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Credit and Money in a Search Model with Divisible Commodities

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This paper examines the competition between money and credit in a search model with divisible commodities. It is shown that fiat money can be valuable even though it yields a lower rate of return than the coexisting credit. The competition between money and credit increases efficiency. The monetary equilibrium with credit Pareto dominates the monetary equilibrium without credit whenever the two coexist. When a credit is repaid with money, the competition also bounds the purchasing power of money from below by that of credit and so eliminates the weak inefficient monetary equilibrium found in previous search models. With numerical examples, three different monetary equilibria are ranked and the properties of the interest rate are examined.

1. INTRODUCTION

Money, as an asset, must compete against credit. The competition imposes strong restrictions on the value of money and monetary policies. In particular, Wallace (1980) has forcefully argued that fiat money must have the same rate of return as other assets when it serves as a store of value. In reality, however, fiat money is dominated by credit in the rate of return. Interest rates are typically positive while the rate of return to money is zero or even negative. Generating this return dominance has been one of the primary objectives of recent theoretical modelling in monetary economics. An important advance has been made by Kiyotaki and Wright (1991, 1993). Following Diamond (1984), these authors replace centralized markets by decentralized trading where pairs of traders are brought together by random matching. Improving upon Diamond's model, Kiyotaki and Wright abandon the cash-in-advance constraint superimposed by Diamond. The resulting model, called the search-theoretic model of money, captures Jevons' observation that money can alleviate the lack of double coincidence of wants in exchange. Their main results are that fiat money can be valuable, dominated in return by durable goods and welfare improving. These promising results have encouraged further explorations of the search model.[1]

The Kiyotaki–Wright model relies on two major restrictions: the absence of credit and the lack of price determination. Prices are made trivial by the restrictions on agents' inventories. Agents are assumed to hold an indivisible object (money or good) and swap their inventories in trade. The real and nominal prices are unity. If credit were introduced

1. Examples include Aiyagari and Wallace (1992), Matsuyama, Kiyotaki and Matsui (1993), Shi (1995), and Trejos and Wright (1995). Kiyotaki and Wright (1993) provide a list of references. Return dominance can also be generated by imposing some *ad hoc* assumptions such as money-in-the-utility-function (Sidrauski (1967)) or the cash-in-advance constraint (Lucas (1980)). Models which do not rely on these assumptions but employ Walrasian markets include Townsend (1980, 1987), Mitsui and Watanabe (1989), Maeda (1991), Engineer and Bernhardt (1991), Hayashi and Matsui (1993), and Alonso (1993).

into the model under the restrictions, a unit of good lent today would be repaid with a unit of good in the future. The interest rate would be zero and return dominance would not exist.

The absence of credit severely limits the search model. As a concern for policy analysis, Diamond (1984, p. 18) emphasized that major extensions to the search model would be required to accommodate credit. He concluded that "(especially without credit) the analysis of monetary policy must be omitting some of the most important elements, and can not satisfactorily consider open market operations." As a theoretical concern, one may argue that credit can also alleviate the lack of double coincidence of wants. Fiat money may cease to be valuable in the presence of credit that delivers a higher rate of return.

Even if fiat money continues to be valuable in spite of return dominance, there is another reason for introducing credit in a search model. The Kiyotaki–Wright model possesses multiple monetary equilibria, where money can be either partially or fully accepted in the economy. Similar multiplicity exists when prices are endogenously determined, as shown by Shi (1995) and Trejos and Wright (1995), where there are two monetary equilibria that differ in the purchasing power of money. It is interesting to see whether introducing competing credit can eliminate the inefficient monetary equilibrium where money has a weak purchasing power.

The two major restrictions in the Kiyotaki–Wright model have been separately but not simultaneously removed in researches that have followed. Shi (1995) and Trejos and Wright (1995) introduced sequential bargaining to determine the purchasing power of money. Diamond (1990) introduced credit into a non-monetary search economy. Hendry (1992) introduced credit into a monetary search economy but restricted trade to be one-to-one swaps of inventories. As pointed out above, this restriction forces the interest rate to be zero. Moreover, Hendry restricted the competition between money and credit by assuming that money holders cannot choose to use credit once they are matched with producers.

This paper simultaneously removes the two restrictions in the Kiyotaki–Wright model. I allow two agents to conduct an IOU trade when they have only a single coincidence of wants. Besides price determination, the model improves upon Hendry's in two other dimensions. First, there is a direct competition between credit and money here as I allow a money holder to choose between using money and IOU to exchange for goods. Second, an IOU can be repaid with either money or goods. In contrast, an IOU is only repaid with goods in Hendry's model.

It is shown that monetary equilibria exist with or without credit. For suitable parameter values there are three types of monetary equilibria with coexisting credit. The equilibria differ from each other in the means of repayment on credit. When an IOU is repaid only with money, the monetary equilibrium requires that the number of money holders in the economy be large. The opposite requirement is needed when an IOU is repaid only with goods. If the number of money holders is moderate, an IOU can be repaid with either money or goods. In all three equilibria, an IOU dominates money in the rate of return. That is, the net interest rate is positive. In fact the interest rate must be positive to induce lending. If the interest rate were zero, a creditor would search for new partners rather than trading with the current partner.

Money is valuable despite return dominance because a monetary exchange allows agents to trade and consume faster. A monetary trade is instantaneous, after which the two agents can match with other agents. In contrast, a credit trade ties the creditor and the debtor for some time until the debt is repaid. On the other hand, credit is valuable despite the useful features of money because not everyone in the economy has money. In particular, when two producers only have a single coincidence of wants, a monetary trade is desirable, but the two do not have money. Because search is necessary for obtaining money, the two agents will carry out a credit trade as long as it generates non-negative surpluses.

The competition between credit and money increases efficiency. The monetary equilibrium with credit Pareto dominates the monetary equilibrium without credit whenever the two coexist. The multiplicity in the purchasing power of money found in previous search models continues to exist when an IOU is repaid with goods. However, insisting on nominal repayments eliminates the weak monetary equilibrium by requiring the purchasing power of money to be bounded below by that of credit.

The interest rate depends on two factors: the purchasing power of money and the length of repayment, which is termed the maturity of the IOU. With numerical examples I show that an increase in the proportion of money holders increases the interest rate when an IOU is repaid only with money, but reduces the interest rate when an IOU is repaid only with goods. It is also illustrated that the monetary equilibrium with only nominal repayments Pareto dominates the monetary equilibrium with only real repayments.

The remainder of this paper is organized as follows. The next section describes the search economy with special tastes and technology that rule out barter. Section 3 establishes the existence of two monetary equilibria, one with credit and one without. It ranks the two equilibria and shows that credit dominates money in the rate of return when they coexist. Section 4 extends the model to allow for barter and examines three monetary equilibria associated with different means of repayment. Section 5 concludes the paper and the appendices provide necessary proofs.

2. THE ECONOMY

I extend Diamond's (1982, 1984) parable to motivate the modelling assumptions. To simplify discussions on the credit arrangement, barter is ruled out by assumptions on tastes and technology that are similar to Kiyotaki and Wright (1989). Section 4 will incorporate barter.[2]

2.1. *Tastes and endowments*

Imagine an economy with N different production opportunities, lying in the set $\mathcal{N}=\{1, 2, \ldots, N\}$. Each opportunity $i\in\mathcal{N}$ can be used to produce only type i good. Goods are perishable once produced. There are also N types of agents with equal population. Call an agent of type i agent i, although many other agents have the same type. Agent i consumes only type $i+1$ goods (with modulus N), which is called agent i's *consumption good.* The utility of consuming q units of consumption good is $u(q)$ where u is twice-continuously differentiable with $u(0)=0$, $u'>0$, $u''<0$, $u'(0)=\infty$ and $u'(\infty)=0$. All agents have the same rate of time preference $r>0$.

Agents are specialized in production. Agent i only knows how to use the production opportunity i. Call good i agent i's *product.* Since agents cannot produce their own consumption goods, exchange is necessary for consumption.[3] It is necessary to restrict $N\geqq 3$

2. I thank Nobu Kiyotaki and a referee for suggesting the simplification that eliminates barter.

3. These assumptions on production and consumption are made to simplify the model. In Shi (1994) and Burdett *et al.* (1995), agents are allowed to produce their consumption goods. A monetary equilibrium exists in Shi (1994) where agents specialize in production and exchange for their consumption goods.

for an interesting discussion; otherwise any two types of agents can directly barter. In fact, a stronger assumption $N \geq 4$ is imposed to simplify discussions on the credit arrangement. To unify notation for future sections, denote $z = 1/N$ to be the probability with which a randomly selected agent can produce the consumption good for a given agent. It is also the probability with which a given agent can produce the consumption good for a randomly selected agent.

Production opportunities are abundant in the economy. It does not take time for an agent to find the opportunity that he can use.[4] However, agents need enough energy to produce. Energy comes from consumption. Consuming once generates enough energy for producing only once. As we will see later, it is possible that an agent consumes twice consecutively, in which case he can produce twice consecutively. An agent can produce any (non-negative) quantity of goods each time when he produces. The unit cost of production in terms of utility is a constant and normalized to one.

Each agent is also endowed with a consumption tool that is necessary for consumption. These tools are agent-specific so that an agent must use his own tool to consume. The general interpretation of such a consumption tool is an asset that is limited in supply and valued more by the agent himself than by others. Such an asset functions as a collateral in a credit arrangement, as discussed fully later.

Time is continuous. The economy begins with a proportion M of hungry agents and the other $1-M$ proportion of agents who have enough energy to produce once. These agents are evenly distributed across the N types. In this starting time, each hungry agent is given one unit of an intrinsically useless, storable object called money. Call these agents the *money holders*. By construction, money holders must consume before they can produce. This feature of the environment will be preserved by the exchanges described below. I restrict the analysis to the case where money is indivisible. That is, each money holder holds only one unit of money and exchanges the entire unit of money in trade (see Section 5 for a discussion).

2.2. *Exchange*

The economy is large, populated by a unit measure of agents. There is no centralized market place. Agents meet each other according to a random matching process. In each period, the arrival rate of trading partners for each agent in the exchange process is a Poisson process with a constant rate $\beta > 0$. The number of agents with whom a given agent is matched is β times the total number of agents involved in exchange. That is, the matching technology exhibits constant returns to scale. There are none of the increasing returns to scale which are crucial to Diamond's (1984) results.

The flow of agents is depicted in Figure 1. Accompanying each arrow in the figure are the probability with which agents successfully make the corresponding transition and the terms of trade, included in parentheses, which are described by bilateral bargaining in Section 2.3. There are four types of agents in the economy: money holders, producers, debtors and creditors. A producer is an agent who can produce and has no contractual agreement. A debtor is an agent who can produce but has not repaid his IOU. A creditor cannot produce and is assumed to stay out of exchange (see discussion below). Let the measure of money holders, producers and debtors be M, N_p and N_d respectively. Because

4. In a previous version of this paper I assumed that it took time for an agent to find a production opportunity. The purpose was to emphasize the difference between exchanging and producing. Such a difference is not the emphasis here so the simplifying assumption is made.

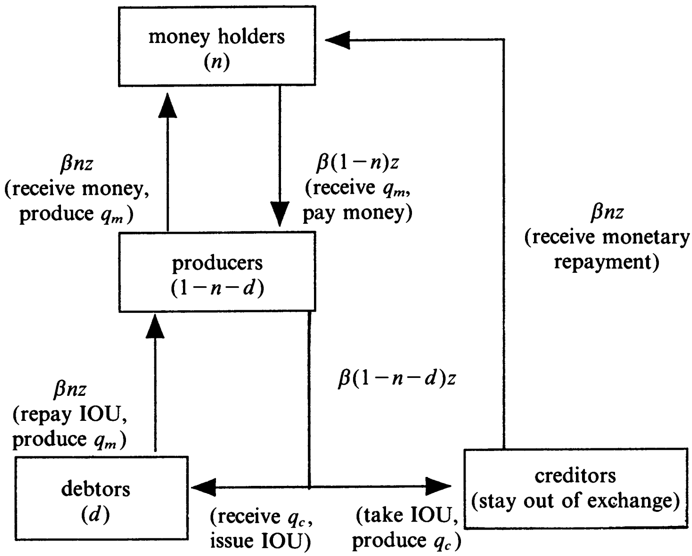

FIGURE 1
Flows of agents

creditors are not in exchange, the measure of agents in exchange is $M+N_d+N_p=1-N_d$. Denote the proportion of money holders in exchange by $n=M/(1-N_d)$ and the proportion of debtors in exchange by $d=N_d/(1-N_d)$.

Because there is no double coincidence of wants, the possible exchanges are monetary trade and credit trade, as summarized in Table 1. There are two types of monetary trade. One is between a money holder and a producer; the other is between a money holder and a debtor. Both types of trade require that the agent who exchanges goods for money produce the money holder's consumption goods. After the exchange the money holder consumes immediately and becomes a producer. In a trade between a money holder and a producer, the producer produces q_m units of goods for money, after which the producer becomes a (hungry) money holder. In a trade between a money holder and a debtor, the debtor exchanges q_d units of goods for money, after which the debtor repays his debt with money and becomes a producer. For reasons described in Section 2.3, we have assumed $q_d=q_m$ in Figure 1.

A credit trade takes place between a pair of producers where one's product is the other's consumption good. For clarity, consider a match between producers i and $i+1$. According to the description of tastes and production technology, agent $i+1$ can produce

TABLE 1

Possible exchanges without barter

	Producers	Debtors	Money Holders
Producers	credit trade	—	monetary trade (credit trade)
Debtors	—	—	monetary trade
Money Holders	monetary trade (credit trade)	monetary trade	—

agent i's consumption good $i+1$, but agent i cannot produce agent $(i+1)$'s consumption good $i+2$. In this case agent i can issue an IOU to agent $i+1$ in exchange for q_c units of good $i+1$. In return, agent i gives agent $i+1$ his consumption tool as collateral and promises to repay with money whenever it becomes available to him. If the two agree on the trade, agent $i+1$ produces immediately for agent i to consume; agent i immediately consumes and gives agent $i+1$ his consumption tool as collateral. Agent i becomes a debtor and agent $i+1$ becomes a creditor. To regain his consumption tool, the debtor participates in exchange to obtain money for the repayment.

The use of collateral induces repayment as long as future consumption yields a positive value to the debtor. This is because the debtor must regain his collateral from the creditor for future consumption. It is the debtor's self-interest to repay the IOU as soon as possible; delaying repayments only reduces utility because of positive discounting. This way of inducing repayment is different from that in Diamond (1990), who assumes that all market participants coordinate to deny the future access to trade by those debtors who fail to repay. It is unclear whether the coordination can be achieved in a random matching economy. In this sense the assumption here is simpler and more natural.[5]

It is desirable to require that the collateral have certain realistic features. First, to repay the IOU the debtor must be able to find the creditor. To ensure this in a random matching economy, I assume that the creditor stays put and hence does not participate in exchange.[6] Second, it is costly for an outsider to evaluate the collateral. To capture this, I assume that an agent must pay a utility cost $\varepsilon>0$ to accept an IOU that is originally issued in another credit trade, where ε can be arbitrarily small. With the cost, a debtor does not accept an IOU from other agents and swap IOU's with his own creditor: the original creditor will not agree on the swap because it reduces his utility by ε. The cost has a similar role as the cost of receiving a commodity money in Kiyotaki and Wright (1993). Without the cost, it is possible that the collateral functions as a commodity money.

Several features of the credit trade are noteworthy. First, an agent can at most issue one IOU at a time. He can issue the second IOU only after repaying the first. This is because a producer has only one consumption tool. The inability to issue multiple IOU's rules out trade between two debtors: for two debtors to trade, at least one of them must issue a second IOU. It also rules out repayments with goods under the assumption $N\geqq 4$. To see why, suppose that agent i is the debtor and agent $i+1$ the creditor. To repay with the creditor's consumption good $i+2$, the debtor must meet agent $i+2$ in exchange. Since agent i cannot issue a second IOU before repaying the first, the only way for him to exchange for good $i+2$ is to produce agent $(i+2)$'s consumption good $i+3$. Because agent i can only produce good i, this is possible if and only if $i=i+3$ with modulus N, i.e. only if $N=3$, which is excluded by the assumption $N\geqq 4$. In Section 4, barter is allowed for so that an IOU can be repaid with either money or goods.

Second, a credit trade enables an agent to consume twice consecutively. A would-be debtor enters a credit trade with enough energy to produce. By issuing an IOU, he consumes the second time without producing a good so he has enough energy to produce twice consecutively, once for a unit of money for the repayment and once after the repayment. That is, a debtor becomes a producer after the repayment. The creditor receives money and becomes a (hungry) money holder.

5. I am indebted to Nobu Kiyotaki for suggesting the use of collateral to induce repayments.

6. The credit trade does not exist when the creditor searches separately from the debtor and when the population is large, because then the debtor has a very small probability to meet his creditor for repayments. Also, as suggested by a referee, one can alternatively assume that the creditor and the debtor always search together. Under the later Assumption 1 on bargaining, this alternative assumption delivers identical results to the ones in this paper.

Third, there is a direct competition between credit and money. In particular, a credit trade is possible between a money holder and a producer whenever a monetary trade is possible. The money holder can issue an IOU instead of using money to exchange for goods. In Table 1 such an optional trade is included in the parentheses. To be valuable money must be the preferable medium of exchange in such exchanges. This direct competition between credit and money is absent in Hendry (1992).

It should be noted that all types of trade require at least a single coincidence of wants. If two agents have no coincidence of wants at all, they would rather search for new matches than trade with the current partner. For example, if two producers had a credit trade when they have no coincidence of wants at all, the debtor would receive a negative surplus (see the discussion on (2.5) later). Furthermore, two money holders do not have any advantageous trade. Since money holders do not have energy to produce, the only possible exchange between the two is to swap their monies, which has no consequence on equilibrium.

2.3. *Terms of trade*

The terms of trade in a match are given by the Nash bargaining solution. To be precise, let V_p be the value function of a producer, V_m of a money holder, V_c of a creditor and V_d of a debtor. Individual agents take these values as given. I examine only the stationary equilibrium where these values are constant. Consider a monetary trade between a money holder and a producer. The terms of trade are the quantity q_m of goods that the producer exchanges for money. The surplus is $V_m - V_p - q_m$ for the producer and $u(q_m) + V_p - V_m$ for the money holder. The specific Nash bargaining problem takes the following form:

$$\begin{aligned} &\max_{q_m} && (V_m - V_p - q_m)^a (u(q_m) + V_p - V_m)^{1-a} \\ &\text{s.t.} && V_m - V_p - q_m \geqq 0;\; u(q_m) + V_p - V_m \geqq 0; \\ & && V_m \text{ and } V_p \text{ given.} \end{aligned}$$

The parameter $a \in [0, 1]$ is the bargaining weight of the producer.[7] To simplify the algebra, let us further simplify the bargaining solution by assuming that the money holder in a monetary trade and the debtor in a credit trade make a take-it-or-leave-it offer.[8]

Assumption 1. In a bilateral trade the agent who exchanges goods for money or credit has a bargaining weight $a = 0$.

Under Assumption 1, the producer receives zero surplus in a monetary trade. Thus

$$q_m = V_m - V_p. \tag{2.1}$$

7. It is well-known that the solution to the Nash bargaining problem coincides with the solution to a particular non-cooperative sequential bargaining problem examined by Rubinstein and Wolinsky (1985) and Wolinsky (1987). For a general discussion and cautions on the coincidence between the two solutions, see Binmore (1987) and Osborne and Rubinstein (1990). For applications to search monetary models, see Shi (1995) and Trejos and Wright (1995). The latter two papers also illustrate that the existence of a monetary equilibrium is robust to variations of the non-cooperative environment from Rubinstein's (1982) framework, where agents do not search between bargaining rounds, to the framework where agents search between bargaining rounds.

8. The extreme value of a is unnecessary for valuable fiat money. As shown by Shi (1995) and Trejos and Wright (1995) in models without credit, a monetary equilibrium exists when $a = 1/2$. Simplification may also be achieved with $a = 1$. However, money will have no value in this case because a money holder obtains zero surplus in every trade. To ensure existence of valuable money, $1 - a$ must be sufficiently large. It is in this sense that Assumption 1 should be understood.

The trade is possible only if it has a positive (total) surplus, i.e. only if

$$u(q_m)+V_p-V_m=u(q_m)-q_m>0. \tag{2.2}$$

Similarly, in a credit trade the debtor's surplus is $(V_d-V_p+u(q_c))$ and the creditor's surplus is $(V_c-V_p-q_c)$, where q_c is the quantity of goods that the creditor exchanges for the IOU. Since the debtor has all the bargaining power, we have:

$$q_c=V_c-V_p. \tag{2.3}$$

A credit trade has a positive surplus if

$$u(q_c)+V_d-V_p>0. \tag{2.4}$$

The third type of trade is a monetary trade between a money holder and a debtor. The terms of trade are the quantity q_d of goods that the debtor exchanges for money. After exchange the money holder becomes a producer; the debtor repays the IOU and becomes a producer. The surplus of trade is $(V_p-V_d-q_d)$ for the debtor and $(u(q_d)+V_p-V_m)$ for the money holder. The debtor's surplus is in general different from that of a producer in a monetary trade and so $q_d\neq q_m$. Nevertheless I assume $q_d=q_m$ in the analysis.

There are three justifications for this assumption. First, the assumption greatly simplifies the analysis and eases the exposition: it enables me to depict some of the existence propositions through two-dimensional diagrams. Second, the analysis with this assumption is a good approximation for the more general case $q_d\neq q_m$. In Appendix C, it is shown that q_d is close to q_m in the parameter region of interest in this paper and that a monetary equilibrium exists with $q_d\neq q_m$ in the same analytical way as with the assumption $q_d=q_m$. Third, and perhaps more importantly, it may be easy to tell whether an agent can produce but difficult to tell whether an agent has an unpaid IOU. That is, a money holder may not be able to distinguish a debtor from a producer, for both can produce. If a money holder must make offers based solely on whether the trading partner can produce, Appendix C shows that the assumption $q_d=q_m$ can be endogenously delivered in some parameter regions that are consistent with the existence regions of the equilibria examined below.[9]

With $q_d=q_m$, the following lemma can be established:

Lemma 2.1. *An IOU is accepted and expected to be repaid if and only if* (2.2) *and the following conditions hold*:

$$V_p-V_d-q_m\geqq 0, \tag{2.5}$$

$$V_m\geqq V_c. \tag{2.6}$$

Proof. To repay an IOU the debtor must trade for money. For a money holder to trade with a debtor, (2.2) is necessary. For a debtor to repay an IOU, (2.5) is necessary. Otherwise the debtor would retain a higher value from not repaying the IOU, V_d, than the value from repaying the IOU, V_p-q_m, in which case a creditor would not expect an IOU to be repaid and hence would not accept an IOU. In addition, for the creditor to

9. This alternative environment is not explicitly adopted for the main text because, as a referee pointed out, it enlarges the equilibrium set. In addition to the equilibrium described below, which satisfies $V_p-V_d\geqq q_m$, there can be another equilibrium which violates this condition, in which case a money holder offers $q=V_p-V_d$ in every match that involves an agent whose product is the money holder's consumption good.

accept the repayment, the surplus $(V_m - V_c)$ must be non-negative. That is, (2.6) must hold. If all conditions in the lemma are satisfied, an IOU is accepted and anticipated to be repaid. ‖

Since $V_d < V_p$ by (2.5), two agents without a coincidence of wants at all do not carry out the credit trade. If they traded, the surplus to the debtor would be $V_d - V_p < 0$. Finally, a money holder can in principle conduct a credit trade with a producer and receive a surplus $(V_d - V_m + u(q_c))$. For a money holder to choose the monetary trade, he must obtain a larger surplus from the monetary trade than from the credit trade. That is,

$$V_p - V_m + u(q_m) > V_d - V_m + u(q_c). \tag{2.7}$$

3. MONETARY EQUILIBRIUM

3.1. *Monetary equilibrium without credit*

There are two types of monetary equilibria in this economy: in one only the monetary trade is conducted; in the other both the monetary trade and the credit trade are conducted. Let us first examine the monetary equilibrium where credit is not accepted. That is, suppose that the values (V_p, V_d, V_m) violate at least one of the two conditions (2.5) and (2.6). It is verified later that these values can be delivered by an equilibrium. Without credit trade, all agents are in the exchange. There are M proportion of money holders and $(1-M)$ proportion of producers (i.e. $n=M$ and $d=0$ in Figure 1). At a rate $\beta(1-M)z$, each money holder meets the producers with whom he can trade. At a rate βMz, each producer meets the money holders with whom he can trade. Thus in a stationary equilibrium, the values (V_m, V_p) are given by

$$\begin{aligned} rV_m &= \beta(1-M)z(V_p - V_m + u(q_m)); \\ rV_p &= \beta Mz(V_m - V_p - q_m) = 0. \end{aligned} \tag{3.1}$$

Definition 3.1. A monetary equilibrium without credit is a vector $(V_m, V_p, V_c, V_d, q_m)$ such that $q_m > 0$, $V_c = V_d = 0$, and

(i) given (V_m, V_p), q_m satisfies (2.1) and (2.2);
(ii) given (2.1), (V_m, V_p) satisfy (3.1);
(iii) one of the conditions (2.5) and (2.6) is violated.

Proposition 3.2. *For all $M \in (0, 1)$, there exists a unique monetary equilibrium without credit.*

Proof. If a monetary equilibrium without credit exists, then $V_c = V_d = 0$ by definition and $V_p = 0$ by (3.1). Also, (3.1) $\Rightarrow$ $V_m = q_m$, where q_m is the positive solution to the following equation:

$$u(q_m) - \left[1 + \frac{r}{\beta z(1-M)}\right] q_m = 0. \tag{3.2}$$

The properties of u specified in Section 2.1 guarantee that there is a unique positive solution to the above equation. By construction, q_m satisfies (2.1) and (2.2); (V_m, V_p) satisfy

(3.1). Moreover, (2.5) is violated because $V_p - V_d - q_m = -q_m < 0$. These values of $(V_m, V_p, V_c, V_d, q_m)$ constitute the unique monetary equilibrium without credit. ||

3.2. *Monetary equilibrium with credit*

Now let us examine the monetary equilibrium where money and credit coexist. In this equilibrium, exchanges involve money holders, producers and debtors. In a stationary equilibrium, the measures of these agents in exchange are constant. Then

$$0 = \dot{N}_d = \beta(1-n-d)zN_p - \beta nzN_d.$$

The first term $\beta(1-n-d)zN_p$ is the measure of producers who become new debtors. In particular, $\beta(1-n-d)z$ is the rate at which a producer meets another producer who can produce his consumption good. The second term βnzN_d in the above equation is the flow of debtors who have repaid their IOU's and become new producers.

From the definitions of n and d, we have

$$N_d = 1 - M/n, \qquad d = n/M - 1,$$
$$N_p = 1 - M - 2N_d = 2M/n - M - 1. \tag{3.3}$$

Substitute these relations into the equation for N_d, we have:

$$n\left(\frac{n}{M} - 1\right) = \left[2 - \left(1 + \frac{1}{M}\right)n\right]^2. \tag{3.4}$$

There is a unique admissible solution $n \in [M, 2M/1+M]$.[10] The solution is an increasing function of M. Also, d is a decreasing function of M.

To describe an equilibrium, the values V must also be determined. They are given by the following equations:

$$rV_m = \beta(1-n)z(V_p - V_m + u(q_m)); \tag{3.5}$$

$$rV_p = \beta(1-n-d)z(V_d - V_p + u(q_c)); \tag{3.6}$$

$$rV_d = \beta nz(V_p - V_d - q_m); \tag{3.7}$$

$$rV_c = \beta nz(V_m - V_c). \tag{3.8}$$

Note that (3.6) has already been simplified under Assumption 1 by the relations $V_m - V_p - q_m = 0$ and $V_c - V_p - q_c = 0$. The explanations for the above equations are similar so only (3.6) is briefly explained. A producer can have three types of trade: a monetary trade, a credit trade where he is a creditor, and a credit trade where he is a debtor. Only the last trade gives him a positive expected surplus, which is represented by the right-hand side of (3.6).

Definition 3.3. A monetary equilibrium with credit is a vector $V = (V_m, V_p, V_d, V_c)$, a distribution of agents (N_p, N_d, n, d), and the quantities of trade (q_m, q_c) such that $q_m > 0$, $q_c > 0$, and

(i) given V and (N_p, N_d, n, d), (q_m, q_c) satisfy (2.1) and (2.3);
(ii) given (2.1) and (2.3), V satisfies (3.5)–(3.8);
(iii) the incentive constraints (2.2), (2.4), (2.5), (2.6) and (2.7) are satisfied;
(iv) (N_p, N_d, n, d) satisfy (3.3) and (3.4).

10. The other solution to the quadratic equation is greater than $2M/(1+M)$ and hence implies $N_p < 0$.

The first step for finding such an equilibrium is to determine V and (q_m, q_c) jointly. Define $R \equiv r/(\beta z)$ to be the effective rate of time preference. Substitute (2.1) and (2.3) into (3.5)–(3.8) to express V as functions of (q_m, q_c) only. Then substitute these functions back into (2.1) and (2.3) to obtain the following equations for (q_m, q_c):

$$q_c = \frac{1+R}{R+n} q_m - \frac{1-n}{R+n} u(q_m); \tag{3.9}$$

$$u(q_c) + \left(1 + \frac{R+n}{1-n-d}\right) q_c = n\left(\frac{1}{1-n-d} + \frac{2}{R+n}\right) q_m. \tag{3.10}$$

(3.9) defines a function $q_c = f(q_m)$ with the properties $f(0)=0$, $f'(0)<0$, $f'(\infty)>0$ and $f''>0$. Similarly (3.10) implicitly defines a function $q_c = g(q_m)$ with the properties $g(0)=0$, $g'>0$ and $g''>0$. The functions f and g are plotted in Figure 2.

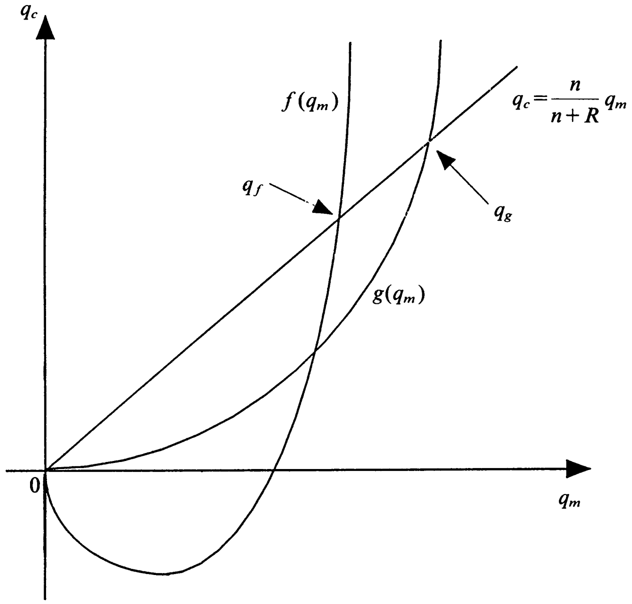

FIGURE 2

Lemma 3.4. *There exist an odd number of positive pairs* (q_m, q_c) *that satisfy* (3.9) *and* (3.10). *All such pairs satisfy the following relation*:

$$q_c < \frac{n}{n+R} q_m. \tag{3.11}$$

The lemma is clear from Figure 2 and its proof is given in Appendix A. It does not exclude the possibility that there can be more than one admissible solution to (3.9) and (3.10). However, this possible multiplicity is unimportant for our analysis, because only the inequality (3.11), which is satisfied by all the solutions, is required for the analysis below. If (3.11) is violated in the form $q_c > n/(n+R) q_m$, then $V_p < 0$ so a monetary equilibrium with credit does not exist. Moreover, the solution is unique for special utility functions such as the one in the later Example 3.6. For these reasons it is assumed that there is a unique positive solution to (3.9) and (3.10).

Remark 1. *The solution to* (3.9) *and* (3.10) *satisfies* (2.2), (2.4) *and* (2.6).

Proof. First, $(2.4) \Leftrightarrow V_p > 0 \Leftrightarrow (3.11)$. Also, (2.2) holds, because $(1-n)(u(q_m)-q_m) = (R+n)(q_m - q_c) > 0$. The equality follows from (3.9) and the inequality from (3.11). Thus by (3.5), $V_m > 0$. Finally, $(3.8) \Rightarrow V_m - V_c = R/(R+n)V_m > 0 \Rightarrow (2.6)$. ||

For an equilibrium with credit to exist, it then suffices to verify (2.5) and (2.7). Since (2.5) implies $V_p - V_d > q_m > 0$ and hence implies (2.7), only (2.5) is left to be verified. The following proposition is proven in Appendix B.

Proposition 3.5. *For sufficiently small R, there exists $M_0 \in (0, 1)$ such that a monetary equilibrium with credit exists for $M \geqq M_0$.*

The conditions for the existence of credit are quite intuitive. The length of time for the repayment on an IOU depends on the proportion of money holders in the economy. If there are few money holders, repayments take a long time because it is difficult for the debtor to find a suitable money holder. M must then be large for credit to be valuable. For the same reason, a high matching rate β and a high chance of coincidence z also make the repayment faster. Furthermore, a small rate of time preference r helps the existence of valuable credit by reducing the utility cost of time that is required for the repayment. These factors are summarized in Proposition 3.5 by a small effective rate of time preference R. Section 4.4 will provide some numerical examples to illustrate the critical level M_0.

3.3. *The interest rate and return dominance*

Credit and money have different rates of return, although both are used in exchange. Let θ be the net interest rate or the internal rate of return to an IOU. To define θ, note that the "nominal" price of the IOU is q_c/q_m, because an IOU costs q_c units of goods and goods can be sold at a nominal price $1/q_m$.[11] An IOU is repaid with a unit of money after a random length of "maturity", say t_r. The interest rate can then be intuitively defined by $1 = (q_c/q_m)e^{\theta t_r}$, i.e.

$$\theta = \frac{1}{t_r} \ln \left(\frac{q_m}{q_c} \right).$$

Since $q_m > q_c$ by Lemma 3.4, the interest rate is positive for all finite realizations of t_r. On the other hand, the net rate of return to money is zero: money costs q_m units of goods and can be used to exchange for q_m units of goods after a random length of time. Therefore credit dominates money in the rate of return when money and credit coexist.

Return dominance is *necessary* for the equilibrium with credit. For a creditor to accept monetary repayments, money must yield a higher value than holding onto the IOU. Holding onto the IOU retains a value V_c which equals the opportunity cost of becoming

11. The quotation marks on the word nominal are needed because there is no direct trade between an IOU and money.

a creditor, V_p+q_c. Accepting the monetary repayment gives the creditor the value of money V_m which equals the opportunity cost V_p+q_m that a producer would pay for the money. Thus for the creditor to accept monetary repayments, it must be true that $V_m > V_c$ so that $V_p+q_m > V_p+q_c$ and $q_m > q_c$. That is, a monetary trade yields a higher surplus to the money holder than a credit trade does to a debtor.[12]

Intuitively, the interest rate is positive because credit is not as good a medium of exchange as money. Credit ties the creditor and the debtor until the IOU is repaid; neither can consume during the repayment period. In contrast, after a monetary exchange the two agents can be matched to any other agents. Furthermore, from Table 1 in Section 2.2, a debtor has fewer suitable exchanges than a producer. Therefore, relative to a monetary trade, a credit trade slows down exchange and consumption for the traders.

If money has such advantages over credit, one wonders why credit is still valued in this economy. Why would the competition between money and credit not drive out credit? The explanation lies in the lack of a centralized marketplace in this economy. Although money is attractive, not everyone has it. In particular, in a match between two producers who have a single coincidence of wants, the one who cannot produce the partner's consumption good could obtain a higher surplus if he had money. But costly search is necessary for acquiring money so the two agents try to make the best out of the current match. That is, they are willing to carry out a credit trade as long as it gives a non-negative surplus. If it were costless to find money whenever there is a single coincidence of wants, money would be used in every trade.

At a general level, the above explanation shares the insight given by Hosios (1990) in a random matching model of unemployment. Explaining why the wage rate in a search model does not approach the competitive equilibrium level when the supply and demand satisfy the competitive conditions, Hosios argues that when matching takes time and takes place before bargaining, the terms of trade determined by bargaining have only a limited allocative or signalling function. In the context of the current model, two matched producers do not have an immediate access to money. The difference between the purchasing power of money q_m and the purchasing power of credit q_c is likely to remain as long as matching is time-consuming.

Let us now examine the dependence of θ on the parameters of the model. Since t_r is random, attention is restricted to the expected interest rate, $E\theta = E(1/t_r)\ \ln(q_m/q_c)$. Notice that the repayment to an IOU follows a Poisson process with a rate βnz, at which a debtor meets a suitable money holder. Thus the expected maturity is $Et_r = 1/(\beta nz)$. However, the expectation $E(1/t_r)$ is not in a tractable form. For simplicity I use the approximation $E(1/t_r) \approx 1/E(t_r)$ that preserves the essence. Then

$$E\theta \approx \beta zn \ln\left(\frac{q_m}{q_c}\right). \tag{3.12}$$

Parameters can affect the expected interest rate either through the nominal price of the IOU, q_c/q_m, or through the expected "maturity" of the IOU, $1/(\beta zn)$. A high price q_c/q_m reduces the interest rate for a given maturity and a long maturity reduces the interest rate for a given price.

The interest rate depends positively on the rate of time preference r. To see such dependence, note that the rate of time preference does not affect the proportions n or d

12. This higher surplus is not necessarily at the expense of the producer in the trade as one might suspect. The producer in a monetary trade is indifferent between producing for money or producing for an IOU. The surplus is zero in both cases under Assumption 1.

and so does not affect the maturity of the IOU. It affects the price q_c/q_m and does so entirely through the effective time preference R. It can be shown from (3.9) and (3.10) that an increase in R shifts up the f curve and shifts down the g curve in Figure 2. Thus the price q_c/q_m falls, although both q_m and q_c fall. This pushes up the expected interest rate. The intuition for the result is as follows. Both monetary trade and credit trade delay consumption by the trader who uses goods to exchange. However, a credit trade delays consumption to the creditor by more than a monetary trade does to the producer. In a monetary trade, the producer immediately receives money; in a credit trade, the creditor must wait a length of time t_r to receive the monetary repayment. By increasing the utility cost of such extra delay, an increase in the rate of time preference reduces the purchasing power of credit q_c by more than it reduces the purchasing power of money q_m. In this way a higher rate of time preference reduces the nominal price of an IOU.

Similarly an increase in the matching rate β or the probability z reduces the effective rate of time preference and hence increases the nominal price of the IOU. This tends to reduce the expected interest rate. However, such an increase also reduces the maturity of the IOU and hence tends to increase the expected interest rate. The overall effect is analytically ambiguous.

An increase in the proportion M of money holders also has an ambiguous effect on the expected interest rate. Although a higher M increases the chance for a debtor to meet a money holder and so reduces the expected maturity of the IOU, it has an ambiguous effect on the purchasing power of the IOU, q_c. This ambiguous effect can be explained as follows. On the one hand, an increase in M increases the proportion of money holders and makes IOU more easily repaid with money. Anticipating such easier repayments, the creditor may be willing to trade a larger quantity of goods for the IOU. That is, "loans" are easier to obtain. On the other hand, a higher M crowds out producers. Since more money holders are chasing each producer, the purchasing power of money q_m tends to be lower. Since the ultimate goal for a creditor to accept an IOU is to use the repaid money to purchase consumption goods, a reduction in the purchasing power of money reduces the purchasing power of credit.

Numerical exercises show that for large M, an increase in M reduces the nominal price of an IOU and increases the expected interest rate. The following example is used.

Example 3.6. $u(q)=q^{\sigma}$, $\sigma=2/3$, $R=0{\cdot}01$.

For $M=0{\cdot}68$, $\ln(q_m/q_c)=0{\cdot}03$ and $E\theta=0{\cdot}0109$. When M increases to $0{\cdot}70$, $\ln(q_m/q_c)$ increases to $0{\cdot}0305$ and $E\theta$ increases to $0{\cdot}0113$.

3.4. *Welfare ranking of the equilibria*

Whenever the monetary equilibrium with credit exists, the monetary equilibrium without credit also exists. In the coexistence region the two equilibria can be ranked according to agents' utility. To distinguish the two equilibria, give a superscript x to the equilibrium without credit. Consider first a producer. In the equilibrium without credit, a producer's present value is $V_p^x=0$. In the equilibrium with credit, $V_p>0$ so a producer is better off ex ante (before match). When $M>M_0$, a producer is also better off ex post (after match) because $V_c>0$ and $V_d>0$.

Now consider a money holder. The present values of a money holder in the two equilibria are:

$$V_m^x=\frac{1-M}{R}[u(q_m^x)-q_m^x], \qquad V_m=\frac{1-n}{R}[u(q_m)-q_m],$$

where q_m^x is the solution to (3.2) and q_m is the solution to (3.9) and (3.10). The existence of credit improves everyone's utility if $V_m > V_m^x$, which holds when the effective time preference is small, as shown below. The intuition is simple. When the effective time preference is small, the utility cost of the delay in consumption caused by a credit trade is small and hence credit brings a Pareto improvement by facilitating trade.

Proposition 3.7. *For sufficiently small R, the monetary equilibrium with credit Pareto dominates the monetary equilibrium without credit.*

Proof. When $R \to 0$, $q_m^x \to q_0$ and $q_m \to q_0$ where q_0 is the positive solution to $u(q) - q = 0$. The function $u(q) - q$ is decreasing in q at q_0 so $u'(q_0) < 1$. With these features, $RV_m^x \to 0$ and $RV_m \to 0$ when $R \to 0$. Furthermore, computation reveals

$$\left.\frac{d(RV_m)}{dR}\right|_{R=0} > q_0 = \left.\frac{d(RV_m^x)}{dR}\right|_{R=0}.$$

The proposition immediately follows. ||

4. EQUILIBRIUM WITH BARTER

As is standard in monetary search models, barter can be introduced by assuming that there is some probability, z^2, of a double coincidence of wants between two randomly-selected agents with production capacities.[13] This extension allows debt to be repaid with goods and so eliminates the necessary reliance of credit on money in previous sections. The extension also allows us to examine whether the competition of credit can eliminate the weak inefficient monetary equilibrium in Shi (1995) and Trejos and Wright (1995) that relies on the existence of barter.

Since barter is possible, an IOU may be repaid with either money or goods or both. Let us refer to repayments with money as *nominal repayments* and repayments with the creditor's consumption goods as *real repayments*. The general possibilities of trade are summarized in Table 2.

TABLE 2

Exchanges when barter is possible

	Producers	Debtors	Money Holders
Producers	barter credit trade	[barter]	monetary trade (credit trade)
Debtors	[barter]	[barter]	monetary trade
Money Holders	monetary trade (credit trade)	monetary trade	—

As before, the trade in ( ) is optional to the money holder. The trade in [ ] takes place only when creditors accept real repayments. Section 4.1 examines only nominal

13. This barter possibility can be delivered in the following economy. There are a continuum of types of goods identified by a circle with circumference 2. Use the same notation $\mathcal{N}$ to denote the points along the circle. There are also a continuum of types of agents with a unit mass. Identify the set of agents by $\mathcal{N}$. Agent i is specialized in producing good i. Agents do not consume their own products. The set of consumption goods of agent i is $\{j \in \mathcal{N} : \widehat{ji} \leq z\}$, where $0 < z < 1$ and $\widehat{ji}$ is the arc length between goods j and i. Every consumption good is equally preferred. In this case, two randomly selected producers can barter with probability z^2.

repayments. Sections 4.2 and 4.3 examine the equilibria where real repayments are possible. The monetary equilibrium without credit also exists for some parameters but is omitted.

4.1. *Monetary equilibrium with only nominal repayments*

This section shows that for suitable parameter values there exists a monetary equilibrium with only nominal repayments on credit. This equilibrium is an extension of the one without barter in Section 3.2. In such an equilibrium, barter occurs only between two producers. The terms of trade in barter must be determined. Since the two agents in barter both produce goods, additional assumptions are needed to pin down the bargaining weights. Because the two producers in barter are completely symmetric, it is reasonable to assume that they have equal bargaining weights. Let q_b be the quantity of goods each agent produces in barter. Since the producers immediately find new production opportunities after barter, the surplus of trade for each producer is $V_p+u(q_b)-q_b-V_p=u(q_b)-q_b$. Therefore q_b is given by

$$\max_{q_b}\{(u(q_b)-q_b)(u(q_b)-q_b): u(q_b)-q_b>0\}.$$

The solution is $q_b=u'^{-1}(1)$ and the surplus of trade is positive.

Now the equation for N_d is revised to

$$0=\dot{N}_d=\beta(1-n-d)z(1-z)N_p-\beta nzN_d.$$

The revision has taken into account that not every trade between two producers is a credit trade. Barter takes place when there is a double coincidence of wants. Only when there is a single coincidence of wants does a credit trade take place. Thus only with probability $z(1-z)$ does a producer become a debtor when meeting another producer.[14] Consequently the equation for n becomes:

$$n\left(\frac{n}{M}-1\right)=(1-z)\left[2-\left(1+\frac{1}{M}\right)n\right]^2.$$

Omitting the zero-surplus terms, the equation for V_p is revised to:

$$rV_p=\beta(1-n-d)z(1-z)(V_d-V_p+u(q_c))+\beta(1-n-d)z^2(u(q_b)-q_b).$$

The equations for (V_m, V_d, V_c) are the same as (3.5), (3.7) and (3.8).

A monetary equilibrium with only nominal repayments can be defined by adapting Definition 3.3, with new conditions being added to exclude real repayments. To specify these conditions, let us examine a single pair of a debtor and a creditor who deviate to accept real as well as nominal repayments, whichever become available first to the debtor. A debtor can obtain real repayments through barter with a producer or another debtor. Since a debtor has a different threat point than a producer, the terms of barter trade between a debtor and a producer are different from q_b. As in treatment of q_d in Section 2.3, I assume away such a difference. Thus the real repayment is q_b units of consumption goods. By accepting real repayments, the deviating creditor increases the expected surplus by $\beta(1-n-d)z^2(V_p-V_c+u(q_b))$; the deviating debtor increases the expected surplus by $\beta(1-n-d)z^2(V_p-V_d-q_b)$. For real repayments not to be accepted to equilibrium, at

14. Imagine that one of the producers is elected to choose between barter and credit trade. If a credit trade is chosen, each of the two has probability 1/2 to be the debtor. The expected surplus of trade for each producer is $\frac{1}{2}(V_d-V_p+u(q_c))$. It can be shown that this surplus is smaller than $u(q_b)-q_b$ when R is small, in which case both producers prefer barter to credit trade.

least one of the following inequalities must hold:

$$V_p - V_c + u(q_b) < 0, \qquad V_p - V_d - q_b < 0. \tag{4.1}$$

The equations for q_m and q_c now are (3.9) and the following:

$$\left(\frac{1}{1-n-d} + \frac{2(1-z)}{R+n}\right) nq_m = (1-z)u(q_c) + \left(1 - z + \frac{R+n}{1-n-d}\right) q_c + z(u(q_b) - q_b). \tag{4.2}$$

The following results can be established. The proofs, similar to Section 3.2, are omitted.

Lemma 4.1. *For sufficiently small z, there exist an odd number of positive solution pairs* (q_m, q_c) *that satisfy* (3.9) *and* (4.2). *At least one such pair satisfies* (2.2), (2.4), (2.6) *and the following*:

$$q_c < \frac{n}{R+n} q_m - \left(\frac{1-n-d}{R+n}\right) z(u(q_b) - q_b).$$

Proposition 4.2. *Let z and R be sufficiently small. There exists* $M_1 \in (0, 1)$ *such that a monetary equilibrium with only nominal repayments exists for* $M \geqq M_1$.

As before the effective rate of time preference R must be small to induce a credit arrangement. The parameter z is required to be sufficiently small to restrict the usefulness of barter. Since there is no barter in Section 3.2, Proposition 4.2 illustrates the continuity of the existence of the monetary equilibrium with credit when z is near zero. The equilibrium also extends return dominance to an economy with barter. The interest rate has the same definition as in Section 3.3 and return dominance is evident from the inequality in Lemma 4.1. The intuition for return dominance is similar to that in Section 3.3.

4.2. *Equilibrium with only real repayments*

A monetary equilibrium with only real repayments is one where an IOU is repaid only with goods.[15] From the discussion leading to (4.1), real repayments are compatible with the creditor's and debtor's incentives if and only if

$$V_p - V_c + u(q_b) \geqq 0, \qquad V_p - V_d - q_b \geqq 0. \tag{4.3}$$

Also, for nominal repayments to be excluded, at least one of the inequalities (2.5) and (2.6) must be violated. That is, one of the following conditions must hold:

$$V_m < V_c, \qquad V_p - V_d - q_m < 0. \tag{4.4}$$

When nominal repayments are not incentive compatible, debtors only barter because there is no monetary exchange between a debtor and a money holder. In this case, the equation for N_d is revised to

$$0 = \dot{N}_d = \beta(1-n-d)z(1-z)N_p - \beta(1-n)z^2 N_d.$$

Consequently the equation for n becomes

$$z(1-n)\left(\frac{n}{M} - 1\right) = (1-z)\left[2 - \left(1 + \frac{1}{M}\right)n\right].$$

15. It can be shown that there is also a non-monetary equilibrium with credit, which is omitted in order to focus on monetary equilibrium.

Since a monetary trade is now possible only between a money holder and a producer, the equations for (V_m, V_p, V_d, V_c) must be revised. Omitting the zero surplus terms, we have:

$$rV_m=\beta(1-n-d)z(V_p-V_m+u(q_m)), \tag{4.5}$$

$$rV_p=\beta(1-n)z^2(u(q_b)-q_b)+\beta(1-n-d)z(1-z)(V_d-V_p+u(q_c)), \tag{4.6}$$

$$rV_d=\beta(1-n)z^2(V_p-V_d-q_b), \tag{4.7}$$

$$rV_c=\beta(1-n)z^2(u(q_b)+V_p-V_c). \tag{4.8}$$

The equilibrium pair (q_m, q_c) is given by the following equations:

$$u(q_m)-\left(1+\frac{R}{1-n-d}\right)q_m=\frac{(1-n)z}{1-n-d}\left[u(q_b)-\left(1+\frac{R}{(1-n)z}\right)q_c\right], \tag{4.9}$$

$$u(q_c)+\left[1+\frac{R+(1-n)z}{(1-n-d)(1-z)}\right]q_c$$
$$=\frac{(1-n)z}{R+(1-n)z}\left\{u(q_b)+\left[1+\frac{R+(1-n)z}{(1-n-d)(1-z)}\right]q_b\right\}. \tag{4.10}$$

Lemma 4.3. *There are either two admissible solution pairs (q_m, q_c) or no solution to* (4.9) *and* (4.10). *The two pairs differ only in q_m. All possible solutions satisfy* (2.2), (2.4), *the inequality $u(q_b)+V_p-V_c>0$ and*

$$q_c<\frac{(1-n)z}{R+(1-n)z}q_b. \tag{4.11}$$

The proof is in Appendix D. Denote the two possible solutions for q_m by q_L and q_H, with $q_L<q_H$. For brevity, let us refer to the pair (q_m, q_c) as an equilibrium if it satisfies the corresponding incentive constraints. To describe existence, denote

$$\alpha=\frac{u(q_b)-q_b}{u(u(q_b))-u(q_b)},\quad z_0=\frac{1}{2\alpha}[1+\alpha-[(\alpha-1)(\alpha+3)]^{1/2}].$$

Note that $\alpha>1$ because the function $u(q)-q$ is decreasing for $q>q_b$ and because $u(q_b)>q_b$. Also $z_0\in(0,1)$. The following proposition is proved in Appendix E.

Proposition 4.4. *For given z and $1-M$ that are bounded strictly above zero, there exists R_0 such that the two solutions q_L and q_H exist for $R<R_0$. Moreover, $q_L<q_c<q_b<q_H$. (q_L, q_c) is a monetary equilibrium with only real repayments. The pair (q_H, q_c) is also such a monetary equilibrium only if $z<z_0$.*

As before, the effective time preference R must be small to make credit valuable. Also, for an IOU to be repaid only with the creditor's consumption goods, the debtor must be able to find barter exchanges for these goods relatively fast. Otherwise nominal repayments are a superior option. Thus Proposition 4.4 requires z and $1-M$ to be bounded strictly above zero.

There can be two monetary equilibria with different purchasing powers of money. Multiple monetary equilibria are a typical feature of search models of money. The earlier model by Diamond (1984) generates multiple equilibria by increasing returns to scale in the matching technology. With constant returns to scale the Kiyotaki–Wright model generates multiple levels of probabilities with which money is accepted. Shi (1995) and Trejos and Wright (1995) have extended the result by showing that multiple equilibrium levels of the purchasing power of money exist. The present analysis extends the result further into an economy with credit.

The multiplicity depends on the means of repayment on credit. When the IOU is repaid only with money there is an odd number of equilibria in general and a unique monetary equilibrium with credit for a class of utility functions (see Example 3.6). Thus insisting on nominal repayments stabilizes the purchasing power of money and the price level. This is so because credit competes against money more strongly when an IOU is repaid only with money. As explained in Section 3.3, insisting on nominal repayments puts a lower bound on the value of money ($V_m > V_c$) and on the purchasing power of money ($q_m > q_c$). When an IOU is repaid only with goods, this dimension of the competition is absent and hence there is a larger room for the purchasing power of money to vary. The equilibrium purchasing power of money can be either smaller or bigger than the purchasing power of credit (see Proposition 4.4).

The equilibrium with only real repayments also exhibits return dominance. Because an IOU is now repaid only with goods, the interest rate is $\theta = (1/t_r) \ln (q_b/q_c)$. Since $q_b > q_c$ by Lemma 4.3, the interest rate is bounded above zero. As before, return dominance is necessary for the equilibrium with credit to exist, but the explanation is slightly different from that in Section 3.3. Here the explanation lies in a creditor's choice between accepting an IOU and terminating the credit trade. If the creditor accepts the IOU, he pays q_c units of goods for a quantity q_b of goods repaid after a random maturity. Because a debtor can only repay the IOU by conducting barter, accepting the IOU slows down the creditor's consumption. If he did not accept the IOU, he could search for both barter and monetary exchanges. To compensate for such loss in the speed of consumption, the repayment must be higher than q_c.

As in Section 3.3 an increase in the rate of time preference r reduces the purchasing power of credit and increases the expected interest rate. The expected maturity is $Et_r = 1/[\beta(1-n)z^2]$. In contrast to Section 3.3, here an increase in the proportion of money holders M increases the maturity of an IOU and reduces the expected interest rate.

4.3. *Equilibrium with both real and nominal repayments*

It is possible that an IOU is repaid with either goods or money, whichever becomes available first to the debtor. The incentive constraints are (2.2), (2.4), (2.5), (2.6), (2.7) and (4.3). In this case, the equation for n is revised to:

$$[n+(1-n)z]\left(\frac{n}{M}-1\right)=(1-z)\left[2-\left(1+\frac{1}{M}\right)n\right]^2.$$

The value V_p is still given by (4.6). The values (V_m, V_d, V_c) are now given by

$$rV_m = \beta(1-n)z[V_p - V_m + u(q_m)];$$
$$rV_d = \beta nz(V_p - V_d - q_m) + \beta(1-n)z^2(V_p - V_d - q_b);$$
$$rV_c = \beta nz(V_m - V_c) + \beta(1-n)z^2[u(q_b) + V_p - V_c].$$

TABLE 3

Coexistence of equilibria E^N and E^{NR}

	V_m	V_p	V_c	V_d
E^N	1·8046	1·3163	1·7839	0·8186
E^{NR}	1·858	1·3956	1·8368	0·9253

TABLE 4

Comparisons between equilibria

	0·5			0·65			0·75		
M	E^N	E^R_L	E^R_H	E^N	E^R_L	E^R_H	E^N	E^R_L	E^R_H
V_m	1·7946	0·6511	1·2046	1·9549	0·4551	0·9049	2·003	0·3469	0·6703
V_p	0·9255	0·6244	0·6268	1·1638	0·4243	0·4332	1·3119	0·3084	0·3104
V_c	1·7636	0·8392	0·8416	1·9272	0·6106	0·6196	1·9776	0·4659	0·4679
V_d	0·0554	0·258	0·2599	0·3674	0·0902	0·0965	0·613	0·0075	0·0087
$E\theta$	0·0103	0·0059	0·0059	0·0124	0·0055	0·0055	0·0146	0·0051	0·0051

The joint equation system for (q_m, q_c) is

$$q_c=\frac{1-n}{R+n+(1-n)z}\left[zu(q_b)-u(q_m)+\left(1+\frac{R+n}{1-n}\right)q_m\right],$$

$$\frac{(2+k)n}{R+n+(1-n)z}q_m=u(q_c)+(1+k)q_c-\frac{(1-n)z}{R+n+(1-n)z}[u(q_b)+(1+k)q_b],$$

where $k\equiv[R+n+(1-n)z]/[(1-n-d)(1-z)]$. As in Section 4.2, there are an even number of solutions to these two equations. However, it is difficult to verify the incentive constraints analytically. The next section provides a numerical example to show that such an equilibrium exists for moderate M and small but not too small z.

4.4. *Coexistence of the equilibria*

This section uses numerical examples to show the coexistence of the three monetary equilibria in Sections 4.1 to 4.3. Denote the equilibrium with only nominal repayments by E^N, the one with only real repayments by E^R, and the one with both real and nominal repayments by E^{NR}. For equilibrium E^R, the purchasing power of money can be either high or low. Use E^R_H and E^R_L to denote this subdivision. The equilibrium E^{NR} may also have two different levels of the purchasing powers, but only the high purchasing power survives to be an equilibrium in the following numerical exercises. The utility function and the values of parameters (σ, R) are taken from Example 3.6. In addition, choose $z=0{\cdot}1$. The value for M varies across different comparisons.

When $M=0{\cdot}85$, the equilibria E^N and E^{NR} coexist. The utilities are listed in Table 3, which indicates that allowing for both types of repayments Pareto-improves agents' utilities. This is hardly surprising because an additional means of repayments speeds up consumption.

The equilibria E^N and E^R coexist for $M\in[0{\cdot}47, 0{\cdot}78]$. Three different levels of M are chosen from this interval. For all three levels, money can have either high or low purchasing power in the equilibrium E^R. The utilities and expected interest rates are listed in Table 4. The following conclusions can be drawn. First, in all three cases the equilibrium E^R_H Pareto dominates the equilibrium E_L^R so that the latter is inefficient. Second, the equilibrium E^N Pareto dominates the equilibrium E_H^R ex ante (before match) because V_p and V_m are higher in equilibrium E^N. For large M the equilibrium E^N even Pareto dominates the equilibrium E_H^R ex post (after match). Finally, the expected interest rate increases with M in equilibrium E^N but decreases with M in equilibria E_H^R and E_L^R.

5. CONCLUSION

When decentralized trading is characterized by random matching, the use of money allows agents to exchange and consume faster. This desirable feature supports a valuable fiat money. Extending the Kiyotaki–Wright search monetary model to incorporate bilateral credit and price determination, this paper shows that money can be valuable despite the fact that its rate of return is dominated by credit. However, when a credit is repaid with only money, the competition with credit bounds the purchasing power of money from below and eliminates the inefficient weak monetary equilibrium found in previous search models.

The search model captures the transaction cost in a specific way. When it is costly to find a suitable trading partner, agents try to make the best deal out of their current matches. The resulting terms of trade deviate from those in a Walrasian market. The deviation depends both on the proportions of different types of agents in the market and on how agents perceive the gains from trade. Different beliefs induce different means of exchange and different forms of repayments on credit. In this sense a search model might provide a novel approach to other issues such as the exchange rate and the purchasing power disparity.

While this paper improves upon the previous ones by simultaneously introducing credit and price determination, it shares their restriction that money holders spend all of their holdings in each trade. To examine some policy issues such as money growth, it is appropriate to allow money holders to spend only part of their holdings. The difficulty involved in this extension is that agents' strategies depend on their money holdings and their histories of trading. In general there will be a distribution of money holdings and hence of prices. The joint determination of the value of money and the distribution of money holdings presents a serious technical difficulty. Diamond and Yellen (1990) have attempted to solve the joint determination under the restrictions of cash-in-advance and indivisible goods. It remains to see whether their technique is tractable without these restrictions. Circumventing the problem by making some simplifications, Shi (1994) has extended the search model to incorporate divisible money and money growth.

APPENDIX

A. *Proof of Lemma 3.4.*

For sufficiently small positive q, $f(q)-g(q)<0$. Since both f and g are convex, there are an odd number of positive solutions to the equation $f(q)-g(q)=0$ if there is any. To show that there exists at least one positive solution and that all such solutions satisfy (3.11), consider the reference line $q_c=sq_m$, which is drawn in Figure 2 for $s=n/(n+R)$. We show that for all $s\in[n/(R+n),\infty)$, the reference line intersects first the curve f and then g in the positive quadrant (if such intersections exist). If this can be done, the two curves f and g must intersect at least once in the positive quadrant and all the intersections must be below the line $q_c=(n/(n+R))q_m$, since f and g are convex and since $f(q)<g(q)$ for sufficiently small q.

To begin, compute:

$$f'(\infty)=\frac{1+R}{R+n} \quad\text{and}\quad g'(\infty)=\frac{n}{R+n}\cdot\frac{2+\dfrac{R+n}{1-n-d}}{1+\dfrac{R+n}{1-n-d}}.$$

It can be verified that $n/(R+n)<g'(\infty)<f'(\infty)$. Since the reference line intersects neither of the two curves in the positive quadrant if $s\geqq f'(\infty)$, we can restrict attention to the case $s\in[n/(R+n), f'(\infty))$. If $s\in[g'(\infty), f'(\infty))$, the reference line intersects only the curve f in the positive quadrant so the desired result is evident. Thus we restrict attention further to the case $s\in[n/(R+n), g'(\infty))$.

Denote the level of q_m at the intersection between the reference line and each of the two curves by q_f and q_g respectively. To show $q_f<q_g$, substitute q_c by sq_m into the defining equations of f and g. Then

$$u(q_f)-k_f q_f=0, \qquad k_f\equiv\frac{1+R-s(R+n)}{1-n},$$

$$u(sq_g)-k_g sq_g=0, \qquad k_g\equiv\frac{n}{s(R+n)}\cdot\left(2+\frac{R+n}{1-n-d}\right)-\left(1+\frac{R+n}{1-n-d}\right).$$

Therefore, $q_f<q_g \Leftrightarrow u(sq_f)-k_g sq_f>0 \Leftrightarrow u(sq_f)>(sk_g/k_f)u(q_f)$. Tedious algebra shows that $k_g<k_f$ and $sk_g<k_f$. Since u is concave, these two inequalities imply:

$$\frac{sk_g}{k_f}u(q_f)<u\left(\frac{sk_g}{k_f}q_f\right)<u(sq_f). \quad \|$$

B. *Proof of Proposition 3.5.*

It suffices to verify (2.5). Substituting (V_p, V_d) and using (3.9) and (3.10), one can show that $(2.5)\Leftrightarrow q_c<(n-R)/(n+R)q_m$. Clearly this inequality is violated if $R\geqq n$. For existence, R must be sufficiently small. Let this be the case. Consider the reference line $q_c=(n-R)/(n+R)q_m$ and now use the notation q_f and q_g to denote the level of q_m at the positive intersection between this line and each of the curves f and g. In particular, q_f is the largest solution to:

$$u(q_f)=\left[1+\frac{2R}{1-n}\right]q_f.$$

Similar to the proof of Lemma 3.4, $q_c<(n-R)/(n+R)q_m \Leftrightarrow q_f<q_g \Leftrightarrow$

$$u\left(\frac{n-R}{n+R}q_f\right)>\left(\frac{1-n+\dfrac{1-n}{1-n-d}R}{1-n+2R}\right)u(q_f). \tag{B.1}$$

To progress, we first show that for M sufficiently close to 1 and R sufficiently close to zero, the following condition holds:

$$\frac{n-R}{n+R}>\frac{1-n+\dfrac{1-n}{1-n-d}R}{1-n+2R}. \tag{B.2}$$

This condition is equivalent to:

$$(1-n)(3n-2)-2d(2n-1)-\left(2+\frac{1-n}{1-n-d}\right)R>0.$$

Consider the function $h(M)\equiv[1-n(M)][3n(M)-2]-2d(M)[2n(M)-1]$, where the dependence of (n, d) on M is specified by (3.3) and (3.4). The following properties can be verified:

$$\lim_{M\to 1} n=1, \quad \lim_{M\to 1} d=0, \quad \lim_{M\to 1}\frac{1-n}{1-n-d}<2, \quad \lim_{M\to 1} n'(M)=1, \quad \lim_{M\to 1} d'(M)=0.$$

Thus, $h(1)=0$, $h'(1)=-1<0$, and so $h(M)>h(1)=0$ for M sufficiently close to 1. Then (B.2) is satisfied for R sufficiently close to zero and M sufficiently close to 1.

Next, note that $q_f > 0$ for $M \in (0, 1)$ and that $u(bq) > bu(q)$ for $b \in (0, 1)$ and $q > 0$. Then for M sufficiently close to 1 and R sufficiently close to zero, (B.1) holds because

$$u\left(\frac{n-R}{n+R} q_f\right) > \frac{n-R}{n+R} u(q_f) > \left(\frac{1-n+\dfrac{1-n}{1-n-d}R}{1-n+2R}\right) u(q_f).$$

On the other hand, (B.1) is clearly violated when $M \to 0$. Therefore, given sufficiently small R, there exists $M_0 \in (0, 1)$ such that (B.1) holds for $M > M_0$ and that the equality version of (B.1) holds for $M = M_0$. ||

C. *On the Assumption* $q_d = q_m$.

This appendix establishes two results: (a) If money holders cannot distinguish a debtor from a producer, the assumption $q_d = q_m$ can be endogenously delivered in some parameter regions that are consistent with the existence region of the equilibrium in Proposition 3.5. (b) If money holders can distinguish a debtor from a producer, the equilibrium with $q_d \neq q_m$ is very similar to the equilibrium in Proposition 3.5 and q_d is close to q_m when R is sufficiently small.

To show result (a), it suffices to show that, given the aggregate values (V, n, d) and given that all other agents make offers as described in Section 2.3, it is not profitable for a money holder to deviate once to an offer $q' \neq V_m - V_p$ in a match with an agent whose product is the money holder's consumption good. In particular, the aggregate values V and offers in the economy satisfy $V_p - V_d \geqq q_m = V_m - V_p$. Clearly, deviating to an offer $q' < q_m$ or $q' > V_p - V_d$ is not profitable to the money holder: in the first case the offer is always accepted but the money holder receives a lower surplus than with the offer q_m; in the second case the offer is always rejected and so the money holder receives a zero surplus. Also, any deviating offer $q' \in (q_m, V_p - V_d)$ is dominated by the offer $q' = V_p - V_d$: both are only accepted by a debtor but the latter yields a higher surplus to the money holder. Thus, it suffices to show that the deviation $q' = V_p - V_d$ is not profitable. Such an offer generates the following expected surplus to the money holder:

$$\frac{d}{1-n}[u(V_p - V_d) + V_p - V_m],$$

where $d/(1-n)$ is the conditional probability that the trading partner in the match is a debtor. Thus, the deviation is not profitable if and only if

$$\frac{d}{1-n}[u(V_p - V_d) + V_p - V_m] \leqq u(q_m) - q_m. \tag{C.1}$$

From (3.6) and (3.7),

$$V_p - V_d = \frac{1}{1+R-d}[(1-n-d)u(q_c) + nq_m].$$

Substituting this equation into (C.1), replacing $u(q_c)$ by (3.10), and noting $V_m - V_p = q_m$, we can rewrite (C.1) as

$$\frac{d}{1-n}\left[u\left(q_m + \left(\frac{n-R}{n+R}q_m - q_c\right)\right) - q_m\right] \leqq u(q_m) - q_m.$$

Since $d/(1-n) < 1$ for $M < 1$ and $(n-R)/(n+R)q_m = q_c$ for $M = M_0$, where M_0 is described in the above proof for Proposition 3.5, the above relation is satisfied with strict inequality when $M = M_0$. Therefore, the relation is satisfied for M larger than M_0 but sufficiently close to M_0, in which case the deviation from the offer q_m is not profitable to the money holder.

To show result (b), assume $q_d \neq q_m$. Under Assumption 1, the money holder makes a take-it-or-leave offer in the trade with a debtor so that $q_d = V_p - V_d$ and $V_d = 0$. The equation for V_p and V_c are the same as (3.6) and (3.8). Omitting the zero surplus terms, V_m is given by

$$rV_m = \beta z(1-n-d)[u(q_m) - q_m] + \beta zd[u(q_d) - q_m].$$

The quantities q_m, q_c and q_d are given by the following equations:

$$u(q_c)=\left(1+\frac{R}{1-n-d}\right)q_d; \tag{C.2}$$

$$nq_m=Rq_d+(R+n)q_c; \tag{C.3}$$

$$(1-n-d)u(q_m)=\left(1+\frac{R}{n}\right)(1+R-n)q_c+(1+R)\frac{R}{n}q_d-du(q_d). \tag{C.4}$$

(C.2) gives q_c as a convex function of q_d. Substituting this function into (C.3) and (C.4), we obtain two equations for q_m and q_d. Result (b) can be shown by establishing the following results.

Lemma C.1. *There are an odd number of positive solution pairs* (q_m, q_d) *to* (C.3) *and* (C.4). *At least one such pair satisfies*

$$q_m>q_c, \quad q_d>q_c, \quad u(q_c)>q_d, \quad u(q_m)-q_m>0;$$
$$\frac{q_d}{q_m}\in\left(\frac{n}{n+2R}, \quad \frac{1-n+R}{1-n}\right). \tag{C.5}$$

Proof. We only outline the procedures of the proof. First, the inequalities $u(q_c)>q_d$ and $q_m>q_c$ are evident from (C.2) and (C.3). Second, define $q_1>0$ by $u(q_1)-(1+R/(1-n-d))q_1=0$. Then $q_d>q_c \Leftrightarrow q_d<q_1$ by (C.2). Using the approach in the proof for (3.11), one can show indeed $q_d<q_1$. The same inequality implies that there is at least one positive solution to (C.3) and (C.4). Similar techniques show (C.5). With (C.5), one can use (C.4) to deduce $u(q_m)-q_m>0$. ||

Remark 2. *When* $R\to 0$, (C.5) *implies* $q_d\to q_m$.

Proposition C.2. *For* $M\geqq 1/2$, *the monetary equilibrium with credit exists.*

Proof. With the above lemma and $V_d=0$, it suffices to verify $u(q_d)>q_m$. Since $q_m<(1+2R/n)q_d$ from (C.5) and $u(q_d)>(1+R/(1-n-d))q_d$ from the proof for $q_d>q_c$ in the above lemma, a sufficient condition for $u(q_d)>q_m$ is $1-n-d\leqq n/2$. This inequality is satisfied for $M\geqq 1/2$. ||

D. *Proof of Lemma 4.3.*

There is a unique solution for q_c from (4.10). Using the property $u(bq)>bu(q)$ for $b\in(0, 1)$, one can also show (4.11) from (4.10). Substitute the solution for q_c into (4.9). Clearly there are either two admissible solutions or no solution for q_m. If the solutions exist, then

$$u(q_b)+V_p-V_c=u(q_b)-q_c>u(q_b)-q_b>0.$$

Also, (2.4) holds because

$$V_d-V_p+u(q_c)=u(q_c)+q_c-\frac{(1-n)z}{R+(1-n)z}(u)q_b)+q_b)$$
$$=\frac{(1-n)z}{(1-n-d)(1-z)}\left[q_b-\frac{R+(1-n)z}{(1-n)z}q_c\right]>0.$$

The first equality follows from substituting V_d and V_p; the second equality from substituting (4.10) and using (4.11). (4.11) implies that the right-hand side of (4.9) is positive. The left-hand side must also be positive and hence (2.2) holds. ||

E. *Proof of Proposition 4.4.*

For given z and M, we show the proposition for $R\to 0$. The solutions q_L and q_H exist if and only if there are some positive values of q_m that make the left-hand side of (4.9) greater than the right-hand side. To show that there are such values, first note that (4.11) $\Rightarrow u(q_c)-q_c>0$. Next, one can establish $(1-n)z<1-n-d$ for all $z<1$. Since $\lim_{R\to 0} q_c=q_b$ if z and $1-M$ are bounded strictly above zero, then at $q_m=q_c$ we have

$$\text{LHS}(4.9)|_{q_m=q_c}=u(q_c)-\left(1+\frac{R}{1-n-d}\right)q_c$$

$$=\text{RHS}(4.9)+\left(1-\frac{(1-n)z}{1-n-d}\right)(u(q_c)-q_c)>\text{RHS}(4.9).$$

The solutions q_L and q_H exist. The same inequality implies $q_L<q_c<q_b<q_H$.

To qualify as a monetary equilibrium with only real repayments, the pair (q_m, q_c) must satisfy (2.7), the inequality $V_p-V_d-q_b\geqq 0$, and one of the inequalities in (4.4). For $R\to 0$, both (q_L, q_c) and (q_H, q_c) satisfy (2.7) and $V_p-V_d-q_b\geqq 0$ because

$$V_p+u(q_m)-V_d-u(q_c)=\frac{(1-n)z}{R+(1-n)z}[u(q_b)+q_b]-u(q_c)-q_c+u(q_m)$$

$$\xrightarrow{R\to 0} u(q_m)>0,$$

$$V_p-V_d-q_b=\frac{(1-n)zu(q_b)-Rq_b}{R+(1-n)z}-q_c\xrightarrow{R\to 0} u(q_b)-q_b>0.$$

Since $V_m<V_c \Leftrightarrow q_m<q_c$ and since $q_L<q_c$, the pair (q_L, q_c) is a monetary equilibrium with only real repayments.

In contrast, the pair (q_H, q_c) violates the constraint $V_m<V_c$. To qualify as a monetary equilibrium with only real repayments, the pair must satisfy $V_p-V_d-q_m<0$. Because

$$V_p-V_d-q_m=\frac{(1-n)z}{R+(1-n)z}(u(q_b)+q_b)-q_c-q_H$$

$$\xrightarrow{R\to 0} u(q_b)-q_H>0,$$

it is necessary that $q_H>u(q_b)$. Since the left-hand side of (4.9) is decreasing in q_m at q_H, the last requirement is equivalent to

$$u(u(q_b))-\left(1+\frac{R}{1-n-d}\right)u(q_b)>\frac{(1-n)z}{1-n-d}\left[u(q_b)-\left(1+\frac{R}{(1-n)z}\right)q_c\right].$$

When $R\to 0$, $q_c\to q_b$ and the above inequality is satisfied if and only if $d/(1-n)<1-\alpha z$. Substituting the solution for n, the condition is equivalent to $z<z_0$. ‖

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