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Heterogeneity and Lotteries in Monetary Search Models

We introduce ex ante heterogeneity into the Berentsen, Molico, and Wright monetary search model with lotteries. We show that their three main results regarding lotteries do not survive this modification of the environment.

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BERENTSEN, MOLICO, AND WRIGHT (2002), hereafter BMW, introduce lotteries into monetary search models to deal with indivisibilities of goods and money. They obtain three key results. First, with indivisible goods and fiat money, goods always change hands with probability one while money can trade with probability less than one. Second, with fiat money, a social planner cares only about lotteries over goods, not money. Third, with divisible goods, the quantity traded is always less than or equal to the efficient quantity, $q \leq q^*$. BMW conjecture that these results are driven by the "fiat nature of money." In this paper, we construct a search model with fiat money to show that none of these results survive the introduction of ex ante heterogeneous agents. Thus, BMW's results are mainly driven by the symmetry of preferences across agents, not the fiat nature of money.

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1. THE MODEL

The environment is Kiyotaki and Wright (1993) with ex ante heterogeneous preferences as in Shevchenko and Wright (2004). There is a continuum of infinitely lived agents on the unit interval. Agents discount at rate r. They are specialized in the production and consumption such that there is the usual double coincidence of wants problems making barter impossible. Agents meet at random according to a Poisson process with arrival rate α. The probability an agent can produce one's desired consumption good is x.

We introduce ex ante heterogeneity across agents by assuming there are two types of agents, $i = H, L$, in the economy whose measures are given by μ_H and μ_L, respectively, with $\mu_H + \mu_L = 1$. Type i agents get utility $u_i(q)$ from consuming q units of their desired consumption good and incur utility cost $c_i(q)$ from producing q units of their production good. We assume $u_i' > 0$, $u_i'' \leq 0$, $u_i(0) = 0$, $c_i' > 0$, $c_i'' \geq 0$, and $u_i'(0)/c_j'(0) = \infty$ for all i, j. When goods are indivisible we assume that $u_i(1) = U_i$ and $c_i(1) = C_i$ where $U_i > C_H > C_L$ for $i = H, L$. This ordering ensures that there are gains from trade and eliminates market participation issues present in Camera and Vesely (2006). While we do not rank order U_H and U_L, we do assume that $C_H/C_L > U_H/U_L$. This simply means the percentage differences in the costs are greater than the differences in utility.

Due to the absence of a double coincidence of wants and anonymity, agents need money to trade. Let M be the fraction of agents with money and they are constrained to hold no more than one indivisible unit of money. Since there are different types of agents in the economy, the fraction of each type holding money can differ in equilibrium. Let m_H denote the fraction of high types holding a unit of money and m_L denote the same for low types. We then have

$$M = \mu_L m_L + \mu_H m_H. \tag{1}$$

Finally, since we only consider steady-state equilibria, let V_i^1 denote the stationary value function for an agent i holding one unit of money, and V_i^0 denote the value function for an agent i without money.

Consider the indivisible good case with buyer-take-all bargaining. This is without loss of generality since all of BMW's results hold, and are strengthened, with this form of bargaining. When a type i ($i = H, L$) buyer meets a type j ($j = H, L$) seller, he makes a take-it-or-leave-it offer to the seller that extracts the entire surplus from the seller. The offer consists of a pair of probabilities $(\lambda_{ij}, \tau_{ij})$ where λ_{ij} is the probability goods are traded between buyer i and seller j, while τ_{ij} is the probability money changes hands.[1] Buyer i's problem becomes

1. In an earlier working paper (Lotz, Shevchenko, and Waller 2004) we prove that this is the optimal lottery.

$$\max_{\lambda_{ij},\tau_{ij}} \lambda_{ij}\left[U_i - \frac{\tau_{ij}}{\lambda_{ij}}\left(V_i^1 - V_i^0\right)\right]$$
$$s.t.\ -\lambda_{ij}C_j + \tau_{ij}\left(V_j^1 - V_j^0\right) \geq 0, \quad \tau_{ij} \leq 1, \quad \lambda_{ij} \leq 1.$$

Since buyer i will extract the entire surplus of seller j, the constraint holds with equality. Using the constraint to substitute out for τ_{ij} in the objective yields:

$$\max_{\lambda_{ij}} \lambda_{ij}\left(U_i - C_j\frac{V_i^1 - V_i^0}{V_j^1 - V_j^0}\right)$$
$$s.t.\ \lambda_{ij}C_j\big/\left(V_j^1 - V_j^0\right) \leq 1, \quad \lambda_{ij} \leq 1. \tag{2}$$

It is clear that $\lambda_{ij} = 1$ is the solution as long as the buyer has positive expected surplus. It then follows that $\tau_{ij} = C_j/(V_j^1 - V_j^0)$ if $C_j \leq V_j^1 - V_j^0$. If $C_j > V_j^1 - V_j^0$, the constraint on τ_{ij} is violated. Nevertheless, the buyer can still offer an acceptable lottery: he offers $\tau_{ij} = 1$ and $\lambda_{ij} = (V_j^1 - V_j^0)/C_j \leq 1$. The buyer will choose to trade as long as his surplus is positive. The seller is indifferent and accepts the offer. Note that the lotteries only depend on the seller's type.

The intuition for these lottery values is clear; if money is valued more than the cost of producing a good, the seller is willing to give up his good for a fraction of the buyer's unit of money. With indivisible money, the buyer is unable to offer a fraction of his money. However, by offering a lottery over his unit of money, he is able to do so in expected value and this increases his surplus from the match. On the other hand, when the good is costly to produce relative to the value of a unit of money, the seller is only willing to give up a fraction of the good for a unit of money. Without lotteries this is not possible, so no trade occurs. But by offering a lottery over the good, a buyer can acquire a fraction of the good on average and this is better than not trading at all.

In a stationary equilibrium the following flow condition must hold

$$m_H(1 - m_L)\tau_{HL} = m_L(1 - m_H)\tau_{LH}. \tag{3}$$

If $\tau_{HL} \neq \tau_{LH}$, then $m_H \neq m_L$, and $m_H > M > m_L$ if $\tau_{LH} > \tau_{HL}$. Heterogeneous lotteries over money will alter the distribution of money balances across agent types. If $\tau_{HL} = \tau_{LH}$, as would be the case in a homogeneous agent model, equation (3) implies $m_H = m_L = M$ and there are no distributional effects from lotteries over money. Finally, note that λ_{ij} does not appear in this expression so lotteries over goods do not affect the distribution of money balances.

The value functions satisfy

$$\rho V_i^0 = \mu_H m_H\left[-\lambda_{Hi}C_i + \tau_{Hi}\left(V_i^1 - V_i^0\right)\right]$$
$$+\mu_L m_L\left[-\lambda_{Li}C_i + \tau_{Li}\left(V_i^1 - V_i^0\right)\right] \tag{4}$$

$$\rho V_i^1 = \mu_H(1 - m_H)\left[\lambda_{iH} U_i - \tau_{iH}\left(V_i^1 - V_i^0\right)\right] \\ + \mu_L(1 - m_L)\left[\lambda_{iL} U_i - \tau_{iL}\left(V_i^1 - V_i^0\right)\right], \tag{5}$$

for $i = H, L$ where $\rho = r/\alpha x$. Since buyers extract the entire expected surplus of the sellers, $V_H^0 = V_L^0 = 0$. The buyers' value functions can be rewritten as

$$V_i^1 = \frac{\mu_H \lambda_{iH}(1 - m_H) + \mu_L \lambda_{iL}(1 - m_L)}{\rho + \mu_H(1 - m_H)\tau_{iH} + \mu_L(1 - m_L)\tau_{iL}} U_i. \tag{6}$$

We can now state the following.[2]

Lemma 1: $\lambda_{ij} = \tau_{ij} = 0$ *is never optimal in any monetary equilibrium so* $\tau_{ij} = \tau_j > 0$ *and* $\lambda_{ij} = \lambda_j > 0$.

In short, lotteries only depend on the seller's type and buyers always find it optimal to trade with all sellers of their consumption good. Furthermore, equation (6) gives $V_L^1 = V_H^1 U_L / U_H$.

Definition 1: *A stationary monetary equilibrium is a list of value functions* V_i^1, *lotteries* τ_i *and* λ_i, *and money holdings* m_i, *for* $i = H, L$, *such that equations (1), (2), (3), and (6) are satisfied.*

2. THREE RESULTS

2.1 Lotteries over Goods

Proposition 1: *For critical values* $\rho_1 < \rho_2$:

1. *If* $\rho < \rho_1$, *then for*
   - *(i)* $M \in (0, \bar{M}_1]$, *a unique monetary equilibrium exists with* $\tau_L, \tau_H \leq 1$ *and* $\lambda_L, \lambda_H = 1$.
   - *(ii)* $M \in (\bar{M}_1, \bar{M}_2]$, *a unique monetary equilibrium exists with* $\lambda_H, \tau_L \leq 1$ *and* $\lambda_L, \tau_H = 1$.
   - *(iii)* $M > \bar{M}_2$, *no monetary equilibrium exists.*
2. *If* $\rho \in [\rho_1, \rho_2)$ *for* $M \in (0, \bar{M}_3]$, *a unique monetary equilibrium exists with* λ_H, $\tau_L \leq 1$ *and* $\lambda_L = \tau_H = 1$.
3. *For* $\rho > \rho_2$, *no monetary equilibrium exists.*

The proof is in the appendix. The key point of this proposition is that, unlike BMW, lotteries over goods can exist in a monetary equilibrium. To understand the intuition behind Proposition 1, consider $\rho < \rho_1$. For sufficiently low values of ρ and M, money

2. The proof involves a lot of tedious algebra and is available from the authors.

is highly valued so all sellers trade goods with probability one and receive money with a probability less than one. This replicates the monetary equilibrium found in BMW. However, for intermediate values of M, money is of moderate value so lotteries over goods occur with high-cost sellers. With $C_H/C_L > U_H/U_L$, high-cost sellers value money less than the cost of producing so they only want to give up a fraction of the good for money. Consequently, buyers have to offer a lottery over goods for exchange to occur. For high values of the money stock, no monetary equilibrium exists. This demonstrates that BMW's result regarding lotteries over goods is not robust when there is ex ante heterogeneity.

2.2 Welfare

We now want to address how heterogeneity affects welfare. Define welfare as the ex ante weighted average of lifetime utilities across agent types:

$$W = \mu_H\left[m_H V_H^1 + (1-m_H)V_H^0\right] + \mu_L\left[m_L V_L^1 + (1-m_L)V_L^0\right],$$

which using (4) and (5), can be written as

$$\rho W = \mu_H^2 m_H(1-m_H)\lambda_{HH}(U_H - C_H) + \mu_L^2 m_L(1-m_L)\lambda_{LL}(U_L - C_L) + \mu_H\mu_L m_L(1-m_H)\left\{\frac{\tau_{LH}}{\tau_{HL}}\lambda_{HL}(U_H - C_L) + \lambda_{LH}(U_L - C_H)\right\}. \quad (7)$$

Consider a planner who can choose lotteries in matches such that agents are willing to trade; i.e., they receive non-negative payoffs. Note that τ_{HH} and τ_{LL} do not appear in equation (7), so the planner does not care about lotteries over money when agents of the same type meet other than that they satisfy incentive constraints. This is similar to BMW's result. However, τ_{HL} and τ_{LH} do appear in equation (7) so the planner cares about lotteries over money as well as goods when different types of agents trade. The reason is they alter the composition of buyers and sellers. The optimal composition is to have most low-cost agents being sellers (m_L is low) and most high-cost agents being buyers (m_H is high), which can be achieved with appropriate lotteries over money. For example, we can show that for $\mu_H = \mu_L$ and values of ρ and M in the neighborhood of zero, the lottery values chosen by the planner are $\lambda_{ij} = 1$, $\tau_{LH} = 1$, and $\tau_{HL} < 1$, which results in $m_L < M < m_H$. Note that from part one of Proposition 1, $\tau_{LH} < 1$ for these parameter values and so the equilibrium allocation differs from the planner's. Hence, BMW's result that the planner does not care about lotteries over money breaks down when there is ex ante heterogeneity across agents.

2.3 Divisible Goods

The final result from BMW to address is their finding that when goods are divisible, the quantity traded is always less than or equal to the efficient quantity. We now show that this result does not hold with ex ante heterogeneity. To do so only requires examining the bargaining problem.

As before, when a type i buyer meets a type j seller, he makes a take-it-or-leave-it offer to the seller. Generally, the offer consists of τ_{ij}, and a pair of conditional probability distributions $[\lambda^0_{ij}(q), \lambda^1_{ij}(q)]$, where τ_{ij} is the probability money changes hands while $\lambda^m_{ij}(q)$ is the conditional probability measure of q that changes hands given $m = 0, 1$. It is easy to show that the quantity of goods that changes hands is degenerate and independent of m. Moreover, $\lambda^0_{ij}(q_{ij}) = \lambda^1_{ij}(q_{ij}) = 1$, where q_{ij} is the solution to the problem described below.

Then the buyer's problem is

$$\max_{q_{ij},\tau_{ij}} u_i(q_{ij}) - \tau_{ij}\left(V^1_i - V^0_i\right)$$

$$s.t. \quad -c_j(q_{ij}) + \tau_{ij}\left(V^1_j - V^0_j\right) \geq 0, \quad \tau_{ij} \leq 1. \tag{8}$$

Necessary and sufficient conditions for a solution are

$$u'_i(q_{ij}) - \eta^1_{ij} c'_j(q_{ij}) \leq 0, \quad = \text{if } q_{ij} > 0, \tag{9}$$

$$-\left(V^1_i - V^0_i\right) + \eta^1_{ij}\left(V^1_j - V^0_j\right) - \eta^2_{ij} \leq 0, \quad = \text{if } \tau_{ij} > 0, \tag{10}$$

where η^1_{ij} and η^2_{ij} are the non-negative multipliers of the constraints (8) and $\tau_{ij} \leq 1$, respectively. We are looking for monetary equilibria, which implies that these two conditions hold with equality. From Expression (9), it follows that $\eta^1_{ij} > 0$—buyer i will extract the entire surplus of seller j—so $V^0_j = 0$ for all j.

Consider the case where $\eta^2_{ij} = 0$, which implies that $\tau_{ij} \leq 1$. Then $\eta^1_{ij} = V^1_i / V^1_j$, and

$$u'_i(q_{ij}) = c'_j(q_{ij}) \frac{V^1_i}{V^1_j}. \tag{11}$$

Since $u_i(0) = c_j(0) = 0$ and $u'_i(0)/c'_j(0) = \infty$, there exists a unique value $\hat{q}_{ij} > 0$ satisfying this equation for any finite value of V^1_i / V^1_j. Thus, if $\hat{q}_{ij}$ is such that $c_j(\hat{q}_{ij})/V^1_j < 1$, then buyer i will give up money with probability less than one but ask for $\hat{q}_{ij}$. Remember that the efficient quantity q^*_{ij} traded between a buyer of type i and a seller of type j solves $u'_i(q_{ij}) = c'_j(q_{ij})$. Therefore, if $V^1_i < V^1_j$, then $\hat{q}_{ij} > q^*_{ij}$ and vice versa.[3] Analogous results hold for $\eta^2_{ij} > 0$, although $\tau_{ij} = 1$ in this case. The efficient quantity is only asked for if $V^1_i = V^1_j$ implying buyers have identical payoffs from holding a unit of money. Obviously, this is true for $i = j$; homogeneous agents will always trade the efficient quantity of goods. This is the reason why BMW find that $q = q^*$ if $\tau_{ij} < 1$ is feasible.

How do our results change if we assume ex post heterogeneity instead of ex ante heterogeneity? It is easy to show that a variation of Proposition 1 still holds, meaning

3. Berentsen, Camera, and Waller (2004) also get this result when agents have heterogeneous money balances. This result generalizes to divisible money models when agents have differing marginal values of money as in Molico (2006).

lotteries over goods will still exist in a fiat monetary equilibrium. However, it should be clear that a planner would not care if lotteries over money are used since all agents are the same ex ante so there is not way to change the composition of buyers and sellers by type. Finally, examining equation (11) reveals that if agents have the same out-of-match valuations of holding money, as will occur with ex post heterogeneity, then $q \leq q^*$ is the only equilibrium outcome as shown in BMW. Thus, results two and three of BMW survive the introduction of ex post heterogeneity while result one does not.

We have thus demonstrated that the three major results of BMW arise from assuming homogenous agents and not from the fiat nature of money.

APPENDIX

Proof. (i) Assume $\rho < \rho_1$ where ρ_1 is defined below. Conjecture $\tau_L, \tau_H < 1; \lambda_L = \lambda_H = 1$. Using equation (6) the steady-state flow condition and the value function for H are given by

$$\frac{m_H}{m_L} = \frac{(1 - m_H)}{(1 - m_L)} z \tag{A1}$$

$$\begin{aligned} \rho V_H^1 &= \mu_H(1 - m_H)(U_H - C_H) + \mu_L(1 - m_L)\frac{U_H}{U_L}(U_L - C_L) \\ &= (1 - M)U_H - M\frac{(1 - m_H)}{m_H}C_H, \end{aligned} \tag{A2}$$

where $z = C_H U_L / U_H C_L > 1$. Using equations (1) and (A1) to eliminate m_L gives

$$f(m_H) = \mu_H(z - 1)m_H^2 - [1 + (z - 1)(M + \mu_H)]m_H + zM = 0. \tag{A3}$$

Since $f(m_H)$ is quadratic with $f(0) > 0, f(1) < 1$, and $f(M) > 0$, there is a unique value $\tilde{m}_H(M)$ solving equation (A3) with $0 < M < \tilde{m}_H(M) < 1$. Thus $\tilde{m}_H(M) > M > \tilde{m}_L(M)$. Totally differentiate equations (1) and (3)

$$\frac{dm_H}{dM} = \frac{1}{\mu_H m_H^2 + z m_L^2 \mu_L} > 0.$$

Then

$$\frac{dV_H^1}{dM} = -\frac{1}{\rho}\left[(U_H - C_H) + \frac{C_H}{m_H}\left(1 - \frac{M}{\mu_H m_H + \mu_L m_L \dfrac{1 - m_L}{1 - m_H}}\right)\right] < 0,$$

since the term in the second round bracket is positive. For this type of equilibrium to exist ($\tau_L < \tau_H = \frac{C_H}{V_H^1} < 1$) we require that $V_H^1|_{M=0} > C_H$, where

$$\rho V_H^1 |_{M=0} = \mu_H(U_H - C_H) + \mu_L \frac{U_H}{U_L}(U_L - C_L).$$

So

$$\mu_H \left(\frac{U_H - C_H}{C_H}\right) + \mu_L \frac{1}{z}\left(\frac{U_L - C_L}{C_L}\right) > \rho. \tag{A4}$$

Let ρ_1 be the value such that expression (A4) holds with equality. Assume $\rho < \rho_1$. When M increases V_H^1 goes down. Then $\tau_H \to 1$ as $V_H^1 \to C_H$, or $M \to \bar{M}_1$, where $\bar{M}_1$ solves

$$\begin{aligned}\rho C_H &= \mu_H[1 - m_H(\bar{M}_1)](U_H - C_H)\\ &\quad + \mu_L[1 - m_L(\bar{M}_1)]\frac{U_H}{U_L}(U_L - C_L) = g(\bar{M}_1),\end{aligned} \tag{A5}$$

where $\bar{m}_L(\bar{M}_1) = M - \mu_H \bar{m}_L(\bar{M}_1)$. We have $g'(\bar{M}_1) < 0$, $\rho C_H < g(0)$ (since expression (A4) holds) and $\rho C_H > g(1)$. Consequently, there is a unique value $0 < \bar{M}_1 < 1$ solving equation (A5).

(ii) Assume $\rho < \rho_1$ and conjecture $\tau_L < 1$, $\tau_H = 1$; $\lambda_L = 1$, $\lambda_H < 1$. The value functions and steady-state flow equation reduce to

$$\begin{aligned}V_H^1 &= \frac{\mu_L(1 - m_L)\frac{U_H}{U_L}(U_L - C_L)}{\rho - \mu_H(1 - m_H)\frac{U_H - C_H}{C_H}}\\ m_H(1 - m_L)C_L &= (1 - m_H)m_L\frac{U_L V_H^1}{U_H}.\end{aligned} \tag{A6}$$

Since $V_L^1 = U_L V_H^1/U_H$, if $V_L^1 > C_L$ then $m_H > M > m_L$, and if $V_L^1 = C_L$ then $m_H = M = m_L$. For $V_H^1 > 0$ we need

$$\rho > \mu_H(1 - m_H)\frac{U_H - C_H}{C_H}. \tag{A7}$$

Condition (A7) is satisfied for all $M \geq \bar{M}_1$. To see this note that $\mu_H(1 - m_H)\frac{U_H - C_H}{C_H}$ is decreasing in M. This means that if $\rho C_H > \mu_H(1 - m_H)(U_H - C_H)$ holds for $\bar{M}_1$, then it holds for $M > \bar{M}_1$. Since V_H^1 is continuous at $V_H^1 = C_H$ (compare equations (A2) and (A6)), expressions (A5) and (A7) imply that if

$$(1 - \bar{M}_1)U_H - \bar{M}_1\frac{1 - m_H}{m_H}C_H > \mu_H(1 - m_H)(U_H - C_H) \tag{A8}$$

then inequality (A7) holds for all $M > \bar{M}_1$. Inequality (A8) can be rewritten as

$$\left[\frac{1-\bar{M}_1}{1-m_H}-\mu_H\right](U_H-C_H)+C_H\left[\frac{1-\bar{M}_1}{1-m_H}-\frac{\bar{M}_1}{m_H}\right]>0.$$

This condition holds because both square brackets are positive.

Now we show that $\frac{dV_H^1}{dM}<0$. Totally differentiating the value function, the steady-state flow equation and equation (1) one can get

$$\frac{dV_H^1}{dM}=-\frac{D}{F},$$

where

$$D\equiv\mu_H\frac{m_H}{m_L}\frac{U_H-C_H}{C_H}V_H^1+\mu_L\frac{1-m_L}{1-m_H}\frac{U_L-C_L}{C_L}\frac{C_LU_H}{U_L}>0,$$

$$F\equiv(1-m_L)\left(\frac{\mu_Hm_H}{m_L}+\frac{\mu_L}{1-m_H}\right)\left[\rho-\mu_H(1-m_H)\frac{U_H-C_H}{C_H}\right]$$
$$+(1-m_L)\mu_L\mu_Hm_H\frac{U_H-C_H}{C_H}>0.$$

Hence V_H^1 is a decreasing function. We have already shown that at $M=\bar{M}_1: V_H^1=C_H$. Then V_H^1 decreases with M until it reaches $\frac{C_LU_H}{U_L}$. At $V_H^1=\frac{C_LU_H}{U_L}$, $\tau_H=\tau_L=1$ so $m_H=m_L=M$. Thus, $V_H^1=\frac{C_LU_H}{U_L}$ at $M=\bar{M}_2$, where

$$\bar{M}_2=1-\frac{\rho}{\mu_H\frac{U_H-C_H}{C_H}+\mu_L\frac{U_L-C_L}{C_L}}. \tag{A9}$$

In order for $\bar{M}_2<1$, we must have

$$\rho<\mu_H\frac{U_H-C_H}{C_H}+\mu_L\frac{U_L-C_L}{C_L}, \tag{A10}$$

which holds since expression (A4) is assumed to hold. Since V_H^1 is continuous in M with $dV_H^1/dM<0$ and $V_H^1(\bar{M}_1)>V_H^1(\bar{M}_2)$ it follows that $\bar{M}_1<\bar{M}_2$. Therefore, for $M\in(\bar{M}_1,\bar{M}_2]$, there exists an equilibrium solution such that $\lambda_H\le 1$, $\tau_H=1$, and $\tau_L\le 1$, $\lambda_L=1$. Notice that at $M=\bar{M}_2$, we have indeterminacy of V_H^1.

Finally, consider $\tau_L=1$, $\tau_H=1$; $\lambda_L<1$, $\lambda_H<1$. In this case

$$\rho V_H^1=\mu_H(1-m_H)\left(\frac{U_H-C_H}{C_H}\right)V_H^1+\mu_L(1-m_L)\frac{U_H}{U_L}\left(\frac{U_L-C_L}{C_L}\right)V_H^1,$$

which implies $V_H^1=0$ unless $M=\bar{M}_2$. So for all $M>\bar{M}_2$, no monetary equilibrium exists. It then follows that if $\rho<\rho_1$, $\tau_L<1$, $\tau_H\le 1$; $\lambda_L=1$, $\lambda_H=1$ is optimal for $M\in[0,\bar{M}_1]$, $\tau_L\le 1$, $\tau_H=1$; $\lambda_L=1$, $\lambda_H\le 1$ is optimal for $M\in(\bar{M}_1,\bar{M}_2]$, and $\tau_i=\lambda_i=0$ for $i=H,L$ when $M>\bar{M}_2$.

(iii) Now consider the case where $\rho \geq \rho_1$. In this case, $\tau_H < 1$ is not feasible even at $M = 0$. The only feasible equilibrium is $\tau_L \leq 1$, $\tau_H = 1$; $\lambda_L = 1$, $\lambda_H \leq 1$. In this case, the value function and flow conditions are still given by equation (A6). Furthermore, $V_L^1 = C_L$ at $\bar{M}_3$ where $\bar{M}_3$ solves equation (A9) at this new value of ρ. No monetary equilibrium exists for $M > \bar{M}_3$. Let the value of ρ that satisfies expression (A10) with equality be denoted ρ_2. Comparing expressions (A4) to (A10) we have $\rho_1 < \rho_2$ since $z > 1$. Thus, for $\rho \in (\rho_1, \rho_2)$ the only monetary equilibrium has $\tau_L \leq 1$, $\tau_H = 1$; $\lambda_L = 1$, $\lambda_H \leq 1$. For $\rho > \rho_2$ no monetary equilibrium exists for any value of M.

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