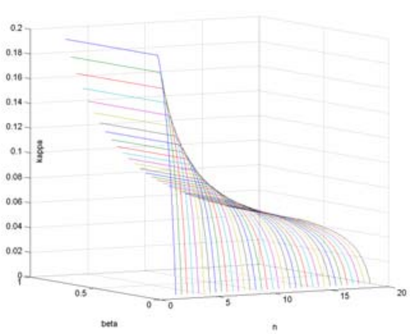

Figure 1: Existence Region

largest values of κ, for different values of n and β, that satisfy the assumptions when $u(c) = \log(1+c)$ and $e = 0.1$. For any given $n > 3$, if households are sufficiently patient (i.e. β is high enough)to support *monetary* exchange, then the only binding constraint is imposed by Assumption 1(c). As β falls, the binding constraint is the one imposed by Assumption 2. In this case, the future utility gain from consumption needs to grow relative to the current disutility from production. The only way to satisfy this is for κ to fall for any given n. Next, as the variety of goods increases (i.e. n grows), since more trading posts are necessary to sustain the symmetric outcome chosen by the mechanism, the cost of operating posts κ must fall. Finally, it is clear that if the cost κ of operating posts is too large, the mechanism may switch to bilateral exchange (e.g. if $n = 2$ and $\kappa = 1$ then bilateral barter exchange will be the optimal outcome). Furthermore, if κ is very small but households are very impatient, multilateral barter exchange may be the optimal outcome.

4 Implementation

In this section, we show that given Assumptions (1) and (2), we can implement the solution to the mechanism design problem as a subgame perfect equilibrium. First, using free entry and the fact that there is a countably infinite number of brokers, we show that at stage (i) of each period, n brokers will open monetary trading posts, $\{i, 0\}$ for $i = 1$ to n, and announce/post non-linear, competitive prices; they buy 1 unit of good i for $\overline{m}$ units of money and sell $1 - n\kappa$ units of good i for $\overline{m}$ units of money. Second, we show that after observing prices, it is a best response for any (i, j) household to send its worker to the $\{i, 0\}$ monetary trading post and its shopper to the $\{j, 0\}$ monetary trading post, for all $(i, j) \in \Theta$ in stage (iii). Third, using the fact that bilateral double coincidence trades are a subgame perfect equilibrium under off-the-equilibrium path beliefs that money has no value, we show it is a best response for brokers to commit to their announced prices even in stage (iv). These results are supported by household strategies that direct search to the best price broker (thereby coordinating household actions) and punish broker deviations from announced prices by reversion to bilateral barter exchange. Finally we show that for some parameterizations, we cannot implement the solution of the mechanism design problem with linear prices, while we can implement it with non-linear prices.

The public history of any broker b includes his stage (i) announced prices as well as his stage (v) executed trades. Let $\mathcal{H}_t^B = 0$ if any positive measure of households has ever announced that a broker has deviated from his announced prices prior to period t and let $\mathcal{H}_t^B = 1$ otherwise. Furthermore, endow all households with beliefs that if $\mathcal{H}_t^B = 0$ then brokers are not trustworthy and money has no value. Consider the following household strategy:

1. (HS1) If $\mathcal{H}_t^B = 0$, then for any household history where $\lambda_t \in [0, \frac{1}{2}]$, the household stays home while if $\lambda_t \in [\frac{1}{2}, 1]$ it goes to $1 - \lambda_t$. If the match results in a double coincidence, the worker of each household produces 1 unit of its good and gives it to the shopper of the other household;

2. (HS2) If $\mathcal{H}_t^B = 1$, then

    (a) in stage (iii) for any household history with $m_t < \overline{m}$, the worker goes to the monetary broker who offers the highest sustainable pay for her type i good and the shopper stays home.[14]

    i. in stage (v), if the broker does not change his announced prices in stage (iv) of that period, then the worker of the household produces for him, otherwise she does not and announces the broker's deviation next period in the first round of stage (i).

    (b) in stage (iii) for any household history with $m_t \in [\overline{m}, 2\overline{m})$ the worker goes to the broker who offers the highest sustainable pay for her type

[14] A "sustainable" price is defined to be a set of menus where if all households who either produce or consume that type good go to the broker announcing those prices, exchange at that post at the announced prices is feasible.

i good and the shopper goes to the broker with the lowest sustainable price for his type j good.

i. in stage (v), if the $\{i, 0\}$ broker does not change either of his announced prices in stage (iv), then the worker of the household produces for him, otherwise she does not and announces the broker's deviation next period. Similarly, if the $\{j, 0\}$ broker does not change either of his announced prices, then the shopper of the household buys from him, otherwise he does not and announces the broker's deviation next period.[15]

(c) in stage (iii) for any household history with $m_t \geq 2\overline{m}$, the worker stays home and the shopper goes to the broker with the lowest sustainable price for his type good.

i. in stage (v), if the broker does not change his announced prices in stage (iv) of that period, then the shopper of the household buys from him, otherwise he does not and announces the broker's deviation next period.

Next consider broker strategies:

1. (BS1) if $\mathcal{H}_t^B = 0$, then all brokers do not open.
2. (BS2) if $\mathcal{H}_t^B = 1$, then

(a) in the first round of stage (i) broker $b \in \{1, ..., n\}$, opens a $\{b, 0\}$ monetary post and announces an exchange menu which is just a pair of functions $(p^{\{b,0\}}(q), \widehat{p}^{\{b,0\}}(q))$ where

$$p^{\{b,0\}}(q) = \begin{cases} 1 - nk & \text{if } q = \overline{m} \\ 0 & \text{otherwise} \end{cases}$$

is the number of units of variety b goods received by the shopper in exchange for paying the shop q of units of money and

$$\widehat{p}^{\{b,0\}}(q) = \begin{cases} \overline{m} & \text{if } q = 1 \\ 0 & \text{otherwise} \end{cases}$$

is the number of units of money received by the worker in exchange for providing the shop with q of units of a variety b good.

(b) Broker $b \in \mathbb{N} \backslash \{1, ..., n\}$ stays closed until round b of stage (i). At round b, if broker b is a monetary broker, and no broker with his type has opened a post with the above menu, then broker b opens a monetary post according to his type and announces the above exchange menu.

[15] As will become apparent, if a broker price change generates lower instantaneous utility to any set of members at the post, then the household strategy implies those members will not participate at the post, thereby making the price unsustainable. Free entry rules out any price change favoring both sides.

i. in stage (iv), brokers do not change their menu.

The next proposition shows that these strategies implement the constrained optimum as a monetary equilibrium with zero profit trading posts.

Proposition 4 (Implementation) *The solution to the mechanism design problem can be implemented as a subgame perfect Nash equilibrium.*

There are two critical parts to the proof. Working backwards, the first part is that once households have arrived at a trading post (after stage iii), the broker is effectively in a monopoly position and may choose to charge a higher price or pay lower wages. The above strategies punish this possible deviation (BS2.a.i) by having workers not produce that period and then revert to economy-wide bilateral barter (HS2.a.i,2.b.i,2.c.i). That bilateral barter is a subgame perfect equilibrium follows from the beliefs we have endowed households with. The second part is that brokers announce "competitive" prices that implement the constrained optimum. Free entry (BS2.b) ensures there are no deviations from those prices.

Proposition 5 (Necessity of Nonlinear Pricing) *There are regions of the parameter space such that the solution cannot be implemented with linear prices.*

If prices are linear, a household may deviate by cutting its consumption in half and working only in even periods. If the utility function exhibits high relative risk aversion, then the decrease in consumption from $(1 - n\kappa)$ to $\frac{1-n\kappa}{2}$ yields a small reduction in utility relative to the utility gain from reduction in effort.

5 Robustness

In this section we discuss the robustness of our results to the assumptions of the environment. It is important to recognize that for a trading post to open given the cost κ, a positive measure of households must visit the post. But if households randomly search for trading posts, the probability that a positive measure of households will arrive at a post is zero. Therefore, if a theory is to address trading posts, we must assume that households can direct their search.

However, without certain restrictions, directed *bilateral* matching can result in the non-existence of trading posts; any (i, j) household could be matched with a (j, i) household in a bilateral barter exchange and economize on κ.[16] Our assumptions on the lack of communication across household residences within a given period, random household relocation, and the fact that a match must include a resident of the location are sufficient to rule out the type of "perfect" bilateral match discussed above and studied more generally in Corbae,

[16]The existence of a perfect matching rule such as this is guaranteed by an isomorphism of separable, non-atomic, normalized measure algebras (See Theorem C, p. 173, in Halmos, P. (1974) Measure Theory).

Temzelides, and Wright [1]. It is easy to see that lack of communication is necessary; if households could communicate after realizing their new location and before directing their members, they could arrange perfect bilateral matches even if households are randomly relocated and subject to the constraint requiring one member of the matches to be the residence of the match's location. However, lack of household communication is not sufficient; if households were not randomly relocated and/or a match was not required to include a resident of that location, then households could eventually learn the location of their perfect match and/or coordinate on a perfect match in a location other than the match members' residence.

Restrictions on communication across locations also play a crucial role for exchange at trading posts. Since we assume posts are costly to operate and at most two goods can be exchanged at them, if brokers could communicate within a period then barter brokers could make payments at one post contingent on payment at another and effectively implement gift exchange at $n/2$ posts rather than monetary exchange at n posts. Furthermore, since brokers can communicate at the begining of each period, if brokers could keep record of their costumers (i.e. households were not anonymous), then following Kocherlakota [5] again $n/2$ barter posts could implement gift exchange conditioning shoppers' consumptions on past production of workers.

Alternatively, we could assume brokers can keep perfect records of their customers, but they cannot communicate these records; that is each broker has a private record of his customers but cannot access the record of other brokers on their customers. This assumption also makes it impossible to form a public record out of a brokers' private record, which we know from Kocherlakota [5] is essential for money to have value. To see this point, realize that if brokers could keep records on their costumers and communicate these records, then they could use each household's first period address as identification and the second period address as a verification code. In this case, each time a worker produces at a broker and announces both her first and second period address, the broker can keep this record and transmit it to the other brokers in subsequent periods. Once the shopper of the household goes to another broker and announces his first and second period address, he can receive his good without the use of money.[17]

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A Appendix

Lemma 1 (Relaxed Program). Given Assumption 1, the solution for the Relaxed Program (16) is given by:

$$\mu_t^{(i,j)} = 1 \text{ and } \phi_t^{(i,j)} = \chi_t^{(i,j)} = 0 \quad \forall i \neq j, t$$

$$C_t^{\{0,i\}} = \kappa \text{ and } C_t^{\{i,j\}} = 0 \quad \forall i \neq j, t$$

Proof. Since TC_t is always less than or equal to 1, we have $u'(TC_t) - e > u'(1) - e > 0$. Therefore, if $\mu_t^{(i,j)} + \phi_t^{(i,j)} + \chi_t^{(i,j)} < 1$, then by increasing $\chi_t^{(i,j)}$, the objective function will increase. Hence in any solution we should have:

$$\overline{\mu}_t^{(i,j)} + \overline{\phi}_t^{(i,j)} + \overline{\chi}_t^{(i,j)} = 1 \quad \forall i \neq j, t \tag{22}$$

Also, if $\exists k$, such that $\overline{\mu}_t^{(i,k)} > 0$, or $\overline{\phi}_t^{(i,j)} > 0$ or $\overline{\phi}_t^{(j,i)} > 0$, then $\overline{\chi}_t^{(i,j)} = 0$. When $\exists k$, such that $\mu_t^{(i,k)} > 0$, then

$$\begin{aligned}
\frac{\partial V_0^R}{\partial \mu_t^{(i,j)}} - \frac{\partial V_0^R}{\partial \chi_t^{(i,j)}} &= \beta^t \left(\frac{1}{n(n-1)} - \frac{1}{n(n-1)} \times \frac{1}{n} \right) (u'(TC_t) - e) \\
&= \beta^t \frac{1}{n^2} (u'(TC_t) - e) > \beta^t \frac{1}{n^2} (u'(1) - e) > 0.
\end{aligned}$$

Therefore, V_0^R will increase by reducing $\chi_t^{(i,j)}$ and increasing $\mu_t^{(i,j)}$ while respecting (22). Similarly, if $\phi_t^{(i,j)} > 0$ or $\overline{\phi}_t^{(j,i)} > 0$, then reducing $\chi_t^{(i,j)}$ and increasing $\phi_t^{(i,j)}$ will increase V_0^R. Hence,

$$\overline{\chi}_t^{(i,j)} > 0 \quad \Longrightarrow \quad \overline{\chi}_t^{(i,j)} = 1 \text{ and } \overline{\mu}_t^{(i,k)} = \overline{\phi}_t^{(i,j)} = \overline{\phi}_t^{(j,i)} = 0 \text{ for } \forall k \neq i. \quad (23)$$

Also, if $\overline{\mu}_t^{(i,j)} > 0$ (which implies $\overline{\chi}_t^{(i,j)} = 0$), then we can set $\overline{\mu}_t^{(i,j)} = 1$, and $\overline{\phi}_t^{(i,j)} = 0$, without decreasing V_0^R, if not increasing it. Moreover, if $\overline{\mu}_t^{(i,j)} > 0$, then we can set $\overline{\mu}_t^{(i,k)} = 1$ for $\forall k \neq i$ (including $\overline{\mu}_t^{(i,j)} = 1$), without decreasing V_0^R, if not increasing it.

Finally if $\overline{\phi}_t^{(j,i)} > 0$, and $\overline{\phi}_t^{(i,j)} = 0$ (that is a barter $\{i,j\}$ shop is open but the workers from the (i,j) households do not go there), we have $\overline{\mu}_t^{(j,k)} = 0$ for $\forall k \neq j$ (i.e. No $\{0,j\}$monetary shop) and by (23) $\overline{\phi}_t^{(j,i)} = 1$ from above. But then by setting $\overline{\mu}_t^{(j,k)} = 1$ and $\overline{\phi}_t^{(j,i)} = 0$, and changing $\left\{C_t^{\{j,k\}}\right\}_{t=1}^{\infty}$ and $\left\{C_t^{\{0,j\}}\right\}_{t=1}^{\infty}$, such that $\left\{C_t^{\{j,k\}} + C_t^{\{0,j\}}\right\}_{t=1}^{\infty}$ remains the same, and (14) and (15) are not violated, the same value of V_0^R can be attained. That is, if in period t, only the workers from the (j,i) households go the the barter $\{i,j\}$ post, then we can close down that post and open a monetary $\{0,j\}$ post instead.

Therefore, without loss of generality we can look for the solutions such that:

$$\left\{\begin{array}{c} \overline{\mu}_t^{(i,j)}, \overline{\phi}_t^{(i,j)} \text{ and } \overline{\chi}_t^{(i,j)} \in \{0,1\} \\ \overline{\phi}_t^{(i,j)} = \overline{\phi}_t^{(j,i)} \\ \overline{\mu}_t^{(i,j)} = \overline{\mu}_t^{(i,k)} \end{array}\right\} \quad \text{for } \forall i \neq j \neq k \neq i, \text{ and } \forall t$$

Denoting the number of open monetary posts in period t by $\Pi_t^m = \sum_t 1_{\left\{\mu_t^i > 0\right\}}$ (which means the workers of $(n-1) \times \Pi_t^m$ types go to the monetary posts in period t) and the number of open barter posts in period t by $\Pi_t^b = \sum_t \sum_{j \neq i} 1_{\left\{\phi_t^{(i,j)} > 0 \text{ or } \phi_t^{(j,i)} > 0\right\}}$ (which means the workers of $2 \times \Pi_t^b$ types go to barter posts in period t).

Therefore the workers of $(n-1)(n - \Pi_t^m) - 2 \times \Pi_t^b$ types in period t will be directed to bilateral matches, and the Relaxed Problem (16) can be rewritten as

$$V_0^R = \max \sum_t \beta^t \left\{ u\left(\left(\frac{\Pi_t^m (n-1) + 2\Pi_t^b + n}{n^2} \right) - \left(C_t^m + C_t^b\right) \right) - e\left[\frac{\Pi_t^m (n-1) + 2\Pi_t^b + n}{n^2} \right] \right\}$$

such that

$$\kappa \sum_t \beta^t \Pi_t^m \leq \sum_t \beta^t C_t^m \quad \text{and} \quad \kappa \sum_t \beta^t \Pi_t^b \leq \sum_t \beta^t C_t^b$$

where C_t^m and C_t^b are the total consumption by the brokers operating the monetary posts and barter posts.

If $n \geq 3 \Leftrightarrow (n-1) \geq 2$, then since $u'(TC_t) - e > u'(1) - e > 0$, (that is producing more is welfare improving) if $\Pi_t^b > 0$, then by changing the number of barter posts to zero, that is $\Pi_t^b = 0$, adding as much as feasible to the number of monetary shops, that is $\Pi_t^m = \min(\Pi_t^m + \Pi_t^b, n)$, and changing the consumptions, that is $C_t^b = 0$ and $C_t^m = C_t^m + C_t^b$, the objective function, V_0^R, will not decrease (and it will increase if $n > 3$.)

So the problem can be rewritten as:

$$V_0^R = \max \sum_t \beta^t \left\{ u\left(\left(\frac{\Pi_t^m (n-1) + n}{n^2} \right) - C_t^m \right) - e \left[\frac{\Pi_t^m (n-1) + n}{n^2} \right] \right\}$$

such that

$$\kappa \sum_t \beta^t \Pi_t^m \leq \sum_t \beta^t C_t^m \tag{24}$$

The first order conditions for Π_t^m and C_t^m are:

$$\beta^t \frac{(n-1)}{n^2} \{u'(TC_t) - e\} - \beta^t \kappa \zeta - \zeta_{\Pi_t^m}^n = 0 \tag{25}$$

$$-\beta^t \{u'(TC_t)\} + \beta^t \zeta + \zeta_{C_t^m}^0 = 0 \tag{26}$$

where ζ is the Lagrangian multiplier for (24), $\zeta_{\Pi_t^m}^n$ is the Lagrangian multiplier for $\Pi_t^n \leq n$, and $\zeta_{C_t^m}^0$ is the Lagrangian multiplier for $C_t^m \geq 0$.

Setting $\Pi_t^m = n$ and $C_t^m = n\kappa$, which imply $TC_t = 1 - n\kappa$, for $\forall t$, and the Lagrangian multipliers as follows:

$$u'(1 - n\kappa) = \zeta$$

$$\begin{aligned} \beta^t \left\{ u'(1-n\kappa) \left(\frac{(n-1)}{n^2} - \kappa \right) - e \frac{(n-1)}{n^2} \right\} &= \zeta_{\Pi_t^m}^n \\ \zeta_{C_t^m}^0 &= 0 \end{aligned}$$

all of the conditions (24), (25) and (26) are satisfied.

Notice in order to have $\zeta_{\Pi_t^m}^n \geq 0$, we should have:

$$u'(1 - n\kappa) > e \frac{(n-1)}{((n-1) - n^2\kappa)}$$

which in addition to $u'(1) > e$ assures that $\Pi_t^m = n$. That is, the solution to the Relaxed Problem will be attained by sending all the agents to the monetary posts every period. ■

Lemma 2 (Participation). Provided Assumptions 1 and 2 hold, for any possible deviation which satisfies (21), the individual participation constraint (4) is satisfied (i.e. $V_t \geq V_t^D$).

Proof. Without loss of generality, consider V_0 and V_0^D. Rewriting (21) as

$$\sum_{s=0}^{t-1}\left(1-1_s^{hw}\right) \leq \sum_{s=0}^{t}\left(1-1_s^{hs}\right) \quad \forall t \tag{27}$$

Assuming $1_{-1}^W = 1$, and using assumption 2, (27) implies $A_t = \sum_{s=0}^{t} a_s \geq 0, \forall t$ where

$$a_s = \left[\left(1-1_s^S\right)u\left(1-n\kappa\right)\right] - \left[\left(1-1_{s-1}^W\right)\frac{e}{\beta}\right] \quad \forall s$$

Now using the *Abel's partial summation formula* (Rudin (1976)Theorem 3.41) we have $\sum_{s=0}^{t}\beta^s a_s = \sum_{s=0}^{t} A_s\left(\beta^s - \beta^{s+1}\right) + A_t\beta^{t+1}$, which is positive given $A_t \geq 0$ and $\beta < 1$. Therefore

$$\begin{aligned} 0 &\leq \sum_{s=0}^{\infty}\beta^s a_s = \sum_{s=0}^{\infty}\beta^s\left\{\left[\left(1-1_s^S\right)u\left(1-n\kappa\right)\right] - \left[\left(1-1_{s-1}^W\right)\frac{e}{\beta}\right]\right\} \\ &= \sum_{s=0}^{\infty}\beta^s\left\{u\left(1-n\kappa\right)-e\right\} - \sum_{s=0}^{\infty}\beta^s\left\{1_t^S \times u\left(1-n\kappa\right) - 1_t^W \times e\right\} = V^P - V^D \end{aligned}$$

■

Proposition 4 (Implementation) The solution to the mechanism design problem can be implemented as a subgame perfect Nash equilibrium.

Proof. Since each household is measure zero, they will truthfully announce broker deviations in stage (iv) of any previous period, so the event that $\mathcal{H}_t^B = 0$ or 1 will be reported truthfully.

First, we show given $\mathcal{H}_t^B = 0$, bilateral barter is a subgame perfect equilibrium. That is for any household, given all other households follow HS1 and brokers follow BS1, it is optimal to follow HS1 every period; also for any broker given households follow HS1, it is optimal to follow BS1 every period. Given $\mathcal{H}_t^B = 0$, by opening a trading post, a broker will incur $-\kappa$ disutility cost but since the households follow HS1, the trading post will have no costumers, hence the broker's consumption will be zero. Therefore it is optimal for the broker to not open a trading post (thereby achieving 0 utility instead of $-\kappa$). In the event of $\mathcal{H}_t^B = 0$, since no trading post will be open a household residing at λ can either stay at its residence or direct its members to another household's residence. Given all other households follow HS1, if the household also follows HS1, then with probability $\frac{1}{n(n-1)}$ it will be able to conduct a bilateral barter exchange and receive utility $u(1) - e > 0$. But if the household deviates, because of the constraint that a match must include a resident of the location, it will not be able to exchange. Moreover, since money has no value, only barter exchange is possible.

In the case of $\mathcal{H}_t^B = 1$, given all households followed HS2 and all the brokers followed BS2 up to period t, we must verify whether any household or any broker deviates from HS2 or BS2, respectively, in period t, when all other agents act according to strategy part 2. Clearly if all households and brokers have been following strategy part 2, all households start period t with $\overline{m}$ units of money. Given all others follow strategy part 2, if in stage (v) a broker has customers at his post, since the households only go to the trading posts with sustainable prices, if the broker does not change his prices in stage (iv), his consumption will be non-negative while if he changes his prices all of his customers will neither produce nor provide money (so that in this event his consumption is zero and obviously his lifetime utility will be zero). Therefore given everybody else is following strategy part 2, a broker has no incentive to deviate from the described strategy in stage (iv). Moreover, if the broker changes his prices, it is optimal for a household at that post to neither produce nor provide money to the broker, since following HS2 the other households will not do so, in which case the household receives nothing in exchange for its good or money. Therefore a household does not deviate from HS2 in stage (v) given the other agents follow HS2 and BS2.

Given all brokers follow BS2, a household can at most consume $1-n\kappa$ units of his consumption good provided he has at least $\overline{m}$ units of money. Hence lemma 2 (participation) assures us it is optimal for a household to follow HS2 in stage (iii) given the other households follow HS2. Finally, provided the other brokers follow BS2 in stage (i), a broker cannot attain positive utility in a period. Thus it is rational for the broker to follow BS2 (which yields 0 utility). ■

Proposition 5 (Necessity of Nonlinear Pricing). There are regions of the parameter space such that the solution cannot be implemented with linear prices.

Proof. If the price menus are linear then a shopper can obtain $\frac{1-n\kappa}{\overline{m}}$ units of goods for each unit of money it spends. One possible deviation for a household is to spend its $\overline{m}$ units of initially endowed money in T periods, each period consuming $\frac{1-n\kappa}{T}$ units of the good, and then producing in period T to obtain $\overline{m}$ units of money. Following this same pattern of consuming $\frac{1-n\kappa}{T}$ units of goods every period, and producing once every T periods, lifetime utility from this deviation will be

$$V^D = \frac{u\left(\frac{1-n\kappa}{T}\right)}{1-\beta} - \beta^{T-1}\frac{e}{1-\beta^T}.$$

In order to implement solution for the mechanism design problem, it is *necessary* to have $V^D \leq V^P$, where

$$V^P = \frac{u\left(1-n\kappa\right)-e}{1-\beta}$$

is the lifetime utility from producing and consuming every period. This condition can be written

$$e\left(\frac{1-\beta^{T-1}}{1-\beta^{T}}\right)<u\left(1-n\kappa\right)-u\left(\frac{1-n\kappa}{T}\right) \tag{28}$$

where the left hand side is the utility gain from working less and the right hand side is the utility loss due to a reduction in consumption.

Now we will provide conditions on parameters, including the utility function $u(\cdot)$, such that without violating Assumptions 1 and 2, (28) does not hold for $T=2$. We can define $\varepsilon>0$ such that

$$\frac{e\left(n-1\right)}{\left(n-1\right)\left(1-nk\right)-nk}<u'\left(1-n\kappa\right)<u'\left(\frac{1-n\kappa}{2}\right)=\frac{e\left(n-1\right)}{\left(n-1\right)\left(1-nk\right)-nk}+\varepsilon. \tag{29}$$

where the first inequality follows from Assumption 1(c) and the second one follows from strict concavity of $u(\cdot)$.

To violate (28) for $T=2$, it is sufficient to have

$$u\left(1-n\kappa\right)-u\left(\frac{1-n\kappa}{2}\right)<\left(\frac{e\left(n-1\right)}{\left(n-1\right)\left(1-nk\right)-nk}+\varepsilon\right)\left(\frac{1-n\kappa}{2}\right)<\frac{e}{1+\beta} \tag{30}$$

where the first inequality follows from concavity of $u(\cdot)$. Thus, if β is sufficiently small, we violate (28). But Assumption 2 bounds how small β can be. In particular, a sufficient condition is

$$\frac{e}{u\left(1-n\kappa\right)}<\beta<1-\frac{2n\kappa}{\left(n-1\right)\left(1-n\kappa\right)} \tag{31}$$

which can be satisfied for large enough $u(1-n\kappa)$ and small enough κ.

Notice that (29) implies that for small ε, $u(\cdot)$ must not increase substantially from $\frac{1-n\kappa}{2}$ to $1-n\kappa$. But in order for $u(1-n\kappa)$ to be large enough to satisfy the left hand side of (31), we should have large $u\left(\frac{1-n\kappa}{2}\right)$. In other words, if the utility gain from consumption increases substantially up to $\frac{1-n\kappa}{2}$ and then only moderately after that, (28) will not hold while Assumption 1 and 2 hold. For example any utility function with high enough relative risk aversion can satisfy this pattern. ■