The Transition from Barter to Fiat Money

By JOSEPH A. RITTER*

How did it become possible to exchange apparently valueless pieces of paper for goods? This paper provides an equilibrium account of the transition between barter and fiat-money regimes. The explanation relies on the intervention of a self-interested government which must be able to promise credibly to limit the issue of money. To achieve credibility, the government must offset the benefits of seigniorage by internalizing some of the macroeconomic externalities generated by the issue of fiat money. The government's patience and the extent of its involvement in the economy are key determinants of whether the transition can be accomplished. (*JEL* E42)

> "The Mongol IlKhans in Persia, impressed by the use of paper money by their suzerain in China, decided to use the same device themselves. Technical advisers were sent from Peking, and an elaborate organization was set up. The Persians, however, had not been accustomed to the use of paper currency by several hundred years of gradual developments. They simply refused to believe that these nicely printed pieces of paper were worth anything, and the experiment was a failure."
>
> —Gordon Tullock (1957)

How did it become possible to exchange apparently valueless pieces of paper—fiat money—for goods, and why did it take so long for pure fiat money to become prevalent, given the obvious benefits to its issuers? The juxtaposition of these questions at first seems paradoxical. This paper argues that the answers to these two questions are fundamentally linked. The first question has occupied monetary theorists for decades, though much of modern monetary theory has avoided it in favor of one that is closely related but more easily answered: Can an artificial economy be structured in a reasonable way so that there are equilibria in which an apparently worthless commodity has a positive price? The second puzzle has received little attention, though on reflection it is striking that widespread use of purely fiat money is a 20th-century development (Milton Friedman and Anna J. Schwartz, 1986), since the idea and physical implementation are simple.

This paper presents an explicit solution to the first puzzle in a simple artificial economy. In formal terms I ask whether there is an equilibrium transition path between a barter equilibrium and a steady-state monetary equilibrium. The paper also suggests a possible solution to the second, less widely considered puzzle. The answer suggested here is that the money issuer or government must attain a critical level of "credibility" before the transition can take place.

Most economists would probably relate a stylized "history of money" something like the following. As specialization caused problems in coordinating trade, societies naturally settled upon certain commodities, usually metals, as media of exchange. Later, the minting of a certain quantity of a metal into coin acted as a signal of the quantity and purity of the metal. The signals of standardization needed credibility, and the process of standardization was therefore frequently undertaken by governments who had established a reputation for some degree of honesty (which they frequently exploited). From coinage, which was commodity money

*Research Department, The Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63116. I thank Joe Haubrich, Dale Stahl, Charles Swanson, David Wheelock, Leslie Young, and the referees for helpful suggestions and discussions. This paper represents the views of the author and not necessarily those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

in most senses, it was a relatively small step to substitute the use of paper representing contracts between the bearer and a bank or government. The contract specified payment on demand of a fixed quantity of a commodity (or coin).

From time to time, particularly during periods of crisis, governments suspended convertibility of paper currency (i.e., they reneged on the contract). Why did the currency continue to have value (though it frequently depreciated relative to goods) during such episodes? The obvious answer to this question focuses on the belief that convertibility would soon be restored.[1] While it is an easy matter to verify whether coinage has been debased, it is not easy to verify a government's good intentions or lack thereof. Therefore the government must have already established credibility in some way *before* the episode of inconvertibility so that economic actors have some reason to believe that the suspension is temporary.

The next stage is troubling for monetary theory, though hints of the trouble have already arisen above. Relatively recently in historical terms, governments have renounced even the intention of restoring convertibility of their currencies. Individuals are expected to accept intrinsically worthless pieces of paper, fiat money, in exchange for goods and services. These pieces of paper no longer represent any contract, and there is no expectation that they will in the future. Paradoxically, they continue to be an acceptable medium of exchange. How can such a situation be supported? Obviously, one requirement is that the supply of paper money be limited. The question of the government's credibility again arises, but the government has the conflicting motive of seigniorage. "Credibility" in this context must mean that individuals believe that the government will not attempt to exploit this source of seigniorage to the point where money becomes worthless.

There are, of course, many possible sources of such credibility.[2] Political scientists would point to notions of the legitimacy and stability of the government. Sociologists might imagine a tacit social pact between the government and citizens. Economists are, of course, always inclined to pursue the implications of agents' rational self-interest.

Without denying the importance of other motivations, I follow the latter course and ask under what conditions a policy of limiting the supply of fiat money will be incentive-compatible, given the government's explicit objectives. In the specific model used here, credibility is bestowed upon the government as a consequence of its size and patience. Before a critical size is reached, the government cannot credibly limit its money issue, so the money it issues is worthless. Similarly, if the government is too impatient for seigniorage revenue, its money is worthless. This extends and formalizes a similar argument by Friedman (1959)[3] and follows in the spirit of Friedman and Schwartz's later call for a public-choice perspective on monetary policy. I argue therefore that the transition from a barter economy to a monetary economy can be formally understood as the result of a self-interested intervention by a government which is large relative to the economy.

Section I outlines the model and the barter equilibrium that is assumed to exist

[1]It could be argued that suspension was a well-understood contingency in the contract, but the point is not important in the present context. Michael D. Bordo and Finn E. Kydland (1992) discuss the role of the gold standard as such a contingent rule.

[2]It is possible, of course, to supplement what I am calling credibility with legal restrictions. There are many historical examples of government attempts to use laws to enforce the use of debased or inflated currency. They have usually been unsuccessful in the short run and always unsuccessful in the long run. Since legal restrictions have a long history and fiat money, for the most part, a short one, they have apparently been notably unsuccessful in establishing fiat-money regimes. Also, see footnote 18.

[3]"...[S]ome external limit must be placed on the volume of a fiduciary currency in order to maintain its value. Competition does not provide an effective limit, since the value of the promise to pay, if the currency is to remain fiduciary, must be kept higher than the cost of producing additional units. The production of a fiduciary currency is, as it were, a technical monopoly, and hence, there is no presumption in favor of the private market as there is when competition is feasible" (p. 7).

in the absence of money. Section II describes steady-state monetary equilibria. The proposition that no monetary equilibrium can exist when rational individuals issue money is established in Section III. Section IV demonstrates that coordinated action by a group of agents which is large relative to the economy—a government—can, if certain conditions are met, allow the economy to circumvent problems which prevent existence of a monetary equilibrium when individuals issue money. The artificial economy described here undergoes a transition directly from barter to a fiat-money equilibrium. However, the analysis also sheds light on the historical question of why fiat money had only a sporadic existence until recently, despite obvious advantages to its issuers. Section V discusses this issue in detail in light of the results in Section IV.

I. The Model and Barter Equilibrium

Fiat money is troubling for textbook Walrasian theory because the theory predicts that it will not have value. This is not surprising given the metaphor of a central Walrasian auctioneer who instantaneously reconciles all excess demands in the economy. It is necessary to alter the model in a fundamental way to admit the possibility that a commodity which does not directly produce utility for any agent will have a positive price. In the most well-known such alteration, the overlapping-generations model, money's ability to facilitate current transactions plays no role; instead it makes possible otherwise impossible intergenerational transactions. Other major approaches, cash-in-advance constraints and inclusion of real balances in the utility function, appeal only indirectly to the ability of money to facilitate transactions, but this aspect is kept in the background for simplicity.[4] Thus none of the standard models is appropriate for the study of a monetary transition.

Recently, Nobuhiro Kiyotaki and Randall Wright (1991, 1993) have constructed search models along the lines of Peter Diamond (1982) in which a fixed stock of fiat money can have positive value in a steady-state equilibrium because it allows agents to circumvent the famous problem of the double coincidence of wants. They show that such a monetary equilibrium is quite robust to changes in the environment, including the presence of assets that dominate money in rate of return.[5] The model used here follows in this vein.

The economy consists of an infinite number of agents who live forever. Each agent is periodically endowed with one indivisible unit of one of J different market goods ($J \geq 2$) and derives utility from only one of the J goods and from a nonmarket good. For convenience below, define $x \equiv 1/J$. Agents are uniformly distributed over the J^2 possible types of agents, and the marginal distributions of endowment type and consumption-good type are independent. All agents must trade in order to consume any market good, including those who consume the good with which they are endowed.[6]

In each period, agents decide whether to attempt to trade. If they do not, they consume one unit of the nonmarket good. Each agent who does enter the market is randomly matched with another agent, and they decide whether they wish to trade. If an agent concludes a trade in which he acquires the type of good from which he derives utility, he consumes it immediately and receives a new endowment at the start of the next period.[7] Otherwise he consumes nothing. The holdings of either goods or money (defined below) by agents who do

[4] Robert C. Feenstra (1986) and Dean Croushore (1993) show that money in the utility-function models can be derived from a more primitive shopping technology which *assumes* that real balances make shopping more efficient.

[5] Diamond (1984) also introduces money into a search-theoretic framework in which all transactions are required to use money.

[6] This assumption reduces algebraic clutter considerably without altering the model in any important way. Kiyotaki and Wright (1993) explicitly relax this assumption in their Appendix A.

[7] The "unemployed" state in Diamond's (1982) model in which agents wait for production opportunities is not central here, so it is collapsed.

not attempt to trade or do not successfully trade are unchanged. Thus agents will always have exactly one unit of either money or a market good at the start of the next period.

At time t the expected lifetime utility of an agent is given by

$$V_t = E_t\left\{ \sum_{\tau=t}^{\infty} \beta^{\tau} C_{\tau} \right\}$$

where $\beta \in (0,1)$ is a discount factor and

$$C_r = \begin{cases} U & \text{if the consumer obtains his market consumption good at time } \tau \\ A & \text{if the consumer consumes the nonmarket good at time } \tau \text{ (autarky)} \\ 0 & \text{otherwise.} \end{cases}$$

The expectation is conditional on the state of the economy and the holdings of the agent at time t.

Money is defined to be an additional commodity which is freely available in indivisible units to any agent at any time. However it may be uniquely differentiated by the issuing agent or group of agents (i.e., counterfeiting is not possible). No agent derives utility directly from money. Money can be issued or goods consumed while an agent carries money or goods, but each agent can carry over only one unit of either goods or money to the next period. As in Kiyotaki and Wright's (1989, 1991, 1993) work, this means that money, if it is used, serves primarily as a medium of exchange and only trivially as a unit of account, since relative prices are constrained by the technology to be either 0 or 1.

Assume initially that there is no money. I will say that a barter equilibrium exists if each agent prefers to attempt trade given that other agents attempt trade. Let V^{B} be the expected utility of an agent in a barter equilibrium. No time subscript is necessary at this point because an agent always goes to the market holding goods and faces the same distribution of potential trading partners. Define b as the probability of meeting another agent with whom barter is possible, that is, the probability an agent of type jk (endowed with j, consumes k) meets an agent of type kj. In a barter equilibrium $b = x^2$ where $x \equiv 1/J$. In a barter equilibrium an agent will successfully trade and get payoff U in the current period with probability b. He returns to the market to play the same game in the following period whether he gets the payoff or not. Thus the following equation determines V^{B}:

$$V^{\mathrm{B}} = b(U + \beta V^{\mathrm{B}}) + (1-b)\beta V^{\mathrm{B}}.$$

This gives

$$V^{\mathrm{B}} = \frac{bU}{1-\beta}.$$

Given that all other agents barter, the agent's Bellman equation is

$$V = \max\{bU + \beta V, A + \beta V\}$$

with the maximization performed over the choice of autarky or barter. I assume that $Ux^2 > A$ so that barter is preferred in every period and $V = V^{\mathrm{B}}$. Consequently, a barter equilibrium exists.

II. Steady-State Monetary Equilibria

A useful starting point in the study of money in this model is to assume an exogenously determined money stock and to ask what conditions allow a steady-state monetary equilibrium to exist. This provides a point of reference with earlier search models (Kiyotaki and Wright, 1991, 1993) and a useful analytical base for the study of equilibria other than steady states. Since the economy is (eventually) to be studied outside steady states, the definition of a monetary equilibrium allows for the explicit treatment of subgames that start in a given period.

Definition: A *monetary equilibrium* (at t) is a sequence of money supplies $\{M_{\tau}\}_{\tau=t}^{\infty}$ and actions and expectations by agents, such that, given the actions and expectations of all other agents, (i) at least one agent attempts to trade in $\tau \geq t$ and (ii) each agent who trades accepts money in exchange for goods in $\tau \geq t$.

Assume for the moment that there is a constant money supply M, measured as the fraction of the population holding money. How the current money supply M may have gotten into circulation is the central focus of the paper and will be addressed subsequently. Let $V^{\mathrm{M}}(M)$ and $V^{\mathrm{G}}(M)$ be the expected lifetime utilities of trading agents holding money or goods, respectively, in a monetary equilibrium with a constant money supply M. Let p_t be the probability per period that an agent holding money will be able to *purchase* his consumption good using money, and let s_t be the probability that an agent holding a good will be able to *sell* it for money. In a steady-state monetary equilibrium the constant transition probabilities are $b=(1-M)x^2$, $p=(1-M)x$, and $s=Mx$.[8] The symmetry of the model guarantees that no agent will be made better off by trading either goods or money for a good that he does not consume; these transition probabilities do not depend on the commodity held or desired. This eliminates the usual reasons for the emergence of a particular commodity money. Robert A. Jones (1976), Kiyotaki and Wright (1989), and Seonghwan Oh (1989) construct models in which commodity money arises endogenously.

If a steady-state equilibrium exists with money supply M, the expected lifetime utilities of agents holding money and goods are given by

$$(1)\quad V^{\mathrm{G}}(M)=b[U+\beta V^{\mathrm{G}}(M)]+s\beta V^{\mathrm{M}}(M)+(1-b-s)\beta V^{\mathrm{G}}(M)$$

$$(2)\quad V^{\mathrm{M}}(M)=p[U+\beta V^{\mathrm{G}}(M)]+(1-p)\beta V^{\mathrm{M}}(M).$$

These equations imply

$$(3)\quad V^{\mathrm{G}}(M)=(1-M)\left[1+\frac{\beta M(1-x)}{1-\beta(1-x)}\right]V^{\mathrm{B}}$$

and a similar expression for $V^{\mathrm{M}}(M)$. It will be useful later to notice that while $V^{\mathrm{G}}(M)$ and $V^{\mathrm{M}}(M)$ are quadratic in M, their difference is linear in M:

$$(4)\quad V^{\mathrm{M}}(M)-V^{\mathrm{G}}(M)=\frac{p-b}{1-\beta(1-p-s)}U=(1-M)\psi$$

where $\psi>0$ depends only on U, x (or J), and β.

The conditions under which a steady-state monetary equilibrium exists are characterized by the following proposition.

PROPOSITION 1: *If the money supply is fixed at M from t onward, there is an $\overline{M}<1$ such that a monetary equilibrium exists at t if and only if $M \leq \overline{M}$.*

PROOF:

In the case of constant money supply it is sufficient to show that for an appropriate $\overline{M}$: (i) goods-holders will enter the market when $M \leq \overline{M}$ and will not enter when $M > \overline{M}$ and (ii) money-holders will enter the market whenever goods-holders choose to enter. (They obviously do not enter when goods-holders do not.)

Since the economy is in a steady state, it is sufficient for part (i) to show that it is not optimal to consume the reservation good in the current period, that is,

$$V^{\mathrm{G}}(M)>A+\beta V^{\mathrm{G}}(M)$$

[8] The probability p, for example, is the product of the probability $(1-M)$ that the agent will meet someone holding goods times the probability x that the agent will meet someone who is endowed with the good he consumes. These formulas become more complicated in later sections, but I keep the same b, p, s notation for barter, purchase, and sale probabilities.

if and only if $M < \overline{M}$ for some $\overline{M}$. Since $V^{G}(M)$ is quadratic and concave in M, greater than $A/(1-\beta)$ when $M = 0$, and equal to zero when $M = 1$, it follows that an appropriate $\overline{M}$ exists.

Part (ii) requires that

$$V^{M}(M) \geq A + \beta V^{M}(M).$$

if $M < \overline{M}$. This follows from the fact that $V^{M}(M) > V^{G}(M) > A/(1-\beta)$ by (4) and part (i) of the proof.

The intuition behind Proposition 1 is simple: When an equilibrium exists, agents accept money because other agents accept money, but as the money supply increases, it becomes easier to trade goods for money but harder to trade money for goods. Thus if the proportion of the population holding money is too high, on average it takes too long to find goods for market activity to be worthwhile, so autarky is preferred. This phenomenon is just a specialized version of "too much money chasing too few goods" in a stylized model where inflation cannot occur.

III. Laissez-Faire Money

Can a monetary equilibrium be generated by the actions of rational individuals issuing money? An agent holding goods who meets someone with whom he cannot barter, but who is holding his consumption good, will always have an incentive to issue money. Not surprisingly, this causes any potential monetary equilibrium to collapse because too much money is issued. Since individual decisions do not result in "the social contrivance of money," this provides a role for the institution—a government—which is introduced in the next section.

PROPOSITION 2: *There is no monetary equilibrium in which agents issue money individually.*

PROOF:

There are two varieties of potential equilibria to be considered. Either all agents issue undifferentiated money, or agents accept differentiated money issued by a fraction α of all agents. The first is formally equivalent to the second when $\alpha = 1$, so no distinction need be maintained. An agent of type jk carrying good j who meets an agent of type ki carrying good k with $i \neq j$ issues money if he is in the subset of agents whose money is accepted since this results in a utility gain of U. The difference equation that governs the evolution of the money supply under this process is:

$$M_{t+1} = M_t + \alpha m_t (1 - M_t) \tag{5}$$

where $m_t = (1 - M_t)x(1-x)$ is the probability that an agent carrying goods will wish to issue money.[9] The last term in (5) is the proportion of agents carrying goods $(1 - M_t)$ who can issue valuable money (α) and who will meet an agent carrying the good they wish to consume, but with whom barter is not possible (m_t). Clearly M_t converges to 1. As M_t converges to 1, m_t, p_t, and b_t each converge to 0. Since the agents consume only if one of the corresponding events occurs, utility of both goods and money-holders converges to 0 as well. Since utility of $A/(1-\beta)$ can be obtained in the reservation activity, at some time goods-holders must cease to enter the market, and money becomes worthless. The well-known backward-induction argument implies that it could not have had value in any previous period.

Proposition 2 can be interpreted as the consequence of a kind of coordination failure: the average agent would be better off with a positive amount of money in circulation,[10] yet because individuals do not consider the aggregate externality they impose by issuing money, this outcome is not supported by individual rationality. Thus valued money cannot arise endogenously with-

[9]This probability m_t later becomes more complicated, but, as for b, p, and s, I will keep the same notation for analogous quantities.

[10]If $2\beta(1-x) > 1$, *every* agent would be made unambiguously better off by the introduction of at least a small amount of money.

out adding more structure to the model. Since the model (which is essentially the same in this respect as Kiyotaki and Wright's [1991] model) contains no mechanism for attaining the steady-state equilibrium, it is incomplete in an important sense. The way in which the model is completed has important implications for existence of a monetary equilibrium and the welfare of agents in the equilibrium.

IV. Government Money

The previous sections showed that while in principle a monetary equilibrium can exist in this economy when there is a small enough fixed money supply, individuals' self-interest cannot accomplish the transition from barter equilibrium to a steady-state monetary equilibrium. This section introduces a government which provides a mechanism that allows the transition to occur.

The government is formally modeled as a coalition of otherwise unremarkable agents.[11] This is not intended as a definitive model of government, but only as a vehicle to give substance to two crucial aspects of the problem which would otherwise be too ill-defined to allow progress: the government's objectives and the notion of credibility. Indeed, there is no theoretical reason why this coalition should be called a government; it serves none of the other functions usually associated with a government. There is no theoretical reason, for example, why this could not represent a large bank whose goal is to maximize the private gains from issuing notes. Friedman and Schwartz (1986 p. 45) report, however, that they "do not know... of any example of the private production of purely inconvertible fiduciary moneys." Moreover, given the obvious benefits of fiat money, it is hardly surprising that governments have given themselves legal monopolies in its production.

To be specific, I assume that, for reasons outside the scope of the model, a coalition of agents exists encompassing a fraction $\alpha > 0$ of the population and uniformly distributed over the J^2 types. This coalition is called "the government." Only one such coalition forms.[12] Such a monopoly might arise for historical or political reasons, and seigniorage would provide one strong incentive to establish and maintain it. The structure and objectives of the government are known to all agents. There is nothing special about the interaction of these agents with the rest of the economy except that they can issue money which is identified with the government and that they act in concert. The government does not levy taxes, and it does not have a monopoly on a particular good. It has, however, the ability to limit the issue of money by individual members. This ability to act collectively to limit money issue (i.e., as a monolithic entity) distinguishes the government from the fraction α of the population hypothesized in the proof of Proposition 2. The fact that the government is large in relation to the rest of the economy suggests that it may have an incentive to limit the amount of money issued by the coalition so that a monetary equilibrium will not collapse.

Naturally, this is a very narrow view of the nature and function of government, but it has at least two benefits. First, its symmetry makes it technically tractable. Second, it gives the government no advantage in the issue of money other than the ability of the coalition members to act collectively or, equivalently, its size (it can be treated as a single agent). In particular, the government has no comparative advantage in the sale or purchase of a particular subset of goods. It

[11] S. Rao Aiyagari and Neil Wallace (1993) use a similar construct to study the effects of government intervention on welfare and uniqueness of equilibrium in a search-theoretic model of money.

[12] Though the government is formally modeled as a coalition of agents, the term I have chosen for this coalition emphasizes that it should be regarded here as a single monolithic political entity whose formation is outside the scope of this paper. An interesting alternative would be to study an equilibrium in which both the number and size of coalitions were allowed to be endogenous as an attempt to understand why there are different national monies.

will become apparent below that size endows the government with a limited kind of credibility. The structure imposed thus emphasizes that it is size (credibility) alone which allows the government to overcome the coordination failure which otherwise prevents the establishment of a monetary equilibrium in this economy.

At the time the government becomes active in monetary affairs ($t = 0$), the economy is still in a barter equilibrium since, as shown in the previous section, a monetary equilibrium cannot be established without it. The government chooses a monetary policy which maximizes the expected welfare of a representative member. *Ex ante* all members are identical except that they hold and consume different goods, so this policy also maximizes individual members' expected utility. Since the government's size and composition are exogenously determined, I do not address the issues surrounding internal incentives; with respect to the issuing of money, the government coalition *by assumption* acts collectively.

The government chooses a sequence of values of μ_t, the proportion of its members permitted to issue money, with money-issuing licenses allocated randomly if $0 < \mu_t < 1$. An agent with a license to issue money will wish to do so if he is not already holding money and meets a trader who has a good he wishes to purchase but with whom barter is impossible.[13] Proposition 3 characterizes the shape of the optimal money-supply path and greatly simplifies later analysis.

PROPOSITION 3: *The government's optimal path of M_t will follow*

$$M_{t+1} = M_t + \mu_t \alpha m_t (1 - M_t)$$

with

$$\mu_t = \begin{cases} 1 & t < T \\ 0 \le \mu_t \le 1 & t = T \\ 0 & t > T \end{cases} \tag{6}$$

for some finite $T \ge 0$. In other words, the government prefers to increase the money supply from zero to a steady-state value M as rapidly as possible. Moreover, $0 \le M \le \overline{M}$.

PROOF:

In this model, seigniorage is the additional consumption that results from the ability to consume sooner than if it were not possible to issue money. Therefore since $\beta < 1$, it is optimal to issue money as soon as possible.[14] The fact that $M \le \overline{M}$ follows from the fact that a steady-state monetary equilibrium exists only if the money supply is less than or equal to $\overline{M}$.

Thus the optimal monetary policy is a transition path of finite length to a steady-state monetary equilibrium. Proposition 3 does not provide any reason to think that $M > 0$ is feasible for the government (that traders will accept money), only that, if it is feasible, money is issued as rapidly as possible. Along the transition path, individual members of the coalition are permitted to issue money at will until the last period of the transition path when only a fraction of the members may issue money. In period T, licenses to issue money are randomly awarded only to the fraction of the coalition's members required to reach the money-supply target. Thus if m_t is the probability that a coalition member will want to issue money, the probability μ_t of a coalition member being permitted to issue money is set so that $M_{t+1} - M_t = \alpha \mu_t m_t (1 - M_t)$.

A. *Monetary Equilibria with Binding Precommitment*

When precommitment is possible, Proposition 3 allows the government's planning

[13]The member of the government would be indifferent between barter and money issue, but the trading partner would not be. I assume therefore that if barter is possible it takes place.

[14]If agents were risk-averse, there would be an incentive to spread out the benefits of issuing money over a longer period.

problem to be reduced to choice of a steady-state money supply M:

$$(7)\quad \max_M W_0(M) = \sum_{t=0}^{\infty} \beta^t [M_t p_t + (1-M_t) \times (b_t + \mu_t m_t)] U$$

subject to

$$M_{t+1} = M_t + \mu_t \alpha m_t (1-M_t)$$

with μ_t defined as in Proposition 3 ($\mu_t = 1$ before M_t reaches M and $\mu_t = 0$ after). Denote the solution to (7) by M^*. A natural conjecture at this point is that the government by virtue of its size has an incentive to limit money issue sufficiently that a monetary equilibrium can exist. This conjecture turns out to be correct. The proof requires the following lemma, whose proof is conceptually straightforward but algebraically tedious and therefore presented in the Appendix.

LEMMA 1: *Along any path of the money supply which satisfies Proposition 3, goods-holders and money-holders will prefer market activity to the reservation activity.*

PROPOSITION 4: *When the government can irrevocably commit itself to a monetary policy, the money-supply path induced by solving (7) is a monetary equilibrium.*

PROOF:

It is sufficient to show that the solution to (7) is characterized by $M^* \leq \bar{M}$ since Lemma 1 guarantees that agents will be willing to attempt trade at any point along the transition path. Since $M^* > \bar{M}$ results in valueless money, the value to the government of such a strategy would be $W_0(M^*) = V^{\mathrm{B}}$; the barter equilibrium would persist. It may be easily verified, however, that $(1-M)V^{\mathrm{G}}(M) + MV^{\mathrm{M}}(M)$ has a positive derivative at $M = 0$, so given that $V^{\mathrm{G}}(0) = V^{\mathrm{B}}$, it is apparent that there is at least one $M > 0$ to which the government could commit itself that would make it better off.

B. *Monetary Equilibria without Precommitment*

The ability to precommit is equivalent to an assumption that the government is unable to issue money once it reaches the money supply that solves (7). If this assumption is relaxed, the nature of the game played between the government and the public is more complicated, because the government's plan may be dynamically inconsistent.

Consider a monetary equilibrium in the model with precommitment. In the subgame that begins in period $T+1$ the money supply is constant at M^*. If precommitment is not possible, it is necessary to determine whether a constant money supply M^* that solves (7) is a Nash equilibrium of this subgame. If M^* is an equilibrium of the subgame, Lemma 1 will then guarantee that the entire path is a monetary equilibrium; that is, the solution to (7) is subgame perfect. By construction M^* is the best value less than $\bar{M}$ and therefore is the government's best option among the class described by Proposition 3. However, the government has another choice: once M^* is achieved, drive the economy back to barter by issuing more money. This is a perfect-foresight model, so in the absence of precommitment this option must not be preferred by the government if money is to have value. Proposition 5 adds the condition required for a monetary equilibrium without precommitment.

PROPOSITION 5: *In the absence of binding precommitment, the path for the money supply induced by solving (7) is a monetary equilibrium if and only if the solution M^* satisfies*

$$(8)\quad V^{\mathrm{B}} \leq W^* \equiv (1-M^*)V^{\mathrm{G}}(M^*) + M^* V^{\mathrm{M}}(M^*).$$

PROOF:

Since M^* solves (7), rational agents foresee that only the prospect of driving the

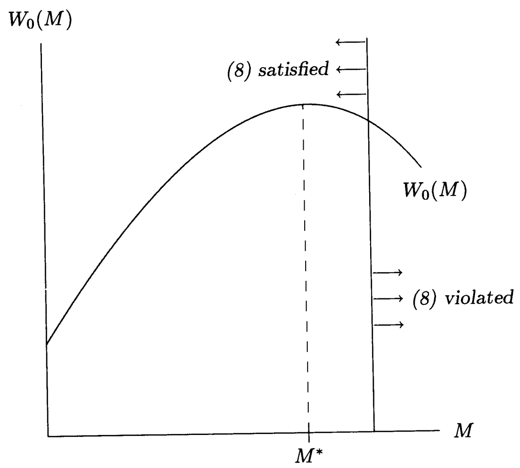

FIGURE 1. MONETARY EQUILIBRIUM EXISTS

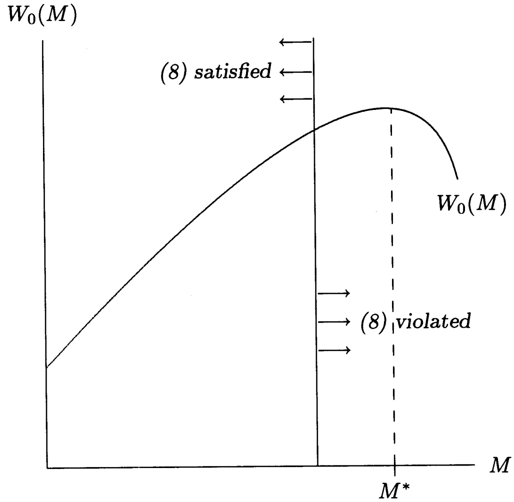

FIGURE 2. MONETARY EQUILIBRIUM DOES NOT EXIST

economy back to barter might induce the government to issue more money. Therefore this is the only kind of deviation from the proposed Nash equilibrium (of the subgame) that must be considered. If the government does issue more money, traders will no longer accept money, since it will ultimately be worthless. W^* is the subgame utility of the government if it does not issue more money. Suppose first that $W^* < V^B$. Since money becomes worthless immediately (so there is no transition path), the government achieves subgame utility V^B, and as $W^* < V^B$, the government prefers this outcome. Hence M^* is not a Nash equilibrium of the subgame, and no monetary equilibrium exists. Suppose instead that $W^* > V^B$. For the same reasons, there would be no transition path if the government chose to issue more money, so again the government achieves V^B by issuing additional money. Now, however, the government has no incentive to deviate from the proposed equilibrium. In this case, Lemma 1 guarantees that the money-supply path is a monetary equilibrium.

Figure 1 shows an example of a monetary equilibrium at M^*. Along the vertical line, (8) is satisfied with equality. Figure 2 illustrates a case in which the monetary equilibrium does not exist. If binding precommitment were possible, the government would choose to issue M^*, but this choice is not dynamically consistent; traders will realize that once the money supply is M^*, it is in the government's interest to force the economy back to barter by making money worthless.

Proposition 5 provides only implicit answers to the questions posed in the Introduction. More explicit statements require a closer examination of the behavior of the solution to (7) as parameter values vary. A necessary condition for $M^* \leq \bar{M}$ to be optimal is that the government does not have an incentive to issue a small additional amount of money. The appropriate derivative is somewhat more complicated than it would be for a simple comparison of steady states. Suppose that M^* is reached at T. Starting from a baseline with $M_T = M_{T+1} = M_{T+2} = \cdots = M$, does the government wish to increase the money supply during period T? This would change $M_{T+1}, M_{T+2}, \ldots$, but not M_T. The appropriate necessary condition is

$$\frac{\partial W_T(M_T, M_{T+1}, M_{T+1}, \ldots)}{\partial M_{T+1}} = 0$$

where the derivative is evaluated at $M_T =$

$M_{T+1} = M^*$, where W_T is the welfare of the government from the standpoint of period T. By exploiting the fact that W_T involves one period of transition followed by a steady state, it is possible to get an explicit expression to differentiate:

$$\begin{aligned}
&W_T(M_T, M_{T+1}, M_{T+1}, \dots) \\
&\quad = M_T W_T^{\mathrm{M}}(M_T, M_{T+1}, M_{T+1}, \dots) \\
&\qquad + (1 - M_T) W_T^{\mathrm{G}}(M_T, M_{T+1}, M_{T+1}, \dots) \\
&\quad = M_T\big[p_T U + p_T \beta V^{\mathrm{G}}(M_{T+1}) \\
&\qquad\qquad + (1 - p_T)\beta V^{\mathrm{M}}(M_{T+1})\big] \\
&\qquad + (1 - M_T)\big[(b_T + \mu_T m_T) U \\
&\qquad\qquad + s_T \beta V^{\mathrm{M}}(M_{T+1}) \\
&\qquad\qquad + (1 - s_T)\beta V^{\mathrm{G}}(M_{T+1})\big] \\
&\quad = \left[M_t p_T + (1 - M_T) b_T + \frac{1}{\alpha}(M_{T+1} - M_T)\right] U \\
&\qquad + \beta V^{\mathrm{G}}(M_{T+1}) \\
&\qquad - M_{T+1}\beta\big[V^{\mathrm{G}}(M_{T+1}) - V^{\mathrm{M}}(M_{T+1})\big]
\end{aligned}$$

where $W_T^{\mathrm{M}}(\cdot)$ and $W_T^{\mathrm{G}}(\cdot)$ are the expected utilities of coalition members holding money and goods, respectively, at time T. (These differ from the utilities of nonmembers because of the possibility of seigniorage.) The last equality makes use of the facts that $\mu_t m_t = (M_{T+1} - M_t)/[\alpha(1 - M_t)]$ and $s_t = xM_t + \alpha\mu_t m_t$ since the probability of a goods-holder meeting the appropriate kind of money-holder is xM_t, and the probability that he will meet a coalition member who wants to and is allowed to issue money is $\alpha\mu_t m_t$. Performing the differentiation and letting ψ be the same constant defined in (4), the necessary condition is:[15]

$$0 = \frac{1}{\alpha}U + \beta V^{\mathrm{G}\prime}(M) + \beta\psi(1 - 2M).$$

Solving the equation for M yields

$$\text{(9)}\quad M^* = \frac{1}{2}\left(\frac{1-2x}{1-x}\right) + \frac{1}{2\alpha}\left(\frac{1}{x(1-x)}\right)\left(\frac{1-\beta}{\beta}\right).$$

Condition (8) is satisfied with equality at $(1-2x)/(1-x) = (J-2)/(J-1)$, where J is the number of goods. According to Proposition 5, an equilibrium exists if and only if $M^* < (J-2)/(J-1)$. Therefore, combining (8) and (9), an equilibrium exists if and only if the parameters α and β lie in the region

$$\text{(10)}\qquad \beta \geq \frac{1}{1 + \alpha\left(\dfrac{J-2}{J^2}\right)}.$$

with $\alpha \leq 1$ and $\beta \leq 1$ as usual. This relationship is shown in Figure 3.[16]

Proposition 6 highlights two particularly interesting applications of (10) which are apparent from Figure 3.

PROPOSITION 6: *(i) For $J > 2$ there is a $\beta_{\min} \in (0, 1)$ such that an equilibrium does not exist for any α if $\beta < \beta_{\min}$. (ii) Given $J > 2$ and $\beta \in [\beta_{\min}, 1)$, there is an $\alpha_{\min}(\beta) \in (0, 1]$ such that an equilibrium exists if and only if $\alpha \geq \alpha_{\min}(\beta)$. (An equilibrium does not exist for $J = 2$.)*

Impatience increases the value of the short-term payoff of seigniorage relative to steady-state payoffs with the result that a government is more likely to desire a money supply that violates (8). Thus even when α is large, an equilibrium exists only for high values of β (low rates of time preference).

[15]It is easily verified that this is a maximum.

[16]Written in terms of the rate of time preference ρ, the boundary is linear; $\rho \leq \alpha(J-2)/J^2$.

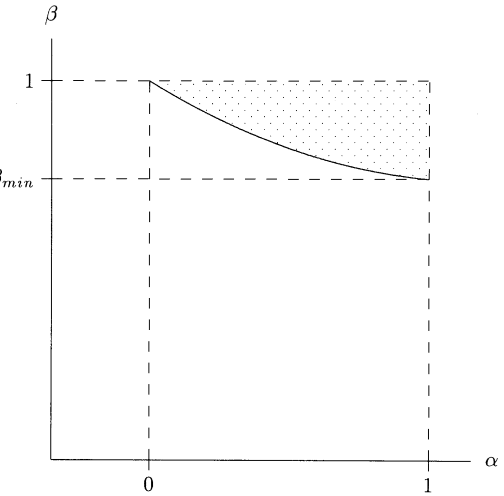

FIGURE 3. MONETARY EQUILIBRIUM EXISTS FOR PARAMETERS IN STIPPLED REGION

Proposition 6 suggests an answer to the second question posed in the Introduction. Whether the government has enough credibility in this economy to issue fiat money is determined by the interplay of its impatience (β) and its size (α). If the government is too impatient, it cannot credibly promise to restrict the money supply, so its currency will have no value regardless of its size [part (i)]. Even if the rate of time preference is low, the steady-state money supply desired by a government that is smaller than $\alpha_{\min}(\beta)$ still does not satisfy the time-consistency constraint (8) [part (ii)]. Since the government cannot credibly limit the money supply, agents will not accept its money. The more general lesson is clear: individuals must believe that the lure of additional seigniorage is offset by some other interest before they will accept unbacked currency. In this paper the internalization of macroeconomic externalities generated by issuing money provides the offsetting interest; the government recognizes that its steady-state utility will be measurably lower if it issues more money.

Even in the restrictive framework developed in this paper, it is not difficult to imagine why fiat currency was a relatively late development. The discount factor plays a powerful role in the model. If the government is too eager to extract resources via seigniorage, other considerations do not even come into play. The absence of effective tax-collection systems, especially when it is a consequence of political instability, can be plausibly interpreted in this way, since the government would be forced to exploit the printing press if it were available. However, a low rate of time preference is not by itself a sufficient condition for existence of an equilibrium. In addition the government must internalize a sufficiently large share of the aggregate externality generated by issuing money. For a strictly self-interested government like the one analyzed, this requirement translates into size, but if, for example, the government were motivated to maximize the expected utility of all agents, its objective function would internalize the externality entirely.

It can be shown that the first term on the right-hand side of (9) is the value of M that maximizes steady-state welfare. Note that the monetary policy that maximizes social welfare is obtained when $\alpha = 1$ (i.e., when the government is the grand coalition and maximizes the welfare of the representative citizen). Equation (9) then contains two important and intuitively appealing welfare implications of this model. First, because society values seigniorage, the socially optimal monetary policy leads to a steady-state money supply that is higher than that which maximizes steady-state welfare. Kiyotaki and Wright (1993), for example, derive the latter. Second, it is obvious from (9) that when $\alpha < 1$, the government will issue more than the socially optimal quantity of money if an equilibrium exists; it does not fully internalize the aggregate externality generated by issuing money but captures all social gains from seigniorage. This is really just a weaker version of the problem, which leads to absence of an equilibrium if α is too small and appears to be a robust implication of a public-choice approach to money.

V. Discussion

The direct transition from barter to fiat money modeled in this paper appears at first glance to be counterfactual in two ways.

The historical evolution of money is generally thought to proceed from barter to commodity money to fiat money, and the transition has been punctuated by many reversals. The theoretical results should, therefore, be interpreted in light of the following observations about the relationship between reality and the theory.

First, a commodity-money regime is really nothing more than a barter equilibrium that is asymmetric in the sense that a particular good is universally acceptable. As Karl Menger (1892), Jones (1976), Kiyotaki and Wright (1989), and Oh (1989) have argued, the transition from barter to commodity money can be understood as a natural outgrowth of individual interests in the exchange process. Fiat-money equilibria, on the other hand, seem very unlikely to arise in such a way. Indeed Proposition 2 demonstrates that, in the present model at least, they are impossible.[17] When the transition to fiat money occurs one faces a puzzle: how does a situation arise in which people willingly accept an item with zero fundamental value in exchange for goods? It seems reasonable that at least some aspects of the answer may be shared by transitions from both symmetric and asymmetric barter.

Second, the irregular transition and the long delay before the appearance of pure fiat money could be interpreted as a symptom of the uncertainty surrounding a fiat-money equilibrium. In the real world, individuals lack perfect foresight, so they face a signal-extraction problem: how to weigh the government's promises to restrict the amount of unbacked currency. A new government may have a "honeymoon period," which it can perhaps extend by a period of conservative monetary policy and by legal restrictions on the use of other monies, but when the government attempts to exploit this source of revenue, perhaps because of changing preferences or circumstances, evidence accumulates against it, and agents' valuation of the currency changes. As the government repudiates its promises, individuals repudiate its currency.[18]

Third, the development of private banking resulted in the use of deposits and bank notes as media of exchange before the widespread appearance of fiat money. While this is a crucial facet of the overall development of modern payments systems, it is important to recognize that private bank notes and deposits represent claims on an underlying monetary standard.[19] Therefore, the same question (once removed) arises for bank money: how did it become possible to exchange claims on apparently valueless pieces of paper—fiat money—for goods?

These consideration would complicate a formal model but do not diminish the central thrust of this paper: a transition to unbacked money can be accomplished, even in a perfect-foresight economy, if and only if the government has some means of making credible its promise to restrict the use of the printing press.

[17]Benjamin Klein (1974) comes to the opposite conclusion, but in a model in which competitive money-issuing firms are able to precommit to a particular money-supply path.

[18]A fascinating example of a punctuated transition to fiat money can be found in the monetary history of ancient China. The Chinese were apparently the first to invent paper money and to make it inconvertible. Tullock (1957) and Lien-sheng Yang (1952) describe the repetition over several hundred years of a cycle in which fiat money became almost worthless through "the use of inflation for budgetary purposes" (Tullock, 1957 p. 395) and was eventually replaced by new money issued by a new dynasty. Chinese governments experimented with a number of techniques to support and promote the use of fiat money by constraining either themselves or the public, but as Tullock notes, "Any effect they may have had, however, was more than canceled out by the activities of the government mint" (p. 399), and "the use of paper money was, in each case, eventually abandoned" (p. 396). Ultimately, after this repeated failure, the Ming dynasty in the 15th century A.D. quit trying to issue fiat money.

[19]To the extent that banks hold only fractional reserves of the prevailing currency or coin, these claims may be risky and therefore not perfect substitutes for currency, but the banks' liability is not limited to reserves. In the absence of deposit insurance, market perception that assets are not sufficient to cover liabilities causes a run on the bank.

VI. Conclusion

This paper provides an equilibrium account of the transition between barter and fiat-money regimes. The explanation relies on the intervention of a self-interested (though not necessarily nonaltruistic) government which must be able to promise credibly to limit the issue of money. In the model used here, a government achieves this credibility by internalizing some of the macroeconomic externalities generated by the use of fiat money, thus offsetting the benefits of seigniorage. The key determinants of the government's ability to issue fiat money are the government's patience for revenue and the extent of the government's involvement in the economy, both of which could take a long time to mature. The paper thus provides, simultaneously, an account of the transition to fiat money and a possible explanation for its delayed appearance. These questions have previously received almost no attention in monetary theory.

Appendix

PROOF OF LEMMA 1:

The proof of Lemma 1 proceeds by using backward induction to demonstrate that $V_t^{\mathrm{G}}(M_t, \ldots, M_{T-1}, M, M, \ldots)$ and $V_t^{\mathrm{M}}(M_t, \ldots, M_{T-1}, M, M, \ldots)$, the time-$t$ expected utilities of goods- and money-holders who attempt trade given that other agents attempt trade, are in fact the maximum attainable utilities for agents holding goods and money, respectively. In other words it will not be optimal in any period t to deviate from the strategy of attempting to trade. The arguments to V_t^{G} and V_t^{M} are suppressed subsequently.

Two strictly algebraic results are established first. The non-steady-state counterparts of (1) and (2) are:

$$V_t^{\mathrm{G}} = b_t U + s_t \beta\left(V_{t+1}^{\mathrm{M}} - V_{t+1}^{\mathrm{G}}\right) + \beta V_{t+1}^{\mathrm{G}}$$

$$V_t^{\mathrm{M}} = p_t U - p_t \beta\left(V_{t+1}^{\mathrm{M}} - V_{t+1}^{\mathrm{G}}\right) + \beta V_{t+1}^{\mathrm{M}}.$$

Subtracting, substituting recursively, and

$$\text{(A1)}\quad V_t^{\mathrm{M}} - V_t^{\mathrm{G}} = (p_t - b_t)U + (1 - p_t - s_t)\beta\left(V_{t+1}^{\mathrm{M}} - V_{t+1}^{\mathrm{G}}\right)$$

$$= (p_t - b_t)U + (1 - p_t - s_t)\beta\{(p_{t+1} - b_{t+1})U$$
$$+ (1 - p_{t+1} - s_{t+1})\beta[(p_{t+2} - b_{t+2})U + \cdots$$
$$+ (1 - p_{T-1} - s_{T-1})\beta(V^{\mathrm{M}}(M) - V^{\mathrm{G}}(M))] \cdots\}$$

$$= (p_t - b_t)U + (1 - p_t - s_t)\beta\left\{(p_{t+1} - b_{t+1})U\right.$$
$$+ (1 - p_{t+1} - s_{t+1})\beta\left[(p_{t+2} - b_{t+2})U + \cdots\right.$$
$$\left.\left.+ (1 - p_{T-1} - s_{T-1})\beta\left((p - b)U \sum_{q=0}^{\infty} [\beta(1 - p - s)]^q\right)\right] \cdots\right\}$$

applying Proposition 3, one obtains equation (A1), at the bottom of the preceding page, where b, p, and s are the respective steady-state transition probabilities (as in Section II) and where the last equality is obtained using (4). Noting that $p - b \leq p_{t+i} - b_{t+i}$ for $t+i \leq T$ and $1 - p_t - s_t \leq 1 - p_{t+j} - s_{t+j}$ for $j \geq 1$, then,

$$\text{(A2)} \quad V_t^{\text{M}} - V_t^{\text{G}} > (p-b)U \sum_{j=0}^{\infty} [\beta(1-p_t-s_t)]^j = \frac{(p-b)U}{1-\beta(1-p_t-s_t)} = \frac{(1-M)x(1-x)U}{1-\beta(1-x)[1-\alpha x(1-M_t)]}.$$

Noting that $p_t - b_t \geq p_{t+i} - b_{t+i}$ for $i > 0$ and $1 - p - s \geq 1 - p_{t+i} - s_{t+i}$ for $t+i \leq T$,

$$\text{(A3)} \quad V_t^{\text{M}} - V_t^{\text{G}} < \frac{(p_t - b_t)U}{1-\beta(1-p-s)} = (1-M_t)\psi.$$

Turning now to the main argument, recall that money supply M is reached in period T. Since $M \leq \overline{M}$, Proposition 1 guarantees that it is optimal for both goods- and money-holders to trade in T. Thus $V^{\text{G}}(M)$ and $V^{\text{M}}(M)$ are the maximum utilities unattainable at T.

As an inductive hypothesis, assume that in $t+1 \leq T$, V_{t+1}^{G} and V_{t+1}^{M} are the maximum attainable utilities for agents who are not part of the government. The Bellman equation at t for a goods-holder not part of the government is

$$V_t = \max\{A + \beta V_{t+1}^{\text{G}}, V_t^{\text{G}}\}$$

where the maximization is over the choice of whether to attempt trade or to consume the reservation good.

Clearly a goods-holder attempts trade if

$$G_t^{\text{G}} \equiv b_t U + s_t \beta\left(V_{t+1}^{\text{M}} - V_{t+1}^{\text{G}}\right) > A.$$

Using (A2), one obtains equation (A4), below. Then, defining $\bar{b}$ and $\bar{s}$ as the respective steady-state transition probabilities when $M = \overline{M}$,

$$\begin{aligned} G_t^{\text{G}} - A &> b_t U + xM_t\beta(1-\overline{M})\psi - \left[\bar{b}U + \bar{s}\beta(1-\overline{M})\psi\right] \\ &= \left(1 - M_t - 1 + \overline{M}\right)x^2 U + \left(M_t - \overline{M}\right)\beta(1-\overline{M})\psi \\ &= \left(\overline{M} - M_t\right)\left[x^2U - x\beta(1-\overline{M})\psi\right] \\ &= \left(\overline{M} - M_t\right)\frac{1}{\overline{M}}\left[\overline{M}x^2U - (A - \bar{b}U)\right] \\ &= \left(\overline{M} - M_t\right)\frac{1}{\overline{M}}\left[x^2U - A\right] \\ &> 0. \end{aligned}$$

$$\begin{aligned} \text{(A4)} \quad G_t^{\text{G}} &> b_t U + s_t\beta\frac{(1-M)x(1-x)U}{1-\beta(1-x)[1-\alpha x(1-M_t)]} \\ &= b_t U + \beta(1-M)x(1-x)U\frac{M_t x + \alpha x(1-x)(1-M_t)}{1-\beta(1-x)+\beta\alpha x(1-x)(1-M_t)} \\ &> b_t U + \beta(1-M)x(1-x)U\frac{M_t x}{1-\beta(1-x)} \\ &= b_t U + xM_t\beta(1-M)\psi \\ &> b_t U + xM_t\beta(1-\overline{M})\psi \end{aligned}$$

Thus V_t^{G} is the maximum utility at t of a trader holding goods.

The Bellman equation for a money-holder not in the government is

$$V_t = \max\{A + \beta V_{t+1}^{\mathrm{M}}, V_t^{\mathrm{M}}\}$$

where again the maximization is performed over the actions of attempting trade and consuming the reservation good. The money-holder chooses to attempt trade if

$$G_t^{\mathrm{M}} \equiv p_t U - p_t \beta(V_{t+1}^{\mathrm{M}} - V_{t+1}^{\mathrm{G}}) > A.$$

Using (A3),

$$G_t^{\mathrm{M}} > p_t U + (1 - M_{t+1})\psi$$

$$> p_{t+1} U + (1 - M_{t+1})\psi.$$

The last quantity is the gain to money-holders from trade in a steady-state equilibrium with money supply $M_{t+1} < \overline{M}$ and is therefore greater than A.

Finally observe that members of the government are always at least as well off as nonmembers.

REFERENCES

Aiyagari, S. Rao, and Wallace, Neil. "Government Transactions, Policy, and the Medium of Exchange." Federal Reserve Bank of Minneapolis Working Paper No. 516, 1993.

Bordo, Michael D. and Kydland, Finn E. "The Gold Standard as a Rule." Federal Reserve Bank of Cleveland Working Paper No. 9205, 1992.

Croushore, Dean. "Money in the Utility Function: Functional Equivalence to a Shopping-Time Model." *Journal of Macroeconomics*, Winter 1993, *15*(1), pp. 175–82.

Diamond, Peter. "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy*, October 1982, *90*(5), pp. 881–94.

______. "Money in Search Equilibrium." *Econometrica*, January 1984, *52*(1), pp. 1–20.

Feenstra, Robert C. "Functional Equivalence Between Liquidity Costs and the Utility of Money." *Journal of Monetary Economics*, March 1986, *17*(2), pp. 271–91.

Friedman, Milton. *A program for monetary stability*. New York: Fordham University Press, 1959.

Friedman, Milton and Schwartz, Anna J. "Has Government Any Role in Money?" *Journal of Monetary Economics*, January 1986, *17*(1), pp. 37–62.

Jones, Robert A. "The Origin and Development of Media of Exchange." *Journal of Political Economy*, August 1976, *84*(4), pp. 757–75.

Kiyotaki, Nobuhiro and Wright, Randall. "On Money as a Medium of Exchange." *Journal of Political Economy*, August 1989, *97*(4), pp. 927–54.

______. "A Contribution to the Pure Theory of Money." *Journal of Economic Theory*, April 1991, *53*(2), pp. 215–35.

______. "A Search-Theoretic Approach to Monetary Economics." *American Economic Review*, March 1993, *83*(1), pp. 63–77.

Klein, Benjamin. "The Competitive Supply of Money." *Journal of Money, Credit, and Banking*, November 1974, *6*(4), pp. 423–53.

Menger, Karl. "On the Origin of Money." *Economic Journal*, June 1892, *2*(6), pp. 239–55.

Oh, Seonghwan. "A Theory of a Generally Acceptable Medium of Exchange and Barter." *Journal of Monetary Economics*, January 1989, *23*(1), pp. 101–19.

Tullock, Gordon. "Paper Money—A Cycle in Cathay." *Economic History Review*, Second series, April 1957, *9*(3), pp. 393–407.

Yang, Lien-sheng. *Money and credit in China: A short history*. Cambridge, MA: Harvard University Press, 1952.