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Search, money, and inflation under private information

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Abstract

I study monetary exchange and inflation when buyers have private information about their willingness to pay for certain goods. Introducing imperfect information in the Lagos–Wright [A unified framework for monetary theory and policy analysis, J. Polit. Economy 113(3) (2005) 463–484] economy shows that the existence of monetary equilibrium is a more robust feature of the environment. In general, my model has a monetary steady state in which only a proportion of the agents hold money. Agents who do not hold money cannot participate in trade in the decentralized market. The proportion of agents holding money is endogenous and depends (negatively) on the level of expected inflation. As in Lagos and Wright's model, in equilibrium there is a positive welfare cost of expected inflation, but the origins of this cost are very different.

JEL classification: D83; E31; E41

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1. Introduction

A significant portion of everyday market transactions in modern economies is executed using money. Furthermore, in most of these transactions the seller does not have a precise sense of how much the buyer values the commodity being traded. In view of these two fundamental facts, it is natural to ask: does it matter for explaining why people use money that the willingness to pay of buyers is, in general, private information? This paper answers this question in the context of the now standard search theory of money. And the answer is that it matters. First, under private information monetary exchange is a more robust feature of the environment. In other words, agents with heterogeneous unobservable tastes are more likely to use money for transactions. Second,

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the fact that monetary exchange depends on the existence of private information has implications for how inflation influences the aggregate welfare of society.

Most existing search models of monetary economies, in pursuit of tractability, disregard the private information issue addressed in this paper. Recently, new developments in the study of monetary policy and inflation (see [12]) provide a more suitable framework to deal with unobservable taste shocks. I exploit this possibility and study monetary exchange and inflation in a model where heterogeneity and information play a crucial role.

In the early contributions to the monetary-search literature, the divisibility of money was difficult to handle because the models usually generated complicated dynamics of the distribution of money holdings. To avoid this problem, most of the work was done assuming indivisibility of money and limits on the amount of money that agents could carry. While this assumption increases the tractability of the models, it renders them unsuitable for the study of inflation. The paper of Lagos and Wright [12] breaks with this trend in the literature by proposing an alternative set of assumptions. [1] In particular, they assume that agents can hold any positive amount of divisible money, but they introduce a subperiod to the model where agents can trade, in a centralized walrasian market, goods for which they have quasilinear preferences. This new combination of assumptions makes the distribution of money holdings (endogenously) degenerate and hence makes policy analysis tractable. One crucial feature of the Lagos–Wright model is that in the interesting cases of the possible (steady state) monetary equilibria, the buyer in the decentralized market, when matched with an appropriate seller, must have some bargaining power that allows her to obtain a positive surplus from the match. When this is the case, the share of the surplus obtained by the buyer is positively associated with her money holdings and this extra payoff provides the incentives necessary for a positive demand for money even when, purely as an asset, money would have an insufficient real rate of return.

This feature of the Lagos–Wright model is shared with the previous money-search literature. In particular, if sellers have the power to propose prices and quantities and buyers can only decide whether or not to trade, most search models of money do not have a monetary equilibrium (see [5]). This result follows from the fact that, by the time trading takes place, the buyer cannot adjust her money holdings and hence the sellers can extract all the surplus from trade. This is an instance of the classic hold-up problem in contract theory. One possibility to get away from the hold-up problem is to assume that sellers can commit to post prices ex ante and buyers can choose which market-post to attend (as in [16]). This is the strategy taken, for example, in Rocheteau and Wright [19].

An alternative possibility is to modify the information structure in the model. In particular, the ability of the seller to extract all the surplus from a match is a direct consequence of the fact that sellers have perfect information about the buyers' willingness to pay. In this paper I use the Lagos–Wright framework to study the role of asymmetric information over buyers' tastes. I show that informational frictions can endogenously limit the ability of sellers to extract surplus from buyers, and in this way, contribute to sustain monetary exchange.

My model is in many aspects very similar to that of Lagos and Wright [12]. The only important difference is in the treatment of the decentralized market activity. First, I assume that agents are ex ante heterogeneous with respect to their preferences for the goods that are being traded in that market: some agents are more likely than others to find those goods very appealing. Second, upon entering a match, the buyer receives a preference shock that determines how much she

[1] There are other modifications of the standard model that have been used to deal with this problem. See, for example, [21]. Lagos and Wright [11] provide a careful discussion of the different approaches.

actually likes the particular good being offered in that match. [2] This shock is private information to the buyer and creates an additional level of (ex post) heterogeneity among agents. The ex ante heterogeneity is related to the *likelihood* of receiving a "favorable" preference shock after entering a match. Ex post, once agents are in a match, some will find out that they like very much the good being offered in the specific match. In such case, the agent will obtain a positive surplus from the match.

The third feature of my model that differs from the Lagos–Wright environment is that the seller has all the bargaining power in the decentralized market. Even though this is the case, the combination of ex post heterogeneity and private information makes a subgroup of the agents willing to hold money. Agents who are more likely to have a high marginal utility of consuming the goods sold in the decentralized market will carry positive money balances. By using those holdings of money when they enter a suitable match, private information will allow such agents to extract a positive surplus from trade.

I show that there exists a monetary equilibrium where a proportion of agents hold a positive amount of money. Yet, as in Lagos and Wright's model, the distribution of money holdings is very simple and easy to track in equilibrium. In particular, the distribution only has two mass points (zero and a positive number). [3] More interestingly, not only is the positive mass-point endogenous (the intensive margin), but also the mass that the distribution puts in each point will depend on equilibrium outcomes (the extensive margin). For example, the level of expected inflation in the economy will determine the proportion of agents that participate as potential buyers in the decentralized market and the amount of money that those agents take to that market. Since higher inflation rates discourage the use of money, fewer agents will decide to carry money to the decentralized market and even those that do, will be able to buy lower quantities of the goods. These two factors create an endogenous welfare cost of inflation in the model that is associated with different channels than those studied in Lagos and Wright [12].

This paper, then, achieves two main objectives. First, it provides a tractable analysis of the trade-frictions-based model of monetary exchange when there is private information about the willingness to pay by buyers. In this respect, it shows that inflation can result in welfare costs to society by reducing the participation of agents in those trades that are typically conducted through monetary exchange. Second, the paper shows that the existence of a monetary equilibrium in this kind of model is a more robust feature than suggested by the previous literature. In particular, under private information monetary exchanges still take place even if the sellers have full power to set prices and quantities.

Williamson and Wright [26] were the first to study some of the potential effects of private information in a random matching model of money. Their environment is, however, very different than mine. They study an economy where there is no problem of absence of double coincidence of wants. Agents may choose to use money to facilitate trade because there is private information about the quality of the goods being bartered. Several papers have followed this tradition (see [1] and the references therein). Berentsen and Rocheteau [1] provide the most recent treatment of this problem using Shi's [21] framework. In contrast to most of this literature (see [22] for a

[2] This is the kind of preference shocks used by Curtis and Wright [5] to study (ex ante) price posting in the Trejos and Wright [23] environment. See also [6,7].

[3] Purely general equilibrium forces eliminate any price dispersion and all agents carrying money hold exactly the same amount and pay the same prices. This feature of the equilibrium significantly increases tractability and stands in contrast with some of the results in [5].

notable exception), in my model money is useful to circumvent an absence of double coincidence of wants problem, like in the traditional random matching models of money [9].

In recent, independent work, Faig and Jerez [7] study private information (heterogeneous) buyers' valuations of goods in the competitive search framework of Rocheteau and Wright [19]. [4] They allow the seller to post and commit to a non-linear pricing scheme ex ante and, those buyers who do not hold enough money to make any one of the possible required payments involved in the pricing scheme, cannot participate in a trade. In my model, no such commitment is possible. This feature is the main driver of the differences on our results. While I allow for the possibility of non-linear prices, this pricing scheme is never observed in equilibrium. In Faig and Jerez [7], on the other hand, equilibrium non-linear pricing is what generates a precautionary demand for money that determines the welfare implications of positive inflation.

An important feature of the equilibrium in my model is the existence of an endogenous extensive margin. There are other papers in the literature that incorporate extensive margin effects to the Lagos–Wright framework. Rocheteau and Wright [19], for example, assume that agents can choose whether (or not) to become sellers. Also, they assume that the probability of getting matched in the decentralized market is a function of the number of sellers (relative to buyers) participating in that market (see also [7,10]). In this paper, no such assumptions are being made. Instead, the endogenous decision not to carry positive money holdings to the decentralized market determines the number of buyers in that market, and hence the movements in the extensive margin.

The paper is organized as follows. Section 2 provides the description of the environment, characterizes the monetary equilibrium in the model, and discusses the effects of positive expected inflation on equilibrium outcomes. Section 3 provides some quantitative estimates of the welfare cost of inflation in a calibrated version of the model and Section 4 concludes.

2. The model

The model is a modified version of that in Lagos and Wright [12]. Time is discrete and there is a continuum of infinitely lived agents with unit mass. Agents discount the future according to the discount factor $\beta \in (0, 1)$. Each period is divided into two subperiods.

In the first subperiod, agents interact in a decentralized market where trade is anonymous and agents get matched in pairs. There are three possible situations in a particular meeting between any two agents, j and k: either agent j likes the goods that agent k can produce, or agent k likes the goods that agent j can produce; or neither of the agents like what the other can produce. Each of the two possibilities for a single coincidence of wants meeting happens with probability σ and the lack of coincidence of wants happens with probability $(1 - 2\sigma)$. If agent j likes what agent k can produce, I will call j the (potential) buyer and k the seller. The buyer's utility is given by $\varepsilon_i u(q)$, where q is the quantity consumed and ε_i can take one of two values, ε_H or ε_L, with $\varepsilon_H > \varepsilon_L$. The seller's cost of production is given by $c(q)$ where q is the quantity produced. The value of ε_i is specific to the match and it is private information to the buyer. I will call agents with preference parameter ε_i in a particular match as agents *subtype* i, with $i = H, L$.

Agents are also heterogeneous with respect to *the probability* of being subtype i. I will say that an agent is of *type* j if, upon entering a match as a buyer, the probability of her preferences being $\varepsilon_H u(q)$ is equal to $\xi_j \in [0, 1]$. Types are also private information. Let us assume that there

[4] Faig and Jerez [6] study price posting under private information in Shi's [21] environment using a central clearing mechanism to settle payments. This mechanism allows them to abstract from monetary issues.

are $K \geqslant 1$ types with $\xi_1 < \xi_2 < \cdots < \xi_K$. For simplicity we assume that all types are, a priori, equally likely; that is, in the economy there is a measure $1/K$ of agents of each type.

In the second subperiod, agents interact in a centralized market and produce and consume a "general" good. Let $U(X)$ be the utility from consuming a quantity X of the general good. These goods can be produced one-to-one with labor, from which agents experience linear disutility.

I maintain the same technical assumptions as Lagos and Wright [12]. That is, u, c, and U are twice continuously differentiable, $u(0) = c(0) = 0$, $u'>0$, $c'>0$, $u'' < 0$, $c'' \geqslant 0$, $U'>0$, and $U'' \leqslant 0$. Also, there exists $q_i^* \in (0, \infty)$ such that $\varepsilon_i u'(q_i^*) = c'(q_i^*)$ for $i = H, L$, and there exist $X^* \in (0, \infty)$ such that $U'(X^*) = 1$ and $U(X^*) > X^*$.

Finally, in this environment there is also an intrinsically useless, perfectly divisible and storable asset that will be called money. The stock of money in period $t = 0, 1, \ldots$ is given by M_t.

We start the analysis of equilibrium by studying the optimal price–quantity pair offered by a seller that is already in a match with a potential buyer. We then use this information to determine the decisions, in the centralized market, of agents that anticipate such optimal behavior by sellers in the decentralized market. In particular, we study the demand for money of the different agents in the economy. Finally, we describe a general equilibrium of the economy and we show how equilibrium variables depend on the level of (expected) inflation.

2.1. The sellers' problem

Upon entering a match, the buyer becomes one of the two possible subtypes, H or L. Then the seller makes a take-it-or-leave-it offer to the buyer without knowing the buyer's subtype. [5] The seller may want to propose an offer that voluntarily "separates" the two subtypes, that is, the seller may want to set two price–quantity pairs such that a buyer chooses to buy one quantity at the given price if she is subtype H and the other quantity at the other price if she is subtype L. [6] Hence, the seller's offer in this case is a price schedule $\{(q_H, d_H), (q_L, d_L)\}$, where q_i is the quantity offered and d_i the monetary payment required to buy that quantity.

To determine his best offer, the seller needs to have an estimate of the likelihood that the particular agent he is matched with is of subtype H. Let $p = \Pr[\varepsilon_i = \varepsilon_H]$ be such an estimate. In an equilibrium, this probability p is endogenous and could be a function of the amount of money m that the buyer carries into the match. In this section we will solve the seller's problem for all arbitrary combinations of p and m. An equilibrium of the complete model is characterized in the next section. Given p and m, the seller chooses a price schedule to maximize his expected surplus from the match. This problem is similar to a standard private information problem where the seller chooses prices and quantities to maximize expected surplus subject to the buyer's participation constraints and incentive compatibility constraints (see [15]). There is, however, one important difference. The buyer enters the match with a given quantity of money balances m and hence the

[5] Here, for simplicity, we will be assuming that the seller can observe the amount of money that the buyer has. In Appendix B, we consider the case of unobservable money holdings and show that the same results obtain in that case.

[6] Camera and Delacroix [2] study a random matching non-monetary economy with private information over valuations of traded goods. Goods are indivisible in their model and there is only one good per match. This rules out any non-linear pricing scheme like the ones studied here. However, under one of the trading mechanism studied by Camera and Delacroix [2], the seller separates different-type buyers by making a combination of sequential offers. This possibility is not studied here.

seller can only demand from the buyer a monetary payment that is at most that amount.[7] Both buyers and sellers take as given the value of money in the economy, ϕ. The seller's problem is then,[8]

$$\max_{(q_H,q_L,d_H,d_L)} p\left[-c\left(q_H\right)+\phi d_H\right]+(1-p)\left[-c\left(q_L\right)+\phi d_L\right] \tag{1}$$

s.t.

$$\begin{aligned}
&q_H, q_L \geqslant 0, \quad d_H \leqslant m, \quad d_L \leqslant m,\\
&\varepsilon_H u(q_H)-\phi d_H \geqslant 0, \quad \varepsilon_L u(q_L)-\phi d_L \geqslant 0,\\[1ex]
&\varepsilon_H u(q_H)-\phi d_H \geqslant \varepsilon_H u(q_L)-\phi d_L,\\
&\varepsilon_L u(q_L)-\phi d_L \geqslant \varepsilon_L u(q_H)-\phi d_H.
\end{aligned}$$

The first set of constraints are associated with feasibility. The second set of constraints are participation constraints for H and L buyers and the last two constraints are incentive compatibility constraints, also for H and L buyers, respectively.

Lemma 2.1. *The solution to problem* (1) *satisfies*:

(1) $\varepsilon_H u(q_H)-\phi d_H=\varepsilon_H u(q_L)-\phi d_L$,
(2) $q_H \geqslant q_L$, $d_H \geqslant d_L$, *and*
(3) $\varepsilon_L u(q_L)-\phi d_L=0$.

These results are standard in the private information literature. (Proofs are provided in Appendix A.) The first result tells us that the incentive compatibility condition for subtype H agents must be binding in the solution to the problem. Otherwise the seller could increase his expected payoff by lowering q_H without violating any of the constraints in the problem. The second result says that subtype H agents will be offered a quantity that is at least as big as the one being offered to subtype L agents. Finally, the last result tells us that subtype L agents will get zero surplus from trade. If this would not be the case, the seller could sell smaller quantities for the same payment while still having the subtype L agent participating in the trade. This move would clearly increase the seller's surplus.

Let $z \equiv \phi m$ and $v(q_H, q_L) \equiv \varepsilon_L u(q_L)+\varepsilon_H\left[u\left(q_H\right)-u\left(q_L\right)\right]$. Then, we can use the equalities (1) and (3) in Lemma 2.1 to rewrite the seller's problem as follows:

$$\max_{(q_H,q_L)} p\left[-c\left(q_H\right)+v(q_H,q_L)\right]+(1-p)\left[-c\left(q_L\right)+\varepsilon_L u(q_L)\right] \tag{2}$$

s.t.

$$v(q_H, q_L) \leqslant z \quad \text{and} \quad q_H \geqslant q_L \geqslant 0.$$

[7] Che and Gale [3] study the problem of a risk-neutral seller facing (potentially) budget-constrained buyers with *linear* (i.e., risk-neutral) heterogeneous valuations over the good being offered. They consider the case where willingness to pay (valuation) and ability to pay (budget) are private information. While similar issues arise in their paper, risk-neutrality makes their analysis very different from mine.

[8] The payoff functions in this problem are a result of the environment. In Section 2.2 it will become clear why in this environment the payoff for sellers and buyers in a match is linear in money balances which all agents value equally.

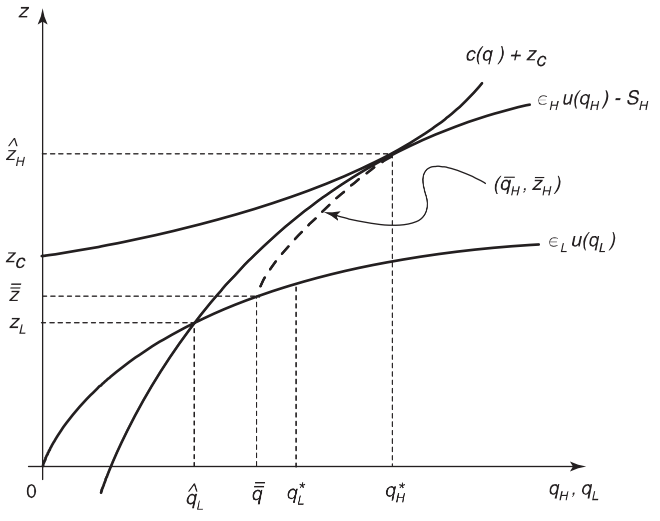

Fig. 1.

Before establishing the properties of the solution to this problem, let us note some preliminary results. The first thing to note is that if $z = 0$ the solution is trivial: all quantities in the solution are zero. Hence, for the rest of this section assume that $z>0$. The second thing to note is that when $z>0$ the solution to problem (2) must have $q_H>0$.[9] Finally, it is important to realize that the problem is general enough to handle the case in which the seller prefers to trade only with sub-type H buyers. Since it is always feasible for the seller to set the quantities q_L and z_L to zero, the problem does allow for the possibility that the seller would prefer to propose a contract that only results in trading with buyers H.[10]

I now characterize the solution to problem (2) when $z>0$. Fig. 1 is a useful reference to understand the solution to this problem. It plots iso-surplus curves for buyers H and L and an iso-cost curve for the seller. The curve labeled $\varepsilon_L u(q_L)$ is the iso-surplus curve for a subtype L buyer obtaining zero surplus from the trade (that is, such that equality (3) in Lemma 2.1 holds). The curve labeled $\varepsilon_H u\,(q_H) - S_H$ is the iso-surplus curve for a subtype H buyer obtaining a constant surplus value equal to S_H. Finally, the remaining (solid) curve is the iso-cost for the seller obtaining a surplus equal to z_C. In the figure, we have also plotted two threshold values for real balances, $\bar{\bar{z}}$ and $\widehat{z}_H$, that will be formally defined below.

The two threshold values $\bar{\bar{z}}$ and $\widehat{z}_H$ delimitate three relevant regions. First, when money holdings are not binding (that is, when $z\geqslant\widehat{z}_H$ in Fig. 1); second, when money holdings are binding but subtype H buyers would buy more quantity of the good than subtype L buyers would (that is, when $z \in \left(\bar{\bar{z}}, \widehat{z}_H\right)$ in Fig. 1); and third, when both H and L buyers would buy the same amount of the good being offered in the match (that is, when $z\leqslant\bar{\bar{z}}$ in Fig. 1).

[9] Setting $q_L = 0$ and q_H small (such that $\varepsilon_H u(q_H)\leqslant z$) but positive gives a positive value for the objective without violating any of the constraints.

[10] Note that the seller cannot provide a contract that selects only subtype L buyers. This is the case because any contract that satisfies the participation constraint for agents subtype L, also satisfies the participation constraint for agents subtype H.

Define $(\widehat{q}_H, \widehat{q}_L)$ as the solution of problem (2) when we ignore the constraint $v(q_H, q_L) \leqslant z$. Note that, in such case, the solution is not a function of z. Then, there exist values of z such that $v(\widehat{q}_H, \widehat{q}_L) \leqslant z$ and the solution to the seller's problem is indeed $(\widehat{q}_H, \widehat{q}_L)$. Define $\widehat{z}_H = v(\widehat{q}_H, \widehat{q}_L)$ as the relevant threshold on values of z (see Fig. 1).

Lemma 2.2. *When $z \geqslant \widehat{z}_H$ the solution $(\widehat{q}_H, \widehat{q}_L)$ to problem* (2) *is given by the following expressions*:

(1) *If $p\varepsilon_H \geqslant \varepsilon_L$ then*

$$\widehat{q}_H = q_H^*, \quad \widehat{q}_L = 0; \text{ and}$$

(2) *if $p\varepsilon_H < \varepsilon_L$ then*

$$\widehat{q}_H = q_H^*,$$

$$c'(\widehat{q}_L) = \frac{1 - p\dfrac{\varepsilon_H}{\varepsilon_L}}{1-p} \varepsilon_L u'(\widehat{q}_L).$$

Recall that q_i^* is such that $\varepsilon_i u'(q_i^*) = c'(q_i^*)$, for $i = H, L$. Going back to Fig. 1, then, we have drawn the iso-surplus for subtype H such that it is tangent to the iso-cost for the seller at the efficient quantity q_H^*. Now note that since $1 - p\frac{\varepsilon_H}{\varepsilon_L} < 1 - p$, we have that $\widehat{q}_L < q_L^* < q_H^* = \widehat{q}_H$. Also note that we can express the solution for $\widehat{q}_L$ as a function of p, which will be determined in equilibrium (in Section 2.2). Define $\widehat{S}_H \equiv \varepsilon_H u(\widehat{q}_H) - \widehat{z}_H$ as the surplus of subtype H agents. Then, we have the following important result:

Lemma 2.3. *If $\widehat{z}_H \leqslant z$ then the equilibrium surplus $\widehat{S}_H$ is a function of p and we denote it $\widehat{S}_H(p)$. Furthermore, if $p\varepsilon_H > \varepsilon_L$ then $\widehat{S}_H(p) = 0$ and if $p\varepsilon_H < \varepsilon_L$ then $\widehat{S}_H(p) > 0$ and*

$$\frac{d\widehat{S}_H(p)}{dp} < 0.$$

The lemma shows that the informational rent earned by H buyers is decreasing in the probability that the seller assigns to the possibility of facing an H buyer (that is, p). When the seller considers more likely to be facing an H buyer, he adjusts the offered bundles so as to extract more surplus from sales to H buyers, at the expense of extracting less surplus from sales to L buyers. In Fig. 1, when p increases, the quantity $\widehat{q}_L$ moves closer to the origin, moving up the relevant iso-surplus curve for H buyers (i.e., the curve labeled $\varepsilon_H u(q_H) - S_H$ that intersects the curve $\varepsilon_L u(q_L)$ at the new quantity $\widehat{q}_L$). The change in the bundles offered, then, implies a decrease in the surplus obtained by H buyers; specifically, a decrease in $\widehat{S}_H$.

Note also that when $p\varepsilon_H < \varepsilon_L$ we have that

$$\frac{d\widehat{q}_L}{dp} < 0, \quad \frac{d\widehat{z}_H}{dp} > 0 \quad \text{and} \quad \frac{d\widehat{z}_L}{dp} < 0,$$

where $\widehat{z}_L = \varepsilon_L u(\widehat{q}_L)$ is the amount of real balances paid by sub-type L agents to buy the quantity $\widehat{q}_L$.

Suppose now that $\widehat{z}_H > z$ so that $(\widehat{q}_H, \widehat{q}_L)$ can no longer be the solution to the seller's problem. In particular, sub-type H agents will spend all their money in the trade. However, sub-type L agents may or may not spend all their money in the trade. Consider first the cases when they do not. Let us call this solution $(\overline{q}_H, \overline{q}_L, \overline{z}_H, \overline{z}_L)$. Clearly, this solution would depend on the level of z. The dashed curve in Fig. 1 plots different combinations of $\overline{q}_H$ and $\overline{z}_H$ that correspond to different values of $z \in \left(\overline{\overline{z}}, \widehat{z}_H\right)$.

In this case, it is easy to see that $\overline{z}_L < \overline{z}_H = z$. Then, from the first-order conditions of problem (2) we have that the values of $(\overline{q}_H, \overline{q}_L)$ in an interior solution satisfy the following system of equations:

$$\begin{aligned} \frac{pc'(\overline{q}_H)}{\varepsilon_H u'(\overline{q}_H)} &= f(\overline{q}_L, p) \\ v(\overline{q}_H, \overline{q}_L) &= z \end{aligned} \tag{3}$$

where $f(q_L, p) \equiv (1-p)[\varepsilon_L u'(q_L) - c'(q_L)]/(\varepsilon_H - \varepsilon_L)u'(q_L)$ and $\overline{q}_H > \overline{q}_L$. Clearly, the solution to this system of equations depends on the values of both z and p. Let us denote it by $\left(\overline{q}_H(z,p), \overline{q}_L(z,p)\right)$.

Before proceeding, we need to consider an important especial case. For some values of z and p the seller may prefer to set $\overline{q}_L = 0$. Whether this situation can occur depends on the value that the function f takes in the neighborhood of $\overline{q}_L = 0$. It is not hard to show that for $q_L < q_L^*$ the function $f(q_L, p)$ is decreasing in q_L and p and its limit when $q_L \to 0$ is given by:

$$f_{LIM} \equiv \frac{(1-p)\,\varepsilon_L}{\varepsilon_H - \varepsilon_L}.$$

If $p\varepsilon_H > \varepsilon_L$, then we have that $f_{LIM} < p$ and for values of z close to $\widehat{z}_H$ the seller will rather set $\overline{q}_L = 0$ and not sell to sub-type L buyers. In fact, we can exactly determine the threshold of z for which, when $p\varepsilon_H > \varepsilon_L$, any value of z greater than such threshold implies that the seller will set $\overline{q}_L = 0$. For this, define q_H^T as the value of q that solves the following equation:

$$\frac{pc'\left(q_H^T\right)}{\varepsilon_H u'\left(q_H^T\right)} = f_{LIM}.$$

Note that when $p\varepsilon_H > \varepsilon_L$ we have that $q_H^T < q_H^*$. Define $z_T = \varepsilon_H u(q_H^T)$. Then, if $p\varepsilon_H > \varepsilon_L$ and $z > z_T$ we have that $\overline{q}_L = 0$.

Fig. 2 illustrates this case. For a given value of p the figure plots the function $f(q; p)$ together with the left-hand side of Eq. (3). For a given value of z, the equation $v(\overline{q}_H, \overline{q}_L) = z$ implies an equilibrium distance between $\overline{q}_H$ and $\overline{q}_L$. Just as an example, plotted in the figure is a set of values $\overline{q}_H$ and $\overline{q}_L$ that solve Eq. (3) for a given value of z. Note also that equation $v(\overline{q}_H, \overline{q}_L) = z$ implies that the equilibrium distance between $\overline{q}_H$ and $\overline{q}_L$ is an increasing function of z. As z increases, $\overline{q}_H$ moves closer to q_H^* and $\overline{q}_L$ moves closer to zero. When $f_{LIM} < p$ there are (high enough) values of z such that $\overline{q}_H$ is greater than q_H^T and the optimal $\overline{q}_L$ is equal to zero.

Fig. 2 also illustrates the fact that, as the value of z gets smaller, the quantities sold to buyers of both sub-types converge to the same amount. We can define $\overline{\overline{z}}$ as the value of real balances that solves $\varepsilon_L u(\overline{q}_L(\overline{\overline{z}}, p)) = \overline{\overline{z}}$. Since we have that $\overline{q}_H(\overline{\overline{z}}, p) = \overline{q}_L(\overline{\overline{z}}, p) \equiv \overline{\overline{q}}(p)$ then $\overline{\overline{q}}(p)$ solves:

$$c'(\overline{\overline{q}}(p)) = \frac{1-p}{1-p\dfrac{\varepsilon_L}{\varepsilon_H}}\varepsilon_L u'\left(\overline{\overline{q}}(p)\right).$$

It is clear from this expression that $\overline{\overline{q}}(p) < q_L^* < q_H^*$. Also, it is straightforward to show that $\overline{\overline{q}}(p)$ is a decreasing function of p (and so is $\overline{\overline{z}}(p)$, of course).

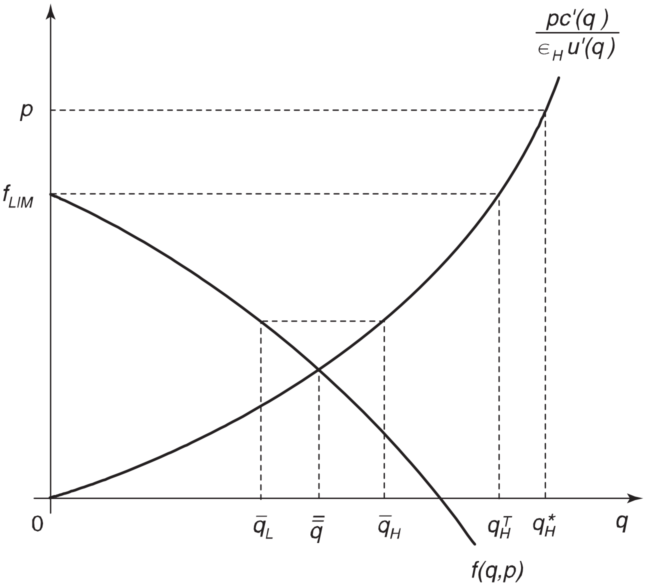

Fig. 2.

From the definition of z_T we have that $0 < \bar{\bar{z}} < z_T$. Then, there always exist values of $z>0$ such that $\overline{q}_L(z, p)>0$. This result is important because it implies that there are always values of z for which the surplus for sub-type H agents, given one such value of z, is positive. The analysis in the next section uses this result.

We now summarize the above discussion in the following lemma:

Lemma 2.4. *When* $\bar{\bar{z}} < z < \widehat{z}_H$ *we have that* $\overline{z}_L < \overline{z}_H = z$ *and the solution* $(\overline{q}_H, \overline{q}_L)$ *to problem* (2) *satisfy the following conditions*:

(1) *if* $p\varepsilon_H>\varepsilon_L$ *and* $z>z_T$ *then*

$$\varepsilon_H u\left(\overline{q}_H\right) = z \quad \textit{and} \quad \overline{q}_L = 0,$$

(2) *if* $p\varepsilon_H>\varepsilon_L$ *and* $z < z_T$ *or* $p\varepsilon_H < \varepsilon_L$ *then we have that* $\overline{q}_L>0$ *and* $(\overline{q}_H, \overline{q}_L)$ *solve the system*

$$\frac{pc'(\overline{q}_H)}{\varepsilon_H u'(\overline{q}_H)} = f(\overline{q}_L, p),$$
$$v(\overline{q}_H, \overline{q}_L) = z.$$

Note that $(\overline{q}_H, \overline{q}_L) \to (\widehat{q}_H, \widehat{q}_L)$ as $z \to \widehat{z}_H$. The next lemma provides some further characterization of this solution.

Lemma 2.5. *The solution* $(\overline{q}_H, \overline{q}_L)$ *is a function of* p *and* z *and satisfies*:

(1) $\widehat{q}_L(p) \leqslant \overline{q}_L(z, p) < q_L^*$,
(2) $\overline{q}_H(z, p) < q_H^*$, *and*
(3) *when* $\overline{q}_L(z, p)>0$, *we have that* $\frac{d\overline{q}_H}{dp} < 0$, $\frac{d\overline{q}_H}{dz}>0$, $\frac{d\overline{q}_L}{dp} < 0$, $\frac{d\overline{q}_L}{dz} < 0$.

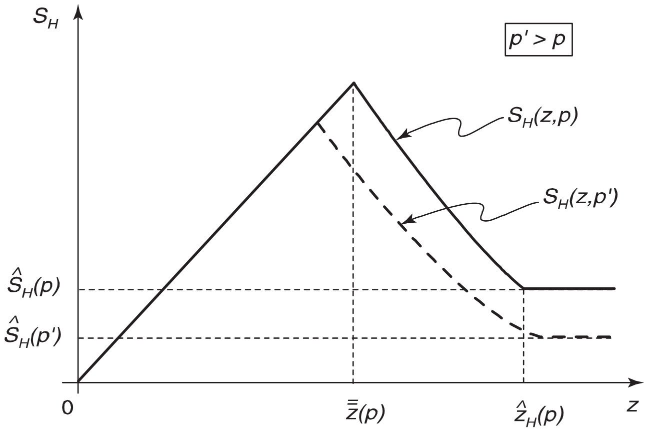

Fig. 3.

In this case, the surplus obtained by agents subtype H is not only a function of p but also a function of z and we denote it by $\overline{S}_H(z, p)$. In particular, it follows directly from the previous lemma that:

Corollary 2.1. *When $\overline{\overline{z}} < z < \widehat{z}_H$, the surplus of subtype H buyers is $\overline{S}_H(z, p) \geqslant \widehat{S}_H(p)$. Furthermore, when $\overline{q}_L(z, p) > 0$ we have that $\overline{S}_H(z, p) > \widehat{S}_H(p)$ and*:

$$\frac{d\overline{S}_H(z, p)}{dp} < 0, \quad \frac{d\overline{S}_H(z, p)}{dz} < 0.$$

For any value of $z \leqslant \overline{\overline{z}}$ the solution to problem (2) has $z_H = z_L = z$ and $q_H = q_L = q(z)$ where $q(z)$ satisfies

$$\varepsilon_L u(q(z)) - z = 0.$$

Finally note that for all $z \leqslant \overline{\overline{z}}$, the surplus from trade obtained by agents subtype H is a linear, increasing function of z (see Fig. 3). The following proposition summarizes the previous findings.

Proposition 2.1. *Given a probability p that the buyer will be sub-type H and an amount of real balances z with which the buyer enters the match, the surplus for a buyer if she is sub-type L is zero and the surplus for a buyer if she is sub-type H is*:

$$S_H(z, p) = \begin{cases} \dfrac{\varepsilon_H - \varepsilon_L}{\varepsilon_L} z & \text{if } 0 \leqslant z \leqslant \overline{\overline{z}}(p), \\ (\varepsilon_H - \varepsilon_L)\, u(\overline{q}_L(z, p)) & \text{if } \overline{\overline{z}}(p) < z \leqslant z_T(p), \\ (\varepsilon_H - \varepsilon_L)\, u(\widehat{q}_L(p)) & \text{if } z > z_T(p). \end{cases} \tag{4}$$

Now, with this description of the outcomes in a prototypical match we are ready to study a (monetary) equilibrium of the overall economy.

2.2. Monetary equilibrium

Let the subindex +1 denote next-period values. We denote with π_{+1} the gross inflation rate between the current period and the next. Then, we have that $\pi_{+1} = \phi/\phi_{+1}$. In a monetary equilibrium of this economy, agents choose money holdings in the centralized market based on their anticipation of next period's interactions in the decentralized market. We assume perfect foresight, so agents know the value of π_{+1} when they make the decision on money holdings in the centralized market. Additionally, agents need to anticipate how their money holdings will be interpreted by a potential seller in the decentralized market. We denote with $\mu(\xi|z)$ the belief of the seller. These beliefs specify the probability that a seller assigns to facing a type ξ agent upon observing that the agent's money holdings are z. This conditional probabilities $\mu(\xi|z)$ are induced from a probability measure μ over the product of the set of different types and the set of possible values of $z>0$. [11]

To solve for the choice of money holdings in the centralized market, we first study the value that each type of agent would get from entering the decentralized market with an arbitrary amount z of real balances. The agent takes as given the sellers' belief probabilities $\mu(\xi|z)$, the distribution of (real) money holdings $F(\widetilde{z})$ in the economy, and the inflation rate π_{+1}. We write these three components as a state vector $s = (\mu(\xi|z), F(\widetilde{z}), \pi_{+1})$.

Let $V_j(z, s)$ be the value function for an agent type j that enters the decentralized market with z units of real balances in state s. Also let $W_j(z, s)$ be the value function for an agent entering the centralized market with z units of real balances in state s. We denote with $q_i(z, s)$ and $z_i(z, s)$, with $i = H, L$, the terms of trade faced by a buyer with z units of money in a match in the decentralized market. It is easy to see that the terms of trade will not depend on the real money holdings of the seller, so we do not introduce it in the notation. Then, we have that:

$$\begin{aligned} V_j(z, s) = {} & \sigma E_j \left\{\varepsilon_i u\left[q_i(z, s)\right] + W_j\left[z - z_i(z, s), s\right]\right\} \\ & + \sigma \int \sum_{k=1}^{K} E_k \left\{-c\left[q_i(\widetilde{z}, s)\right] + W_j\left[z + z_i(\widetilde{z}, s), s\right]\right\} \mu(\xi_k|\widetilde{z})\, dF(\widetilde{z}) \\ & + (1 - 2\sigma) W_j\,(z, s)\,, \end{aligned}$$

where E_k denotes the expectation over $i = H, L$, for the binomial probability distribution with parameter $\xi_k = \Pr\left[i = H\right]$, $k = 1, 2, \ldots, K$ (and similarly for E_j).

Denote by h the amount of work dedicated to the production of the "general" good in the centralized market. Then, the value function $W_j(z, s)$ satisfies:

$$W_j(z, s) = \max \left\{U(X) - h + \beta V_j(z_{+1}, s_{+1})\right\}$$

s.t.

$$X = h + z - \pi_{+1} z_{+1},$$

and an equilibrium law of motion for s that is taken as given by the individual. From this expression for $W_j(z, s)$ it is easy to see that in equilibrium X equals X^*, z_{+1} does not depend on z, and W_j is linear in z. Recall here that X^* is such that $U'(X^*) = 1$. [12] Note that the linearity of W_j implies that the net payoff from trading in the decentralized market is quasilinear for both the seller and

[11] It is easy to find a finite upper bound for the set of possible values of z which would be non-binding in equilibrium.

[12] It is possible to give conditions so that the natural non-negativity of h is non-binding in equilibrium. See [12].

the buyer. This simplification is what justifies the choice of quasilinear payoff functions in our study of the seller's problem in the previous section.

To solve the agent's problem we need to specify the terms of trade $q_i(z, s)$ and $z_i(z, s)$, $i = H, L$. For this purpose we use some of the insights that we obtained in the previous section as part of our characterization of the seller's problem. In the previous section, the seller's problem was solved for arbitrary combinations of z and p. Now, we restrict attention to only those combinations of z and p such that $p = \sum_{k=1}^{K} \xi_k \mu(\xi_k|z) \equiv p(z)$.

First, we know that agents with a low preference for the good in a particular match (i.e., agents with $\varepsilon_i = \varepsilon_L$) will get zero surplus from that match. [13] So, we only need to consider the surplus for agents sub-type H in the expression for $V_j(z, s)$. We also know that the terms of trade do not depend on the distribution of money holdings in the economy; nor on inflation. Hence, the surplus $S_H = \varepsilon_H u(q_H) - z_H$ is only a function of z and the belief function $\mu(\xi|z)$, or equivalently $p(z)$. We denote this surplus function as $S_H[z, p(z)]$. Note that the function $S_H(\cdot, \cdot)$ is the same as in the previous section. However, the surplus S_H as a function of z has to now take into account that when z changes, the beliefs of the seller may change according to $p(z)$.

With the definition of the surplus function $S_H[z, p(z)]$, we can now rewrite the value function $V_j(z, s)$ as follows:

$$V_j(z, s) = \sigma\xi_j S_H[z, p(z)] + v(s) + z + \max_{z_{+1}} \left\{-\pi_{+1} z_{+1} + \beta V_j(z_{+1}, s_{+1})\right\},$$

where $v(s) = U(X^*) - X^* + \sigma \int \sum E_k \{-c[q_i(\widetilde{z}, s)] + z_i(\widetilde{z}, s)\} \mu(\xi_k|\widetilde{z})\, dF(\widetilde{z})$. Because the optimal choice of z_{+2} does not depend on z_{+1}, we can find the optimal value of z_{+1} by solving the following problem:

$$\max_{z_{+1}} \left\{-\pi_{+1} z_{+1} + \beta z_{+1} + \beta\sigma\xi_j S_H\left[z_{+1}, p_{+1}(z_{+1})\right]\right\}. \tag{5}$$

The solution to this problem depends on the function $p_{+1}(z)$, which is an equilibrium object. However, some preliminary results can be established without knowledge of the function $p_{+1}(z)$. Since we are interested in studying monetary equilibria, we assume that $\pi_{+1} \geqslant \beta$ (see [11, Lemma 3]). Now, recall from the previous section that the function $S_H(z, p)$ was always less than or equal to $[(\varepsilon_H - \varepsilon_L)/\varepsilon_L]\, z$, for all z and p. Then, define the threshold value ξ^s as

$$\xi^s = \frac{\varepsilon_L}{\sigma(\varepsilon_H - \varepsilon_L)} \left(\frac{\pi}{\beta} - 1\right),$$

and let k^s be the threshold type such that for all $k \geqslant k^s$ we have that $\xi_k \geqslant \xi^s$ and for all $k < k^s$ we have that $\xi_k < \xi^s$. Since for all $k < k^s$ we have that

$$-(\pi - \beta)z + \beta\sigma\xi_k S_H[z, p(z)] \leqslant -(\pi - \beta)z + \beta\sigma\xi^s \frac{\varepsilon_H - \varepsilon_L}{\varepsilon_L} z = 0,$$

where we dropped the time sub-indexes just to simplify notation, then we have that all agents with $k < k^s$ must choose $z_k^* = 0$ in equilibrium, regardless of $p(z)$.

[13] One possibility for the seller would be to, for example, offer some surplus to subtype L agents, in an attempt to differentiate ("separate") agents' types ξ. However, implementing such a scheme would require some degree of commitment by the seller (combined with some form of directed search). If the seller cannot commit, as we assume here, then once the buyer is in a match, the seller's optimal contract is to give zero surplus to subtype L buyers.

In general, we will consider economies with a large number of types such that more than one type chooses positive money holdings in equilibrium. For this reason, in what follows we will concentrate on the case when $k^s \leqslant K - 1$. Of course, the other case is trivial.

When agents choose to hold positive amounts of money holdings they anticipate that they will participate in a (static) signaling game with a future potential seller in the decentralized market.[14] We study sequential equilibrium of this signaling game, which can be described as follows:

Definition 2.1. A sequential equilibrium of the buyer–seller signaling game is a pair $\left(\{z_k^*\}_{k=1}^K, \mu^*\right)$ such that for $k = 1, 2, \ldots, K$:

(1) z_k^* solves problem (5) given the beliefs μ^* (and hence the function $p^*(z)$); and
(2) the beliefs function μ^* satisfies

$$\mu^*(\xi_k|z) = \frac{\gamma^*(z|\xi_k)}{\sum_{j=1}^K \gamma^*(z|\xi_j)}$$

if the denominator is positive, where $\gamma^*(z|\xi_j) = 1$ if $z_j^* = z$ and zero otherwise.

Note that the best response for the seller is implicit in our definition of the surplus function S_H, which in turn implies that $\mu(\xi|z)$ is well defined (over the possible ξ) for all z (even those not played in equilibrium). This last property constitutes, of course, the sequentiality requirement. See, for example, Mailath et al. [14] for details.

As in many other signaling games, in general, there may be multiple sequential equilibria. In principle, sequential equilibria can be of two kinds, separating or "semi-pooling." We call the pooling equilibrium semi-pooling because there is always the chance that some low-type agents are below the threshold k^s and hence choose zero money holdings in equilibrium. The following lemma deals with the existence of separating equilibria.

Lemma 2.6. *There is no separating equilibrium of the buyer–seller signaling game.*

To understand this result it is useful to refer to Fig. 3. Note that, in a (complete) separating equilibrium, the lowest $\bar{\bar{z}}(\xi_k)$ is that corresponding to $k = K$, that is, the one corresponding to the type with the highest ξ. It is then easy to see that z_K^* must be equal to $\bar{\bar{z}}(\xi_K)$ because, otherwise, if z_K^* were different from $\bar{\bar{z}}(\xi_K)$ then type K agents would want to deviate to $\bar{\bar{z}}(\xi_K)$. Note also that all the types choosing positive money holdings would not choose real balances lower than $\bar{\bar{z}}(\xi_K)$. So, it must be that higher money holdings signal lower values of ξ. But in that case, due to the linearity of the surplus function for values of real balances lower than $\bar{\bar{z}}(\xi)$, agents type K would want to imitate the lower types and linearly increase their surplus. The proof in the appendix exploits this logic and generalizes it to show that there cannot be an equilibrium where agents of different types choose different (positive) levels of money holdings which (partially) signal their types (i.e., a separating equilibrium).

However, there may be many semi-pooling equilibria. Basically, the concept of sequential equilibrium places no restriction on the off-equilibrium beliefs of the seller (that is, the value

[14] Okumura [17] studies signaling in a search model of money in which goods and money are indivisible. In Okumura's model there are high and low quality producers, and quality is not perfectly observable. Money holdings are observable. As it is easier for high quality producers to accumulate money, how much money a seller holds can signal the quality of the goods sold by such seller. This type of effect is not present in my model.

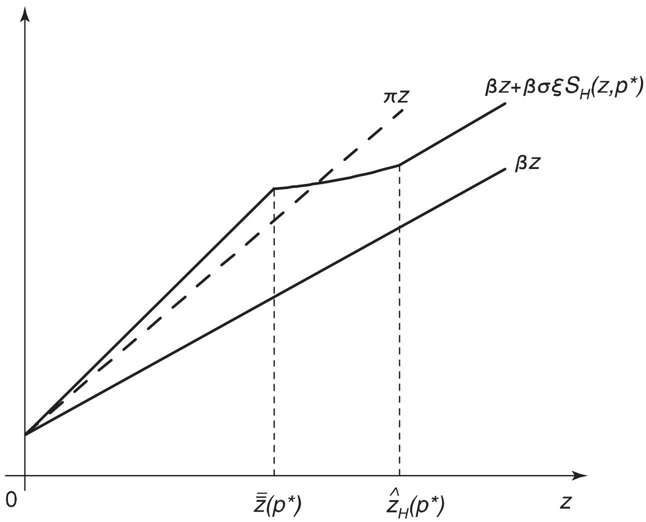

Fig. 4.

of $\mu^*(\xi_k|z)$ for $z \neq z_k^*$ for all k). As a consequence, for most cases, there exist a continuum of belief functions that produce a continuum of different semi-pooling equilibrium outcomes. As it turns out, though, all but one of these equilibria can be regarded as originated in unreasonable off-equilibrium beliefs; or in other words, all but one of these equilibria can be refined away. To formalize the idea of refinement we use the concept of *undefeated equilibrium* introduced by Mailath et al. [14].

In the next lemma, we describe a particular semi-pooling equilibrium which we call the "best" semi-pooling equilibrium because, in this equilibrium, buyers get the highest equilibrium surplus (comparing with other semi-pooling equilibria). After that, we show that the best semi-pooling equilibrium is the only undefeated equilibrium. [15]

Lemma 2.7. *(The best semi-pooling equilibrium.) The signaling game has a semi-pooling equilibrium with* $z_k^* = \bar{\bar{z}}(p^*)$ *for* $k \geqslant k^s$ (*and zero otherwise*); *and, for all* $z>0$, *the equilibrium beliefs are* $\mu^*(\xi_k|z) = 1/(K - k^s + 1)$ *for* $k \geqslant k^s$ (*and zero otherwise*), *where*

$$p^* = \sum_{k=k^s}^{K} \xi_k \mu^*(\xi_k|z) = \frac{1}{K - k^s + 1} \sum_{k=k^s}^{K} \xi_k,$$

and $\bar{\bar{z}}(p^*)$ *maximizes* $S_H(z, p^*)$ (*see Proposition* 2.1).

To see why the lemma is true, note that under the proposed beliefs the problem of the seller is given by problem (2), where p is independent of z and equal to p^*. In such case, the surplus function $S_H(z, p^*)$ is given by expression (4) and the solution to problem (5) is either $\bar{\bar{z}}(p^*)$ or zero (see Fig. 4). In particular, agents with $\xi \geqslant \xi^s$ choose real balances $z = \bar{\bar{z}}(p^*)$ and agents

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[15] Assumption 3 in Mailath et al. [14], the "monotonicity" (single-crossing) condition, does not hold in this model. Hence, we cannot directly apply their theorems.

with $\xi < \xi^s$ choose $z = 0$. These choices, then, are consistent with the semi-pooling beliefs that were used to originate them. In other words, these beliefs support the semi-pooling equilibrium outcome that was proposed.

There are other possible semi-pooling equilibria. Consider, for example, an arbitrary value of the real money holdings $z^P > 0$ such that for all $k \geqslant k^s$ we have:

$$-(\pi - \beta)z^P + \beta\sigma\xi_k S_H(z^P, p^*) \geqslant -(\pi - \beta)\bar{\bar{z}}_K + \beta\sigma\xi_k S_H(\bar{\bar{z}}_K, \xi_K), \tag{6}$$

where $\bar{\bar{z}}_K$ maximizes $S_H(z, \xi_K)$. Then, the following belief function supports $z_k^* = z^P$ for $k \geqslant k^s$ (and zero otherwise) as a semi-pooling equilibrium outcome: for z^P let $\mu(\xi_k|z^P) = 1/(K - k^s + 1)$ for $k \geqslant k^s$ (and zero otherwise); and for $z > 0$ with $z \neq z^P$ let $\mu(\xi_k|z) = 1$ for $k = K$ (and zero otherwise).

The idea behind the claim that these other semi-pooling equilibria are based on unreasonable beliefs follows a forward induction argument, common in the refinement literature for signaling games, in which off-equilibrium messages are interpreted as signals (see [4]). Specifically, if the seller were to observe the off-equilibrium value of z given by $\bar{\bar{z}}(p^*)$ (different from z^P) and he were to believe that any type $k \geqslant k^s$ could be the deviator, then it would indeed be in the interest of all types $k \geqslant k^s$ to deviate to that other value of z (that is, to $\bar{\bar{z}}(p^*)$). Reasonable seller's beliefs, then, would put equal probability in all types $k \geqslant k^s$ upon observing $\bar{\bar{z}}(p^*)$. But in that case, the equilibrium where all agents type $k \geqslant k^s$ play z^P brakes down. The only equilibrium that survives this process is, of course, the best semi-pooling equilibrium. The following lemma formally establishes this result. [16]

Lemma 2.8. *The best semi-pooling equilibrium defeats* (*in the sense of Mailath et al.* [14]) *all other semi-pooling equilibria.*

For this reason, in the rest of the paper we will concentrate our attention on the best semi-pooling equilibrium of the buyer–seller signaling game. Note that in equilibrium the value of ξ^s is a continuous function of the parameters of the model, and more importantly, of the value of inflation, π. However, with a finite number of types, the value of p^* as a function of π would actually be a step function (with jump discontinuities at the points where ξ^s crosses the fixed values of ξ_k for different k's). To avoid cumbersome computations we will consider the limit case of a large number of types, which can be approximated by considering a continuum of types between 0 and 1 with agents uniformly distributed across types. In such case, the value of p^* is determine as follows:

$$p^* = \int_{\xi^s}^{1} \frac{\xi}{1 - \xi^s}\, d\xi = \frac{1 + \xi^s}{2}.$$

Having described the likely outcome of the strategic situation faced by potential buyers we are now ready to study the full monetary equilibrium of the economy. The following proposition provides a simple dynamical system that describes this monetary equilibrium.

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[16] It is easy to see that all these pooling equilibria survive the Intuitive Criterion proposed by Cho and Kreps [4]. The Mailath et al. refinement, while based on a similar logic than the intuitive criterion, has more desirable properties (see [14] for details).

Proposition 2.2. *A sequence* $\{\xi_t^s, p_t, q_t, z_t(\xi), \pi_t\}_{t=0}^{\infty}$ *satisfying the following equations describes a perfect foresight monetary equilibrium for this economy*:

$$\xi_t^s = \frac{\varepsilon_L}{\sigma(\varepsilon_H - \varepsilon_L)}\left(\frac{\pi_t}{\beta} - 1\right),$$

$$p_t = \frac{1+\xi_t^s}{2},$$

$$q_t = \overline{\overline{q}}(p_t), \text{ where } \overline{\overline{q}}(p_t) \text{ solves } c'\left[\overline{\overline{q}}(p_t)\right] = \frac{1-p_t}{1-p_t\frac{\varepsilon_L}{\varepsilon_H}}\varepsilon_L u'\left[\overline{\overline{q}}(p_t)\right],$$

$$z_t(\xi) = \begin{cases} \overline{\overline{z}}(p_t) & \text{if } \xi \geqslant \xi_t^s, \\ 0 & \text{if } \xi < \xi_t^s, \end{cases} \quad \text{where } \overline{\overline{z}}(p_t) = \varepsilon_L u\left[\overline{\overline{q}}(p_t)\right], \text{ and}$$

$$\pi_{t+1} = \frac{M_{t+1}}{M_t}\frac{\left(1-\xi_t^s\right)\overline{\overline{z}}(p_t)}{\left(1-\xi_{t+1}^s\right)\overline{\overline{z}}(p_{t+1})},$$

where the sequence $\{M_t\}_{t=0}^{\infty}$ *is such that* $\pi_t \geqslant \beta$ *and* $\xi_t^s < 1$ *for all* t.

It is interesting to note that the equilibrium distribution of money holdings at the beginning of time t has only two mass-points, $\overline{\overline{z}}(p_t)$ and 0, and that $\varphi_t = 1 - \xi_t^s$ is the mass of agents holding positive real money balances. Also note that even though the general problem for the seller allows for different prices and quantities to be offered with the objective of "separating" the subtypes of buyers, in equilibrium only one combination of price and quantity is offered and observed.[17]

Also note that Bayesian updating on the equilibrium path determines a relevant restriction on the equilibrium belief function $p(\pi)$. For this reason, rational expectations play an important role in this equilibrium. Given the predicted inflation rate π_{t+1}, agents anticipate that next-period sellers will use $p_{t+1} = \left[1 + \xi^s(\pi_{t+1})\right]/2$ as their estimate of the probability of facing a buyer with $\varepsilon = \varepsilon_H$. Knowing this, agents with $\xi > \xi^s(\pi_{t+1})$ decide to hold $\overline{\overline{z}}(p_{t+1})$ real balances to use in the decentralized market next period (this is the amount of real balances that allows them to maximize their expected surplus from an eventual match given that the sellers will be using p_{t+1} as their estimated probability of ε_H). But then, since actual inflation ϕ_t/ϕ_{t+1} depends on the choice of real balances $\overline{\overline{z}}(p_{t+1})$, indirectly it also depends on the value of the expected inflation and, in a perfect foresight equilibrium, the two values should be equalized.

One important feature of the equilibrium described in Proposition 2.2 is that not all agents in the economy carry money. Agents that do not carry money, then, do not participate as buyers in the decentralized market. This participation decision is what in the literature has been called the extensive margin. The reason why there is an extensive margin in this paper is purely endogenous to the model and originates in the properties of the trading contract offered under private information (there are no fixed costs driving the participation decision). This trait is in stark contrast with the previous literature where the extensive margin in monetary exchange is directly introduced by giving the agents an extra dimension of choice (see for example [7,10,19,21]).

Consider now the case when $M_t = M$ for all t. In this case, for β large enough, there exists a steady-state monetary equilibrium with no inflation that solves the following set of equations:

$$\xi^{SS} = \frac{\varepsilon_L}{\sigma(\varepsilon_H - \varepsilon_L)}\left(\frac{1}{\beta} - 1\right),$$

[17] In consequence, we also have that in equilibrium $v(s) = U(X^*) - X^* + \sigma\varphi\left(-c(\overline{\overline{q}}) + \overline{\overline{z}}\right)$, where $\varphi = 1 - \xi^s$.

$$p^{SS} = \frac{1+\xi^{SS}}{2},$$
$$c'(q^{SS}) = \frac{1-p^{SS}}{1-p^{SS}\frac{\varepsilon_L}{\varepsilon_H}}\varepsilon_L u'(q^{SS}),$$
$$z^{SS} = \varepsilon_L u(q^{SS}).$$

Obviously, in this equilibrium money is *neutral* since q^{SS} and ξ^{SS} do not depend on M. However, we show in the next subsection that in this economy money is not *superneutral*.

It is straightforward to obtain the following comparative statics results with respect to σ:

Lemma 2.9. *In a steady-state monetary equilibrium with no inflation we have that*: [18]

$$\text{(i)}\,\frac{\partial \xi^{SS}}{\partial \sigma} < 0, \quad \text{(ii)}\,\frac{\partial p^{SS}}{\partial \sigma} < 0, \quad \text{(iii)}\,\frac{\partial q^{SS}}{\partial \sigma} > 0 \quad \textit{and} \quad \text{(iv)}\,\frac{\partial z^{SS}}{\partial \sigma} > 0.$$

The lemma shows that less frictions in the decentralized market (i.e., high values of σ) result in a steady-state monetary equilibrium where (i) a higher proportion of agents hold money; (ii) it is less likely for a seller who has met a potential buyer to be facing a subtype H buyer; (iii) the quantities traded in a given match are higher; and (iv) the value of money is higher (or in other words, the price level is lower). The comparative statics with respect to β are the same.

Note that there is an *intensive margin effect* present in the model. In the case of the lemma, an increase in σ changes the quantities traded in a given match. This intensive margin effect happens because, when search frictions increase, agents take to the decentralized market lower individual money holdings ($\partial z^{SS}/\partial\sigma>0$). Once the agent enters the match with lower money holdings it is optimal for the seller to offer lower quantities.

What is important to note here is that agents carry lower money holdings anticipating that the seller will increase his estimate of the conditional probability of facing a subtype H agent (a composition effect, as less agents participate in the market). If the probability p^{SS} would remain unchanged, then the quantities traded would not change either. However, given that the agent anticipates that p^{SS} will change, she realizes that if she were to carry the same amount of money as before, the seller would offer a higher quantity (to subtypes H) at a higher price and would effectively lower the buyer's expected net surplus from trade.

In summary, in this model the intensive margin effect is a direct consequence of the existence of an extensive margin effect. In the next section we show that similar effects are present when the economy experiences changes in the inflation rate.

2.3. Changes in the money supply

Suppose now that the money supply is growing at a constant rate. Since in equilibrium not all agents hold money when they enter the decentralized market, we assume that monetary injections take place while agents are still in the centralized market. In this way, money gets redistributed among agents so that only those with high values of ξ exit the centralized market with money. [19]

[18] The proof involves just simple algebra and, hence, is omitted.

[19] Lagos and Wright [12] assume that the monetary injections happen after agents leave the centralized market. This alternative timing would put money in the hands of agents that chose not to carry money to the decentralized market, and would create distributional effects that we wish to abstract from in this paper. For a version of the Lagos–Wright framework where redistribution associated with monetary policy plays a crucial role, see [25].

We consider an equilibrium with a constant inflation rate; that is, $\phi_t/\phi_{t+1} = \pi$ for all t. Since the quantities traded are also constant, it is easy to show that in a monetary equilibrium inflation will equal the money growth rate. Then, in such an equilibrium the proportion of agents not holding money is given by:

$$\xi_\pi = \frac{\varepsilon_L}{\sigma(\varepsilon_H - \varepsilon_L)}\left(\frac{\pi}{\beta} - 1\right),$$

unless the right-hand side is greater than one in which case $\xi_\pi = 1$. In particular, for a high enough inflation rate the threshold ξ_π is unity, no agent chooses to hold money, and there is no monetary equilibrium. Now, using Proposition (2.2), we obtain the following lemma:

Lemma 2.10. *In the set of constant-inflation monetary equilibria indexed by* π, *if* $\xi_\pi < 1$, *then we have that*:[20]

$$\text{(i)}\,\frac{\partial \xi_\pi}{\partial \pi} > 0, \quad \text{(ii)}\,\frac{\partial p_\pi}{\partial \pi} > 0 \quad \textit{and} \quad \text{(iii)}\,\frac{\partial q_\pi}{\partial \pi} < 0.$$

The lemma tells us that in economies with higher levels of inflation, (i) fewer agents hold money, (ii) it is more likely for a seller who has found a potential buyer to be facing a subtype H buyer, and (iii) the quantities traded in a given match are lower.

Since the quantities produced and traded in the centralized market do not change with inflation, it is clear that higher levels of inflation imply lower equilibrium welfare. This is the case for two reasons. First, in economies with higher inflation fewer agents hold money in the decentralized market, decreasing the number of matches where trade takes place (the extensive margin effect). Second, since each agent holds less money in equilibrium, even in those matches where trade does take place the quantities traded are lower and hence the benefit from the match is reduced (the intensive margin effect).

Let us define the real interest rate in this economy as r and the nominal interest rate as i. Then, we have that in equilibrium $1 + r = 1/\beta$ and $1 + i = \pi/\beta$. The optimal monetary policy in this economy is to set the nominal interest rate i as low as possible, eventually to zero, as in the Friedman Rule. At that level, the threshold ξ_π equals zero and all agents hold money. The equilibrium quantity traded in a match in the decentralized market is given by $\overline{\overline{q}}(1/2) < q_H^*$, away from the constrained optimum. The Friedman rule, then, eliminates the inefficiency in the extensive margin but cannot eliminate the inefficiency in the intensive margin. This second inefficiency is a consequence of the power of sellers to extract surplus from buyers carrying extra units of money in a bilateral exchange.[21] This is an important feature of the present model that contrasts with the standard cash-in-advance model where the Friedman rule implements the social optimum.[22]

[20] The proof involves just simple algebra and, hence, is omitted.

[21] Aruoba, Rocheteau, and Waller [20] present a related result for the standard Lagos–Wright framework. See also [19].

[22] When $\varepsilon_H = \varepsilon_L$, there is a continuum of steady-state monetary equilibria under the Friedman rule. This feature is shared with the cash-in-advance economy (see [27, Proposition 1]). However, since the buyer has no incentives to carry any specific amount of real balances, it is still possible that the equilibrium would be inefficient, i.e., $q < q^*$.

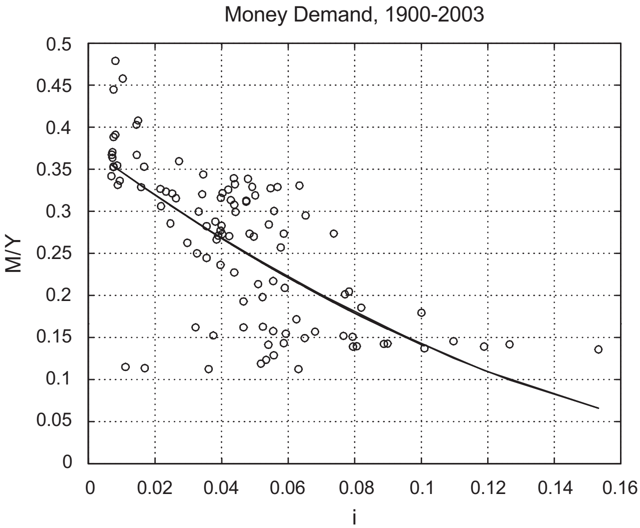

Fig. 5.

3. The welfare cost of inflation

In this section I use the model to provide some estimates of the welfare cost of inflation based on data for the US economy. In the calibration of parameter values, to facilitate comparisons, I follow as close as possible the strategy in Lagos and Wright [12]. Normalize $\varepsilon_H = 1$ and $\varepsilon_L = \varepsilon < 1$. The utility function for goods in the centralized market is $U(X) = B\log(X)$ and that for goods in the decentralized market is $u(q) = \left[(q+b)^{1-\eta} - b^{1-\eta}\right]/(1-\eta)$. Also assume that $c(q) = q$. From the equilibrium conditions in Proposition (2.2) we then have that:

$$\xi(i) = \frac{\varepsilon}{\sigma(1-\varepsilon)} i,$$

$$p(i) = \frac{1+\xi(i)}{2},$$

$$q(i) = \left[\frac{(1-p(i))\varepsilon}{1-\varepsilon p(i)}\right]^{\frac{1}{\eta}} - b,$$

and real money demand is $L(i) = \varepsilon u(q(i))(1-\xi(i))$. Total output per period, $Y(i)$, is the sum of the equilibrium output in the centralized market, B, and in the decentralized market, $\sigma(1-\xi(i))q(i)$. We consider the case when $b \approx 0$ and hence we have four free parameters: B, σ, ε, and η.

Using annual data for the nominal interest rate on commercial paper and the ratio of M_1 to nominal GDP, we choose parameter values so as to minimize the sum of square errors between the observed levels of the money demand and those predicted by the model. Fig. 5 shows the result for the 1900–2003 sample.

The model produces a reasonable fit. In the data there appears to be a lower bound in the level of monetization that the model seems to miss (especially at relatively high interest rates). Note that in the model, a sufficiently high, yet possibly observable, interest rate would make $\xi^s = 1$ and

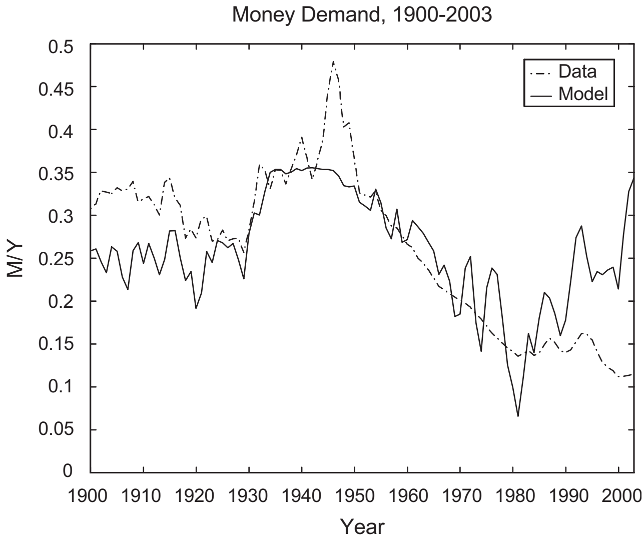

Fig. 6.

hence drive the demand for money to zero, something that seems at odds with the data. [23] Aside from this shortcoming, there is another important limitation of this kind of exercise. Specifically, the statistical relationship between money demand, output, and interest rates in the US has been quite unstable over the years (see, for example, [8]). Clearly, innovations like credit cards and other alternative means of payments have contributed significantly to this instability. Fig. 6 shows that the model systematically underestimates the demand for money early in the sample period and systematically overestimates it late in the period. [24]

The estimated parameter values are not very precisely identified. The figures were constructed with the set of parameter values that produce the best fit of the data in a search over a discrete grid of points. [25] However, there are several different combinations of parameter values that produce essentially the same fit. Different parameter values, however, imply different welfare costs of inflation. For this reason, I provide the welfare calculations for two combinations of parameter values (Set 1 and Set 2) that produce equivalent fits of the data (with coefficients of determination that differ in less than 0.001 from each other, see Table 1). [26]

[23] Here we have maintained the assumption that agents' types are uniformly distributed. A more flexible distribution of types could help to fit the data better. We kept the uniform distribution assumption for the sake of parsimony. Another alternative would be to modify the model to introduce a very high third value of ε and some agents with high probability of obtaining such a high level of utility from a match. In that case, these agents would be very reluctant to hold no money, even at relatively high interest rates. This modification could potentially produce a lower bound in the aggregate demand for money (for all potentially observable interest rates) which would help the model to fit the data in Fig. 5 better.

[24] The model generates larger short-term fluctuations than those observed in the data. This is a standard feature on this kind of exercise. See [13] for a discussion.

[25] The search was done with a grid of size 44 for each parameter in the interval [0,1]. The value of B does not have to be in the interval [0,1] but unrestricted searches show that to fit the data B has to be small (and certainly smaller than 1).

[26] For reference, the R^2 for a standard linear OLS regression of the data was 0.4455.

Table 1

Parameters	η	B	σ	ε	R^2
Set 1	0.289	0.089	0.289	0.467	0.4734
Set 2	0.267	0.133	0.400	0.556	0.4728

In equilibrium, all agents receive a real money transfer $T = \tau\phi_t M_t = \tau(1 - \xi(i))z(i)$ upon entering the centralized market. The equilibrium value for an agent of type $\xi \geqslant \xi(i)$ of being in the decentralized market at the start of the day (when the steady state nominal interest rate is i) is given by:

$$(1-\beta)V_\xi(i) = \sigma\xi S_H(i) - \xi(i)\tau z(i) + v(i), \tag{7}$$

where $v(i)=U(X^*)-X^*+\sigma(1-\xi(i))[-q(i)+\varepsilon u(q(i))]$, $X^*=B$, and $S_H(i) = (1-\varepsilon)\, u(q(i))$. The second term on the right-hand side of expression (7) is the cost (net of the money transfer) associated with carrying $z(i)$ units of (real) money balances in an inflationary environment; that is:

$$z(i) + T - \frac{\phi}{\phi_{+1}} z(i) = z(i) + \tau(1-\xi(i))z(i) - (1+\tau)z(i) = -\xi(i)\tau z(i).$$

Agents with $\xi < \xi(i)$ obtain only $v(i) + T$. To compute the steady-state welfare cost of inflation I use the average value of V across types ξ:

$$(1-\beta)\Omega(i) \equiv \int_0^{\xi(i)} \left[v(i) + (1-\xi(i))\tau z(i)\right] d\xi + \int_{\xi(i)}^1 (1-\beta)V_\xi(i)\, d\xi.$$

Note that inflation has a redistributive effect (agents not holding money get a positive net transfer).[27] However, due to the quasi-linearity properties of preferences, this redistribution has no real implications for total average welfare, which is given by:

$$(1-\beta)\Omega(i) = v(i) + \sigma\frac{\left[1-\xi(i)^2\right]}{2} S_H(i).$$

Define now the following alternative measure of welfare where the quantities consumed at the zero-interest-rate equilibrium are multiplied by the fraction Δ_F:

$$(1-\beta)\Omega_{\Delta_F}(0) = v(0) + B\log(\Delta_F) + \sigma\frac{1}{2}(1-\varepsilon)u(\Delta_F q(0)).$$

Then, I use as a proxy for the welfare cost of inflation the value of $1 - \Delta_F^i$, where Δ_F^i is such that $\Omega_{\Delta_F^i}(0) = \Omega(i)$. We can interpret this value as the amount of consumption agents would be willing to give up to have the nominal interest rate equal zero (the Friedman rule), rather than i. Similarly, I compute $1 - \Delta_0^i$, the amount of consumption agents would be willing to give to move from an economy with a nominal interest rate equal to i to one with a nominal interest rate equal to 3 %, which we interpret as implying zero inflation (with $\beta = 0.97$). For the calculations, I will

[27] One alternative strategy that avoids redistribution is to assume that newly printed money is transferred only to those agents holding the existing balances. This is the standard approach in overlapping generations' models of money (see for example [24]).

Table 2

Param.	$\zeta(0.04)$	$\chi(0.04)$	$1-\Delta_F^{0.04}$	$1-\Delta_0^{0.04}$	$\zeta(0.13)$	$\chi(0.13)$	$1-\Delta_F^{0.13}$	$1-\Delta_0^{0.13}$
Set 1	1.045	0.033	0.025	0.011	1.019	0.009	0.073	0.061
Set 2	1.057	0.050	0.029	0.013	1.024	0.013	0.084	0.072

use as the benchmark values of i the average level of the nominal interest rate during the period, $i = 0.04$, and a higher value, $i = 0.13$, which corresponds to a steady-state inflation of 10%. Table 2 shows the values of $1-\Delta_F^i$ and $1-\Delta_0^i$ for the two different combinations of parameters presented in Table 1.

Table 2 also shows the equilibrium values of the proportion of output traded in the decentralized market, $\chi(i)$, and the average mark-up, $\zeta(i)$, which can be calculated using the following two expressions:

$$\chi(i) = \frac{\sigma(1-\xi(i))q(i)}{Y(i)},$$
$$\zeta(i) = (1-\chi(i)) + \chi(i)\frac{(1-\xi(i))\varepsilon u(q(i))}{(1-\xi(i))q(i)}.$$

The value of $\zeta(i)$ is the weighted average of the ratio of price over marginal cost in the centralized market (equal to one) and in the decentralized market. The equilibrium values of $\chi(i)$ and $\zeta(i)$ give us a sense of how reasonable the parameter values are and also are helpful for comparison with the findings in Lagos and Wright [11,12]. For example, the value of the equilibrium mark-up tends to be relatively low compared with the value 1.1 usually used in calibration.

Note also that the welfare cost of inflation and the mark-up are positively associated with the size of the decentralized sector. The model fits the data best when the decentralized sector is relatively small (less than 5% of the economy) at the nominal interest rate of 4%. This number is similar to the one reported in Lagos and Wright [11]. For the lowest bargaining power of the buyer reported by Lagos and Wright (equal to 0.35) the size of the decentralized sector is around 5.8% of the economy.

Overall, the welfare cost of inflation implied by the model in this paper is relatively high (comparing with, for example, [13]) but the numbers are in the range of those obtained by Lagos and Wright [12] in their calibration. The main difference in terms of calibration is that in the Lagos–Wright model the bargaining power of buyers is a free parameter. In my model, sellers have all the bargaining power but the difference in willingness to pay among buyers (indexed by ε) is a free parameter.

In the Lagos–Wright model, the welfare cost of inflation is higher the lower the bargaining power of buyers (that is, the higher the hold-up problem). When buyers and sellers have the same bargaining power (which implies an average mark-up equal to 1.04) they estimate $1-\Delta_F^{0.13}$ to be 0.041 and $1-\Delta_0^{0.13}$ to be 0.032. When the bargaining power is calibrated to generate a mark-up equal to 1.1, the bargaining power of buyers decreases to 0.35 and the welfare cost estimates increase to 0.068 and 0.046, respectively. [28]

Rocheteau and Wright [18] further study the quantitative implications of the Lagos–Wright model by introducing an extensive margin effect based on the ability of agents to choose whether

[28] Table 2 shows that a similar effect is present in my model. Set 2 of parameter values implies a smaller information friction since $\varepsilon_L(=\varepsilon)$ is closer to $\varepsilon_H(=1)$. Then, for these parameter values the seller can exercise more of his hold-up power and hence the welfare cost of inflation is higher.

to be buyers or sellers in the decentralized market. Again, the bargaining power of the buyers is a key parameter in the calibrations and the existence of an extensive margin tends to increase the welfare cost of inflation. In particular, for the lowest level of the buyer's bargaining power that they report (equal to 0.20), the welfare cost estimates are $1 - \Delta_F^{0.13} = 0.101$ and $1 - \Delta_0^{0.13} = 0.074$.

In general, the welfare cost estimates presented in Table 2 are well within the range of estimates that result from the original Lagos–Wright framework. To the extent that both models are calibrated to match the same data, these common estimates make sense. The assumption in this paper is that sellers have all the bargaining power, which would tend to increase sellers' hold-up power, reduce money demand, and increase the welfare cost of inflation. However, the presence of private information limits the ability of sellers to exercise their hold-up power and, hence, justifies a higher money demand. In the end, money demand is similar in both model and the welfare estimates stay within the same range of values.

4. Concluding remarks

The economy studied in this paper reduces to the one in Lagos and Wright [12] when tastes are homogeneous (i.e., when $\varepsilon_H = \varepsilon_L$). However, since we are assuming that sellers have all the bargaining power, no monetary equilibria with positive nominal interest rates exist in that economy. The reason is that the seller can exercise in full his hold-up power, depriving potential buyers of any incentive to carry money into a match. When this is the case, there is no trade (nor consumption) in the decentralized market. If sellers could somehow commit not to extract the entire surplus from future matches, this would provide incentives for potential buyers to hold money. However, commitment is not possible in the model and, due to their ability to extract too much surplus from a potential match, the sellers are left with no surplus at all (because, to generate any surplus, buyers must carry money to the decentralized market).

Introducing unobservable heterogeneous tastes (i.e., the case of $\varepsilon_H \neq \varepsilon_L$ presented in this paper) limits the ability of the seller to extract surplus from the buyer and hence provides the necessary incentives for the buyer to carry money to the decentralized market. This modification, in turn, translates into some trading in that market and an increase in welfare. In conclusion then, aside from being realistic and tractable, the assumption of imperfect information makes existence of monetary equilibrium a more robust feature of the environment.

The model in this paper also generates an *endogenous* extensive margin on the participation of agents in monetary exchange. When inflation increases, agents that are unlikely to benefit from monetary exchange decide to stop participating in those trades for which money is essential. In this sense, the model captures the simple idea that when agents reduce their money holdings as a response to inflation they may become more exposed to the possibility of not being able to finalize certain beneficial transaction that need to be settled with money. The quantitative section of this paper suggests that the welfare consequences of this kind of effects may be fairly large.

Acknowledgments

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Appendix A. Proofs

This appendix provides the proofs for the lemmas and propositions in the paper.

Lemma 2.1. *The solution to problem* (1) *satisfies*:

(1) $\varepsilon_H u(q_H) - \phi d_H = \varepsilon_H u(q_L) - \phi d_L$,
(2) $q_H \geqslant q_L$, $d_H \geqslant d_L$, *and*
(3) $\varepsilon_L u(q_L) - \phi d_L = 0$.

Proof. (1) To prove that this equality holds in the solution, suppose it does not, that is, suppose that $\varepsilon_H u(q_H) - \phi d_H > \varepsilon_H u(q_L) - \phi d_L$. Note that $\varepsilon_H u(q_L) - \phi d_L \geqslant \varepsilon_L u(q_L) - \phi d_L (=$ when $q_L = 0)$. Since $\varepsilon_L u(q_L) - \phi d_L \geqslant 0$ we have that under the hypothesis $\varepsilon_H u(q_H) - \phi d_H > 0$. This in turn implies that $q_H > 0$. Now, by choosing a smaller q_H, the seller will increase the value of his objective function and still satisfy all the constraints. In particular, for small decreases in q_H the participation constraint of subtype H buyers will still be satisfied (because it holds with strict inequality under the hypothesis). Also, the incentive constraint for subtype H will still be satisfied and the incentive constraint of subtype L will actually be relaxed. Hence, a lower q_H is feasible and it is better for the seller. Hence the incentive constraint of a subtype H agent must be binding in the solution. (2) Plugging the equality in point (1) of the lemma in the incentive constraint of subtype L agents we get that $(\varepsilon_H - \varepsilon_L)u(q_H) \geqslant (\varepsilon_H - \varepsilon_L)u(q_L)$ and hence $q_H \geqslant q_L$. Point (1) then implies that $d_H \geqslant d_L$ holds. (3) To prove that the participation constraint of subtype L agents must be binding in the solution, suppose not. Suppose the solution has $\varepsilon_L u(q_L) - \phi d_L > 0$. This implies that $q_L > 0$. Combining this result with point (2) in the lemma we know that $q_H \geqslant q_L > 0$. Also note that when $q_L > 0$ we have that $\varepsilon_H u(q_H) - \phi d_H \geqslant \varepsilon_H u(q_L) - \phi d_L > \varepsilon_L u(q_L) - \phi d_L \geqslant 0$ which shows that $\varepsilon_H u(q_H) - \phi d_H > 0$. Then, the seller could offer an alternative contract decreasing q_H and q_L to, say q'_H and q'_L, in a manner such that the equality in point (1) of the lemma still holds. By doing so, the seller could increase the value of his objective function without violating any of the constraints: If q'_H and q'_L are sufficiently close to the initial q_H and q_L then the participation constraints would still hold (since under the hypothesis being tested both hold with strict inequality for the initial quantities). Furthermore, because q'_H and q'_L satisfy the incentive constraint for subtype H with equality, we have that $\varepsilon_L \left[u(q'_H) - u(q'_L)\right] < \varepsilon_H \left[u(q'_H) - u(q'_L)\right] = \phi(d_H - d_L)$ which implies that the incentive constraint for agents subtype L still holds. In other words, reducing the quantities sold to q'_H and q'_L is feasible and better for the seller, which shows that the initial q_H and q_L could not be the solution of the problem. □

Remark. When the solution has $q_H > q_L$ it must hold that $\varepsilon_L u(q_H) - \phi d_H < 0$. To see this, plug the equality in point (1) of the lemma into the equality of point (3) and sum and subtract $\varepsilon_L u(q_H)$. Then, we get $\varepsilon_L u(q_H) - \phi d_H + (\varepsilon_H - \varepsilon_L)\left[u(q_H) - u(q_L)\right] = 0$ which implies that if $q_H > q_L$ then $\varepsilon_L u(q_H) - \phi d_H < 0$ must hold. (see also Fig. 1.)

Lemma 2.2. *When $z \geqslant \widehat{z}_H$ the solution $(\widehat{q}_H, \widehat{q}_L)$ to problem* (2) *is given by the following expressions*:

(1) *If $p\varepsilon_H \geqslant \varepsilon_L$ then*

$$\widehat{q}_H = q_H^*, \ \widehat{q}_L = 0; \quad \text{and}$$

(2) *if $p\varepsilon_H < \varepsilon_L$ then*

$$\widehat{q}_H = q_H^*,$$
$$c'(\widehat{q}_L) = \frac{1 - p\frac{\varepsilon_H}{\varepsilon_L}}{1-p}\varepsilon_L u'(\widehat{q}_L). \tag{A.1}$$

Proof. Assume that the participation constraint for agent subtype H and the incentive constraint for agent subtype L are nonbinding. Then, the first-order conditions for an interior solution to problem (2) are:

$$q_H : p\left[-c'(q_H) + \varepsilon_H u'(q_H)\right] = 0,$$
$$q_L : -p(\varepsilon_H - \varepsilon_L)u'(q_L) + (1-p)\left[-c'(q_L) + \varepsilon_L u'(q_L)\right] = 0.$$

From the first condition we get that $\widehat{q}_H = q_H^*$. If $p\varepsilon_H \geqslant \varepsilon_L$ there is no $q_L > 0$ that solves the second equation and $\widehat{q}_L = 0$. If $p\varepsilon_H < \varepsilon_L$, reorganizing the second condition we obtain expression (A.1). The solution to this expression has $\widehat{q}_L > 0$. When $\widehat{q}_L > 0$ and the incentive constraint for agents subtype H holds it is straightforward to see that the participation constraint for subtype H agents is not binding. Finally, since $1 - p\frac{\varepsilon_H}{\varepsilon_L} < 1 - p$, we have that $\widehat{q}_L < q_L^* < q_H^* = \widehat{q}_H$ and from the remark above it follows that the incentive constraint for agent subtype L is also not binding. □

Lemma 2.3. *If $\widehat{z}_H \leqslant z$ then the equilibrium surplus $\widehat{S}_H$ is a function of p and we denote it $\widehat{S}_H(p)$. Furthermore*, *if $p\varepsilon_H > \varepsilon_L$ then $\widehat{S}_H(p) = 0$ and if $p\varepsilon_H < \varepsilon_L$ then $\widehat{S}_H(p) > 0$ and*

$$\frac{d\widehat{S}_H(p)}{dp} < 0.$$

Proof. Expression (A.1) defines an implicit function $\widehat{q}_L(p)$. It is then easy to check that $\widehat{q}_L$ is a decreasing function of p:

$$\frac{d\widehat{q}_L(p)}{dp} = \frac{(\varepsilon_L - \varepsilon_H - p\varepsilon_H)\, u'(\widehat{q}_L)}{(1-p)c''(\widehat{q}_L) - \left(1 - p\frac{\varepsilon_H}{\varepsilon_L}\right)\varepsilon_L u''(\widehat{q}_L)} < 0.$$

Using the third equality in Lemma 2.1 we have that $\widehat{S}_H(p) = (\varepsilon_H - \varepsilon_L)u(\widehat{q}_L(p))$. Hence:

$$\frac{d\widehat{S}_H(p)}{dp} = (\varepsilon_H - \varepsilon_L)u'(\widehat{q}_L(p))\frac{d\widehat{q}_L(p)}{dp} < 0. \qquad \square$$

Remark. The other endogenous variables' comparative statics with respect to p are:

$$\frac{d\widehat{z}_H}{dp} = (\varepsilon_L - \varepsilon_H)u'(\widehat{q}_L)\frac{d\widehat{q}_L(p)}{dp} > 0,$$

and

$$\frac{d\widehat{z}_2}{dp} = \varepsilon_L u'(\widehat{q}_L)\frac{d\widehat{q}_L(p)}{dp} < 0.$$

Lemma 2.5. *The solution* $(\overline{q}_H, \overline{q}_L)$ *is a function of p and z and satisfies*:

(1) $\widehat{q}_L(p) \leqslant \overline{q}_L(z, p) < q_L^*$,
(2) $\overline{q}_H(z, p) < q_H^*$, *and*
(3) *when* $\overline{q}_L(z, p) > 0$, *we have that* $\frac{d\overline{q}_H}{dp} < 0$, $\frac{d\overline{q}_H}{dz} > 0$, $\frac{d\overline{q}_L}{dp} < 0$, $\frac{d\overline{q}_L}{dz} < 0$.

Proof. When $\overline{q}_L = 0$ the results are obvious. When $\overline{q}_L > 0$ we have that $(\overline{q}_H, \overline{q}_L)$ must solve the following system of equations:

$$\frac{pc'(\overline{q}_H)}{\varepsilon_H u'(\overline{q}_H)} = f(\overline{q}_L, p),$$
$$v(\overline{q}_H, \overline{q}_L) = z,$$

where $f(q_L, p) \equiv (1-p)[\varepsilon_L u'(q_L) - c'(q_L)]/(\varepsilon_H - \varepsilon_L)u'(q_L)$ and $\overline{q}_H \geqslant \overline{q}_L$. Since from the first equation $f(\overline{q}_L, p) = pc'(\overline{q}_H)/\varepsilon_H u'(\overline{q}_H) > 0$ we have that by the definition of $f(\overline{q}_L, p)$ it must be the case that $\varepsilon_L u'(\overline{q}_L) - c'(\overline{q}_L) > 0$ which in turn implies that $\overline{q}_L(z, p) < q_L^*$ must hold. From the first-order conditions, and specifically the complimentary slackness condition for $z_H \leqslant z$, we have that when this constraint is binding $f(\overline{q}_L, p) < p$. Hence $\overline{q}_H < q_H^*$. Also, since $f(\widehat{q}_L, p) = p$ and f is decreasing in q_L it holds that $\widehat{q}_L \leqslant \overline{q}_L$. To proof point (3) of the lemma define the following two functions:

$$G(\overline{q}_H, \overline{q}_L, p) = f(\overline{q}_L, p)\varepsilon_H u'(\overline{q}_H) - pc'(\overline{q}_H) \equiv 0,$$
$$J(\overline{q}_H, \overline{q}_L, z) = \varepsilon_H u(\overline{q}_H) - z - (\varepsilon_H - \varepsilon_L)u(\overline{q}_L) \equiv 0.$$

Also let G_i and J_i be the partial derivatives of the functions G and J with respect to the ith argument. Then, the signs for the comparative statics can be found by solving the following (matrix) equation:

$$\begin{bmatrix} G_1 & G_2 \\ J_1 & J_2 \end{bmatrix} \begin{bmatrix} \frac{d\overline{q}_H}{dp} & \frac{d\overline{q}_H}{dz} \\ \frac{d\overline{q}_L}{dp} & \frac{d\overline{q}_L}{dz} \end{bmatrix} = \begin{bmatrix} -G_3 & 0 \\ 0 & 1 \end{bmatrix}.$$

Corollary 2.1. *When* $\overline{\overline{z}} < z < \widehat{z}_H$, *the surplus of subtype H buyers is* $\overline{S}_H(z, p) \geqslant \widehat{S}_H(p)$. *Furthermore, when* $\overline{q}_L(z, p) > 0$ *we have that* $\overline{S}_H(z, p) > \widehat{S}_H(p)$ *and*:

$$\frac{d\overline{S}_H(z, p)}{dp} < 0, \quad \frac{d\overline{S}_H(z, p)}{dz} < 0.$$

Proof. Note that $\overline{S}_H(z, p) \equiv \varepsilon_H u(\overline{q}_H(z, p)) - z = (\varepsilon_H - \varepsilon_L)u(\overline{q}_L(z, p))$. Hence we have that

$$\frac{d\overline{S}_H(z, p)}{dp} = \varepsilon_H u'(\overline{q}_H)\frac{d\overline{q}_H}{dp} < 0,$$

and

$$\frac{d\overline{S}_H(z, p)}{dz} = (\varepsilon_H - \varepsilon_L)u'(\overline{q}_L)\frac{d\overline{q}_L}{dz} < 0.$$

Since $\overline{q}_L \geqslant \widehat{q}_L$ and $\overline{S}_H(z, p)$ is increasing in $\overline{q}_L$ (and equal to $\widehat{S}_H(p)$ when $\overline{q}_L = \widehat{q}_L$) we have that $\overline{S}_H(z, p) \geqslant \widehat{S}_H(p)$. □

Remark. We defined $\overline{\overline{z}}$ as the value of z below which the subtype L agents are cash constrained. Since when $z = \overline{\overline{z}}$ the cash constraint is just binding we still have that $G(\overline{q}_H, \overline{q}_L, p) = 0$ when

$z = \bar{\bar{z}}$. We know that for values of $z \leqslant \bar{\bar{z}}$ the subtype H agents also spend all of their money. Hence, we have that $z_H = z_L = z$. But then, it must be that $q_H = q_L = q$. Define $\bar{\bar{q}}$ to be the value of q that satisfies $G(q, q, p) = 0$. It is easy to check that $\bar{\bar{q}}$ satisfies

$$c'(\bar{\bar{q}}) = \frac{1-p}{1-p\frac{\varepsilon_L}{\varepsilon_H}}\varepsilon_L u'(\bar{\bar{q}}).$$

We can then use $\bar{\bar{q}}$ to obtain the value of $\bar{\bar{z}}$ since it must be the case that $\bar{\bar{z}} = \varepsilon_L u(\bar{\bar{q}})$. Since it is clear from the equation above that $\bar{\bar{q}}$ is a decreasing function of p, then $\bar{\bar{z}}$ is also a decreasing function of p.

Lemma 2.6. *There is no separating equilibrium of the buyer–seller signaling game.*

Proof. We start with the case of $k^s = K - 1$ and later extend the result to the general case. For simplicity suppose that there are two types of agents (i.e., $K = 2$), with $\xi_2 > \xi_1 > \xi^s$; so both agents have incentives to choose positive money holdings. Now suppose that there is a separating equilibrium with $z_1 \neq z_2$. In such an equilibrium, the following incentive compatibility constraints must hold:

$$-(\pi-\beta)z_1 + \beta\sigma\xi_1 S_H(z_1, \xi_1) \geqslant -(\pi-\beta)z_2 + \beta\sigma\xi_1 S_H(z_2, \xi_2),$$

and

$$-(\pi-\beta)z_2 + \beta\sigma\xi_2 S_H(z_2, \xi_2) \geqslant -(\pi-\beta)z_1 + \beta\sigma\xi_2 S_H(z_1, \xi_1).$$

We need to consider two cases. First, suppose that $z_2 < z_1$. From the incentive compatibility constraint for type 1 we have that $S_H(z_1, \xi_1) > S_H(z_2, \xi_2)$. But then we have

$$\begin{aligned}&-(\pi-\beta)(z_1-z_2) + \beta\sigma\xi_2\left[S_H(z_1,\xi_1) - S_H(z_2,\xi_2)\right]\\&\quad > -(\pi-\beta)(z_1-z_2) + \beta\sigma\xi_1\left[S_H(z_1,\xi_1) - S_H(z_2,\xi_2)\right] \geqslant 0,\end{aligned}$$

which implies that incentive compatibility for type 2 cannot hold. So, there is no separating equilibrium with $z_2 < z_1$.

Now, suppose that $z_2 > z_1$. Let us define $\bar{\bar{z}}_2$ the value of z that maximizes $S_H(z, \xi_2)$. Then, we know that $S_H(z, \xi_2) = S_H(z, \xi_1)$ for all $z \leqslant \bar{\bar{z}}_2$ and $S_H(z, \xi_j)$ is increasing in z for $z \leqslant \bar{\bar{z}}_2$ and $j = 1, 2$ (see Fig. 3). So, if $z_j < \bar{\bar{z}}_2$ agents type j will choose to deviate to a value of $z \geqslant \bar{\bar{z}}_2$; that is, $z_j < \bar{\bar{z}}_2$ cannot be an equilibrium. Hence, in equilibrium $z_2 \geqslant \bar{\bar{z}}_2$. Also, since $S_H(z, \xi_2)$ is decreasing in z for $z > \bar{\bar{z}}_2$ and $S_H(z, \rho_2) \leqslant S_H(z, \rho_1)$ for any $\rho_2 \geqslant \rho_1$, if $z_2 > \bar{\bar{z}}_2$ then agents type 2 will choose to deviate to $z < z_2$ because the seller beliefs can only be such that the assigned probability of facing a type 2 agent is equal or lower than when the agent plays z_2. We conclude then that in equilibrium $z_2 = \bar{\bar{z}}_2$ must hold. Lastly, we have by hypothesis that $z_2 > z_1$ and we have shown that $z_1 \geqslant \bar{\bar{z}}_2 = z_2$ which implies a contradiction. Hence, we conclude that there is no separating equilibrium with $z_2 > z_1$.

Let us now go back to the general case with $k^s \leqslant K - 2$, such that more than two types of agents want to choose positive money holdings in equilibrium. Suppose that $z_K \neq z_{k'}$ for all $k' \in \{k^s, \ldots, K-1\}$, where $z_{k'}$ is the real money balances held by agents type k' in equilibrium. Since $\xi_K > \xi_k$ for all $k \in \{k^s, \ldots, K-1\}$, in equilibrium we have that

$$\xi_K > p(z_{K-1}) = \sum_{j=k^s}^{K-1} \xi_j \mu(\xi_j | z_{K-1}),$$

(i.e., $p(z_{K-1})$ is a weighted average of smaller numbers than ξ_K). Again, we need to consider two cases. If $z_K < z_{K-1}$ then $S_H(z_{K-1}, p(z_{K-1})) > S_H(z_K, \xi_K)$ or otherwise agents type ξ_{K-1} would prefer to choose z_K. But then, for the same reasons that in the case with $k^s = K-1$, we have that if agents type $K-1$ want to choose z_{K-1} then agents type K also want to choose z_{K-1}. Now suppose that $z_K > z_{K-1}$. Since $\xi_K > p(z_{K-1})$ we have that $S_H(z, \xi_K) = S_H(z, p(z_{K-1}))$ for all $z \leqslant \bar{\bar{z}}_K$. Then, type K agents must be choosing $z_K = \bar{\bar{z}}_K$ in an equilibrium that separates them. But, we also have that in equilibrium $z_{K-1} \geqslant \bar{\bar{z}}_K$ because $\xi_{K-1} > \xi^s$. This implies a contradiction. We conclude then that z_{K-1} must be equal to z_K and no *full* separation can take place in equilibrium.

Let us call z_{KK-1} the level of real balances held by types ξ_K and ξ_{K-1} in equilibrium (as we have just seen, this is the only possibility compatible with equilibrium). If $p(z_{KK-1}) > p\,(z_{K-2})$ then the same logic as before shows that $z_{K-2} = z_{KK-1}$ in equilibrium. But, in more complicated partial separation arrangements it could be that $p(z_{KK-1}) < p\,(z_{K-2})$. This is the case if there is a type k with $k < K-2$ choosing z_{KK-1} in equilibrium. The proof that z_{K-2} cannot be greater than z_{KK-1} follows the same logic as before. So we have that $z_{K-2} \leqslant z_{KK-1}$ must hold. Now, suppose that $z_{K-2} < z_{KK-1}$ and let k^o be the type choosing z_{KK-1} with $k^o < K-2$. Then, by the incentive compatibility conditions we know that $S_H(z_{KK-1}, p(z_{KK-1})) > S_H(z_{K-2}, p(z_{K-2}))$ and that

$$
\begin{aligned}
&-(\pi-\beta)\,(z_{KK-1}-z_{K-2}) + \beta\sigma\xi_{K-2}\left[S_H(z_{KK-1}, p(z_{KK-1})) - S_H(z_{K-2}, p(z_{K-2}))\right] \\
&\quad > -(\pi-\beta)\,(z_{KK-1}-z_{K-2}) + \beta\sigma\xi_{k^o}\left[S_H(z_{KK-1}, p(z_{KK-1})) - S_H(z_{K-2}, p(z_{K-2}))\right] \\
&\quad \geqslant 0,
\end{aligned}
$$

which implies that the incentive compatibility condition for type $K-2$ would be violated. Hence, we must have that $z_{K-2} = z_{KK-1}$. By induction, it follows that for any k and j greater than k^s the condition $z_k \neq z_j$ cannot be part of an equilibrium. □

Lemma 2.8. *The best semi-pooling equilibrium defeats* (*in the sense of Mailath et al.* [14]) *all other semi-pooling equilibria.*

Proof. We start by defining the concept of defeated equilibrium. Let $U(\sigma, \xi_k)$ be the payoff of a type-k buyer in equilibrium σ. Then, an equilibrium $\sigma^* = \left(\{z_k^*\}_{k=1}^K, \mu^*\right)$ defeats another equilibrium $\sigma' = \left(\{z_k'\}_{k=1}^K, \mu'\right)$ if there exists a value of z such that: (1) $z_k' \neq z$ for all $k \geqslant k^s$ and there exists a k such that $z_k^* = z$; (2) for all $k \geqslant k^s$ we have that $U(\sigma^*, \xi_k) \geqslant U\left(\sigma', \xi_k\right)$ (with strict inequality for at least one k); and (3) there exist $k \geqslant k^s$ such that

$$
\mu'(\xi_k|z) \neq \frac{\pi_k}{\sum_{j \geqslant k^s} \pi_j}
$$

for $\{\pi_j\}_{j \geqslant k^s}$ satisfying that if $z_j^* = z$ and $U(\sigma^*, \xi_j) > U\left(\sigma', \xi_j\right)$ then $\pi_j = 1$ and if $z_j^* \neq z$ then $\pi_j = 0$.

Now consider the best semi-pooling equilibrium $z_k^* = \bar{\bar{z}}(p^*)$ for $k \geqslant k^s$ (and zero otherwise) and $\mu^*(\xi_k|z) = 1/(K-k^s+1)$ for $k \geqslant k^s$ and all $z > 0$ (and zero otherwise). Call this equilibrium σ^*. We want to compare σ^* with some other arbitrary semi-pooling equilibrium σ' with $z_k' = z^P \neq \bar{\bar{z}}(p^*)$ for $k \geqslant k^s$ (and zero otherwise) and $\mu'(\xi_k|z^P) = 1/(K-k^s+1)$ for $k \geqslant k^s$ (and zero otherwise) and $\mu'(\xi_k|z) = 1$ for $z > 0$, $z \neq z^P$ and $k = K$ (and zero otherwise). Clearly, the candidate value of z to evaluate in this comparison is $\bar{\bar{z}}(p^*)$, for which condition (1) above is obviously satisfied.

Since $\overline{\overline{z}}(p^*)$ maximizes $S_H(z, p^*)$ we have that $U(\sigma^*, \xi_k) > U\left(\sigma', \xi_k\right)$ for all $k \geqslant k^s$ (condition (2) above) and then $\pi_k = 1$ for all $k \geqslant k^s$. But then

$$\mu'(\xi_K | \overline{\overline{z}}(p^*)) = 1 \neq \frac{\pi_K}{\sum_{j \geqslant k^s} \pi_j} = \frac{1}{K - k^s + 1}.$$

We conclude then that the best semi-pooling equilibrium σ^* defeats any other arbitrary semi-pooling equilibrium σ'. □

Appendix B. Unobservable money holdings

In the text we have assumed that the seller can observe the amount of money that the buyer has. The objective in this appendix is to discuss the implications of assuming that money holdings are not fully observable and to argue that under some reasonable assumptions the analysis provided in the text is still valid.

Suppose that the seller cannot observe the buyer's money holdings. Then, given the timing of decisions, when making an offer to the buyer, the seller must base such offer on a conjecture about the buyer's money holdings. Since the heterogeneity of buyers is private information, the seller finds all buyers indistinguishable. Hence, we assume that he forms a single conjecture denoted by z^*. In principle, the conjecture could be inaccurate. The key to this argument lies in the choice of assumptions regarding the sequence of play in such an event.

First note that the conjecture will always be less than or equal to $\widehat{z}_H$ (i.e., the maximum amount ever charge to a buyer; see Lemma 2.2.) and the buyer will never carry more than $\widehat{z}_H$. Let us denote by z_{B} the amount of money the buyer is actually holding. Now, suppose $z_{\mathrm{B}} < z^*$ and that we are in the situation where the seller asks the buyer for a payment that is greater than z_{B}. In this case, there is a feasibility problem. One alternative is to assume that trade cannot take place. Such an assumption results in a multiplicity of equilibria. In particular, any conjecture $z^* \in (0, \widehat{z}_H)$ would result in an equilibrium with $z_{\mathrm{B}} = z^*$ for those agents choosing to carry money.

However, it seems unreasonable to assume that no trade will take place if the buyer has less money than the amount asked by the seller. Instead, it seems more natural to assume that the buyer can always show some amount of money to the seller at the time of trade and ask for a feasible offer based on such amount. Clearly, the seller would prefer this procedure over missing the trade. (Financial-disclosure requirements of this sort appear also in Che and Gale's [3] analysis).

Let us then call z_{R} the amount of real balances reported by the buyer at the time of trade. Since the offer will be contingent on the report, it is easy to see that the buyer will never report less than what she is carrying (no agent would carry money that does not intend to use). Here, it is important to realize that even after the buyer has observed her subtype ε_i, she does not want to send a report that signals her subtype. To see this, recall that the seller has all the bargaining power and hence, if the buyer reveals her subtype, then she necessarily gets zero surplus from the trade. There is no gain for the buyer from signaling her subtype.

Now, once we allow the buyer to show some amount of money to the seller on which the offer will be based, we have that $z_{\mathrm{R}} = z_{\mathrm{B}}$ and the problem of the seller reduces to that in expression (1). For this case, then, the rest of analysis provided in the text applies.

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