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Government Transaction Policy, Media of Exchange, and Prices*

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We study government transaction policies in search-theoretic models of money. We model government as a subset of agents, who are subject to the same random matching technology and other constraints as private agents, but who behave in an exogenous way regarding which objects they accept in trade and at what price. The objective is to see how these policies affect private agents' strategies, and hence the set of equilibria. We analyze how the effects depend on factors like the size of government, the intrinsic properties of money, and the availability and efficacy of substitutes like barter or foreign currency. *Journal of Economic Literature* Classification Numbers: C78, E31, E42.

1. INTRODUCTION

It is a venerable notion that what government accepts in payment in its transactions can have an impact on what private agents do. Smith [15], for instance, suggested that "A prince, who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind, might thereby give a certain value to this paper money; even though the term of its final discharge and redemption should depend altogether upon

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the will of the prince." Lerner [10] further argued that "The modern state can make anything it chooses generally acceptable as money and thus establish its value quite apart from any connection, even of the most formal kind, with gold or with backing of any kind. It is true that a simple declaration that such and such is money will not do, even if backed with the most convincing constitutional evidence of the state's absolute sovereignty. But if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done."

Can a government really make "anything it chooses" generally acceptable as money, simply by accepting it in its own transactions? How does the answer depend on the size or importance of the government in the economy, on the nature of the proposed money, and on factors like the existence of alternatives to the proposed money such as barter opportunities or other potential media of exchange? Historically, it is not uncommon for governments to adopt policies meant to influence whether one object or another circulates as money, and at what relative price, including policies that favor the object by accepting only it in payment for taxes or in other transactions, by announcing its legal tender status, and so on. The record indicates that these policies sometimes worked and other times failed, as we discuss in Section 6 below.

The objective of this paper is to study the effects of government transaction policies in search-theoretic (or random matching) models of money, and to make precise how these effects depend on factors like the size and influence of the government, the intrinsic properties of money, and other parameters. A search model is appropriate for this analysis because it generates an endogenous transactions pattern, and this allows one to determine in equilibrium which objects are accepted in which trades at what relative prices. We model the government here as a subset of the agents in the economy, who are subject to the same random matching technology and other constraints as private agents, but who behave in a specific way. For example, they may always accept a particular money, they may accept only money (i.e., they may refuse to barter), they may accept money but only at a predetermined exchange rate, and so on. The objective is to see how their transaction policy, which we specify exogenously, affects the trading strategies that private agents choose endogenously, and hence how it affects the set of equilibria.

As an example of the kind of results we derive, we show that a policy whereby the government always accepts money in exchange can guarantee the existence, and also the uniqueness, of an equilibrium where money is universally accepted by the private sector, if and only if the size of the government exceeds a certain threshold (i.e., it is involved in a large enough proportion of economic activity). This threshold depends on several factors, including the amount of money in the economy, its properties, the availability and efficacy of substitutes like direct barter or foreign currency, and whether the government also accepts other objects in exchange. In particular, a sufficiently big government may be able to establish the existence of a monetary equilibrium even if one does not exist without intervention. Also, by refusing to accept a particular money, the government can rule out equilibria where that money is accepted by private agents, if and only if the government is sufficiently big.

Our analysis extends the work of Aiyagari and Wallace [2], who also model government transaction policy by specifying exogenously the trading strategies of a subset of agents. Some differences in the papers are the following. First, we build on the model in Kiyotaki and Wright [9], instead of the relatively more complicated model in Kiyotaki and Wright [7]. Given a more tractable basic model we can include some interesting features absent from the Aiyagari and Wallace environment (e.g., a double coincidence problem, multiple currencies, general storage costs and production costs), and we can consider a more general class of policies (e.g., we can vary the probability with which government agents trade goods for money, money for goods, and goods for goods, while they set the first two probabilities to unity and vary only the third). We also analyze dynamics, while Aiyagari and Wallace consider only steady state equilibria. Additionally, we are able to fairly completely characterize the way in which the set of equilibria depends on policy and other parameters, while they focus on an example that is meant to illustrate one particular effect.[1]

Perhaps the main difference between the current paper and the analysis of Aiyagari and Wallace is that they consider only a model with indivisible goods, which means that prices are exogenous, while here we follow Shi [14] and Trejos and Wright [16] and endogenize prices using bilateral bargaining. This allows us to do two new things. First, we can analyze the effects on prices of policies that specify which objects the government accepts. Second, we can analyze policies that specify not only which objects the government accepts, but the price at which it accepts them. For example, without intervention there are often multiple monetary equilibria with different price levels; but, by committing to accept money at a certain price a sufficiently big government can rule out all but a unique monetary equilibrium. As another example, even if a monetary equilibrium does not exist without intervention, a monetary equilibrium may exist if a sufficiently big government commits to accept money at a certain price (although note that this requires the government sell its output for less than the market price, and means that it must give out more goods than it takes in).

[1] The effect on which they focus is this: A policy whereby government favors money by accepting trades of goods for goods with a low probability can have both a benefit (it rules out an inferior nonmonetary equilibrium) and a cost (it slows down trade).

The rest of the paper is organized as follows. Section 2 considers the simplest version of the model, with indivisible commodities. Section 3 considers divisible commodities and introduces bargaining to determine prices. Section 4 considers economies with two monies, such as a domestic and a foreign currency. The analysis to that point focuses on steady state equilibria, but in Section 5 we go on to characterize the set of dynamic equilibria. Section 6 concludes with a summary of the main results, a discussion of some historical evidence, and suggestions for future work.[2]

2. A MODEL WITH INDIVISIBLE GOODS

We begin with a model where goods and money are indivisible, and hence every trade is a one-for-one swap. This model obviously has nothing to say about prices, but it does make predictions about which objects are accepted in exchange, and, in particular, about when money will have value. Although this is basically the same issue addressed by Aiyagari and Wallace [2], it is worth reconsidering here because the present model is both simpler and more general along certain dimensions, as discussed above.

Time is continuous. There is a [0, 1] continuum of infinite-lived agents who meet bilaterally according to an anonymous matching process with Poisson arrival rate α. Specialization, which motivates gains from trade, is modeled here as follows. Let S be the set of goods. Each agent i consumes goods in a subset $S_i \subset S$ and cannot consume goods not in S_i. He derives utility U from consuming any good in S_i. He can produce just one good, which is not in S_i, at a cost in terms of disutility C. The goods are nonstorable, which means that they must be produced for immediate delivery to a consumer. Also, the model is symmetric in the sense that for any agent i, whenever i meets someone, there is a probability x that he consumes what i produces; and, conditional on this event, there is a probability y that he also produces what i consumes. Thus, the probability of a "double coincidence of wants" is xy, and it is y that measures the difficulty of direct barter.[3]

[2] In addition to Aiyagari and Wallace [2], some other related papers are the following. Li [11, 12] models government agents in a similar way and allows them to tax (confiscate) private money holdings. Ritter [13] studies a model where the role of government is to issue fiat money in the first place. Green and Weber [4] have government agents whose role is to detect and confiscate counterfeit notes. Aiyagari, Wallace, and Wright [3] have government agents who can issue bonds.

[3] More details concerning specialization can be found in the literature. For example, Kiyotaki and Wright [8] derive from first principles the case where $y = x$ is endogenous; Kiyotaki and Wright [7] and Aiyagari and Wallace [1] analyze models with $N \geqslant 3$ goods where $x = 1/N$ and $y = 0$; and Aiyagari and Wallace [2] use a version of that model with $N = 2$, which implies $x = 1/2$ and $y = 1$, so that there is no double coincidence problem in the sense that every private agent who has a good that you want also wants the good that you have.

In addition to the goods described above, there is another indivisible but storable object that no one produces or consumes called *money*. A fraction $M \in [0, 1]$ of the agents are endowed with money, while the rest are endowed with production opportunities. We assume that, unless an agent is initially endowed with a production opportunity, he must consume before producing. This assumption means that two agents with money cannot trade (neither has a production opportunity). Furthermore, it means that every trade is either a trade of a good for a good, or a trade which involves an agent with 1 unit of money giving it to an agent with 0 units of money in exchange for a good; hence, as long as no one disposes of money, there will always be M agents with one unit of money, called *buyers*, and $1-M$ agents without money, called *sellers*. We allow buyers to freely dispose of money, but note that if they do they do not acquire a production opportunity, but simply drop out of the economy.

Agents are interested in maximizing utility by their choice of trading strategies. Clearly, an agent is willing to accept a good in trade if and only if it is in S_i, and all that remains to be determined is whether he accepts money. Let Π be the probability that a random agent in the economy accepts money, and let π be the best response of a utility maximizing individual. If V_0 and V_1 denote the value functions of sellers and buyers, standard dynamic programming arguments lead to

$$rV_0 = v_0 + \alpha(1-M)\,xy(U-C) + \alpha Mx \max_{\pi} \pi(V_1 - V_0 - C), \tag{1}$$

$$rV_1 = v_1 + \alpha(1-M)\,x\Pi(U + V_0 - V_1), \tag{2}$$

where r is the rate of time preference, v_1 is an instantaneous utility from holding money, and v_0 is an instantaneous utility from holding a production opportunity. The v_j's can be interpreted in a variety of ways; for example, if $v_1 < 0$ then money has a "storage cost" that one may want to loosely interpret in terms of inflation.[4]

The maximization problem in (1) implies:

$$\pi = \begin{cases} 0 & \text{if} \quad V_1 - V_0 - C < 0 \\ [0, 1] & \text{if} \quad V_1 - V_0 - C = 0. \\ 1 & \text{if} \quad V_1 - V_0 - C > 0 \end{cases} \tag{3}$$

[4] Equation (1) says that the flow value to being a seller, rV_0, is the sum of three terms. The first is the instantaneous utility v_0. The second is the probability of meeting a seller, $\alpha(1-M)$, times the probability he wants what you produce and you want what he produces, xy, times the gain from trading. The final term is the probability of meeting a buyer, αM, times the probability he wants what you produce, x, times the gain from trading with probability π where π is chosen optimally. Equation (2) has a similar interpretation.

In what follows we normalize $\alpha x = 1$, with no loss in generality as long as we adjust r and v_j. Then manipulation of (1) and (2) implies that $V_1 - V_0 - C$ takes the same sign as

$$\Delta = (1 - M)(\Pi - y) - K, \tag{4}$$

where $K = (rC + v_0 - v_1)/(U - C)$.

An equilibrium can now be defined as a value of Π satisfying

$$\Delta < 0 \Rightarrow \Pi = 0; \qquad \Delta > 0 \Rightarrow \Pi = 1; \qquad \Pi \in (0, 1) \Rightarrow \Delta = 0, \tag{5}$$

where Δ is given by (4). If $\Pi = 0$ the equilibrium is called nonmonetary, and if $\Pi > 0$ the equilibrium is called monetary.[5] In Fig. 1 we depict the best response correspondence $\pi = \pi(\Pi)$ under the assumption that $0 < \hat{\Pi} < 1$, where

$$\hat{\Pi} = y + \frac{K}{1 - M}. \tag{6}$$

When $0 < \hat{\Pi} < 1$, as shown, there are exactly three equilibria: $\Pi = 0$, $\Pi = 1$, and $\Pi = \hat{\Pi}$. If $\hat{\Pi} > 1$, however, the only equilibrium is $\Pi = 0$; and if $\hat{\Pi} < 0$ the only equilibrium is $\Pi = 1$.

The model illustrates how there are two requirements for money to be valued. First, fundamentals have to be right for a monetary equilibrium to exist, which means y, M, r, $v_0 - v_1$, or $C - U$ cannot be too big (barter cannot be too easy, money too plentiful, individuals too impatient, money too costly to store, or production too expensive). Second, for at least some parameter values, even if a monetary equilibrium exists there also exists a nonmonetary equilibrium, and money is only valued if agents coordinate on the former. Note further that V_0 and V_1 are increasing in Π, and so $\Pi = 1$ Pareto dominates $\Pi \in (0, 1)$ and $\Pi \in (0, 1)$ Pareto dominates $\Pi = 0$.

[5] In principle one also has to check the incentive constraints that guarantee that buyers and sellers want to trade, and the participation constraints $V_j \geqslant 0$ that guarantee agents do not want to drop out of the economy. Consider first a pure strategy monetary equilibrium ($\Pi = 1$). The incentive constraint for buyers is $U + V_0 \geqslant V_1$, which one can show holds if and only if $1 + r - (1 - M)(1 - y) + K \geqslant 0$; a sufficient condition for this is that $v_1 - v_0$ is not too big (if v_1 is too big, e.g., then it is better to hoard money than to spend it). The incentive constraints for sellers are never binding. The participation constraint $V_j \geqslant 0$ holds as long as v_j is not too big a negative number. Hence, these constraints all holds as long as $|v_0|$ and $|v_1|$ are not too big. Similarly, one can show that in the mixed strategy monetary equilibrium ($0 < \Pi < 1$) all of the relevant constraints hold as long as $-v_0$ is not too big. And, finally, in the nonmonetary equilibrium ($\Pi = 0$) the constraints also hold as long as $-v_0$ is not too big. When $\Pi = 0$, note that $v_1 < 0$ implies agents endowed with money dispose of it while $v_1 > 0$ implies they do not, but in either case we have a nonmonetary equilibrium in the sense that money does not circulate.

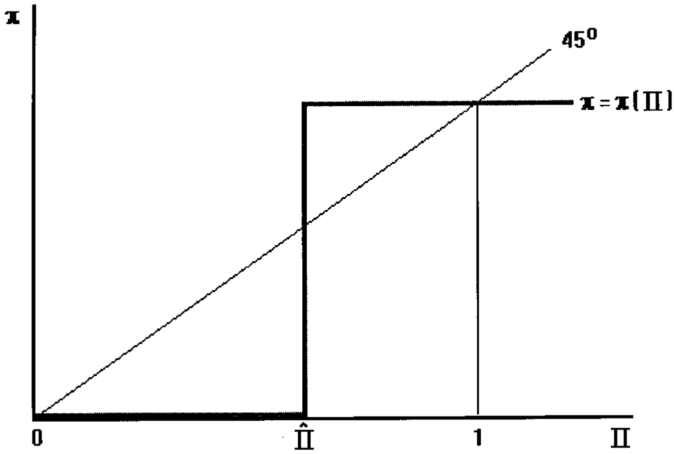

FIG. 1. Equilibria in the fixed price model.

We are interested in whether government transaction policy can be used to guarantee the existence of the equilibrium with $\Pi = 1$ or to rule out other equilibria. To this end, assume that a fraction of the population γ constitutes a special class called *government agents*. They are in all respects like private agents, except that they adopt exogenous trading rules rather than strategies based on maximizing behavior.[6] Private agents continue to use the individually-maximizing trading strategies described above: type i trades his production good for a good in S_i with probability 1; trades money for a good in S_i with probability 1; and trades his production good for money with probability π. But a government agent trades his production good for a good in S_i with probability T_{gg}; trades money for a good in S_i with probability T_{mg}; and trades his production good for money with probability T_{gm}. A government trading policy is specified by $T = (T_{gg}, T_{mg}, T_{gm})$.

We need to keep track of who holds the money. Let $m = (m_p, m_g)$, where m_p is the fraction of private agents and m_g the fraction of government agents with cash. Given the normalization $\alpha x = 1$, the rate at which a private agent switches from buyer to seller is $\gamma(1 - m_g) T_{gm} + (1 - \gamma)(1 - m_p) \Pi$, and the rate at which he switches back is $\gamma m_g T_{mg} \Pi + (1 - \gamma) m_p \Pi$. Combining

[6] One should interprert γ as capturing the importance of government in the economy, or the frequency with which private agents interact with the public sector, rather than measuring number of individuals who "work for the government." Note that we interpret our government agents as consuming and producing, while Aiyagari and Wallace [2] interpret their government agents as simply storing various objects; this is not a particularly important distinction for anything we do in this paper.

these and using the identity $\gamma m_g + (1-\gamma)\, m_p = M$, the steady state value of m_p solves

$$\Pi T_{mg} M - [\Pi T_{mg} + (T_{gm} - \Pi T_{mg})(\gamma - M)]\, m_p - (T_{gm} - \Pi T_{mg})(1-\gamma)\, m_p^2 = 0. \tag{7}$$

The dynamic programming equations for a private agent can now be written

$$r V_0 = v_0 + A_1(U - C) + A_2 \max_{\pi} \pi(V_1 - V_0 - C), \tag{8}$$

$$r V_1 = v_1 + A_3(U + V_0 - V_1), \tag{9}$$

where $A_1 = \gamma(1-m_g)\, y T_{gg} + (1-\gamma)(1-m_p)\, y$ is the probability of a barter opportunity; $A_2 = \gamma m_g T_{mg} + (1-\gamma)\, m_p$ is the probability of an opportunity to trade goods for money; and $A_3 = \gamma(1-m_g)\, T_{gm} + (1-\gamma)(1-m_p)\,\Pi$ is the probability of an opportunity to trade money for goods. The individual maximization condition is still described by (3), but now $V_1 - V_0 - C$ is proportional to

$$\Delta = \gamma(1-m_g)(T_{gm} - y T_{gg}) + (1-\gamma)(1-m_p)(\Pi - y) - K. \tag{10}$$

Given policy T, a steady state equilibrium is a pair (m, Π) satisfying the steady state condition (7) and the best response condition (5) with Δ now given by (10).

For an arbitrary policy T, one can solve for m, insert the answer into (10), and check (5). We begin by studying the effects of a policy with $T_{gm} = 1$ (government agents always accept money). We first analyze the following issue: suppose that $\Pi = 0$ is an equilibrium without government intervention (i.e., with $\gamma = 0$); then, for which values of γ is it still an equilibrium when $T_{gm} = 1$? In other words, when does the government policy of always accepting money eliminate the nonmonetary equilibrium?

First note that $\Pi = 0$ and $T_{gm} = 1$ imply the steady state is:

$$(m_p, m_g) = \begin{cases} \left(\dfrac{M-\gamma}{1-\gamma}, 1\right) & \text{if} \quad \gamma \leqslant M; \\[2ex] \left(0, \dfrac{M}{\gamma}\right), & \text{if} \quad \gamma > M. \end{cases} \tag{11}$$

If, on the one hand, $\gamma \leqslant M$, then setting $T_{gm} = 1$ does not affect the existence of the nonmonetary equilibrium at all. If, on the other hand, $\gamma > M$, then the condition for the existence of the $\Pi = 0$ equilibrium is

$$\Delta = (\gamma - M)(1 - y T_{gg}) - (1-\gamma)\, y - K \leqslant 0.$$

This is violated if and only if

$$\gamma > \gamma_1 = \frac{K + y + M(1 - yT_{gg})}{1 + y(1 - T_{gg})}.$$

If $K < (1-M)(1-yT_{gg})$ then $\gamma_1 < 1$, and there is an nonempty interval such that $\gamma \in (\gamma_1, 1)$ rules out the $\Pi = 0$ equilibrium when $T_{gm} = 1$. Also note that the nonmonetary equilibrium is less likely to exist under $T_{gm} = 1$ when we make T_{gg} small; that is, a policy that favors the use of money is more effective in combination with a policy that disfavors barter.

The next question to ask is: when will $\Pi = 1$ be an equilibrium under a $T_{gm} = 1$ policy? For simplicity, consider the case where we also set $T_{mg} = 1$, so that government agents not only accept money but also spend it whenever they can. This implies that in steady state $m_p = m_g = M$, and the condition for the existence of the $\Pi = 1$ equilibrium is

$$\Delta = (1-M)[\gamma(1 - yT_{gg}) + (1-\gamma)(1-y)] - K \geqslant 0.$$

If $K \leqslant 0$ then $\Delta \geqslant 0$ for all $\gamma \geqslant 0$; but if $K > 0$ then $\Delta \geqslant 0$ if and only if

$$\gamma \geqslant \gamma_2 = \frac{K - (1-M)(1-y)}{(1-M)\, y(1 - T_{gg})}.$$

If $K < (1-M)(1-yT_{gg})$, then $\gamma_2 < 1$, and there is an nonempty interval such that $\gamma \in (\gamma_2, 1)$ guarantees the $\Pi = 1$ equilibrium exists under the policy $T_{gm} = 1$. And note that as long as $K \geqslant (1-M)(1-y)$ the $\Pi = 1$ equilibrium does not exist without government. Note also that reducing T_{gg} makes $\Delta \geqslant 0$ more likely; again, a policy that favors the use of money is more effective in combination with a policy that disfavors barter.

We conclude that a policy of $T_{gm} = 1$ can guarantee the existence of monetary equilibrium, as well as rule out the existence of nonmonetary equilibrium, if and only if the government is sufficiently big relative to K, y, M, and T_{gg}. We turn now to a second type of policy, with $T_{gm} = 0$. Given this, can $\Pi = 1$ still be an equilibrium? That is, can money still circulate among private agents if government agents do not accept it? If $\Pi = 1$ then under this policy the steady state is:

$$(m_p, m_g) = \begin{cases} \left(\dfrac{M}{1-\gamma}, 0\right) & \text{if} \quad \gamma \leqslant 1 - M \\ \left(1, \dfrac{M + \gamma - 1}{\gamma}\right) & \text{if} \quad \gamma > 1 - M. \end{cases} \qquad (12)$$

Consider the case $\gamma \leqslant 1 - M$ (the other case is similar). The condition for the existence of the $\Pi = 1$ equilibrium is

$$\Delta = (1 - M - \gamma)(1 - y) - \gamma y T_{gg} - K \geqslant 0,$$

which is violated if and only if

$$\gamma > \gamma_3 = \frac{(1 - M)(1 - y) - K}{1 - y(1 - T_{gg})}.$$

If $K > -M(1 - y) - yT_{gg}$ then $\gamma_3 < 1$, and there is a nonempty interval such that $\gamma \in (\gamma_3, 1)$ eliminates the $\Pi = 1$ equilibrium under the policy $T_{gm} = 0$. And note that as long as $K \leqslant (1 - M)(1 - y)$ the $\Pi = 1$ equilibrium always exists without government. Also notice that a larger value of T_{gg} makes it more likely that the monetary equilibrium is eliminated.

One can also analyze polices where the government accepts money with some probability $T_{gm} \in (0, 1)$. For simplicity, consider the case where $T_{gg} = T_{mg} = 1$, so that the only instrument is the probability with which government agents accept money. Then one can show $\partial \hat{\Pi} / \partial T_{gm} < 0$, which means that an increase in the probability with which government agents accept money shifts the best response correspondence in Figure 1 to the left. If government is big enough, then as T_{gm} increases eventually $\hat{\Pi}$ becomes negative.[7] Hence, by increasing T_{gm} towards 1 the government makes it more likely that there is a unique equilibrium and it is $\Pi = 1$.

We conclude this section with a summary of some of the main results.

Proposition 1. *If and only if γ is big enough, a policy whereby government agents accept money with high probability can eliminate equilibria with $\Pi < 1$ that exist without government intervention, and can guarantee the existence of a unique equilibrium with $\Pi = 1$. This is true even if the $\Pi = 1$ equilibrium does not exist without government intervention. Such a policy is more effective if T_{gg} is small. Also, if and only if γ is big enough, a policy whereby government agents accept money with low probability can eliminate a monetary equilibrium that exists without intervention. This policy is more effective if T_{gg} is big.*

3. A MODEL WITH BARGAINING

In this section we assume goods are divisible. When agent i consumes q units of a good in S_i he enjoys utility $u(q)$, and when he produces q units

[7] To be more precise, suppose T_{gm} is given. Then one can show the following: If $K < (T_{gm} - y)(1 - M)$ then $\hat{\Pi} < 0$ if and only if $\gamma > [K + MyT_{gm} + (1 - M)y]/T_{gm}$; and if $K > (T_{gm} - y) \times (1 - M)$ then $\hat{\Pi} < 0$ if and only if $\gamma > [K + MyT_{gm} + (1 - M)y](1 - M)/[K + y(1 - M)]$.

of his production good he suffers disutility $c(q)$. We normalize $c(q)=q$ with no loss of generality, and assume that $u(0)=0$, $u'(q)>0$ and $u''(q)<0$ for all $q>0$. Also, there is a $\hat{q}>0$ such that $u(\hat{q})=\hat{q}$. Otherwise, everything is the same as in the previous section. In particular, we still assume that money is indivisible and that agents need to consume in order to produce, so that they always have either 1 or 0 units of currency. We continue to concentrate for now on steady state equilibria, and, as in the previous section, we begin with the economy without government as a benchmark.

When agents meet and there are potential gains from trade, they bargain. When two sellers meet and there is a double coincidence of wants we adopt the symmetric Nash bargaining solution, which implies that each produces q^* for the other where q^* satisfies $u'(q^*)=c'(q^*)$ (nothing important here depends on symmetric Nash, and other bargaining solutions would work fine for our purposes). When a buyer and seller meet, it turns out to simplify things a lot to assume that the buyer gets to make a take-it-or-leave-it offer, which allows him to extract the entire surplus. If a unit of money buys q units of output the nominal price level is $p=1/q$.

It facilitates comparison with the previous section to analyze the model as follows. Assume all agents expect that there is a "going market price" that implies one unit of money buys Q units of output, and when any individual buyer and seller meet the former chooses q taking Q as given (Q is analogous to Π and q is analogous to π in the model with indivisible goods). Then the value functions satisfy

$$rV_0=v_0+(1-M)\,y[u(q^*)-q^*]+M\max_{\pi}\pi[V_1-V_0-Q] \tag{13}$$

$$rV_1=v_1+(1-M)\Pi\max[u(Q)+V_0-V_1,0]. \tag{14}$$

Notice that the maximization problem in (14) implies the buyer chooses whether to spend his money or not when he meets a seller. However, in any monetary equilibrium, take-it-or-leave-it offers implies $Q=V_1-V_0$, which means $u(Q)+V_0-V_1=u(Q)-Q\geqslant 0$ for all $Q\leqslant\hat{q}$. Here we assume parameter values are such that $Q<\hat{q}$ in equilibrium, and so buyers are always willing to spend their money (a sufficient condition for this is that v_1 is not too big). Also notice that, given $Q\in[0,\hat{q}]$, a buyer's take-it-or-leave-it offer will be $q=q(Q)=\max[D(Q),0]$, where $D(Q)=V_1-V_0$. In equilibrium, then, either $Q=V_1-V_0>0$, or $V_1\leqslant V_0$ and $Q=0$, which implies $\pi=0$. In either case, the last term in (13) vanishes: $\pi[V_1-V_0-Q]=0$.

A steady state equilibrium is a fixed point of $q(Q)$. If $D(Q)<0$ then the equilibrium is nonmonetary and $\Pi=0$; if $D(Q)>0$ then the equilibrium is monetary and $\Pi=1$.

Assuming we are in a monetary steady state, manipulation of (13) and (14) yields

$$D(Q) = \frac{(1-M)\,u(Q) + MQ - k}{1+r}$$

where $k = (1-M)\,y[u(q^*) - q^*] + v_0 - v_1$. To reduce the number of cases, we assume here that $k > 0$ (but see below). Then $D(0) < 0$, and since $D(Q)$ is increasing and concave the best response correspondence is as depicted in Figure 2. There is always a nonmonetary equilibrium. Whether there exist other equilibria depends on parameters: there is a critical $\tilde{k}$ such that $k > \tilde{k}$ implies $D(Q)$ is below the 45° line for all Q and there are no monetary equilibria; and $k < \tilde{k}$ implies $D(Q)$ intersects the 45° line twice and there are two monetary equilibria, as shown in Fig. 2. In both monetary equilibria money is accepted with probability 1, but one has a low q_l and hence a high nominal price level, while the other has a high q_h and hence a low nominal price level.[8]

When multiple equilibria exist, it is easy to show that the q_h equilibrium Pareto dominates the q_l equilibrium and both dominate the nonmonetary equilibrium. However, even the best equilibrium is not necessarily efficient. For example, suppose that a planner can impose any q and seeks to maximize welfare, as given by $W = MV_1 + (1-M)\,V_0$. Then he chooses the q that solves $u'(q^*) = c'(q^*)$. In general, q may be bigger or smaller than q^* in equilibrium.

We now re-introduce government. For now, set $T = (1, 1, 1)$, so that in particular government agents always trade goods for money and money for goods, but not necessarily at the market price. Rather, assume that they supply q^s units of output in exchange for a unit of money and demand q^d units of output in exchange for a unit of money, where q^s and q^d are exogenous. Then we have

$$rV_0 = v_0 + (1-M)\,y[u(q^*) - q^*] + \gamma m_g \max(V_1 - V_0 - q^d, 0) \quad (15)$$

$$rV_1 = v_1 + (1-\gamma)(1-m_p)\Pi \max[u(Q) + V_0 - V_1, 0]$$
$$+ \gamma(1-m_g)\max[u(q^s) + V_0 - V_1, 0]. \quad (16)$$

[8] For completeness, we report the following results for $k < 0$. First, $k < 0$ implies all equilibria are monetary. Then there are two cases to consider. On the one hand, if $0 > k > -r\hat{q}$ then $D(0) > 0$ and $D(\hat{q}) < \hat{q}$, so there is a unique intersection of $q(Q)$ with the 45° line and a unique monetary equilibrium. On the other hand, if $k < -r\hat{q}$ then $D(Q)$ lies above the 45° line for all $Q \in [0, \hat{q}]$. This means that sellers are willing to accept money, but buyers prefer to hoard their crash and enjoy lifetime utility v_1/r.

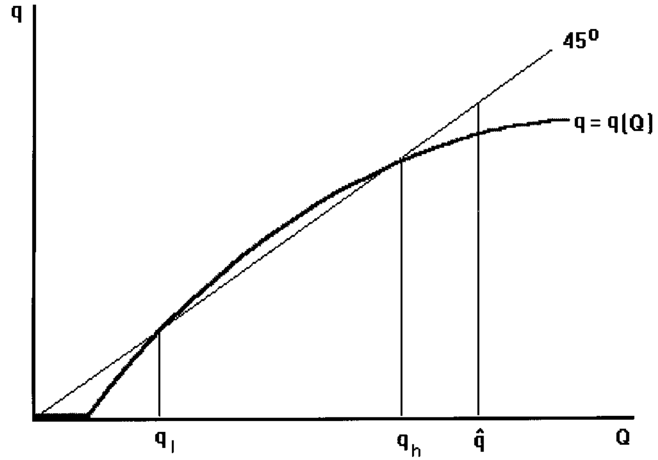

FIG. 2. Equilibria in the bargaining model ($0 < k < \tilde{k}$).

Notice that sellers will reject trades with government agents if q^d exceeds the market price, $q = V_1 - V_0$, since this is the most a seller would be willing to produce for a unit of money (by virtue of the take-it-or-leave-it offer assumption). Similarly, buyers may reject trades with government sellers (e.g., if q^s is small) or with private sellers (e.g., if q^s is big, in which case they may prefer to hold out for a government seller). Other than this, there are no restrictions on q^s or q^d, although we do point out that government agents consume more than they produce, which one might want to interpret in terms of a government deficit, if and only if $q^d > q^s$.

It is convenient to analyze q^s and q^d policies one at a time. First, consider an equilibrium with $\Pi = 1$, assume that government agents demand the same quantity as the market, $q^d = Q = V_1 - V_0$, and consider the effects of fixing q^s. Assume that q^s is such that $u(q^s) > V_1 - V_0$ (this is verified below). Then $m_p = m_g = M$, and manipulation of (15) and (16) yields

$$D(Q) = \frac{(1-M)[(1-\gamma)\,u(Q) + \gamma u(q^s)] + MQ - k}{1+r}.$$

Suppose that without government intervention there exist two monetary equilibria, which we now denote (q_l^o, q_h^o). We are interested in how the q^s policy changes things, depending on the size of government. To this end, notice that as we increase γ from 0, $D(Q)$ rotates clockwise around the point $[q^s, D(q^s)]$ as shown in Fig. 3. For $\gamma > k/(1-M)\,u(q^s)$ we have $D(0) > 0$, and therefore $q(Q)$ has a unique fixed point q^γ, as shown.

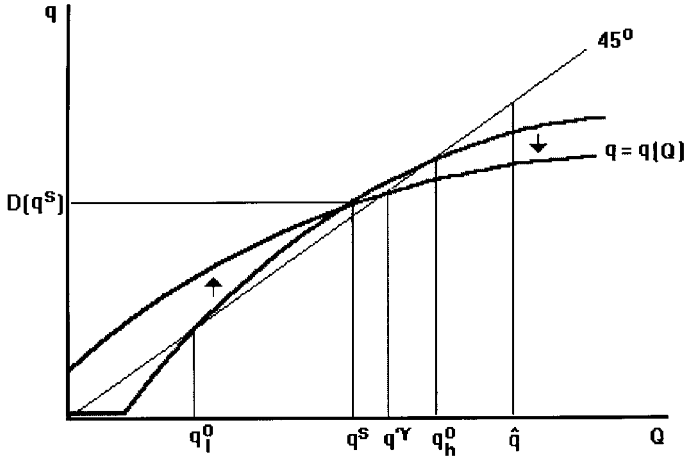

FIG. 3. Equilibrium with government policy.

We conclude from this that, when there are multiple equilibria, a sufficiently big government can eliminate all but one of them by committing to a particular q^s. In particular, suppose there are initially two monetary steady state equilibria without government intervention, (q_l^o, q_h^o), and set $q^s = q_h^o$. Then $\gamma \geqslant k/(1-M)\, u(q_h^o)$ implies that $q = q_h^o$ is the unique monetary steady state equilibrium. Thus, a sufficiently big government can insure that the economy coordinates on the best equilibrium simply by playing the appropriate equilibrium strategy itself. Notice that in this case total government consumption equals total government production. Moreover, in this case government agents enjoy the same utility as private agents, and even though we are not really taking their incentives into account in this analysis, they have no incentive to "defect" to any q other than q^s.

Now suppose that k is so big that monetary equilibria do not exist with $\gamma = 0$. It is still the case that $\gamma > k/(1-M)\, u(q^s)$ implies $q(Q)$ has a fixed point $q^\gamma \in (0, q^s]$. Hence, a q^s policy can also establish a monetary equilibrium that would not have existed without intervention. However, for such a result we must have $q^s > q^\gamma = q^d$, which implies that government agents are producing more than they consuming. In other words, the only way for the government to establish a monetary equilibrium where one did not exist without intervention is to have it give out more goods than it takes in. Moreover, in this case government agents enjoy lower utility than private agents. Still, the monetary equilibrium established under this policy may provide more utility to both government and private agents than the nonmonetary outcome (e.g., if y is small).

We now briefly consider the other policy, where government agents demand q^d in exchange for a dollar, but otherwise act exactly like private agents, in the sense that they accept money with whatever probability private agents choose, $T_{gm} = \Pi$, and always supply the market quantity, $q^s = q$. As indicated above, q^d must be below the equilibrium value of q for private sellers to agree to trade with government buyers. Now suppose, for example, we are initially in the q_h^o monetary equilibrium with $\gamma = 0$ and set $q^d < q_h^o$. As γ increases $D(Q)$ rotates clockwise around $[q^d, D(q^d)]$ and the equilibrium value of q_h decreases. If γ is big enough and q^d small enough, this policy can eliminate monetary equilibria. Intuitively, if government buyers demand a very low q^d then sellers will give private buyers only a very low q; but if q is too low private buyers may prefer to hoard their money. Hence, by committing to a very low q^d the government may be able to drive money out of circulation.

We conclude this section by summarizing some of the main results.

PROPOSITION 2. *If and only if γ is big enough, a policy whereby government agents supply q^s in exchange for money can guarantee the existence and uniqueness of a particular monetary equilibrium. In particular, if there does exist multiple monetary equilibria without government, setting $q^s = q_h^0$ implies $q = q_h^0$ is the unique equilibrium if γ is big. A sufficiently big government can also establish the existence of a monetary equilibrium even if one does not exist without intervention, although this implies that government agents must give out more goods than they take in. Also, a policy whereby government agents demand q^d in exchange for money can eliminate monetary equilibria that exist without intervention if q^d is low and γ big enough. Even if it does not affect whether or not money circulates, by fixing q^s or q^d the government can affect equilibrium prices.*

4. MULTIPLE CURRENCIES

Suppose there are two types of monies in the economy, called money 1 and money 2, with stock M_i of currency i and $M_1 + M_2 = M < 1$. As in previous sections, let us begin with the model without government as a benchmark. All agents take as given the probability Π_i that money i will be accepted as well as the market price as given by $p_i = 1/Q_i$. If the value function for an agent holding money i is V_i, then

$$rV_0 = v_0 + (1 - M)\, y[u(q^*) - q^*]$$

$$rV_1 = v_1 + (1 - M)\Pi_1 \max[u(Q_1) + V_0 - V_1, 0]$$

$$rV_2 = v_2 + (1 - M)\Pi_2 \max[u(Q_2) + V_0 - V_2, 0].$$

Note that there is no currency exchange in this model.[9] This gives the model a simple recursive structure, since we can solve for the value of each money independently.

Given $Q_i \in (0, \hat{q})$, a buyer with money i will demand $q_i = q_i(Q_i) = \max[D_i(Q_i), 0]$, where $D_i(Q_i) = V_i - V_0$. In an equilibrium with $\Pi_1 = \Pi_2 = 1$, we have

$$D_i(Q_i) = \frac{(1-M)\, u(Q_i) - k_i}{1+r}$$

where $k_i = (1-M)\; y[u(q^*) - q^*] + v_0 - v_i$, $i = 1, 2$. Depending on parameter values (k_1 and k_2 in particular), $q_i(Q)$ can have zero, one, or two fixed points in $(0, \hat{q})$. Each possible combination of q_1 and q_2 is a different equilibrium.

Various policies can be considered, but here we assume that government accepts currency i with probability T_{gi}, and, if it accepts it, supplies q_i^s in exchange. The value functions with government then satisfy

$$\begin{aligned}
rV_0 &= v_0 + (1-M)\; y[u(q^*) - q^*] \\
rV_1 &= v_1 + (1-\gamma)(1 - m_{p1} - m_{p2})\; \Pi_1 \max[u(Q_1) + V_0 - V_1, 0] \\
&\quad + \gamma(1 - m_{g1} - m_{g2})\; T_{g1} \max[u(q_1^s) + V_0 - V_1, 0] \\
rV_2 &= v_2 + (1-\gamma)(1 - m_{p1} - m_{p2})\; \Pi_2 \max[u(Q_2) + V_0 - V_2, 0] \\
&\quad + \gamma(1 - m_{g1} - m_{g2})\; T_{g2} \max[u(q_2^s) + V_0 - V_2, 0].
\end{aligned}$$

Following the previous section, we assume that parameter values are such that $Q_i < \hat{q}$, and that q_i^s is such that private agents do not reject trades with government sellers.

Consider first the case where government always accepts both monies at the exogenously fixed terms q_1^s and q_2^s. When $T_{gi} = \Pi_i = 1$, for $i = 1, 2$, we have $m_{pi} = m_{gi} = M_i$, and

$$D_i(Q_i) = \frac{(1-M)[(1-\gamma)\, u(Q_i) + \gamma u(q_i^s)] - k_i}{1+r}.$$

Following the analysis in the previous section, we can eliminate non-monetary and low q monetary equilibria if and only if the government is

[9] If $V_1 = V_2$ then currency exchange is irrelevant; and if, for example, $V_1 > V_2$, then a holder of money 1 would never exchange it for money 2. Note that this depends on our assumption that agents with money cannot produce until after they consume, since otherwise an agent with money 2 could potentially exchange it *plus* some output for the higher valued money 1, as in Aiyagari, Wallace, and Wright [3]. See Zhou [17] for a search model where currency exchanges occur for a different reason.

big enough. In particular, if we set $(q_1^s, q_2^s) = (q_{1h}^o, q_{2h}^o)$, where q_{ih}^o is the high value of money i without government intervention, $(q_1, q_2) = (q_{1h}^o, q_{2h}^o)$ is the unique equilibrium if and only if

$$\gamma > \max\left[\frac{k_1}{(1-M)\,u(q_1^s)}, \frac{k_2}{(1-M)\,u(q_2^s)}\right].$$

One can also think about using policy to drive one of the currencies out of circulation. Suppose, for example, that the government rejects one of them, say money 2. Thus, set $(T_{g1}, T_{g2}) = (1, 0)$ and $q_1^s = q_{1h}^o$ (obviously, q_2^s is irrelevant when $T_{g2} = 0$). By the usual reasoning, we can rule out equilibria with $\Pi_2 = 1$ if γ is sufficiently big. Then the only possible equilibria involve $\Pi_2 = 0$, so we are essentially back to an economy with only one money, and there will again be unique equilibrium with $\Pi_1 = 1$ if γ is big enough. We conclude that a sufficiently big government can establish a unique circulating currency with $(T_{g1}, T_{g2}) = (1, 0)$.

Policy can also be used to affect the market exchange rate, as defined by $e = q_1/q_2$. For example, suppose that we start in an equilibrium with $(q_1, q_2) = (q_{1h}^o, q_{2h}^o)$, and the government announces the official values of the monies to be $q_1^s = q_{1h}^o$ and $q_2^s = q_{2h}^o - \varepsilon$ for $\varepsilon > 0$. Then the equilibrium value of q_1 stays the same while q_2 falls, which moves the exchange rate e in favor of money 1. Notice, however, there will be a difference between the official and the market exchange rates, because the equilibrium q_2 exceeds q_2^s.

Summarizing some of these results, we have:

PROPOSITION 3. *If and only if γ is big enough, a policy whereby government agents accept both monies at fixed terms can guarantee the existence and uniqueness of a particular equilibrium where both monies circulate. A big government can also eliminate one money from circulation. Even if it does not affect which monies circulate, by fixing q_1^s and q_2^s, the government can affect market exchange rates.*

5. DYNAMICS

In general dynamic models can have multiple equilibria even if there is a unique steady state, and vice-versa. Hence, in this section, we analyze dynamic equilibria in the model with divisible goods and one fiat money. As above, we start with the model without government. Also, as above, we assume that buyers get to make take-it-or-leave-it offers and that sellers always produce q^* for each other in all barter trades.

Let V_{0t} and V_{1t} denote expected lifetime utilities for sellers and buyers at date t and let $D_t = V_{1t} - V_{0t}$. Then the dynamic generalizations of (13) and (14) are

$$rV_{0t} = v_0 + (1-M)\, y[u(q^*) - q^*] + \dot{V}_{0t} \tag{17}$$

$$rV_{1t} = v_1 + (1-M)\, \Pi_t \max[u(Q_t) - D_t, 0] + \dot{V}_{1t}. \tag{18}$$

Take-it-or-leave-it offers imply $Q_t = D_t$, where we assume that $D_t \leqslant \hat{q}$ (we will check this below), which is the incentive condition for buyers to be willing to spend their money. Also, we will write $\Pi_t = \Pi(D_t)$ below to indicate that the best response condition for a seller is $\Pi_t = 1$ $(=0)$ if $D_t > 0$ (<0) for all t.

One can subtract (17) and (18) to yield

$$\dot{D}_t = \varphi(D_t) = k + rD_t - (1-M)\, \Pi(D_t) \max[u(D_t) - D_t, 0]. \tag{19}$$

The value functions (V_{0t}, V_{1t}) satisfy the system given by (17) and (18), or equivalently by (17) and (19), which is more convenient because (19) depends only on the difference D_t. Hence, instead of looking for paths for (V_{0t}, V_{1t}), we define an equilibrium to be a bounded path for D_t satisfying $\dot{D}_t = \varphi(D_t)$.[10] There is no initial condition in the definition of equilibrium because D_0 can assume any value, as long as the implied time path from this initial value stays bounded.

It is easy to see that $\varphi(0) = k$, $\varphi'(D_t) = r$ for $D_t < 0$, $\varphi'(0^+) = -\infty$, and $\varphi''(D_t) > 0$ for $D_t > 0$. As we already know from previous analysis, if we assume $k \in (0, \tilde{k})$ then there exist two monetary steady states, denoted (q_l^o, q_h^o), as well as the nonmonetary steady state. This case is shown in Fig. 4, from which the following is clear: the set of equilibria includes the three steady states, plus a continuum of non-steady state equilibria indexed by initial beliefs, in the sense that for any $D_0 \in (-k/r, q_h^o)$ there exists an equilibrium, and in all such equilibria $D_t \to q_l^o$. In particular, there exist paths where D starts negative, so that money is initially not accepted, but then D becomes positive and converges to q_l^o.[11]

[10] For an equilibrium D_t must be bounded, because V_{0t} and V_{1t} are bounded by the following argument. At any date, instantaneous utility for sellers cannot exceed $v_0 + (1-M)\, y[u(q^*) - q^*]$, and for buyers cannot exceed $\max[v_1, u(\hat{q})]$ because $q_t \leqslant \hat{q}$ (if in equilibrium $q_t = D_t > \hat{q}$ then buyers will not trade). Since instantaneous utility is bounded and agents discount, the value functions must be bounded.

[11] In principle one needs to check the participation constraints, $V_{jt} \geqslant 0$ for all t, which will hold under conditions like those discussed in footnote 5. Obviously, we can still have $D_t < 0$ when these constrants are satisfied, so that buyers hold on to their money (rather than disposing of it) while $\Pi_t = 0$, waiting for the date at which we switch to $\Pi_t = 1$.

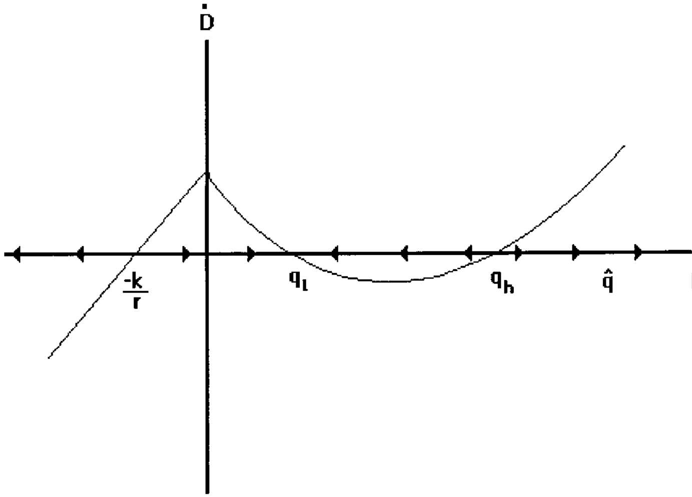

FIG. 4. Dynamic equilibrium without government.

We now re-introduce government. We set $T=(1, 1, 1)$ and consider the policy $q^s \in (0, \hat{q})$, where we suppose that in a monetary equilibrium $q^s < \hat{q}$ and $u(q^s) < D_t$ (of course, this has to be checked once we find a candidate equilibrium). The value functions are

$$rV_{0t} = v_0 + (1-M)\, y[u(q^*) - q^*] + \dot{V}_{0t}$$

$$rV_{1t} = v_1 + (1-\gamma)(1-m_{pt})\Pi \max[u(Q_t) - D_t, 0]$$

$$+ \gamma(1-m_{gt}) \max[u(q^s) - D_t, 0] + \dot{V}_{1t}.$$

Following the above analysis, we have $\dot{D}_t = \psi(D_t)$ where

$$\psi(D_t) = k + rD_t - (1-\gamma)(1-m_{pt})\Pi \max[u(Q_t) - D_t, 0]$$

$$- [\gamma - M + (1-\gamma)\, m_{pt}] \max[u(q^s) - D_t, 0]. \tag{20}$$

The model is now complicated by the fact that the measure of private agents holding money can change over time, and so we need to keep track of m_p as well as D. The law of motion is

$$\dot{m}_p = \begin{cases} M - m_p & \text{if} \quad \Pi = 1; \\ -[1 - M - (1-\gamma)(1-m_p)]\, m_p & \text{if} \quad \Pi = 0; \end{cases} \tag{21}$$

(ignoring the time subscripts from now on when there is no risk of confusion). Note that m_p must lie in an interval $[\underline{m}, \bar{m}]$ for all t.[12] An equilibrium is defined to be a bounded path for (D_t, m_{pt}) satisfying (20) and (21), plus an initial condition $m_{p0} \in [\underline{m}, \bar{m}]$, which is given at $t = 0$.

Suppose for now that there are two monetary steady states, denoted (q_l, q_h), in addition to the nonmonetary steady state. We start with a standard local analysis by linearizing (20) and (21). The determinant and trace of the Jacobian matrix evaluated at a monetary steady state are given by

$$\det = (1-\gamma)(1-M)\, u'(D_t) - (r+1-M)$$

$$\mathrm{tr} = r - M - (1-\gamma)(1-M)\, u'(D_t).$$

One can show that at q_h we have $\det < 0$, so q_h is a saddle point; and at q_l we have $\det > 0$ and $\mathrm{tr} < 0$, so q_l is a sink. The determinant evaluated at the nonmonetary steady state is

$$\det = -[1 - M - (1-\gamma)(1-2\underline{m})][r + \gamma - M + (1-\gamma)\underline{m}].$$

Since this is negative the nonmonetary steady state is a saddle point.

The locus of points in (m_p, D) space along which $\dot{m}_p = 0$ is given by $m_p = M$ if $D > 0$ and $m_p = \underline{m}$ if $D < 0$. The locus of points along which $\dot{D} = 0$ depends on q^s. Figures 5 and 6 show the case where $q^s > q_h^0$ and the case where $q_l^0 < q^s < q_h^0$ (other cases are similar except for the slopes of the different branches of $\dot{D} = 0$). In any case, it can be seen that for any initial m_p, there is a unique initial D on the saddle path leading to the steady state with high D; there is a unique initial D on the saddle path leading to the nonmonetary steady state; and for any initial D between the two saddle paths the system converges to the steady state with low D. All of these paths constitute equilibria. Any initial D that is not between the two saddles paths generates a path that is unbounded and hence not an equilibrium.

In particular, consider the case where the policy is $q^s = q_h^0$. Then the high D steady state is given by $D = q_h^0$, and the saddle path leading to it corresponds to the $\dot{D} = 0$ locus and is horizontal at $D = q_h^0$. Hence, for any initial m_p, there exists an equilibrium where $q_t = q_h^0$ for all t.

As is known from the analysis in the previous sections, if the government is big enough then all steady states except the one with the highest D will disappear. In this case, for all initial m_p, there is a unique equilibrium with

[12] The interval $[\underline{m}, \bar{m}]$ is defined as follows. If $\gamma < 1/2$ then: (1) $[\underline{m}, \bar{m}] = [0, M/(1-\gamma)]$ if $M < \gamma$; (2) $[\underline{m}, \bar{m}] = [(M-\gamma)/(1-\gamma),\ M/(1-\gamma)]$ if $\gamma < M < 1-\gamma$; and (3) $[\underline{m}, \bar{m}] = [(M-\gamma)/(1-\gamma), 1]$ if $1-\gamma < M$. If $\gamma > 1/2$ then: (1) $[\underline{m}, \bar{m}] = [0, M/(1-\gamma)]$ if $M < 1-\gamma$; (2) $[\underline{m}, \bar{m}] = [0, 1]$ if $1-\gamma < M < \gamma$; and (3) $[\underline{m}, \bar{m}] = [(M-\gamma)/(1-\gamma), 1]$ if $\gamma < M$. Note that $\underline{m} < M < \bar{m}$ in all cases.

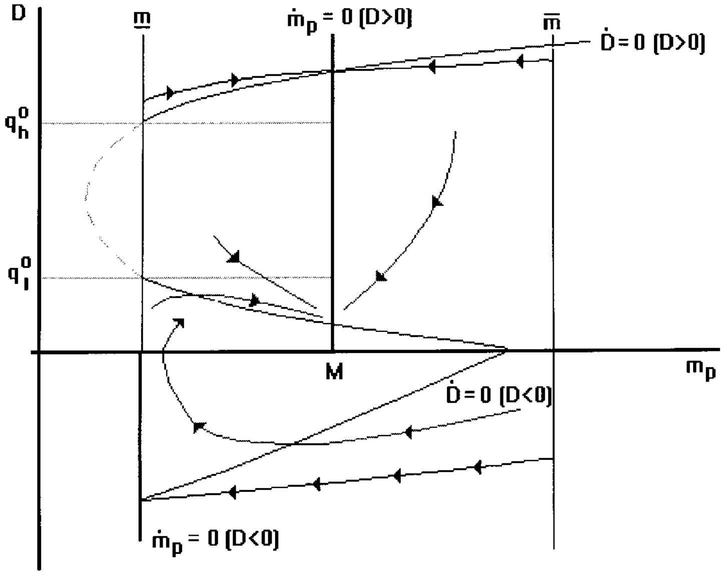

FIG. 5. Dynamic equilibria with government: $q^s > q_h^0$.

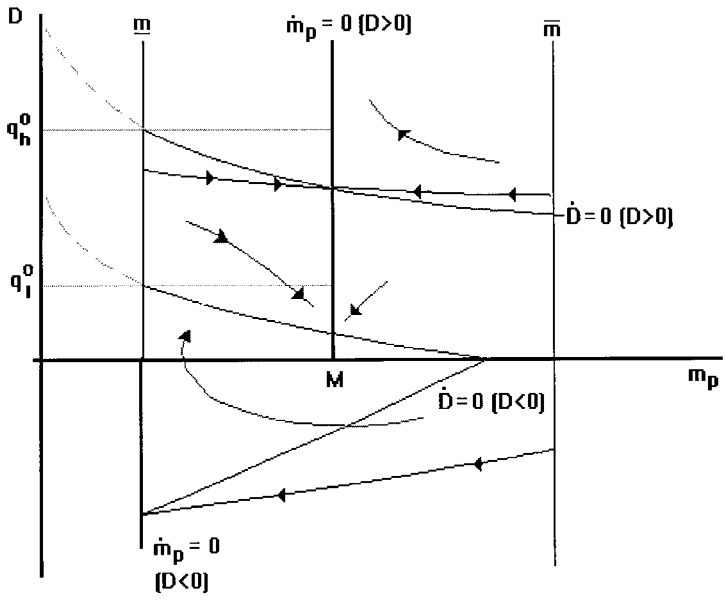

FIG. 6. Dynamic equilibria with government: $q_l^0 < q^s < q_h^0$.

D starting on the saddle path converging to the unique steady state (all other choices for starting values of D generate unbounded paths). This is shown in Fig. 7, where we set $q^s = q_h^o$, which implies that the saddle path is horizontal at $D = q_h^o$, and the unique equilibrium has constant prices. Notice that in this case each government agent consumes in trades with private agents the same amount that he produces in trades with private agents. But if we start with $m_p > M$ then out of steady state there will be more private buyers than government buyers, and so total government production exceeds total government consumption, while if we start with $m_p < M$ then the opposite is true.

We summarize these results on dynamic equilibria in the following proposition.

PROPOSITION 4. *If γ and q^s are such that there are two monetary steady states and a nonmonetary steady state, then for any initial condition for $m_p = m_{p0}$, the following equilibria exist: one with D_0 on the saddle path converging to (M, q_h); one with D_0 on the saddle path converging to the nonmonetary steady state; and a continuum starting at any D_0 in between the two saddle paths and converging to (M, q_l). If γ and q^s are such that (M, q_h) is the unique steady state, then for any initial condition $m_p = m_{p0}$ there is a unique equilibrium and it converges to (M, q_h). In particular, if γ is big*

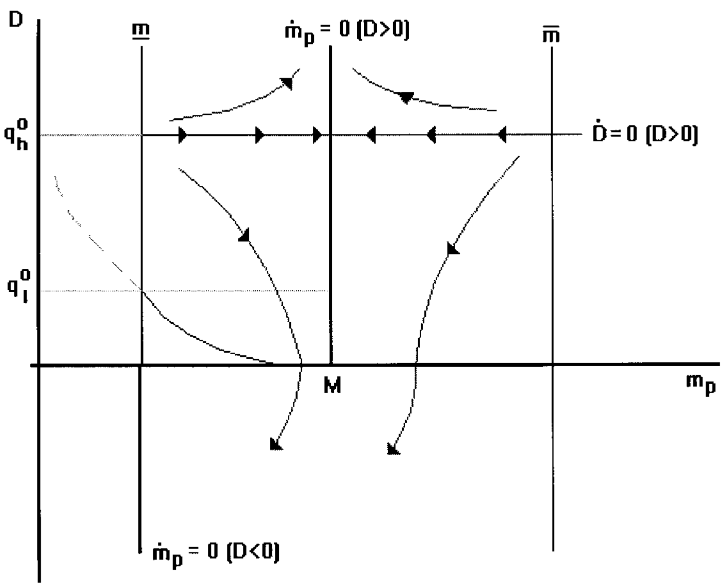

FIG. 7. Unique dynamic equilibrium with government.

enough and we set $q^s = q_h^0$, *then there will be a unique equilibrium and it will entail* $q_t = q_h^0$ *for all* t.

6. CONCLUSION

This paper has studied the effects of policies that specify what types of transactions the government makes and at what prices. It has been established that when the government is sufficiently big, accepts money with a sufficiently high probability, and gives a sufficient amount of output for that money, it can guarantee the existence and the uniqueness of an equilibrium where the money is universally accepted in private transactions, whether or not such an equilibrium exists without intervention. How big is sufficiently big depends on several factors, including properties of the money, the presence of alternative means of payment, and other aspects of policy. Also, by refusing to accept a certain money, or by accepting it only at a particular price, a sufficiently big government can preclude an equilibrium where it circulates. Even when government is not so big, so that these policies do not affect the number of equilibria, they will generally affect equilibrium prices and exchange rates.

Historically, government policies designed to influence which objects were used as money and at what prices have sometimes worked and sometimes failed. Friedman [5] describes a case that worked: France's success in maintaining a stable bimetalic commodity money system from 1785–1873. He argues that this was due to the combination of France's economic importance in the world and the large amounts of metal in the country at that time, and concludes that "These two factors made France a major participant in the market for silver and gold, an important enough participant to be able to peg the price ratio despite major changes in the relative production of silver and gold."

Kitayama [6] describes an example of a policy that failed. During the early stage of the colonial period in Taiwan (circa 1895), the Japanese colonial government required that taxes had to be paid in Japanese money, rather than the copper coins that were being used in private transactions at the time, with the purpose of establishing Japanese currency as medium of exchange. The effect was that people deferred paying taxes, decreasing revenue by so much that the policy had to be abandoned! It seems that the Japanese government was not such a major participant in the local economy at the time, and was therefore not able to exert enough influence, to determine which objects were used as money. Later, when its presence became more important in the Taiwanese economy, the money favored by the Japanese government did begin to circulate. Although we do not have taxation explicitly in our model, these examples are consistent with the basic prediction that size matters.

We did not spend much time discussing why the government might choose to follow particular policies; rather, here we have been content to describe the correspondence from exogenous government trading strategies to the endogenous strategies of the private sector and hence to the set of equilibria. Moreover, we did not necessarily insist on "balancing the budget" in the sense of constraining government agents to consume and produce the same amount on average. It would be interesting to consider maximizing some objective function, perhaps subject to some government budget constraint. We leave this to future research.

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