Vol. 66 (1997), No. 2, pp. 103–125

Journal of Economics
Zeitschrift für Nationalökonomie

Can Ice Cream Be Money?: Perishable Medium of Exchange

Xavier Cuadras-Morató

Received February 4, 1997; revised version received May 20, 1997

The aim of this paper is to demonstrate that a perishable good may be used as commodity money, even in economies in which perfectly durable commodities are available. This is shown in the general context of a search-theoretical model of a decentralized economy.

Keywords: money, perishable commodities, search theory.

JEL classification: C73, D83, E00.

1. Introduction

The aim of this paper is to demonstrate that a perishable good may be used as medium of exchange, even in economies in which perfectly durable commodities are also available. We present a search-theoretical model of a simple decentralized economy with three different goods and three different types of agents who are specialized in production and consumption. The model is close to the ones described in Kiyotaki and Wright (1989) and Cuadras-Morató (1994a) (see also Aiyagari and Wallace, 1991), but, in contrast with these models, where goods have respectively different storage costs and quality homogeneity, in our setting goods have different durability (measured as the number of periods during which a good can be consumed with no loss of utility). In particular, we show that a perishable good may play the role of commodity money when the rest of the goods of the economy are perfectly durable.

One of the first questions addressed by monetary economists was under what circumstances a particular object may have come to be used as medium of exchange. Jevons (1875) gives a list of requirements that any object should have in order to be suitable to perform the functions of money. Among others, portability, homogeneity, divisibility, stability of value, cognizability, and indestructibility are regarded by him as desirable qualities of any commodity performing the role of money. Nevertheless, as Jevons himself was aware, the functions performed by money are of very different nature and they do not have to be necessarily performed by a unique asset with all those characteristics. For instance, in order to be used as store of value, it seems quite obvious that a commodity should have a high degree of durability, although this may not be so important when the role to be played is that of a medium of exchange.[1] Therefore, it may seem intuitively plausible that a perishable object may appear as commodity money playing the role of medium of exchange but, to date, this question has not been explicitly addressed in the literature about monetary exchange. Instead, most models identify very closely money with durable objects.

As a matter of fact, there has been a number of cases reported by anthropologists and historians in which perishable goods appear to be used as medium of exchange. Eggs in Guatemala, cocoa beans[2] in Mexico and Central America, butter in Norway, tobacco, rice, grain, beef, peas, pork, dairy products, etc. in the United States are only a few significant examples.[3] As Einzig himself points out, "one of the gravest defects of commodity-currencies was that they were perishable. Sooner or later their quality deteriorated to such an extent that they ceased to be taken as currencies and the last receiver had to sell them as merchandise at their low market price" (Einzig, 1966, p. 284).

Models of the exchange process based on search theory have been used to study the characteristics of objects that make them more or less likely to be used as money. This is because this type of model illustrates well the trade-off between endogenous acceptability and intrinsic properties of goods, such as perishability or recognizability. In previous related work, Cuadras-Morató (1994a) and Y. Li (1995) analyze the issue of homogeneity or recognizability in the context of commodity money, Burdett et al. (1995) look at portability, and Cuadras-Morató

1 "We come to regard as almost necessary that union of functions which is at the most a matter of convenience and may not always be desirable. We might certainly employ one substance as medium of exchange and a second as a measure of value, etc." (Jevons, 1875, p. 16).

2 Users of that kind of money were perfectly aware of its perishable nature. Thus, Pedro Martir Anghiera (1457–1526), one of the earliest writers on the New World made the following observation: "Oh, blessed money which yieldeth sweete and profitable drinke for mankinde, and preserveth the possessors thereof free from the hellish pestilence of avarice because it cannot be long kept or hid underground" (quoted in Einzig, 1966, p. 175).

3 See Einzig (1966) for more information about different types of objects used as money in different historical contexts.

and Wright (1997) examine the role of two other features of money: scarcity (relative low supply) and intrinsic utility (relative high demand). In general terms, the conclusion of this type of literature can be stated as follows: intrinsic properties of goods matter (so goods with very bad properties will not be used as money) but money is also determined by convention (so goods with relatively worse features can be used as money if agents decide to accept them).[4]

A large number of models focus on the analysis of fiat money and its role of medium of exchange (see Lucas, 1980; Wallace, 1980; Townsend, 1980; and Kiyotaki and Wright, 1991, for different approaches tackling this same issue). In these models, fiat money is an object which has been endowed with all the desirable qualities of money, durability, recognizability, portability, divisibility, etc., and takes on value because of its role as a medium of exchange. In this sense, these models of exchange economies are not particularly interesting for our investigation. For our purposes, we are more interested in the existing models of commodity money, although they do not address our question very directly either. In particular, all models of commodity money we know (among others, King and Plosser, 1986; Iwai, 1988; Jones, 1976; Oh, 1989; Harris, 1979; Kiyotaki and Wright, 1989) assume durability of all commodities. This particular assumption precludes the type of analysis which is the focus of the present paper. We regard the essential nature of money as being strategic. The extrinsic beliefs of agents about the acceptability of the different goods in the economy play an important role in the determination of which goods are going to be used as medium of exchange, together with the intrinsic qualities of those goods. In this context, it is clear that durability may be a desirable quality of money but it is not, by any means, an essential characteristic of money.

Our modelling strategy is very simple. We take a well-known model of commodity money in the literature and introduce the possibility that goods are perishable. Then, we show that a perishable good may well appear as commodity money (even in the case in which the rest of

4 These are not merely theoretical issues. In studying currency substitution during inflationary processes, economists have asked why inflationary currencies were only partly substituted by foreign currencies with much more stable value. Apart from explanations relying on legal restrictions (which are not always perfectly enforceable), one possible answer is that foreign currencies were not acceptable. What this stresses is the character of social convention of the circulation of money. As Kiyotaki and Wright (1992, p. 19) put it, "acceptability may not actually be a property of an object as much as it is a property of social convention."

commodities in the economy are durable). The chosen model (in our opinion the model that best reflects what we think is the true nature of money as medium of exchange) is the search-theoretical model of money of Kiyotaki and Wright (1989). In this model, agents choose optimal trading strategies and commodity money appears endogenously as an equilibrium outcome. The goods appearing as commodity money will only be partially determined by the intrinsic qualities of the different goods of the economy (fundamentals). In fact, the extrinsic beliefs held by the agents about acceptability of goods play a major role in the determination of the goods appearing as commodity money. That is, the nature of money is basically that of pure social convention, and its essential characteristic is its acceptability. Other desirable physical characteristics like durability, homogeneity, or storability are not necessary features for the use of money as a medium of exchange.

The structure of the paper is as follows. In Sect. 2 a general model of a decentralized economy with perishable goods is described. Section 3 examines a particular case of the model. Our main result is to prove the existence of equilibria in which perishable commodities play the role of medium of exchange. Section 4 concludes the paper.

2 The Economy

In this section a model of a simple exchange economy with decentralized trade and nondurable goods is specified. The general structure of the model is like the one described in Kiyotaki and Wright (1989) (see also Cuadras-Morató, 1994a, for a version of the model with goods of heterogeneous quality). The crucial modification here is that we do not assume that goods are perfectly durable and, indeed, we allow for the existence of goods which have different durability over time (durability being defined as the number of periods of time during which a good can be consumed with no loss of utility).

2.1 General Environment

Time is discrete. There are three different types of indivisible goods: good 1, good 2, and good 3. There is a continuum of infinitely-lived agents who are, in equal proportion, of type I, type II, and type III. Agents of type i (i = I, II, III) are specialized in consumption in such a way that they only consume goods of type i ($i = 1, 2, 3$). They are also specialized in production with the following pattern: agents of type i produce only goods of type $i + 1$ (modulo 3). The characteristics of goods of a certain type vary with their age. More precisely, goods of type i are apt for consumption during a given number of periods, n_i, ($n_i > 0$) and after that, they expire and their consumption adds no utility to the agent who consumes them. In general, $n_i \neq n_j$ ($\forall i, j = 1, 2, 3$ and $i \neq j$). In order to identify goods of the same type but of different age, the following notation is used to refer to goods of type i: $i_0, i_1, \ldots, i_t, \ldots$ (t indicating the number of periods that have passed since the good was produced).

Consuming a good i_r adds U_i units of utility to agent of type i if $r < n_i$ and no utility at all if $r \geq n_i$. After consuming a good of type i, agents of type i immediately produce a good $(i+1)_0$, with production costs in terms of disutility being denoted by D_i ($U_i - D_i > 0$). Also, agents of type i can dispose at any time and at no cost of the good they hold in inventory and produce a new good $(i+1)_0$ at the same cost D_i. All goods can be stored by agents at no cost, but only one at one time. Every agent is assumed to be perfectly informed about the type, age, and conditions of consumption of both the good he is holding and the goods held by the other agents in the economy. Given the above structure, agent i's expected discounted lifetime utility is given by (see Kiyotaki and Wright, 1989, p. 930) $\mathrm{E}\sum_{t=0}^{\infty} \beta^t [I_i^U(t)U_i - I_{i+1}^D(t)D_i]$, where U_i and D_i are defined as above, β is the discount factor ($0 < \beta < 1$), $I_i^U(t)$ is an indicator function that equals one if the agent consumes his consumption good, zero otherwise; and $I_{i+1}^D(t)$ is an indicator function that equals one if the agent produces good $(i+1)_0$, zero otherwise.

The structure of the economy is assumed to be decentralized. No centralized market exists and agents only meet through a random matching process. Every period of time, agents, who always hold a good, meet in pairs and decide about whether to exchange their respective inventories or not (according to trading strategies about which details are provided below) and also about consumption, disposal, and production of commodities. Exchange takes place only by common agreement of the two paired agents and it is always quid pro quo. As it is obvious from the above setting, this economy is such that no agent produces the good he consumes and, also, there is no double coincidence of wants of the goods produced by any two agents. This means that, in order to consume, agents will have to exchange goods previously, and, also, that this exchange cannot be pure barter between the goods produced by two agents. Some form of monetary exchange pattern must necessarily emerge if there is going to be exchange at all.

In this economy agents must make decisions about trade, consumption, disposal, and production of goods. Nevertheless, in order to make things more tractable, we shall restrict the analysis to equilibria with the property that agents use very simple strategies to decide about consumption, production, and disposal of goods. This will allow us to concentrate on the trading strategies.

In particular, first we look for equilibria in which agents will always accept in exchange, consume immediately, and hence never hold, their own consumption good whenever they are offered it and provided that it has not perished, producing immediately after a new good to be held in inventory ("consume if possible").

Second, it will be the case that, in equilibrium, agents will never dispose of any good which has not perished yet to replace it producing a new good ("never dispose").

Finally, the information structure of this economy implies that the value of holding a good that has already perished and is not apt for consumption is zero. This is because nobody will be willing to accept a good like this in exchange, for the simple reason that it has no final consumption value to anyone. Nevertheless, we want the equilibria to be such that it will be optimal for an agent in this contingency to dispose of the good which has expired and produce a new good ("participation constraint"). This means that even in this worst possible case, agents will not drop out of the economy, because they always have the option of getting rid of the expired good and produce a new good, these two actions yielding positive value.[5] It will be shown that there exist equilibria with these properties for a large set of the parameter space.

In order to clarify further the structure underlying our setting, it is worthwhile to present a summary of the sequence of the events in this economy. This will be done by examining what happens to a representative commodity of type i from the moment it is produced until the moment it is consumed or it perishes and is disposed of by some agent. In order to simplify the exposition, we will assume that $n_i = 2$, so that good i can be consumed with no loss of utility for two periods after it was produced. Good i_0 is produced by an agent $i-1$ (mod 3) at the end of period t. Period $t+1$ starts and agent $i-1$ is paired randomly with another agent. Both agents recognize mutually their respective holdings and make decisions about trade. Basically, two situations may arise: first, good i_0 is acquired by an agent of type i who consumes it, which implies its physical destruction; alternatively,

5 In the present model, this possibility of disposal is equivalent to what Einzig refers to as "the last receiver had to sell them as merchandise at their low market prices" (see Sect. 1). Given the structure of this economy, agents will only use as media of exchange goods which have low consumption value to them (zero consumption value as a matter of fact). Once the good is no longer valid for consumption and, hence, cannot be passed on (zero exchange value), the best available option in the current setting is to dispose of it.

the good remains in agent $i - 1$'s hands or is bought by any other agent who does not want it for consumption. In this case, those agents will find themselves holding i_1 at the end of the period $t + 1$, because one period of time has passed since the good was produced. Period $t + 2$ starts and, again, two situations may accrue: good i_1 may be exchanged and consumed before the end of the period or, alternatively, will be disposed of by the agent who holds it before the end of the period. This is simply because at the end of period $t + 2$, two periods of time have passed since the good was produced and the agent holding it would find himself holding i_2, which is a good with no value for consumption or exchange. That is, whenever an agent is unsuccessful in his search for a trading partner who wishes to take good i before it perishes, it will be optimal for him to dispose of the good and produce a new one.

Following the notation advanced in Kiyotaki and Wright (1989) (although slightly changed to adapt it to our particular model), let V_{ij_s} be the payoff function (optimal value) for an agent of type i when he walks out of a trade meeting holding good j_s. In general, this payoff function is equivalent to the following expression: $V_{ij_s} = \max \beta\, \mathrm{E}(V_{ih_r} \mid j_s)$. This latter expression is a standard Bellman's equation of dynamic programming, where $\mathrm{E}(V_{ih_r})$ is the expected indirect utility of agent i at next-period random state h_r, conditional on j_s, and the maximization is over strategies about exchange, consumption, disposal, and production of commodities. It is worth emphasizing a couple of points that can help us to better understand the previous expression. First, the good held by an agent is what characterizes his current state (strategies will define actions to be taken by agents depending upon their states). Second, the random element comes simply from the assumed matching technology.

Before ending this subsection, some more notation is introduced. Let $p_{ij_s}(t)$ be the proportion of type i agents who are holding good j_s in inventory ($\forall i$, j and $s \le n_j$) at time t. By definition, $0 \le p_{ij_s}(t) \le 1$ and $\sum_j \sum_s p_{ij_s}(t) = 1$. Considering that each individual has exactly the same probability of meeting an agent of any type, the probability of being paired with another agent of type i holding good j_s at time t can be simply characterized by the vector $p(t) = (\ldots p_{ij_s}(t) \ldots)$ which will be called the distribution of inventories at t.

2.2 *Trading Strategies and Equilibrium*

The behavior of agents in this economy is determined by their chosen strategies about trade, consumption, production, and disposal of goods. In Sect. 2.1 we restricted the analysis to equilibria in which agents use simple strategies for consumption, production, and disposal of commodities. Nevertheless, the important strategic elements in this economy occur at the level of the exchange process. Therefore, the keynote for understanding whether monetary exchange can arise and how it is characterized are the trading strategies of agents. A trading strategy is a rule defining the conditions under which an agent of type i is intending to trade. Specifically, this will depend on the good held by the agent himself and the good being held by the agent with whom he has been matched. The following notation is used. Let $\tau_i(j_r, k_s) = 1$ if agent of type i wants to trade j_r for k_s and $\tau_i(j_r, k_s) = 0$ otherwise. It follows from this that when type i with good j_r meets type h with good k_s, they only trade if $\tau_i(j_r, k_s) \cdot \tau_j(k_s, j_r) = 1$. A trading strategy for any agent will be a rule that specifies the actions of the agent (trade denoted by 1, no trade denoted by 0) in all possible states. States are characterized by the goods being held in inventory by the agent and his trading partner. Formally, a trading strategy for an agent of type i is a vector w_i of dimension $(n_1+n_2+n_3)^2$ composed of elements 0 and 1 as follows, $w_i = (\ldots, \tau_i(j_n, h_{n'}), \ldots)$, $\forall j, h = 1, 2, 3$, $n < n_j$, $n' < n_h$.

This trading strategy completely characterizes all actions of agents in all possible states of the world. It has been specified generally, but it can be simplified recalling the strategies for consumption, disposal, and production of goods of the agents in this economy. First, it is never possible for an agent to be holding his consumption good (it is optimal to consume it immediately). This means that it is not necessary to specify the elements $\tau_i(i_s, j_r)$ of the vector describing the trading strategy of agent i, because they are only relevant for a hypothetical situation that simply will never arise in our model. It is also known that $\tau_i(j_r, i_s) = 1$ is always optimal, since agent i will always be willing to get his consumption good and consume it immediately.

At this point we will assume that agents do not randomize between strategies and do not change them over time. Consequently, we are only looking at pure and steady-state strategies. Also, since we only consider symmetric equilibria, we can summarize the strategies of the continuum of agents by simply stating a strategy for each type of agent.

Given an initial distribution, the strategies of the different agents and the realizations of matchings will determine the resulting distribution of inventories at any time t [i.e., $p(t) = p(t, w_1, w_2, w_3)$]. Given a strategy vector (w_1, w_2, w_3), we can define a steady-state distribution of inventories $p(w_1, w_2, w_3)$ as an inventory distribution that satisfies the following condition: $p(t, w_1, w_2, w_3) = p(t+1, w_1, w_2, w_3)$.

Finally, let an equilibrium be a vector of strategies (w_1^*, w_2^*, w_3^*), a steady-state distribution of inventories, p^*, and the corresponding optimal value functions $V_{ij_s}(w^*, p^*)$ such that for each agent of type i:

1. w_i^* maximizes individual expected discounted lifetime utility of agent i given the strategies of the other agents and the steady-state distribution of inventories, or, in other words, it is a best response for agent i given those strategies and the distribution of inventories,
2. $p(w_1^*, w_2^*, w_3^*) = p^*$, and
3. $U_i - D_i + V_{i,(i+1)_0} > V_{ij_s}, \forall i, j = 1, 2, 3, i \neq j, s < n_j$ (consume if possible); (1)
   $V_{ij_s} > -D_i + V_{i,(i+1)_0}, \forall i, j = 1, 2, 3, i \neq j, s < n_j$ (never dispose); (2)
   $D_i + V_{i,(i+1)_0} > V_{ij_r}, \forall i, j = 1, 2, 3, i \neq j, r \geq n_j$ (participation constraint). (3)

Condition 1 is the usual condition for Nash equilibrium (optimality of trading strategies). Condition 2 is a consistency condition that states that given the vector of strategies (w_1^*, w_2^*, w_3^*), p^* is a resulting steady-state distribution. The conditions in 3 ensure optimality of the conjectured consumption, disposal, and production strategies and basically imply restrictions on the values of the parameters for which equilibria will exist.

3 Equilibrium Results

The main objective of this section is to present results which prove the existence of equilibria in which a nondurable good appears as commodity money. In order to do this, it will be convenient to analyze a particular case of the economy described in Sect. 2. Specifically, only the case will be examined in which goods 1 and 2 are perfectly durable ($n_1, n_2 = \infty$) and good 3 perishes two periods after its production took place ($n_3 = 2$). The fact that we only examine a particular case should not cast too much doubt about the generality of our results. This is due to the fact that what we have actually done is to choose a tractable case which is quite extreme in the following sense: there is only one perishable good in our economy, the rest being perfectly durable goods; and it is a commodity of a very short life (the minimum required to be able to appear as commodity money). Even so, there is a region of the parameter space for which an equilibrium can be found in which the perishable good plays the role of commodity money. Hence, there should be a large number of economies of the type described in Sect. 2 where nondurable goods may play the role of commodity money.

There will be four different goods in this particular economy: 1, 2, 3_0, and 3_1. The same notation introduced above is maintained, although for goods 1 and 2 no further information about their age is necessary, because they are perfectly durable and their characteristics do not change over time. Moreover, it is not possible for agents of type I to hold commodity 3_0. This is because agent I cannot produce good 3 himself and can only get it by trade after at least one period of time has gone. Consequently, the distribution of inventories characterizing this particular economy can be expressed as $p = (p_{12}, p_{13_1}, p_{23_0}, p_{23_1}, p_{21}, p_{31}, p_{32})$ which, due to the fact that probabilities should add to one, can be reduced to $p = (p_{12}, p_{23_0}, p_{21}, p_{31})$.

Next, we present equilibrium results referring to the simple economy described above and show that there is a region of the parameter space in which there exists an equilibrium in which a perishable good emerges as commodity money. We will also characterize conditions of existence for the rest of pure-strategy equilibria existing in this model and show that, when mixed strategies are allowed, there exist exchange equilibria for almost all values of the parameters U_i and D_i.

In this simple economy, optimal exchange strategies for agents can be characterized in the following way. Value functions and equilibrium strategies must satisfy the following incentive-compatibility constraints. Agent of type I will play the strategy "use good 3 as money" [i.e., $\tau_1(2, 3_0) = 1$] iff $V_{13_1} > V_{12}$; vice versa, he will play the alternative strategy "do not use good 3 as money" [i.e., $\tau_1(2, 3_0) = 0$] iff $V_{12} \geq V_{13_1}$. The first constraint guarantees that it is optimal for agent I to accept good 3 to use it as a medium of exchange, while the second alternative means that the agent prefers to hold the good he produced until he can swap it for the good he wants to consume.[6] Note the asymmetry between the two equilibrium conditions for the two strategies. In the second case, it is necessary and sufficient for good 2 (which is the good produced by agent I) to be held that it is at least as good as the alternative possibility. Instead, in the first case, we have a strict inequality since to exchange good 2 for good 3 requires that the latter is strictly preferred to the former. The reason for this asymmetry is that mixed strategies are not considered and it is assumed that trade does not take place when one of the agents is indifferent between his good and the good held by his trading partner.[7] This means that, in steady state, whenever agent I is indifferent between good 2 and 3, he will always

6 The situation in which agent of type I holds good 3_0 and is offered good 2 (consequently he also must compare V_{12} and V_{13_1}) will never arise simply because agents of type I can never hold good 3_0 (only producers of good 3 can).

7 This seems the natural assumption to make. You need only to consider explicitly the existence of an arbitrarily small cost of transaction to get it as a result of the model.

Table 1: Trading decisions of agent of type I

		Good held by trading partner							
		"Use good 3 as money"				"Do not use good 3 as money"			
		1	2	3_0	3_1	1	2	3_0	3_1
Good held by type I	1	–	–	–	–	–	–	–	–
	2	1	0	1	0	1	0	0	0
	3_0	1	0	0	0	1	1	0	0
	3_1	1	1	1	0	1	1	1	0

keep in inventory good 2, which is the good he produced (see Cuadras-Morató, 1994a, for similar equilibrium conditions in the context of a different model). Equivalently, the agent of type II will bring into play strategy "use good 1 as money" [i.e., $\tau_2(3_0, 1) = 1$] iff $V_{21} > V_{23_1}$; and vice versa, he will play "do not use good 1 as money" [i.e., $\tau_2(3_0, 1) = 0$] iff $V_{23_1} \geq V_{21}$.[8] Finally, the agent of type III will play "use good 2 as money" [i.e., $\tau_3(1, 2) = 1$ and $\tau_3(2, 1) = 0$] iff $V_{32} > V_{31}$ and "do not use good 2 as money" [i.e., $\tau_3(1, 2) = 0$] iff $V_{31} \geq V_{32}$.[9]

Table 1 presents a summary of this discussion. We present there an outline of the trading decisions taken by agents of type I. Notice that the two trading matrices differ only in two elements. The first row (type I holds good 1) is a situation that simply never occurs, since type I will consume immediately good 1 whenever he gets it. The first and the last column are also trivial: type I will always accept good 1 (consumption good) and never accept good 3_1, which is a good that will perish before anything can be done with it. Also, we assumed that agents do not trade when they are indifferent, so the elements of the diagonal are 0. So, we are left with the relevant decision: whether to accept good 3_0 (to use it as medium of exchange) or keep good 2 (production good) in storage.

The following notation will be used to denote a vector of strategies (one for each type of agents): $w = (w_1, w_2, w_3)$ where

8 A situation in which an agent of type II holds good 1 and is offered good 3_0 will only arise if his trade partner is also of type II (producer of good 3). Since we know that no mutual benefits from trade can be realized when traders are of the same type, this is an irrelevant case for our analysis.

9 If an agent of type III is using this latter strategy, it will never happen that he holds good 2 and is offered good 1. This is because it was not optimal to accept good 2 in the first place (agent III produces good 1 and can only get good 2 in the trading process).

$$w_i = \begin{cases} 1 & \text{iff agent of type } i \text{ plays} \\ & \text{strategy "use good } i+2 \text{ as money,"} \\ 0 & \text{otherwise.} \end{cases}$$

As can be easily seen, the problem of finding the equilibria of this model has now become more tractable. The procedure to be implemented to carry out this task is as follows: first, a vector of strategies, w, is conjectured. Second, the steady-state distribution implied by those strategies of the agents is computed. This is done simply by computing the steady-state of the stochastic Markov process defined by the strategies of the agents and the assumed matching technology in this environment. Third, it has to be checked that the strategies conjectured in the first place effectively satisfy the incentive-compatibility conditions of equilibrium. Finally, it has to be verified that, in equilibrium, the conjectured strategies for consumption, disposal, and production of goods are optimal for some values of the parameters.

The following proposition summarizes the main equilibrium result of our model. There exist two parameter regions each of which generates a unique equilibrium in pure strategies, one of them involving the use of good 3 (the only nondurable good in our economy) as medium of exchange.

Proposition 1: In the economy described above, for values of the parameters such that U_i/D_i and β are large enough, there exist only the following two pure-strategy equilibria: (a) in the region of the parameter set for which $U_1/D_1 > 5.2301$, there is a unique equilibrium in which goods 1 and 3 are used as commodity money; and (b) in the region of the parameter set for which $U_1/D_1 \leq 5$, there is a unique equilibrium in which only good 1 is used as commodity money.

Both these equilibria coincide (in the sense that the equilibrium strategies are identical and, consequently, also the media of exchange circulating in the economy) with the equilibria found in Kiyotaki and Wright (1989, theorem 1) and Cuadras-Morató (1994, proposition 1). In particular, the equilibrium strategies are $w = (1, 1, 0)$ for equilibrium a and $w = (0, 1, 0)$ for equilibrium b. The region of the parameter space for which they exist is characterized in Fig. 1a–c. In both equilibria a and b, the restrictions on the parameters U_1/D_1 are the incentive-compatibility conditions for the conjectured trading strategies to be optimal, while the general restriction on U_i/D_i and β ensures that strategies for con-

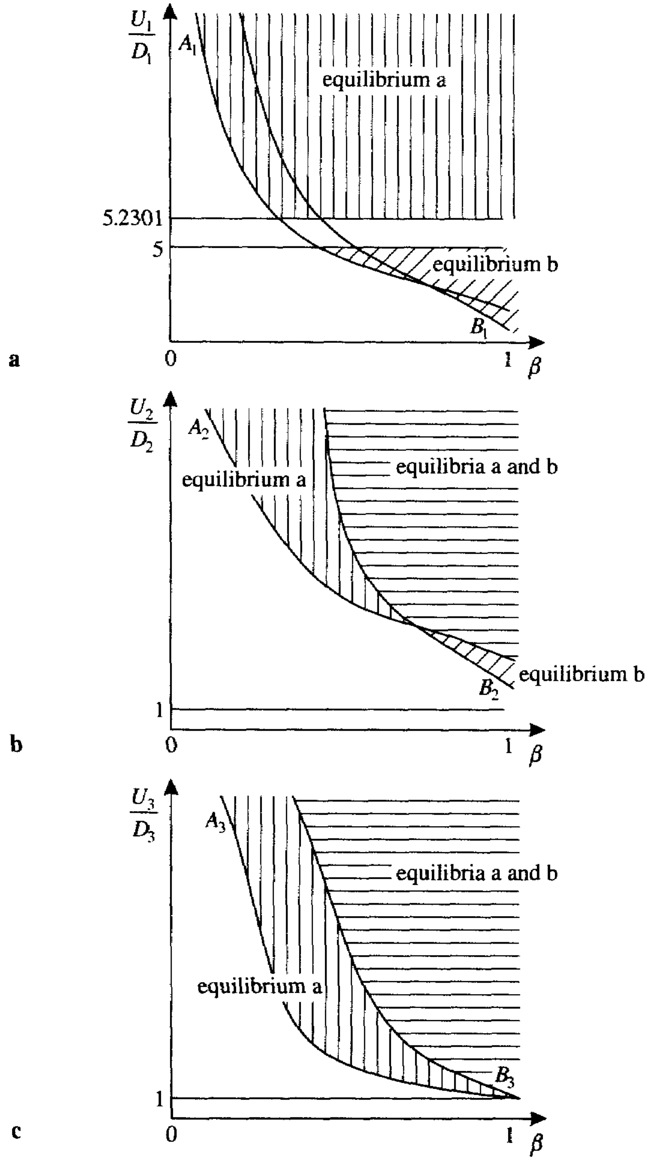

Fig. 1a–c: Parameter regions for equilibria a and b

sumption, disposal, and production of goods are also optimal[10] (that is, Condition 3 holds).

Proof: To prove the existence of equilibria a and b we must follow the methodology outlined above: first, conjecture the corresponding strategy; second, work out the probability distribution of inventories; third, check that the conjectured trading strategies satisfy equilibrium conditions; and fourth, check that the strategies for consumption, disposal, and production of goods are also optimal for some values of the parameters. To prove uniqueness we simply have to repeat exactly the same procedure with all the rest of possible strategies combinations and discard them as equilibria. Since the number of possible strategy vectors, w, is finite (there are only eight possible combinations), this is a relatively simple task.

In order to avoid repeating identical arguments several times, we will only give full details of the derivation of the conditions of existence for equilibrium a. The rest of the proof is nothing more than repeating the same procedure for all different possible strategy vectors. Consequently, we first conjecture the following strategy vector, $w = (1, 1, 0)$. Next, the strategies for each type of agent contained in w plus the assumed matching technology generate a Markov process the steady-state probability distribution of which is equivalent to the steady-state distribution of inventories in our model. In the particular case of the vector of strategies considered here, this is $p = (0.8967, 0.3456, 0.5272, 1)$. (Details of the above computation are provided in the appendix.) Thirdly, it has to be checked that the strategies conjectured above satisfy the equilibrium conditions. Thus, given the strategies of other agents, the strategy conjectured for an agent of type I would imply that

$$V_{13_1} = b[-D_1 + V_{12} + p_{21}(-D_1 + V_{12}) + p_{23_0}(-D_1 + V_{12}) + p_{23_1}(-D_1 + V_{12}) + p_{31}(U_1 - D_1 + V_{12}) + p_{32}V_{12}] \,, \tag{4}$$

$$V_{12} = b[V_{12} + p_{21}(U_1 - D_1 + V_{12}) + p_{23_0}V_{13_1} + p_{23_1}V_{12} + p_{31}V_{12} + p_{32}V_{12}] \,, \tag{5}$$

10 In equilibrium a, $U_1/D_1 > (9 - 6.3816\beta)/(1.5816\beta + 0.3456\beta^2) \equiv A_1(\beta)$, $U_2/D_2 > (27 - 36.9297\beta + 9.9297\beta^2)/(8.0703\beta - 11.0383\beta^2 + 2.9680\beta^3) \equiv A_2(\beta)$, and $U_3/D_3 > (1 - 0.80797\beta)/0.19203\beta \equiv A_3(\beta)$ are sufficient conditions for optimality of consumption, disposal, and production strategies, while $U_1/D_1 > (6-5\beta)/\beta \equiv B_1(\beta)$, $U_2/D_2 > (27-18\beta)/(3\beta^2+2\beta^3) \equiv B_2(\beta)$, and $U_3/D_3 > (6 - 5\beta)/\beta \equiv B_3(\beta)$ are the equivalent conditions in equilibrium b (see the proof for a derivation of these expressions).

where $b = \beta/3$. In order to understand better what is happening in the trade meetings taking place at each period of time, an explanation about how expression (4) is derived follows. Agent of type I holding good 3_1 has a payoff (optimal value) function equivalent to the sum of the following terms. (1) Whenever he meets another agent of type I no trade takes place, because there cannot be trade mutually beneficial between two agents of the same type. Consequently, agent I would find himself holding good 3_2 (a perished good) at the end of the period and so it is optimal for him to dispose of this good and produce a new good 2 at a cost equivalent to D_1. (2) With probability p_{21} he meets an agent of type II holding good 1. Trade does not take place because it goes against the strategy conjectured for an agent of type II. Therefore, an agent of type I disposes of the good and produces a new good. (3) With probabilities p_{23_0} and p_{23_1}, agent I meets an agent of type II holding good 3. Trade does not take place because both agents are holding the same type of good. (4) With probability p_{31} agent I meets an agent of type III who is holding good 1. In this case trade takes place and agent I consumes good 1 and produces a new good 2. (5) With probability p_{32}, an agent of type I meets an agent of type III holding good 2. In this case trade takes place because agent III wants to get good 3_1 in order to consume it and agent I prefers holding good 2 to holding a perished good (which involves having to dispose of it and produce a new good at a disutility cost). The explanation for expression (5) follows a similar argument.

From (4) and (5),

$$V_{13_1} - V_{12} = \frac{(p_{31} - p_{21})(U_1 - D_1) - 2D_1}{1 + bp_{23_0}} .$$

Substituting for the p's and rearranging, $V_{13_1} - V_{12} > 0$ iff $U_1/D_1 > 5.2301$. That is, for this region of the parameter space the strategy conjectured for agents of type I is the best response given the strategies played by other agents. In similar terms it can be shown that

$$V_{21} - V_{23_1} = b[p_{12}U_2 + (1 + p_{13_1})(V_{21} + D_2 - V_{23_0})] > 0 .$$

This last expression is always positive because the first term of the sum is obviously positive and so is the second term (recall that it has been conjectured that it is never optimal for an agent to dispose of a good not perished to produce a new good).

Finally, for agents of type III, $V_{31} - V_{32} = 0$, so $w_3 = 0$ also satisfies the equilibrium condition.

We should find the space of the parameters for which the strategies for consumption, disposal, and production are optimal. This is equivalent to finding for which values of the parameters the inequalities of Condition 3 in the definition of equilibrium hold. In order to do that, we have to obtain values for the V's as function of the parameters U_i, D_i, and β. This will be done by substituting the values for the p's in the system formed by Eqs. (4) and (5) (analogously for types of agents other than type I) and solving the system for the V's. This gives

$$V_{12} = \frac{U_1(1.5816\beta + 0.3456\beta^2) - D_1(1.5816\beta + 1.0368\beta^2)}{9 - 7.9632\beta - 1.0368\beta^2},$$

$$V_{13_1} = \frac{U_1(3\beta - 1.0728\beta^2) - D_1(9\beta - 6.3816\beta^2)}{9 - 7.9632\beta - 1.0368\beta^2}.$$

It is a matter of simple algebra to derive the following necessary and sufficient conditions for (1), (2), and (3) in Condition 3 of the equilibrium definition to be satisfied,

$$U_1 - D_1 + V_{12} > V_{13_1} \quad \text{iff} \quad \frac{U_1}{D_1} > \frac{9 - 15.3816\beta + 6.3816\beta^2}{9 - 9.3816\beta + 0.3816\beta^2}, \tag{1}$$

$$V_{13_1} > -D_1 + V_{12} \quad \text{iff} \quad \frac{U_1}{D_1} > \frac{-9 + 15.3816\beta - 6.3816\beta^2}{1.4184\beta - 1.4184\beta^2}, \tag{2}$$

$$-D_1 + V_{12} > 0 \quad \text{iff} \quad \frac{U_1}{D_1} > \frac{9 - 6.3816\beta}{1.5816\beta + 0.3456\beta^2}. \tag{3}$$

It is easy to check that, when $U_1/D_1 > 5.2301$, (1) holds for all values of β ($0 < \beta < 1$). This does not happen with (2) and (3), although (2) always holds when (3) does. Following exactly the same procedure for agents of type II and III, it is possible to derive the other constraints to be satisfied by the parameters in equilibrium to make sure that the strategies being used by the agents for consumption, production, and disposal are optimal.

In order to show that there is another equilibrium in the model in which good 1 emerges as the only medium of exchange, the same previous procedure would have to be repeated, now for the strategy vector $w = (0, 1, 0)$. It is a matter of simple algebra (available from the author upon request) to repeat the same steps as before to show that $V_{12} \geq V_{13_1}$ iff $U_1/D_1 \leq 5$, and that $V_{21} > V_{23_1}$, and $V_{31} = V_{32}$ for all values of the parameters. Equally, to make sure that equilibrium condition 3 is satisfied in equilibrium b, we need the rest of the constraints on the parameters stated in footnote 8.

Finally, to show that no other equilibria exist in the model, it is just a matter of repeating the procedure for the rest of strategy vectors and checking that the equilibrium conditions are not satisfied for all three types of agents. Details are not provided for the sake of brevity, but are available upon request. □

Focusing the analysis on the parameters U_1 and D_1, Proposition 1 shows that different single equilibria exist for different regions of the parameter space in this model. Also, there are values of the parameters U_1 and D_1 for which there is no pure-strategy equilibrium, specifically when $5.2301 \geq U_1/D_1 > 5$. Proposition 2 proves that, when mixed strategies are considered, there exists a steady-state equilibrium for all parameters U_1 and D_1 of the economy. This is done by constructing a mixed-strategy equilibrium which naturally connects equilibria a and b in Proposition 1, in a similar fashion to what was done in proposition 2 in Cuadras-Morató (1994a).

Proposition 2: In this model, when mixed strategies are taken into consideration and for some values of β, there is a steady-state equilibrium for all values of the parameters U_1 and D_1.

For a proof, see appendix.

This proposition nicely fills the gap in Proposition 1, where there was a region of the parameter space formed by U_1 and D_1 for which no pure-strategy equilibrium could be found. Proposition 2 ensures that there is a steady-state equilibrium in which exchange takes place and commodity money emerges for all the values of the parameters U_1 and D_1 of the economy (for chosen values of β).[11]

The following lines are intended to provide with an intuition of the results described in Propositions 1 and 2. In equilibrium, it is always optimal for agents of type II and III to play respectively the strategies "use good 1 as money" and "do not use good 2 as money." This simply means that agent II always finds it optimal to use good 1 (a commodity that he neither produces nor consumes) as a medium of exchange.

---

11 Proposition 2 also ensures that there is an exchange equilibrium for almost all values of the parameters U_i and D_i. The only exception would be when $1.8 > U_2/D_2$, as can be seen from Fig. 1b. In that case, neither equilibrium a, nor equilibrium b exist.

Equally, agent III always holds the good he produces until he can exchange it for his consumption good and uses no medium of exchange to carry out his trade. An agent of type I, however, will find it optimal to use a perishable commodity as commodity money only if $p_{31}(U_1 - D_1) - 2D_1 > p_{21}(U_1 - D_1)$, that is when the expected utility of holding good 3 is greater than the expected utility of holding good 2. The expected utility of holding good 3 is the level of utility obtained from consuming good 1 plus producing a new good $(U_1 - D_1)$ times the probability of being matched with an agent of type III holding good 1 (p_{31}) (in which case exchange will take place), taking into account the additional cost of being left after the random matching with a useless good that has to be thrown away and replaced at an expected cost of $2D_1$. In other words, agent I will find it optimal to use good 3 as medium of exchange when the liquidity advantage of doing so, $(p_{31} - p_{21})(U_1 - D_1)$, is greater than the expected costs of accepting a perishable good $2D_1$.

Finally, the results reported in this section are robust to changes in the assumptions about the information available to agents. In particular, the same type of equilibria in which perishable goods are used as media of exchange exists when we assume that agents have imperfect information about the age of perishable goods. Also, it can be shown that the value of holding money is decreasing over time when the medium of exchange is a perishable good (this is in contrast with many other search-theoretical models of money where the value of money is constant).[12]

4 Conclusions

Although it has been included in the catalogue of necessary characteristics of money many times, we have shown that durability is not an indispensable feature for an object to be used as medium of exchange. This is so because, in our model, money has a strategic nature. To a large extent, what determines which good appears as a medium of exchange are the extrinsic beliefs of agents about acceptability of goods, more than the intrinsic qualities of those goods. To put it in Einzig's words (1966, p. 323), "provided that a currency is freely acceptable its limited durability need not necessarily disqualify it, since its recipients may assume that they may pass it on before it deteriorates; and holders can always consume it or turn it over if they feel that it is approaching the limit of its durability." As a result of this, we have found equilibria in which perishable goods may be used as media of exchange, even

12 See a more extensive version of this paper (Cuadras-Morató, 1994b) for a detailed derivation of these results.

when other, perfectly durable, goods are available in the economy. Interestingly, one of the characteristics of these equilibria is that the value of holding a perishable good which performs the role of medium of exchange decays over time. Intuitively, this is what inflation is about: the decreasing value of money. Nevertheless, we are well aware that to analyze inflation properly a very different model which includes fiat money and endogenous price formation is required.[13] Obviously, a perishable fiat object with no intrinsic utility would not circulate at all, but an alternative setup can help to overcome this difficulty. For instance, V. Li (1995) analyzes inflation in a search model with fiat money by introducing a government that can tax (confiscate) the money holdings of randomly chosen agents with an exogenous given probability.

Appendix

Computation of Steady-state Distribution of Inventories

Strategy vector $w = (1, 1, 0)$, together with the assumed matching technology, generates a Markov process characterized as follows. For agents of type I, the distribution of inventories can be defined as the vector $p_1 = (p_{12}, p_{13_1})$, and the matrix of transition probabilities, Π_1, as follows

$$\Pi_1 = \begin{pmatrix} \dfrac{p_{12} + p_{13_1} + p_{21} + p_{23_1} + p_{31} + p_{32}}{3} & \dfrac{p_{23_0}}{0} \end{pmatrix} .$$

The resulting steady-state distribution of inventories p_1^* (remember, condition for steady state is $p_1^* \Pi_1 = p_1^*$) is characterized by the equation $p_{12}p_{23_0} = 3(1 - p_{12})$.

Following the same procedure for agents of type II and III, we would get the following system of equations,

$$p_{12}p_{23_0} = 3(1 - p_{12}), \quad (1 - p_{21})p_{31} = p_{21}(1 + p_{12} - p_{31}) ,$$
$$3(1 - p_{21} - p_{23_0}) = p_{23_0}(2 - p_{12}), \quad p_{31} = 1 .$$

The solution of this system of nonlinear equations gives us the steady-state distribution of inventories in the economy, which is what we need

13 Trejos and Wright (1995) and Shi (1995) are the most relevant contributions to the analysis of prices and inflation in the context of search-theoretical models of money.

to proceed with the proof of the proposition, $p = (0.8967, 0.3456, 0.5272, 1)$.

Proof of Proposition 2

Let r_i be the probability that agents of type i play the strategy $w_i = 1$ $(0 \leq r_i \leq 1)$ and let $r = (r_1, r_2, r_3)$. With mixed strategies, the assumption of no trade when agents are indifferent between holding their good or the good held by their trading partners will be modified and it will be assumed that agents may randomize between trade and no trade whenever they are indifferent between two goods. Then, best-response mixed strategies will be characterized as follows:

$$r_i \in \begin{cases} \{0\} & \text{if } V_{i\,i+1} > V_{i\,i+2}, \\ [0, 1] & \text{if } V_{i\,i+1} = V_{i\,i+2}, \\ \{1\} & \text{if } V_{i\,i+1} < V_{i\,i+2} \end{cases}$$

(note that in this particular model for $i+1, i+2 = 3$, the notation concerning the payoff functions will only be complete including the superscript $s = 1$).

In order to construct a mixed-strategy equilibrium that connects the pure-strategy equilibrium found in Proposition 1, the following strategy vector is conjectured: $r_1 \in [0, 1]$, $r_2 = 1$, and $r_3 = 0$. This vector of strategies, together with the matching technology, generates a Markov process, the steady-state probability distribution of which is equivalent to the steady-state distribution of inventories in our economy. It can be shown that, in this particular case, the steady-state distribution of inventories will be given by the following system of equations:

$$r_1 p_{12} p_{23_0} = 3(1 - p_{12}), \quad 3(1 - p_{21} - p_{23_0}) = p_{23_0}(2 - r_1 p_{12}) ,$$
$$1 - p_{21} = p_{21} p_{12}, \quad p_{31} = 1 .$$

In order to simplify notation, let $p_{12} = x$. The resolution of the previous system implies finding the roots of the third-order equation $r_1 x^3 - (5 + r_1)x^2 - r_1 x + 5 = 0$. It can be shown easily that the discriminant of this equation is positive, and consequently, it has three different real roots. It can also be proved that one of these roots has a value between zero and one. However, it is not possible to give a general expression for x as a function of r_1 (using Cardano's method) because it leads to calculations that require the cube root of an imaginary number, the so-called irreducible case of the cubic. Nevertheless, it is not difficult

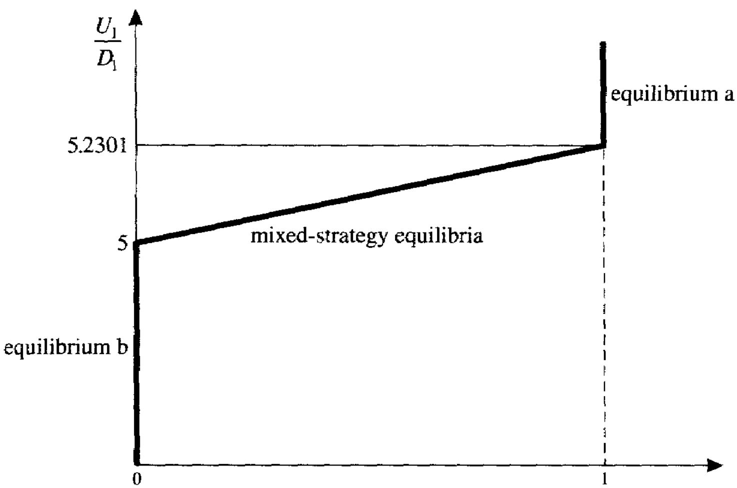

Fig. 2

to compute values of x between zero and one for the different values of r_1 by using simple numerical methods. An illustration of this follows.

As an illustration, let r_1 be equal to 0.5. Solving the previous third-order equation by simple numerical methods, the value $p_{12} = 0.9490$ will be obtained, and substituting in the system of equations the following vector p represents the steady-state inventory distribution, $p = (0.9490, 0.5131, 0.3224, 1)$. Computing the payoff functions for an agent of type I, and given the equilibrium incentive-compatibility constraint, we have the following condition,

$$V_{13_1} - V_{12} = b[-r_1 p_{23_0}(V_{23_1} - V_{12} + (p_{31} - p_{21})(U_1 - D_1) - 2D_1] = 0 .$$

Substituting for the p's and rearranging, it can be shown that the previous expression only holds iff $U_1/D_1 = 5.1076$. Following a similar procedure, we can show that there is a continuum of points in the parameter space for which a mixed strategy for agents of type I with different values of r_1 between zero and one satisfies the equilibrium condition (for instance, for $r_1 = 0.25$ the equilibrium condition is satisfied iff $U_1/D_1 = 5.0518$). Figure 2 maps the set of best responses for agents of type I (values of r_1) with the values of the ratio U_1/D_1, given the strategies of agents of type II and III, $w_2 = 1$ and $w_3 = 0$.

Showing that the conjectured strategy is best response for agents of type II and III involves repeating exactly the same argument as in Proposition 1, to show that $V_{21} > V_{23}$, and $V_{31} = V_{32}$. Again, it is not difficult to prove that there is a region of the parameter space for which this type of equilibria exists (and for which the strategies for consumption, disposal, and production are optimal). This, together with the results of Proposition 1 completes the proof. □

Acknowledgements

I wish to thank Irasema Alonso and Pierre de Trenqualye for many helpful conversations and ideas. I am also grateful to Ramon Faulí-Oller, Gianni de Fraja, Ramon Marimon, Peter Simmons, Randall Wright, and two anonymous referees. All errors are my own.

References

Aiyagari, S. R., and Wallace, N. (1991): "Existence of Steady States with Positive Consumption in the Kiyotaki–Wright Model." *Review of Economic Studies* 58: 901–916.

Burdett, K., Coles, M., Kiyotaki, N., and Wright, R. (1995): "Buyers and Sellers: Should I Stay or Should I Go?" *American Economic Review* 85: 281–286.

Cuadras-Morató, X. (1994a): "Commodity Money in the Presence of Goods of Heterogenous Quality." *Economic Theory* 4: 579–591.

—— (1994b): "Perishable Medium of Exchange (Can Ice Cream Be Money?)" Economics Working Papers 80, Universitat Pompeu Fabra, Barcelona.

Cuadras-Morató, X., and Wright, R. (1997): "On Money as a Medium of Exchange when Goods Vary by Supply and Demand." *Macroeconomic Dynamics* (forthcoming).

Einzig, P. (1966): *Primitive Money*. Oxford: Pergamon Press.

Harris, M. (1979): "Expectations and Money in a Dynamic Exchange Model." *Econometrica* 47: 1403–1419.

Iwai, K. (1988): "The Evolution of Money: a Search-theoretic Foundation of Monetary Economics." CARESS Working Paper 88-03, University of Pennsylvania, Philadelphia.

Jevons, S. (1875): *Money and the Mechanism of Exchange*. London: Henry S. King and Co.

Jones, R. (1976): "The Origin and Development of Media of Exchange." *Journal of Political Economy* 84: 757–775.

King, R. G., and Plosser, C. I. (1986): "Money as the Mechanism of Exchange." *Journal of Monetary Economics* 17: 93–115.

Kiyotaki, N., and Wright, R. (1989): "On Money as a Medium of Exchange." *Journal of Political Economy* 97: 927–954.

—— (1991): "A Contribution to the Pure Theory of Money." *Journal of Economic Theory* 53: 215–235.

—— (1992): "Acceptability, Means of Payment, and Media of Exchange." *Quarterly Review Federal Reserve Bank of Minneapolis* Summer: 18–21.

Li, V. (1995): "The Optimal Taxation of Fiat Money in Search Equilibrium." *International Economic Review* 36: 927–942.

Li, Y. (1995): "Commodity Money under Private Information." *Journal of Monetary Economics* 36: 573–592.

Lucas, R. E. (1980): "Equilibrium in a Pure Currency Economy." In *Models of Monetary Economies*, edited by R. Kareken and N. Wallace. Minneapolis: Federal Reserve Bank of Minneapolis.

Oh, S. (1989): "A Theory of a Generally Acceptable Medium of Exchange and Barter." *Journal of Monetary Economics* 23: 101–119.

Shi, S. (1995): "Money and Prices: a Model of Search and Bargaining." *Journal of Economic Theory* 67: 467–496.

Townsend, R. (1980): "Models of Money with Spatially Separated Agents." In *Models of Monetary Economies*, edited by R. Kareken and N. Wallace. Minneapolis: Federal Reserve Bank of Minneapolis.

Trejos, A., and Wright, R. (1995): "Search, Bargaining, Money and Prices." *Journal of Political Economy* 103: 118–141.

Wallace, N. (1980): "The Overlapping Generations Model of Fiat Money." In *Models of Monetary Economies*, edited by R. Kareken and N. Wallace. Minneapolis: Federal Reserve Bank of Minneapolis.

Address of author: Xavier Cuadras-Morató, Department of Economics, Universitat Pompeu Fabra, Trias Fargas 25–27, E-08005 Barcelona.