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A dynamic model of settlement

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Abstract

We investigate the role of settlement in a dynamic model of a payment system where the ability of participants to perform certain welfare-improving transactions is subject to random and unobservable shocks. In the absence of settlement, the full information first-best allocation cannot be supported due to incentive constraints. In contrast, this allocation can be supported if settlement is introduced, provided that it takes place with a sufficiently high frequency.

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1. Introduction

A distinguishing feature of payment systems (PSs) is *settlement*, or the discharge of past obligations through the transfer of an asset. [1] Actual settlement has three defining properties. First, it is not a welfare-improving activity by itself. Rather, settlement involves a mere transfer of an asset between participants in order to fulfill the obligations created by previous transactions. Second, it takes place periodically. For instance, debit card transactions are generally settled on a monthly basis, while settlement of interbank transactions generally takes place daily. Finally, settlement gives the opportunity to all participants in the system to start afresh since, after settling their obligations, they are no longer liable to the system.

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[1] For references to different notions of settlement, we refer the reader to BIS [4].

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In this paper we employ the dynamic model of a PS developed in Koeppl et al. [16] and use it in order to study the role of settlement. Our main finding is that settlement is an essential part of an optimal PS as it enables agents to engage in beneficial transactions that would otherwise not be realized. We employ a version of the model of exchange developed by Kiyotaki and Wright [12,13]. Our model, however, departs from monetary economics and, instead, emphasizes the role of private information in a way related to the dynamic contracting literature. [2] The use of a dynamic framework is essential since some of the questions that optimal PS design poses are inherently dynamic and, therefore, very hard or impossible to study within the existing literature, which is almost exclusively static. [3]

The environment in our model assumes a periodic pattern in which each *transaction stage* consisting of a finite number of bilateral trades is followed by a *centralized round*, in which a general good can be produced using a linear technology. The transaction stage is assumed to be subject to a private information friction. Thus, incentives are needed in order to induce truthful revelation. This implies that, in the absence of settlement, it is impossible to support the full information first-best allocation. In contrast, we demonstrate that, if settlement is introduced in the centralized stage, the full information first-best allocation is attainable in this setup.

We decentralize the efficient allocation via a PS. This involves assigning balances to individual agents during the transaction stage and optimally adjusting these balances given the agents' histories of transactions. Agents can trade these balances against the general good in the centralized round which we model as a competitive market. [4] Having settlement of balances in the centralized round is essential since the first-best allocation cannot be supported in its absence.

Our model displays the following properties. First, periodic settlement in centralized rounds does not increase welfare by itself. The increase in welfare is accomplished indirectly through the interplay between settlement and intertemporal incentives. Second, the first-best is supportable only if settlement takes place with a sufficiently high frequency. Finally, PS participants exit the settlement stage with identical balances. In this sense, agents start afresh as their history does not affect their future transactions.

The reason for settlement being essential in achieving the first-best can be summarized as follows. In the presence of private information, in order for any transactions to take place, the PS must provide intertemporal incentives. This is costly and, in the absence of settlement, these costs can only be borne directly, creating a distortion during the bilateral transactions stage. Settlement allows accumulating and shifting these costs to the centralized stage. This is efficient since incentives in bilateral transactions can then be set properly. More precisely, under a linearity assumption, balance adjustments in the settlement round do not create direct welfare gains or losses on average. In addition, periodic settlement limits the obligations an agent can accumulate over time. Hence, when settlement occurs frequently enough, and the net value of future transactions is high enough, agents will choose to participate in the system.

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[2] See, for example, Green [6]. Other classic references include Spear and Srivastava [22] and Atkeson and Lucas [2].

[3] See Kahn and Roberds [10,11] for two papers in this literature. Kahn [9] provides an excellent summary of the current literature and outlines some of the main open questions in PSs research. Temzelides and Williamson [23] contains a dynamic model of payments under private information. Their work is complementary to ours since it concentrates on the role of credit limits and insurance against the inability to settle transactions. See Berentsen et al. [3] for another recent dynamic model that involves banking and credit. Finally, Andolfatto [1] considers the role of record-keeping and of monetary exchange in an environment related to ours. None of these models, however, address issues related to settlement.

[4] A novelty of this approach is that it involves non-cooperative implementation together with a Walrasian equilibrium aspect.

The paper proceeds as follows. In Section 2, we present the model. The next section discusses optimal allocations. In Section 4, we study decentralization through a PS and the role of settlement. A brief conclusion follows. The Appendix discusses an extension of the model to the case where both production and consumption are verifiable.

2. The model

Time, t, is discrete and measured over the positive integers. There is a [0, 1] continuum of infinitely lived agents. The common discount factor is $\beta \in (0, 1)$. To generate a role for transactions, we assume that in certain periods, agents are randomly matched bilaterally. Randomness in payments is then captured by assuming that in such periods an agent can transact with the agent he is matched with a certain probability. More precisely, we assume a periodic pattern of length $n + 1$ in which each transaction stage, consisting of n bilateral transaction rounds, is followed by a centralized round. In each period during the transaction stage an agent is in a *trade meeting* with probability 2γ. In this case, he is a potential producer or a potential consumer with equal probability. With probability $(1 - 2\gamma)$ the agent is in a no-trade meeting. We assume that production of goods is perfectly divisible and that goods are non-storable. Producing q units implies disutility $-e(q)$.

There also exists a general non-storable commodity that can be produced and consumed by all agents only during centralized rounds. No other commodity can be produced or consumed during such rounds. Producing ℓ units of the general good implies disutility $-\ell$, while consuming ℓ units gives utility ℓ.

Transactions are subject to private information and commitment frictions. More precisely, we will assume that whether a meeting is a trade meeting (i.e., whether a producer and a consumer are matched) is not observable outside the meeting. In addition, agents cannot pre-commit to producing in such meetings. While the opportunity to produce cannot be verified, we assume that production itself, when it takes place, is verifiable. For simplicity, we assume that consumption is not verifiable.[5]

3. Optimal allocations

Our efficiency benchmark is the full information first-best allocation, in which the efficient level of production takes place in *all* trade meetings and all transaction rounds.[6] The difficulty in implementing this allocation lies in that, due to the private information friction, the planner cannot verify whether a trade meeting has taken place. For example, consider a distinguished agent who, say, for the kth time in a row, reports that he did not produce since he had k consecutive no-trade meetings. Given the information structure, the planner can verify that the agent did not produce in any of the last k meetings. What the planner cannot verify, however, is whether the agent had an opportunity to produce and simply declined, or whether he did not have any trade meetings (an event of probability $(1 - 2\gamma)^k$).

First, let us consider the case where the production and consumption during each centralized round does not depend in any way on the past history of transactions. We will later identify this case with an absence of settlement. A moment's reflection should convince the reader that the

[5] Assuming that consumption is verifiable does not affect our main results regarding the efficiency of settlement or the structure of the optimal PS. We discuss this issue further in the Appendix.

[6] Since the utility from consuming any amount of the general commodity equals the resulting disutility to the producer, the amount l produced in each centralized round is indeterminate.

first-best allocation is not implementable in this case. Indeed, if an agent always receives q^* independent of his history of reports, there is no incentive for him to ever produce. Thus, since there is no other way to induce such an agent to produce, he must receive a quantity less than q^* in at least some occasions.

Next, we study allocations in which the planner can condition production and consumption during centralized rounds on the past history of transactions. In our implementation section we will refer to this case as PSs that use settlement. We will demonstrate that the full information first-best allocation is supportable in this case. Suppose that the planner can condition trades during centralized rounds on the past history of transactions. We can imagine that agents report their state (producer p, consumer c, neither n) in each round of the transactions stage to a planner who instructs them how much to produce (consume) in each bilateral transaction. In addition, the planner instructs agents how much to produce (consume) in the subsequent centralized round. The planner's recommendations will, in general, depend on the agents' past history of reports. An allocation, therefore, specifies the quantity produced (consumed) in each transaction as well as in the centralized round. An allocation is *incentive feasible* if it respects incentive, ex ante participation, and resource feasibility constraints. Throughout, we restrict attention to outcomes that are stationary and symmetric across agents.

We concentrate here on the case where $n = 1$. That is, we impose a periodic pattern in which each bilateral transactions round is followed by a centralized stage.[7] Provided that β is sufficiently high, we show next that the first-best allocation is incentive feasible and, hence, ex ante efficient.

Proposition 1. *Suppose that $n = 1$. The first-best allocation where consumption of q^* occurs in all bilateral trade meetings and where $E[\ell^*] = 0$ in each centralized stage is incentive feasible if and only if $\beta u \geqslant e$.*

Proof. It suffices to show that the first-best can be supported by repeating a static allocation. Following the transaction stage, there are only two possible relevant histories. Either the agent was a producer, in which case we assume that the planner instructs him to produce ℓ_p, or the agent was a consumer, in which case the planner instructs him to produce ℓ_c during the centralized round. Since consumption is not verifiable, agents who were in a no-trade meeting will also be instructed to produce ℓ_c, i.e., $\ell_n = \ell_c$.

Consider the problem of a planner who maximizes ex ante expected utility subject to resource, participation, and incentive constraints. The planner's problem at the beginning of a transactions round is

$$\max_{q,\ell_p,\ell_c,\ell_n} \quad \frac{1}{1-\beta}[\gamma(u(q)-e(q)) - \gamma\ell_p - \gamma\ell_c - (1-2\gamma)\ell_n] \tag{1}$$

s.t.

$$\gamma\ell_p + \gamma\ell_c + (1-2\gamma)\ell_n = 0, \tag{2}$$

$$-e(q) - \ell_p + \frac{\beta}{1-\beta}[\gamma(u(q)-e(q)) - \gamma\ell_p - \gamma\ell_c - (1-2\gamma)\ell_n] \geqslant 0, \tag{3}$$

$$u(q) - \ell_c + \frac{\beta}{1-\beta}[\gamma(u(q)-e(q)) - \gamma\ell_p - \gamma\ell_c - (1-2\gamma)\ell_n] \geqslant 0, \tag{4}$$

$$-\ell_n + \frac{\beta}{1-\beta}[\gamma(u(q)-e(q)) - \gamma\ell_p - \gamma\ell_c - (1-2\gamma)\ell_n] \geqslant 0, \tag{5}$$

$$-e(q) - \ell_p \geqslant -\ell_n. \tag{6}$$

[7] The general case ($n \geqslant 1$) is analyzed below when we decentralize efficient allocations.

The first constraint captures aggregate feasibility. The second, third, and fourth constraints are participation constraints for a producer, a consumer, and a no-trader, respectively. Finally, since the planner cannot observe whether an agent has an opportunity to produce or not during the transaction round, the last constraint is an incentive compatibility constraint. It is easy to verify that $q = q^*$, $\ell_p = -(1-\gamma)e$, and $\ell_c = \ell_n = \gamma e$ solve the above problem provided that $\beta u \geqslant e$. In addition, the resulting allocation satisfies the ex ante participation constraint

$$\frac{1}{1-\beta}[\gamma(u(q) - e(q)) - \gamma\ell_p - (1-\gamma)\ell_c] \geqslant 0 \tag{7}$$

which completes the proof. □

A few remarks are in order. First, note that the expected (average) ℓ equals zero, which implies that, as mentioned earlier, there is no ex ante direct gain from the centralized round. Second, the condition that $\beta u \geqslant e$ is necessary in order to support a positive volume of transactions under full information. Thus, it is not possible to improve over the conditions of the above proposition, in the sense of supporting the first-best allocation for a wider range of parameter values.

To summarize, when the allocation in the centralized round can depend on the past history of transactions, and provided that production in the centralized round involves costs that enter the agents' expected utility in a linear fashion, the full information first-best allocation is incentive feasible in our model and, hence, the first-best can be supported as the ex ante efficient allocation. This is even though the planner is subject to a private information friction in the transaction stage. Next, we demonstrate how the efficient allocation for this environment can be supported via a PS. This involves assigning balances to individual participants and specifying rules for how these balances are updated in order to satisfy incentive and participation constraints. In addition, we will decentralize the settlement stage by assuming that it operates as a competitive market.

4. Payment systems

In the previous analysis we assumed that, subject to a participation constraint, the planner has the ability to re-allocate the output produced in the centralized stage. Here we will discuss how the first-best allocation can be decentralized. As in [16], we assume that agents can participate in a PS which assigns *balances* to participants. The PS specifies rules for how the balances are updated given the histories of reports regarding bilateral transactions. Settlement implies that participants can periodically trade balances for the general, non-storable good. Here the settlement stage is modelled as a competitive market in which agents that are "low" can increase their balances by producing, while those with high balances end up as consumers. The price, p, at which balances are traded, is determined by market clearing conditions which require that $\ell = 0$ on average (aggregate).

To briefly recall our setting, agents engage in bilateral transactions in each of the first n periods of a cycle. Whether an agent is a potential producer in a trade meeting is not observable. We begin again by assuming that $n = 1$ and proceed to study the optimal PS given our setup. In what follows, we analyze a generic period, t, and work backward, first considering the agent's problem in the settlement round, and then moving on to the transactions stage.

Let $V(d, p)$ denote the value function of an agent that exits the transaction round with balance d, given that the anticipated price in the following settlement round is p. Let $v(\widehat{d}, \Psi)$ denote the value of an agent who exits the settlement round with balance $\widehat{d}$, given that the resulting distribution of balances is denoted by Ψ. We describe $v(\widehat{d}, \Psi)$ in detail below. Given p and Ψ, agents at the beginning of the settlement round solve then the following problem.[8]

$$V(d, p) = \max_{\ell, \widehat{d}} \quad \{-\ell + \beta E v(\widehat{d}, \Psi)\} \tag{8}$$

s.t.

$$p\widehat{d} = pd + \ell, \tag{9}$$

$$\widehat{d} \geqslant \underline{d}. \tag{10}$$

We now turn to the problem faced by the agents during the transactions round. We assume that the balances of any two agents that are in a meeting during the transactions round are observable. As before, agents make reports about the type of the meeting that they are in (c, p, or n). Those that report a trade meeting as producers receive instructions from the PS on how much to produce. Consumers report the quantity they consumed. The PS subsequently makes balance adjustments depending on these reports. We rule out that the PS can condition jointly on individual reports of agents in a meeting.

Not taking into account the agents' balances, there are three possibilities. An agent can be in a consumption, a production, or a no-trade meeting. The vector of policy rules (L_t, K_t, B_t, q_t) determines the respective balance adjustments and the quantity produced in a trade meeting, q_t. These functions in general may depend on the agents' histories of transactions, as summarized by their current balances, as well as on the distribution of balances, Ψ. More precisely, $L_t(K_t)$ is the adjustment for an agent who consumes (produces), while B_t is the adjustment for an agent who does not transact. Recall that balances are represented by real numbers not restricted in sign, while production of goods during trade meetings is restricted to be positive. After each transaction stage, agents enter the settlement round knowing their new balances.

We will concentrate on arrangements that satisfy certain incentive and participation constraints. Incentive constraints (ICs) require that the following inequalities hold:

$$V(d + L, p) = V(d + B, p), \tag{11}$$

$$-e(q) + V(d + K, p) \geqslant V(d + B, p). \tag{12}$$

The first constraint equates the continuation utility of a consumer to that of an agent in a no-trade meeting. The equality captures the fact that consumption is not verifiable. In addition, participation constraints require that producers, consumers, and agents in no-trade meetings, respectively, are better off staying in the system; i.e.,

$$-e(q) + V(d + K, p) \geqslant 0, \tag{13}$$

$$u(q) + V(d + L, p) \geqslant 0, \tag{14}$$

$$V(d + B, p) \geqslant 0. \tag{15}$$

We are now ready to formally define a PS.

Definition 2. A PS, **S**, is an array of functions $\mathbf{S} = \{L_t, K_t, B_t, q_t\}$. **S** is incentive feasible if it satisfies the incentive and participation constraints. **S** is simple if balance adjustments do not

[8] We impose a lower bound $\underline{d}$ on the balances in the settlement round. This ensures a solution in which the value function is finite, since $v(\widehat{d}, \Psi)$ will be bounded from below. Imposing such a limit is innocuous when implementing efficient allocations.

depend on the agents' current balances. An incentive feasible **S** is optimal if it can implement the efficient allocation.[9]

Given an incentive feasible PS, the value function of an agent with balances d at the end of the settlement round, $E[v(d, \Psi)]$, is then given by

$$E[v(d,\Psi)] = \int_{d'} \big\{\gamma[u(q(d,d')) + V(d + L(d,d'), p)] + \gamma[e(q(d,d')) + V(d + K(d,d'), p)] + (1-2\gamma)V(d + B(d,d'), p)\big\}\, d\Psi. \tag{16}$$

Note that the balance adjustments are in general functions of the agent's own balance, d, and the balance of his trading partner, d'. We next investigate properties of an optimal PS.

4.1. Decentralization with settlement

We let $q_t(\widehat{d}_{t-1}, \widehat{d}'_{t-1})$ stand for the quantity that the PS requires to be produced in a trade meeting, as a function of the balances of the two trading partners. The next proposition presents a necessary and sufficient condition for an optimal simple PS to exist in the case where each transaction round is followed by settlement ($n = 1$).

Proposition 3. *There exists an optimal simple PS if and only if $\beta u \geqslant e$.*

Proof. Let $q_t(\widehat{d}_{t-1}, \widehat{d}'_{t-1})$ be defined by

$$q_t(\widehat{d}_{t-1}, \widehat{d}'_{t-1}) = \begin{cases} q^* & \text{if } \widehat{d}_{t-1} = \widehat{d}'_{t-1} = 0, \\ 0 & \text{o.w.} \end{cases} \tag{17}$$

Hence, if the balances of both agents are equal to 0, the producer is instructed to produce q^*; otherwise, no production takes place. For all t, define the balance adjustments (K_t, L_t, B_t) by the following:

$$B_t = L_t, \tag{18}$$

$$-e + V(\widehat{d}_{t-1} + K_t, p_t) = V(\widehat{d}_{t-1} + B_t, p_t), \tag{19}$$

$$\gamma K_t + \gamma L_t + (1-2\gamma)B_t = 0. \tag{20}$$

The first two equations express the ICs. The thirst equation requires that nominal balances remain constant. Note that the second equation implies that

$$V(\widehat{d}_{t-1} + K_t, p_t) > V(\widehat{d}_{t-1} + B_t, p_t). \tag{21}$$

We guess that, given balance adjustments (K_t, L_t, B_t) defined above, every agent chooses balances $\hat{d}_t = 0$ in all settlement rounds. To verify that this is consistent with an equilibrium, we need to verify that all participation constraints (PC) hold. These are given by

$$V(\hat{d}_{t-1} + B_t, p_t) \geqslant 0, \tag{22}$$

$$u + V(\hat{d}_{t-1} + B_t, p_t) \geqslant 0, \tag{23}$$

$$-e + V(\hat{d}_{t-1} + K_t, p_t) \geqslant 0. \tag{24}$$

[9] To simplify notation, we suppress the dependence of **S** on t.

Let $X_t \in \{K_t, B_t, L_t\}$ denote the resulting balance adjustment in period t. Since the second inequality above is satisfied whenever the first one holds, and the third inequality holds whenever the IC conditions hold, it remains to verify the participation constraint which requires that $V(\hat{d}_{t-1} + X_t, p_t) \geqslant 0$. Using the linearity of V, the stationarity of d and p, and the fact that $E[X_{t+1}] = 0$, we have

$$\begin{aligned} V(\hat{d}_{t-1} + B_t, p_t) &= -p_t\hat{d}_t + p_t\hat{d}_{t-1} + p_t B_t + \beta E[v(\hat{d}_t, \Psi)] \\ &= p_t B_t + \beta\gamma(u-e) + \beta E[V(\hat{d}_t + X_{t+1}, p_{t+1})] \\ &= p_t B_t + \beta\gamma(u-e) + \beta V(\hat{d}_t, p_{t+1}) + \beta p_{t+1} E[X_{t+1}] \\ &= p_t B_t + \beta\gamma(u-e) + \beta V(\hat{d}_t, p_{t+1}). \end{aligned} \tag{25}$$

Given the linearity of V and since $\hat{d}_{t-1} = \hat{d}_t$, this yields

$$V(\hat{d}_t, p_t) = \frac{\beta}{1-\beta}\gamma(u-e). \tag{26}$$

From the definition of (K_t, L_t, B_t), we have that setting $K_t = (1-\gamma)\frac{e}{p}$ and $B_t = -\gamma\frac{e}{p}$, for all t, satisfy the IC and the constant-balances constraint. Hence, $V(\hat{d}_{t-1} + K_t, p_t) > V(\hat{d}_{t-1} + B_t, p_t)$ and, again by the linearity of V,

$$\begin{aligned} V(\hat{d}_{t-1} + B_t, p_t) &= p_t B_t + V(\hat{d}_{t-1}, p_t) \\ &= -\gamma e + \frac{\beta}{1-\beta}\gamma(u-e) \geqslant 0, \end{aligned} \tag{27}$$

which holds if $\beta u \geqslant e$.

For the converse, suppose that $\beta u < e$. Since $B_{t+1} = L_{t+1}$, for all t, the following three conditions must hold for a simple PS to be optimal:

$$\gamma K_{t+1} + (1-\gamma)B_{t+1} = d_{t+1} - d_t, \tag{28}$$

$$p_{t+1}K_{t+1} - e \geqslant p_{t+1}B_{t+1}, \tag{29}$$

$$(d_t - d_{t+1})p_{t+1} + \frac{\beta}{(1-\beta)}\gamma(u-e) \geqslant -\, p_{t+1}B_{t+1}. \tag{30}$$

The last inequality becomes $V(d_t + B_{t+1}, p_{t+1}) \geqslant 0$, which is the participation constraint. We can now replace $p_{t+1}K_{t+1}$ everywhere to obtain two inequalities that involve only B_{t+1}:

$$(d_{t+1} - d_t)p_{t+1} - \gamma e \geqslant p_{t+1}B_{t+1} \tag{31}$$

and

$$(d_t - d_{t+1})p_{t+1} + \frac{\beta}{(1-\beta)}\gamma(u-e) \geqslant -\, p_{t+1}B_{t+1}. \tag{32}$$

Combining these two we obtain

$$\frac{\beta}{(1-\beta)}\gamma(u-e) \geqslant \gamma e. \tag{33}$$

This is a contradiction. It is easy to check that the above condition, which can be simplified to give $\beta u \geqslant e$, also ensures that no agent chooses to exit the economy after selling balances in the settlement round when the lower bound $\underline{d}$ is set equal to B_{t+1}. □

Note that the above scheme requires that all agents exit the settlement round with $d = 0$, in all periods. Therefore, the PS requires that Ψ is degenerate.[10] While this adds an additional constraint on the agents, the above proposition demonstrates that this leads to the first-best being supported under the same condition as in the full information case.

Using the incentive and participation constraints, we can derive additional properties of the optimal **S**. Since consumption is not verifiable, in order for an agent that consumed to report truthfully, **S** needs to treat him the same way as if he reported a no-trade meeting; i.e., $B = L < 0$, which means that irrespective of whether agents can consume q^* or are in a no-trade meeting, they are penalized with decreasing balances. In addition, it must be that $p_t(K - B) \geqslant e$ for all t. In other words, agents are rewarded for producing. Finally, the aggregate net production of the general good is zero in the settlement round; i.e., $E[\ell] = 0$ for all agents. It should be clear, however, that the ICs imply that different agents actually end up with different values of ℓ during the settlement round.

4.2. Settlement frequency

The case where $n = 1$ implies that settlement takes place after every transaction. We assume now that a settlement round occurs after each transaction stage of length $n > 1$. In particular, we will derive conditions for a PS to support the efficient allocation with respect to $(q^*, E[\ell^*])$ when $n > 1$. We will also assume that the agents' discount factor between transactions is $\tilde{\beta}$, but that there is no discounting between the last transaction and the settlement round. We consider two cases. In the first case, there is equal discounting within and across transaction stages ($\tilde{\beta} = \beta$). In the second case, there is no discounting within the transaction stage ($\tilde{\beta} = 1$).

Definition 4. A PS is history-independent if the balance adjustments in any round s, $1 < s \leqslant n$, of the transaction stage do not depend on the transactions in the previous $s - 1$ rounds.

In what follows we will concentrate on a particular class of history-independent PS, which we term *simple repeated PS*. Such PS employ identical discounted balance adjustments during each transaction round. Formally, let $X_t \in \{K_t, L_t, B_t\}$ be the balance adjustment for a PS when $n = 1$. A simple repeated PS has balance adjustments equal to

$$X_{t+s} = \frac{X_{t+n}}{\tilde{\beta}^{n-s}} \tag{34}$$

for all $s = 1, \ldots, n$, where $n - s$ represents the number of transaction rounds until the next settlement stage. The next proposition gives conditions that are necessary and sufficient for an optimal simple repeated PS to exist.

Proposition 5. *Assume that $\tilde{\beta} = \beta < 1$. There exists an optimal simple repeated PS if and only if $\beta^n u \geqslant e$. Thus, there exists $\bar{n} \geqslant 1$ such that a simple repeated PS is optimal if and only if $n < \bar{n}$.*

[10] Lagos and Wright [17] study a monetary model in which trade is periodically centralized. They assume quasi-linearity and bargaining in order to obtain a degenerate distribution of money holdings. Since we do not impose bargaining in the transactions round, their result does not readily apply to our model.

Proof. Assuming a simple repeated PS, let balances within the transaction rounds be given by

$$X_{t+s} = \frac{X}{\beta^{n-s}} \tag{35}$$

for all $s = 1, \ldots, n$, and all t, where $X \in \{K, L, B\}$ is the balance adjustment for the optimal PS when $n = 1$.

First, note that for a simple repeated PS, all participation constraints are satisfied if the participation constraint for the nth period holds in the worst-case scenario. Hence, all participation constraints are satisfied if the constraint resulting from consuming n times in a row is satisfied, or if

$$(d_t - d_{t+n})p_{t+n} + \frac{\beta}{1-\beta^n}\left(\sum_{s=0}^{n-1} \beta^s\right)\gamma(u-e) \geqslant -p_{t+n}\sum_{s=1}^{n} B_{t+s}. \tag{36}$$

Using $B_{t+s} = L_{t+s}$, for all $s = 1, \ldots, n$, the aggregate law of motion on balances is given by

$$\left[\gamma\sum_{s=1}^{n} K_{t+s} + (1-\gamma)\sum_{s=1}^{n} B_{t+s}\right] = E\left[\sum_{s=1}^{n} X_{t+s}\right] = d_{t+n} - d_t. \tag{37}$$

Finally, in every period, the incentive compatibility constraint is given by

$$-e + \beta^{n-s} p_{t+n} K_{t+s} \geqslant \beta^{n-s} p_{t+n} B_{t+s} \tag{38}$$

for all $s = 1, \ldots, n$. Using the definition of X_{t+s}, this can be written as

$$-e + p_{t+n} K_{t+n} \geqslant p_{t+n} B_{t+n}. \tag{39}$$

Since we are employing a simple repeated PS, the last equation implies that incentive compatibility constraints are identical for all periods of the cycle. Using the fact that

$$\sum_{s=1}^{n} X_{t+s} = \frac{1-\beta^n}{\beta^{n-1}(1-\beta)} X, \tag{40}$$

we obtain for the law of motion

$$p_{t+n} K_{t+n} = \frac{1}{\gamma}\frac{\beta^{n-1}(1-\beta)}{1-\beta^n}(d_{t+n} - d_t)p_{t+n} - \frac{1-\gamma}{\gamma} p_{t+n} B_{t+n}. \tag{41}$$

Replacing $p_{t+n}K_{t+n}$ in the incentive compatibility and the participation constraints, we obtain two inequalities that involve only the term $\sum_{s=1}^{n} B_{t+s}$:

$$(d_{t+n} - d_t)p_{t+n} - \gamma e\frac{1-\beta^n}{\beta^{n-1}(1-\beta)} \geqslant p_{t+n}\sum_{s=1}^{n} B_{t+s}, \tag{42}$$

$$(d_t - d_{t+n})p_{t+n} + \frac{\beta}{(1-\beta)}\gamma(u-e) \geqslant -p_{t+n}\sum_{s=1}^{n} B_{t+s}. \tag{43}$$

The efficient allocation is supportable if and only if both inequalities are satisfied. This is the case whenever

$$\frac{\beta}{1-\beta}\gamma(u-e) \geqslant \gamma e\frac{1-\beta^n}{\beta^{n-1}(1-\beta)}, \tag{44}$$

or, whenever $\beta^n u \geqslant e$. Finally, note that if n is large, the left-hand side of the above condition converges to zero. Thus, for every $\beta < 1$, there exists N such that for $n > N$ an optimal simple repeated PS does not exist. □

The following discusses the case where there is no discounting within the transaction stage. It readily follows from the above proposition.

Corollary 6. *Assume that* $\tilde{\beta} = 1$. *For any* $n \in N$, *an optimal simple repeated PS exists if and only if there exists a simple optimal PS for* $n = 1$.

Returning to the general case where $\tilde{\beta} < 1$, it should be clear that, if the optimal allocation can be supported for a given $\tilde{n}$, then it can also be supported for all $n < \tilde{n}$. Since the converse is not true, our model suggests that a high frequency of settlement can be beneficial. The intuition behind the case where an optimal simple PS does not exist is related to the one obtained in Levine [18], but comes from a very different model. More precisely, if settlement is sufficiently infrequent, a positive fraction of agents experience a long sequence of no-production opportunities during the transaction round, resulting in a high balance that eventually needs to be settled. Sufficiently impatient agents will choose to exit the economy rather than going through the settlement process; i.e., the participation constraint will fail.

5. Conclusion

We studied simple optimal PSs in a dynamic model in which the ability of agents to perform certain welfare improving transactions is subject to random and unobservable shocks. Our main finding in this paper can be summarized as follows. In the absence of settlement, ICs imply that the first-best allocation is not incentive feasible. The first-best is incentive feasible, however, if settlement is introduced, provided that it takes place with a sufficiently high frequency.

The linearity assumption was necessary in order for settlement to affect welfare only indirectly.[11] If the disutility from balance adjustments was non-linear, it would not be possible to achieve the first-best allocation in both the bilateral and the settlement stage. Thus, linearity is necessary in order to implement the first-best in our model. One interpretation of this assumption is that settlement costs are linear. For example, if settlement involves the liquidation of certain assets, such as government bonds, such costs are linear in the number of assets liquidated. Alternatively, if settlement involves central bank money, the cost of settlement involves the opportunity cost from holding such money, which corresponds to the interest rate on an equivalent number of risk-free bonds. A second interpretation is that if settlement involves the need for a sale of indivisible (large-denomination) assets or the supply of extra work hours, linearity captures the fact that it is typically beneficial to use lotteries in order to randomize over the amount of assets that are liquidated, or over the extra work hours supplied.[12]

[11] Other examples in which some form of linearity is invoked in static mechanism design environments include the classic papers by Clarke [5] and Groves and Loeb [7]. See also Jarque [8] for a more recent reference that deals with an intertemporal environment.

[12] See Rocheteau et al. [20] for a discussion of how the linear structure of lotteries can be beneficial in that regard.

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Appendix. Verifiable consumption

We briefly discuss the case where—like production—consumption is also verifiable. This is important for two reasons. First, in the context of PS it is simply natural to study the case where production and consumption are treated symmetrically. Second, one would like to know whether the structure of optimal PS is robust to changes in the information structure. Somewhat surprisingly, it turns out that verifiability of consumption does neither affect our analysis nor our results. In particular, the property that agents that consume are treated symmetrically by the PS to those that do not trade carries over to the case of verifiable consumption.

The intuition for this result is simple. The only binding IC is the one for producers which receive positive balance adjustments in order to reimburse their production costs. Market clearing in the settlement stage requires then that some non-producers (non-traders and/or consumers) have negative adjustments. For a (repeated) simple PS that supports the first-best allocation for the widest range of parameter values only the participation constraint for the worst (i.e., most negative) adjustment matters. But for this constraint to be satisfied, there is no reason for the PS to treat consumers differently from agents that did not trade.[13] This is summarized in the following result which can be easily generalized to the case of simple repeated PS when $n > 1$.

Proposition 7. *Assume that consumption and production are both verifiable. All simple PS that implement the first-best for the widest range of parameters have the same balance adjustments for non-producers irrespective of whether they consume or not.*

Proof. For any simple PS to implement the first-best, it must fulfill four conditions. These are the aggregate law of motion for balances that is consistent with market clearing, the IC for producers and consumers, and the participation constraint with the minimal balance adjustment. These conditions are given by

$$\left[\gamma K + \gamma L + (1-2\gamma)B\right] = 0, \tag{45}$$

$$pK - e \geqslant pB, \tag{46}$$

$$pL + u \geqslant pB, \tag{47}$$

$$p\min\{K, B, L\} + \frac{\beta}{1-\beta}\gamma(u-e) \geqslant 0. \tag{48}$$

[13] With consumption being verifiable, if β is sufficiently high and n sufficiently low, it can be shown that there are simple repeated PS that can support the efficient allocation with negative adjustments only for consumers, while leaving balances unchanged for agents who do not trade. Furthermore, any PS that implements the first-best in dominant strategies must treat consumers and no-traders differently.

The third inequality replaces the restriction $L = B$ since consumption is now verifiable. Note that $K > B$. Hence, $\min X \in \{B, L\}$ and $\min X < 0$. The PS that allows us to implement the first-best for the widest range of parameters (u, e, β) can then be expressed as the solution to the following problem:

$$\max\min \quad \{B, L\} \tag{49}$$

s.t.

$$-e - pL \geqslant \frac{1-\gamma}{\gamma} pB, \tag{50}$$

$$u + pL \geqslant pB, \tag{51}$$

$$p \min\{B, L\} + \frac{\beta}{1-\beta} \gamma(u-e) \geqslant 0, \tag{52}$$

where we have used the fact that

$$K = -\left[\frac{1-2\gamma}{\gamma} B + L\right]. \tag{53}$$

For any solution, one of the first two inequalities must be binding, as otherwise one could increase $\min\{B, L\}$ by ε and still get all constraints to hold. Suppose first that the second inequality binds. Then, $B > L$. This cannot be optimal, as one can increase L by ε sufficiently small and decrease B by $\frac{\gamma}{1-\gamma}\varepsilon$ without violating any of the constraints. Thus, the second inequality cannot bind, while the first must bind. Hence,

$$\gamma pL + (1-\gamma) pB = -\gamma e, \tag{54}$$

which implies that $L = B = -\gamma e$. Thus, non-producers have the same balance adjustments *irrespective* of whether they consume or not. □

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