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Money and specialization*

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Summary. This paper examines the relationship between specialization and the use of money in two versions of the search-theoretic monetary model. The first version establishes a surprising result that specialization is more likely to occur in a barter economy than in a monetary economy. The result is reversed in the second version where a different specification of preferences is adopted to limit the scope of barter. This contrast between the results provides a concrete illustration of the general argument that money encourages specialization only when it enlarges the extent of the market.

JEL Classification Numbers: E40, D40.

1 Introduction

The relationship between the use of money and specialization has been an intriguing issue in economics. The discussion on this issue dates back at least to Adam Smith [16, pp 21–33]. Smith argued that specialization is limited by the extent of the market and that the use of money encourages specialization by enlarging the extent of the market. Despite the intuitive appeal, formally modeling this argument presents a challenge on the theory of monetary economics. The difficulty is as follows. Smith's argument seems to hold it as a premiss that markets are not well organized so that money can possibly improve the organization. Well-known monetary models do not share this premiss (e.g. [19], [11] and [17]). They typically employ Walrasian markets, although some frictions are added to make fiat money valuable. To investigate Smith's argument, a nonwalrasian monetary model seems more appropriate.

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A useful candidate for such investigation is newly developed by Kiyotaki and Wright [8, 9, 10] after Diamond [3]. Such a model, called the *search-theoretic monetary model*, maintains that exchanges are decentralized so that pairs of traders are brought together by random matching. Not all matches create trade, because some pairs of agents may fail to have the double coincidence of wants in barter. Money can be valuable in this environment because it can alleviate the lack of double coincidence of wants.[1]

This paper uses the search monetary model to examine Smith's argument. It shows that money does not necessarily encourage specialization. Money encourages specialization only if it sufficiently enlarges the extent of the market. I establish this result by comparing two versions of the search model. Money *discourages* specialization in the first version of the model but can encourage specialization in the second version. The second version differs from the first in that its specification of agents' preferences limits barter more severely than in the first version. The limitation on the barter opportunity allows money to enlarge the extent of the market more significantly in the second version of the model than in the first version.

The ambiguous relationship between money and specialization is in contrast to the unambiguous relationship found by Kiyotaki and Wright [10]. Also using a search model, these authors have shown that increasing the money supply unambiguously increases specialization, provided money is valuable. The difference results from the following two improvements which the present model makes upon the Kiyotaki–Wright model. First, Kiyotaki and Wright define the degree of specialization as the range of commodities that an agent is willing to consume, not as that an agent is willing to produce. In fact, the range of products each agent possibly produces in their model is independent of the degree of specialization. In common discussion, however, high specialization is usually meant to be a narrow range of commodities produced by each agent. I adopt this more intuitive definition in this paper. Second, Kiyotaki and Wright prohibit agents from consuming their own products. This restriction forces agents to exchange in the market and to be specialized in producing what other agents want. Thus the restrictions limits the scope of the autarkic production a priori. In this paper, I allow agents to produce some products which generate positive utility to themselves.[2]

The allowance for the autarkic production is a separate contribution to the search model. With the exception of [1] (see footnote 2), previous search models of money exclude the possibility of the autarkic production. One might suspect that the existence of valuable fiat money established in those

[1] There are other nonwalrasian monetary models such as [15]. Different from those alternative models, the search model captures the time-consuming nature of the trading proess.

[2] In a similar framework, Burdett, et al. [1] also allow agents to derive utility from their own products with positive probability, but agents are specialized in production as in [10]. Thus there is no discussion on specialization in that model.

models is sensitive to such a restriction. To the contrary, this paper shows that valuable fiat money exists despite the opportunity of the autarkic production.

There are other types of models that emphasize the relationship between money and specialization, such as King and Plosser [7] and Cole and Stockman [2]. These models differ from the present one in that they are Walrasian. There are other differences. King and Plosser focus on the informational role of money and on commodity money. The present paper focuses on the medium-of-exchange role of money and on fiat money. Cole and Stockman analyze how money growth changes the degree of specialization in a cash-in-advance economy with some credit goods. They also use a costly transaction technology to represent barter. However it is not apparent from their model how money facilitates exchange by alleviating the lack of double coincidence of wants.[3]

The remainder of this paper is organized as follows. Section 2 characterizes search equilibria in an economy without money. Section 3 characterizes the monetary equilibrium and shows that money discourages specialization. Section 4 shows the sensitivity of the result by specifying a different preference that limits the scope of barter. Section 5 provides concluding remarks for this paper.

2 Barter economy

2.1 Tastes and technologies

Consider a disrete-time economy with $N(\geq 3)$ agents and N types of goods. All goods are perishable and agents can hold no more than one indivisible good at any given time.[4] Agent $i \in \{1. \dots, N\}$ is endowed with the technology that can be used to produce one indivisible unit of good i without cost in each period. The same technology can be used to produce good $j \neq i$ with a cost cu in terms of utility where $0 < c < 1$. In the latter case, the agent's production is said to be costly.

Agents are different in tastes. Agent's tastes are determined by random shocks. A shock randomly and uniformly selects a good i^* among the N goods to be agent i's ideal good. Good i^* is called agent i's *consumption good*. An agent derives utility u from one unit of the consumption good and nothing from anything else. The taste shocks are drawn every time after an agent consumes. Thus an agent's consumption good changes over time in a random fashion as he continues to consume. The probability with which a specific type of good becomes an agent's consumption good is $1/N$.

[3] In that model, the cost of such transactions is summarized by a cost function which increases in the units of good exchanged through those technologies.

[4] Assuming that goods are perishable eliminates commodity money. This helps us to focus on fiat money. The same objective can be achieved by assuming that accepting goods in exchange incurs a transaction cost (see [10]). The current assumption is simpler. Indivisibility is imposed also for simplicity. Divisible goods can be allowed as in [12,13] and [18].

The possibility of specialization arises as follows. Consider agent i whose consumption good is i^*. If $i = i^*$, the agent is lucky and he can use his technology to produce the consumption good and consume it. If $i \neq i^*$, he has two options. One is to use the technology to produce the consumption good with the cost cu and consume it; the other is to produce commodity i and use it to exchange for the consumption good i^*. In the first case, the agent is autarkic. If all agents do so, there is neither specialization nor exchange. In the second case, the agent is specialized in producing good i. The advantage of specialization is to save the production cost. The cost is that it takes time to find suitable trading partners and hence consumption may not be as quick as in the autarkic case. Therefore, the relative advantage of specialization depends on the extent of the market.

For clarity, let us specify the timing of events. Taste shocks arrive at the beginning of a period. Only those agents who have just consumed in the last period receive new taste shocks. Those agents who have not consumed since the arrival of the last taste shock do not recieve new taste shocks because they are still trying to obtain the consumption good to satisfy their desire. After the taste shocks, agents decide whether to enter the market. Trading takes up the entire period. Going to the market is costly in the sense that an agent in the market cannot produce goods other than i for the period.[5] Production and consumption take place at the end of each period. The discount factor between two adjacent periods is $R(< 1)$.

Trading takes place in the form of random matching. Each agent in the market is matched to exactly one other agent. If there is a mutually agreeable trade between two matched agents, each produces one unit of his specialized good and gives it to his partner to consume. When two matched agents have no mutually agreeable trade, they depart and enter the next period with their taste unchanged. Because goods are perishable, agents in the market will always try to find mutually agreeable traders first before producing. Agents are anonymous so that credit cannot be arranged.[6]

A mutually agreeable trade requires a double coincidence of wants. Consider agent i who has a costly production (i.e., $i \neq i^*$). Let us compute the probability of trade perceived by agent i. The only chance of trade that agent i has is when he meets someone who likes good i and also produces good i^*. Since a partner's taste is randomly drawn from the N goods, the probability with which the partner likes good i is $1/N$. This partner must be unable to produce good i costlessly with the technology, otherwise he will not come to the market. That is, the partner must have the technology to produce one of the $N - 1$ goods $\{1, \ldots, i-1, i+1, \ldots N\}$. Therefore, the

[5] If an agent can exercise his costly autarkic production even when he is in the market, he is clearly better off by always going to the market. The restrictions used here can be naturally satisfied by an alternative way of modelling that requires agents to produce their specialized goods before going to the market. However, the equations are simpler with the current assumptions.

[6] See [4], [6] and [13] for examinations on credit in search models.

partner can produce agent i's consumption good i^* with probability $1/(N-1)$. Conditional on the presence of other agents in the market, agent i will have a probability $[N(N-1)]^{-1}$ to have a trade. Let Π be the probability that other agents go the market when their technologies are costly to produce their consumption goods. Then the unconditional probability of trade for agent i is $\Pi/[N(N-1)]$.

2.2 Decision problems

To describe agents' decision problems, let p, w, a and e be states which agents experience. For clarity consider agent i. State p is when agent i's consumption good is i. State a is when $i^* \neq i$ but the agent has decided not to go to the market (be autarkic). State e is when $i^* \neq i$ but the agent has entered the market to exchange. State w is a pseudo state, describing the situation where $i^* \neq i$ and the agent yet has to decide whether to enter the market. Let V_k be present value of being in state $k(= p, w, a, e)$. Those values are given by the market and hence exogenous to each other. In this paper, I only examine the stationary equilibrium where all such values are constant.

In state p, the agent produces and consumes. Consumption generates utility u. The agent enters the next period with a new taste shock that will place him in state p with probability $1/N$ and in state w with probability $1 - 1/N$. With a discount factor R, the following equation can then be easily understood:

$$V_p = u + R\left[\frac{1}{N} V_p + \left(1 - \frac{1}{N}\right) V_w\right]. \tag{1}$$

If the agent is in the state a, he produces the consumption good with a cost cu, consumes and then enters the next period. Hence

$$V_a = V_p - cu. \tag{2}$$

If the agent is in the state e, he is able to trade with probability $\Pi/[N(N-1)]$. If he fails to trade, he enters the next period with unchanged taste. Then he makes the decision again on whether to enter the market. That is, he will be in state w next period. Therefore,

$$V_e = \frac{\Pi}{N(N-1)} V_p + \left[1 - \frac{\Pi}{N(N-1)}\right] R V_w. \tag{3}$$

Finally, with the notation Π, the value in state w is given by

$$V_w = \Pi V_e + (1 - \Pi) V_a. \tag{4}$$

The probability Π must be determined in equilibrium. To do so, let us specify the decision of an individual agent in state w. Taking Π and $V_k\,(k = p, a, e, w)$ as given, such an agent decides whether to enter the market. Let π be the probability with which he enters the market. Then π is the solution to the following problem:

$$\max_{\pi}\{\pi V_e + (1-\pi)V_a\}.$$

Therefore, $\pi = 0$ if $V_e < V_a$; $\pi = 1$ if $V_e > V_a$; and $\pi \in [0,1]$ if $V_e = V_a$. Because the values of V depend on Π, this decision implicitly defines a best response correspondence $\pi(\Pi)$.

To explicit express the correspondence $\pi(\Pi)$, solve V_p from (1)–(4):

$$V_p = \frac{u}{1-R} \cdot \frac{1 - R\Pi\{1 - \Pi/[N(N-1)]\} - cR(1-1/N)(1-\Pi)}{1 - R\Pi N^{-1}\{1 - \Pi/[N(N-1)]\}}. \quad (5)$$

Substitute this back into (2)–(4) to solve for V_e, V_a and V_w. Then the condition $V_e > V_a$ is equivalent to

$$\Pi > \Pi_0 \equiv N(N-1) \cdot \frac{1 - c(1 - R/N)}{1 + cR/N}. \quad (6)$$

Thus the best correspondence is

$$\pi(\Pi)\begin{cases} = 0, & \text{if } \Pi < \Pi_0 \\ \in [0,1], & \text{if } \Pi = \Pi_0 \\ = 1, & \text{if } \Pi > \Pi_0. \end{cases} \quad (7)$$

In words, an individual agent chooses to enter the market if others are likely to do so.

2.3 Equilibrium

In equilibrium, $\pi = \Pi$. Denote this common probability by π^*. If $\pi^* = 0$, there is no exchange and agents are autarkic. If $\pi^* = 1$, all agents are completely specialized. If $\pi^* \in (0,1)$, specialization is incomplete. A *barter equilibrium* is a list $\{V_p, V_e, V_a, V_w; \pi, \Pi\}$ such that $\pi(\Pi) = \Pi = \pi^* \in [0,1]$ and that the vector V satisfies (1)–(4). An equilibrium exists if and only if the common value $\pi^* \in [0,1]$ exists.

It is not difficult to verify that $\pi^* = 0$ is always a solution to $\pi(\Pi) = \Pi$ and hence autarky is always an equilibrium. The existence of such an equilibrium is intuitive. If an agent believes that no other agent goes to the market, he cannot do better than producing the consumption goods by himself. If all agents believe so, the belief $\Pi = 0$ is supported in equilibrium.

A more interesting equilibrium is such that agents are completely specialized (i.e. $\pi^* = 1$). It can be verified that such an equilibrium exists if $\Pi_0 \leq 1$. The existence of such an equilibrium is also intuitive. If an agent believes that other agents go to the market for sure, the value of going to the market exceeds the value of the autarkic production (since $\Pi = 1 \geq \Pi_0$ in this case). Then it is best for him to go to market for sure. If all agents believe so, the belief $\Pi = 1$ is supported by equilibrium. The condition

$\Pi_0 \leq 1$ is equivalent to

$$c \geq c^b(N,R) \equiv \left(\frac{1}{1-[N(N-1)]^{-1}} - \frac{R}{N}\right)^{-1}. \tag{8}$$

Since $c \in (0,1)$, this condition can be satisfied only if $c^b < 1$ and hence only if $N - 1 - N^{-1} < R^{-1}$.

It can be verified that $\pi^* = \Pi_0$ is also an equilibrium when $\Pi_0 < 1$. We summarize the result in the following proposition.

Proposition 1. *In a barter economy, an autarkic equilibrium exists for all* $c \in [0,1]$*; a completely specialized equilibrium exists if* $c \geq c^b(N,R)$ *and* $N - 1 - N^{-1} < R^{-1}$*; an incompletely specialized equilibrium* $\pi^* = \Pi_0$ *exists if* $c > c^b(N,R)$ *and* $N - 1 - N^{-1} < R^{-1}$.

Because specialization saves agents the the autarkic production cost, it is intuitive that a specialized barter equilibrium exists when the autarkic production cost is large. It is also intuitive that a completely specialized equilibrium requires that the number of goods be small. This is because the probability of having a successful barter trade is $1/[N(N-1)]$, which decreases with the number of goods. If N is too large, market exchange loses its appeal as successful exchanges become difficult.

Note that whenever the incompletely specialized exists, the completely specialized equilibrium also exists. For the parameter values that satisfy $c = c^b(N,R)$, the completely specialized equilibrium exists but the incompletely specialized equilibrium does not. For this reason and for simplicity, I will ignore the incompletely specialized equilibrium in the remainder of this paper and use the term "specialized equilibrium" to refer to the completely specialized equilibrium.

Different types of equilibria can be Pareto ranked whenever they coexist. It can be shown that in the parameter region $c \geq c^b(N,R)$ and $(N - 1 - N^{-1} < R^{-1})$ where equilibria coexist, the specialized equilibrium always Pareto dominates the autarkic equilibrium. That is, V_p is larger in the specialized equilibrium and $V_e > V_a$.

3 Monetary economy

Now suppose money is invented. A proportion $M\,(< 1)$ of agents are given money. Call these agents the money holders and other agents the producers. As in [10], I assume that each money holder is given one indivisible unit of money and can hold up to one unit.[7] Also as in [10], money holders cannot produce. Because money is intrinsically useless, there always exists an equilibrium where producers do not accept money. This nonmonetary equilibrium exists if all agents believe that money has no value. There might

[7] It is possible but complicated to allow fiat money to be divisible in a search model (see Shi [14] and Green and Zhou [5]).

also be an equilibrium where money is accepted but only partially. In this paper, I will ignore these two types of equilibria and focus on the more interesting monetary equilibrium where agents accept money with probability one.[8]

Since the focus of this paper is on the completely specialized equilibrium, the procedure for finding such an equilibrium can be simplified as follows. The analysis in Section 2 indicates that a specialized equilibrium exists if and only if the inequality $V_e > V_a$ holds for $\Pi = 1$. One can start with $\Pi = 1$, calculate the values of V_e and V_a, and then impose the inequality $V_e > V_a$ to find the existence conditions. We follow this procedure below.

Some new notation is needed. Let state m be the situation where an agent is holding money. Let V_m be the present value of holding money. Let N_p and N_e be the measures of agents in states p amd e respectively. Since $\Pi = 1, N_e + M$ is the measure of traders and $N_p = 1 - (N_e + M)$. Let $n = M/(N_e + M)$ be the proportion of money holders in the exchange process.

It is clear that V_p and V_a are still given by the formulas (1) and (2) with $V_w = V_e$. However the formula for V_e now must incorporate the possibility of monetary exchange:

$$V_e = \frac{(1-n)}{N(N-1)} V_p + \frac{n}{N} R V_m + \left[1 - \frac{(1-n)}{N(N-1)} - \frac{n}{N}\right] R V_e. \tag{9}$$

The three terms on the right-hand side represent, respectively, the value of a successful barter trade, of a successful monetary trade and of an unsuccessful trade. Let us interpret the second term. A producer in the market has the probability n to be matched with a money barter. If the money holder likes the producer's specialized production good, which happens with probability $1/N$, trade is successful. Receiving money, the producer expects the present value RV_m from future exchange. Thus a producer has an expected value $\frac{n}{N} R V_m$ from being matched with a money holder. Conditional on being matched, a producer in the market has a higher probability $(1/N)$ of a successful monetary trade than the probability $([N(N-1)]^{-1})$ of a successful barter trade. In this way the use of money alleviates the difficulty of double coincidence of wants.

Similar to (9), the value of holding money is given by:

$$V_m = \frac{1-n}{N} V_p + \left(1 - \frac{1-n}{N}\right) R V_m. \tag{10}$$

The value of holding money must be bounded below because accepting money must be a rational choice for each producer. If a producer accepts money, he expects a value RV_m. But if he rejects money he expects RV_e. Thus for producers to accept money with probability one in exchanges, we must have:

$$V_m > V_e. \tag{11}$$

[8] See [10] for the existence of nonmonetary equilibrium with partially acceptable money.

To completely determine a monetary equilibrium, one must determine the measures N_e and N_p. In a stationary equilibrium, N_e is such that the flows of agents in and out of state e equal each other. Thus

$$0 = \dot{N}_e = \left(1 - \frac{1}{N}\right) N_p - \frac{n}{N} N_e - \frac{1-n}{N(N-1)} N_e. \tag{12}$$

This equation can be explained as follows. The flow into state e is $(1 - N^{-1})N_p$, the producers who have just recieved unlucky shocks. The flow out of state e, given by the last two terms of (12), consists of the producers who have either a successful monetary trade or a barter trade. With the definition of n and the relation $N_p = 1 - (N_e + M)$, (12) can be rewritten as a quadratic equation of $1 - n$:

$$M\left(1 - \frac{1}{N-1}\right)(1-n)^2 - (N - 1 + M)(1-n) + (N-1)(1-M) = 0. \tag{13}$$

It is easy to show that (13) has a unique solution for n that lies in (0,1). Denote this solution by $n(M)$ to emphasize its dependence on M. It can also be shown that $n(M)$ is an increasing function. The number N_e can be recovered from the definition of n.

A *completely specialized monetary equilibrium* is a collection $\{V_p, V_a, V_e, V_m; \pi, \Pi; n\}$ such that (i) the values V given by (1), (2), (9) and (10) satisfy the restrictions (11) and $V_e > V_a$; (ii) $n = n(M)$.

With (1), (2), (9) and (10), one can solve for the value of V_p as follows:

$$V_p = \frac{u}{1-R} \cdot \frac{1 - R + R(\frac{n}{N} + \frac{1-n}{N(N-1)})}{1 + \frac{R}{N}(\frac{n}{N} + \frac{1-n}{N(N-1)} - 1) + \frac{R}{N}\frac{n(1-N^{-1})}{1-R+R(1-n)/N}}. \tag{14}$$

The values of V_a, V_e and V_m can be recovered from (2), (9) and (10). It can then be verified that $V_e > V_a$ if and only if

$$c > c^m(n, N, R) \equiv \left\{ \frac{1 - R + R/N}{\frac{n}{N} + (1 - R + R\frac{1-n}{n})(1 - \frac{n}{N} - \frac{1-n}{N(N-1)})} - \frac{R}{N} \right\}^{-1}. \tag{15}$$

In addition to (15), a monetary equilibrium requires (11). Calculations show that (11) is always satisfied when (15) is satisfied. The existence of a completely specialized monetary equilibrium is given by the following proposition, whose proof is delayed to Appendix A.

Proposition 2. *There exists an $M_0 \in (0,1)$ such that $c^m(n, N, R) < 1$ if and only if $M < M_0$ and $N - 1 - N^{-1} < R^{-1}$. When the latter two conditions are satisfied, a completely specialized monetary equilibrium exists if and only if $c > c^m(n, N, R)$.*

Proposition 2 states intuitively that the number of money holders must be small in order for a monetary equilibrium to exist. This is because money crowds out production opportunities in this economy. Having more money holders in the economy implies fewer producers. If there are too many holders, this crowding-out effect dominates the positive exchange-facilitating role of money and makes money undesirable. Note that the condition $N-1-N^{-1} < R^{-1}$ is also necessary for $c^m < 1$. Because the probabilities of both successful monetary trade and barter depend negatively on N, this condition makes market exchange desirable relative to the autarkic production by ensuring a high probability of successful trade.

We now explain whether specialization is more likely to occur in monetary economy. A particular approach is to examine whether specialization can occur in the monetary economy for a smaller cost of the autarkic production than in the barter economy. That is, whether $c^m < c^b$. The examination leads to the following result.

Proposition 3. *For given parameters (c, z, R), specialization is more likely to occur in a barter economy than in a monetary economy.*

Proof. We show that $c^m > c^b$ for all $n \in (0,1)$ whenever the condition $N-1-N^{-1} < R^{-1}$ holds. Direct algebraic manipulation implies the following:

$$\begin{aligned} c^m > c^b \Leftrightarrow 0 &< R(N-2)(1-n)^2 - (N-R)(1-n) \\ &\quad + \frac{N[(c^b)^{-1}-1]+R}{N(c^b)^{-1}+R} N(N-1)[R+(1-R)N] \\ &= R(N-2)(1-n)^2 - (N-R)(1-n) + [R+(1-R)N] \\ &= n[R+(1-R)N - R(N-2)(1-n)]. \end{aligned}$$

The last expression is greater than $n[R+(1-R)N-R(N-2)]$, which is positive under the conditions $N \geq 3$ and $N-1-N^{-1} < R^{-1}$. □

It is surprising that a specialized barter equilibrium is more likely to exist than a specialized monetary equilibrium. The explanation lies in the condition $N-1-N^{-1} < R^{-1}$. As explained earlier, this is a restriction on market exchange imposed by the autarkic production. For market exchanges to be attractive when the autarkic production is possible, N must be small to induce a high probability of trade (monetary or barter). But if N is small, the chance of a double coincidence of wants, $1/[N(N-1)]$, is large. Thus the same condition that induces market exchanges also limits the attractiveness of monetary exchange relative to barter. For agents to hold money in equilibrium, the cost of the autarkic production must be sufficiently large and possibly larger than what is required for the existence of a specialized barter equilibrium.

The restriction induced by the possibility of the autarkic production has important welfare consequences. To examine these consequences, use a superscript m to indicate the specialized monetary equilibrium and a superscript b to indicate the specialized barter equilibrium. The following can be directly shown:

Proposition 4. *$V_p^m < V_p^b$ and $V_e^m < V_e^b$. However, $V_m > V_e^b$ for sufficiently small M.*

The first part of Proposition 4 states that a specialized monetary cannot Pareto dominate a specialized barter equilibrium. Those agents who have a lucky taste shock are worse off in the specialized monetary equilibrium than in the specialized barter equilibrium. Those producers who enter the exchange process are also worse off. Despite these negative welfare consequences on the producers, money can be valuable. The second part of Proposition 4 provides the reason. Those agents who are given money are strictly better off in the monetary equilibrium than in the barter equilibrium when the money supply is small.

4 An alternative model

The main reason for specialization to be more likely to occur in the barter economy than in a monetary economy in the last section is that the chance of barter is relatively high. In this section we show that specialization can be more likely to occur with money than without if we limit the usefulness of barter. The limitation of barter is generated by an alternative preference structure.

As in previous sections, agent i produces good i without cost. Producing any other good cost cu in terms of utility. Different from previous sections, agent i's possible consumption goods are goods i and $i+1$ only. Each time after consumption, a randon shock selects good i with probability z and good $i+1$ with probability $1-z$ to be the next consumption good. $z \in (0,1)$. Under our earlier assumption that agents accept a good in exchange only when the good is a consumption good, barter cannot exist. To illustrate this, suppose that an agent goes to the exchange process whenever his autarkic production is costly. If agent i goes to the market, his consumption good is $i+1$. The only agent in the market who can supply good $i+1$ is agent $i+1$. However, agent $i+1$ needs good $i+2$ which agent i cannot supply. Thus the double coincidence of wants does not exist. The only equilibrium is the autarkic equilibrium. Use a superscript b to indicate this economy without money, we can calculate the following:

$$V_p^b = u + R[zV_p^b + (1-z)V_a^b]; \tag{16}$$

$$V_a^b = V_p^b - cu. \tag{17}$$

Now consider an economy where a proportion M of agents hold money. As before let n be the proportion of money holders in the exchange process. Because barter is impossible, the only form of market exchange is the monetary exchange. The value functions in a completely specialized monetary equilibrium can be given as follows:

$$V_p = u + R[zV_p + (1-z)V_e]; \tag{18}$$

$$V_e = \frac{n}{N} R V_m + \left[1 - \frac{n}{N}\right] R V_e; \tag{19}$$

$$V_m = \frac{1-n}{N} V_p + \left(1 - \frac{1-n}{N}\right) R V_m. \tag{20}$$

In particular the probability for a producer, say of type i, to have a (monetary) exchange is n/N. It is so because the only money holder who likes good i is the money holder of type $i-1$.[9] Since money holders are uniformly distributed across the N types, a particular money holder is type $i-1$ with probability $1/N$.

It is clear from (19) that $V_m > V_e$. That is, a producer accepts money in exchange if all other producers do so. For agents to specialize, we need $V_e > V_p - cu$. Solving V_p, V_e and V_m from (18)–(20), we can equivalently write this condition as

$$c > c^a(n, z, N, R) \equiv \frac{1 - R + R/N - Rn(1-n)/N^2}{(1-Rz)(1 - R + R/N) + R^2 zn(1-n)/N^2}. \tag{21}$$

A necessary conditions is $c^a < 1$, which can be rewritten as

$$n(1-n) > \frac{Nz}{1+Rz}[R + (1-R)N]. \tag{22}$$

To see whether (21) and (22) can be satisfied, we need to calculate the proportion of money holders in the market, n. In a stationary equilibrium the flow of producers into the market, $(1-z)N_p$, equals the flow out of the market, $(n/N)N_e$. Since $N_p = 1 - M - N_e$ and $N_e = (1/n - 1)M$, we can solve for n:

$$n = \frac{1}{2}\left\{1 - (1-z)\frac{N}{M} + \left[\left(1 + (1-z)\frac{N}{M}\right)^2 - \frac{4N}{M}(1-z)(1-M)\right]^{\frac{1}{2}}\right\}.$$

Denote this solution for n by $n(z, M)$ to emphasize its dependence on (z, M).

Proposition 5. *When z is sufficiently small, $c^a < 1$, in which case a completely specialized monetary equilibrium exists in the alternative economy if and only if $c > c^a$. Under the same conditions, the monetary equilibrium Pareto dominates the equilibrium without money.*

Proof. Clearly when z is sufficiently small, n is in the interior of (0,1). Since the right-hand side of (22) approaches zero as $z \to 0$, (22) can be satisfied for sufficiently small z. That is, there are values of $c \in (0,1)$ that satisfy $c > c^a$. In this case the specialized monetary equilibrium exists. To show that the monetary equilibrium Pareto dominates the equilibrium without money, it

[9] A producer of type i accepts money only when he cannot produce good i, that is, only when he needs good $i+1$. Eventually every money holder in the economy, say type i, desires for the good $i+1$.

suffices to show $V_p > V_p^a$. This condition can be shown to be equivalent to $V_e > V_p - cu$ and hence to $c > c^a$. □

Note that agents are not specialized in the economy without money because each agent produces two types of goods over time. Therefore the existence of a specialized monetary equilibrium indicates that money can encourage specialization. For specialization to be possible, Proposition 5 states that not only the cost of the autarkic production must be large but also the probability of having a costless production must be sufficiently small. Both conditions induce agents to exchange in the market by limiting the gain from the autarkic production. Since market exchanges increase the rate at which agents consume, they improve the utility of all agents.

This alternative economy delivers a result on the relationship between money and specialization that differs from that in previous sections because it imposes a stronger restriction on the possibility of barter. In fact, barter is impossible in this alternative economy. Such a strong restriction is illuminating but may not be necessary for the result. The essential element of this alternative economy is that one can restrict the possibility of the cheaper autarkic production independently of the restriction on the possibility of barter. The possibility of a cheaper autarkic production is restricted through the parameter z while the possibility of barter is restricted through the particular specification of preferences. One can vary the parameter z to increase the appeal of the monetary exchange without increasing the appeal of barter. This was not possible in previous sections. There, these two aspects of the model were both parameterized by N. The very condition which increased the appeal of market exchanges also increased the appeal of barter, which gave rise to the surprising result in Section 3.

5 Conclusion

The general message of the exercise in this paper is that whether money encourages specialization depends on the extent to which money enlarges the market. When the extent of the market is already large, as in the barter economy in Section 2, introducing money merely redistributes value from producers to money holders (see Proposition 4). When the extent of the market is limited in the sense that barter is very unlikely as in Section 4, introducing money can increase the speed of exchange and hence increase the rate of consumption. Although the result is familiar from Smith's [16] verbal argument, this paper provides a concrete model to illustrate to what extent the argument can be supported.

The nonwalrasian approach of this paper to the issue of trade and specialization deserves further attention. The particular nonwalrasian feature captured by the search model is the cost of time-consuming trading. The trading cost is endogenous, depending on the proportions of different types of agents in the market. In essence money can encourage specialization because it can reduce such trading cost by speeding up the transaction. This nonwalrasian approach is fundamentally different from the traditional international trade theory and its variations which take transaction cost into account. In the present approach, money is an important device to facilitate trade. Further research along this line might be able to integrate transaction cost into the theory of trade and specialization in an endogenous fashion. A necessary step toward this goal is to allow price determination by relaxing the restrictions on agents' trading strategies. The research along this line has already begun (see [12, 13, 14] and [18]).

Appendix

A Proof of Proposition 2

Proof. Since (11) is satisfied, a completely specialized monetary equilibrium exists if and only if $c \geq c^m(n, N, R)$. Since $c < 1$, it is necessary that $c^m < 1$. This condition can be shown to be equivalent to

$$0 > f(1-n) \equiv R(N-2)(1-n)^2 - (N-R)(1-n) + \frac{RN(N-1)}{R+N}[R + (1-R)N]. \tag{23}$$

The following properties of f can be verified:

$$f(0) > 0, f'(0) < 0, f(1) = [R + (1-R)N]\left[\frac{RN(N-1)}{R+N} - 1\right].$$

Thus $f(1) < 0 \Leftrightarrow N - 1 - N^{-1} < R^{-1}$. If $N - 1 - N^{-1} < R^{-1}$, $f(1-n)$ has a unique root in (0,1). Let this root be $1 - n_0$ and define M_0 by $n(M_0) = n_0$. Since $n(\cdot)$ is an increasing function, $f(1-n) < 0 \Leftrightarrow 1 - n > 1 - n_0 \Leftrightarrow M < M_0$. Thus the three conditions in the proposition ensure existence.

If $N - 1 - N^{-1} \geq R^{-1}$, then $f(1) \geq 0$. Define n_1 by $f'(1-n_1) = 0$. Then $n_1 = 1 - \frac{N-R}{2R(N-2)}$. For $f(1-n) < 0$ for some $1 - n \in (0, 1)$, it is necessary that $n_1 > 0$, i.e. $R > \frac{N}{2N-3}$. But in this case, the following is true for all $n \in (0, 1)$:

$$\begin{aligned} f(1-n) &> R(N-2)(1-n)^2 - (N-R)(1-n) + \frac{RN(N-1)}{N+R} \\ &> \left[R(N-2)(1-n)^2 - (N-R)(1-n) + \frac{RN(N-1)}{N+R}\right]_{R=N/(2N-3)} \\ &= \frac{N(N-2)}{2N-3}\left[n^2 + \frac{1}{2(N-2)}\right] > 0. \end{aligned}$$

The first inequality follows from $R + (1-R)N > 1$; the second from the fact that the expression is an increasing function of R and that $R > N/(2N-3)$ in the current case. Therefore $c^m(n, N, R) > 1$ for all $n \in (0, 1)$ in this case and there is no specialized monetary equilibrium. □

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