EVOLUTION, COORDINATION, AND BANKING PANICS[1]

Theodosios Temzelides

Research Department

Federal Reserve Bank of Philadelphia

Philadelphia, PA 19106

e-mail: temzelid@frbphil.org

June, 1994

This draft: October 1, 1995

Abstract

I study equilibrium selection by an evolutionary process in an environment with multiple equilibria, one of which involves a banking panic. The analysis is built on a repeated version of the Diamond-Dybvig (1983) model. The optimal (run free) equilibrium is uniquely selected if it is also "risk dominant." Furthermore, the probability of observing a panic increases as the size of the banks decreases. I discuss local interaction and contagion effects that allow for a bank run to spread first among banks in the same geographic location and then throughout the entire population.

[1] I want to thank participants of the money workshop at the University of Minnesota, especially V.V. Chari, Ed Green, John Kareken, Nobu Kiyotaki, Andy McLennan, Loretta Mester, Neil Wallace, and Warren Weber for comments and disscussions. All errors are mine. The views expressed in this paper are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

I. Introduction

Increasing the stability of the banking system has been one of the main goals of bank regulation since the Federal Reserve System was established. Correspondingly, understanding why some banking regimes have been more stable than others has been a major focus of research.

The Diamond and Dybvig (D-D 1983) model of suspension of convertibility versus public deposit insurance has widely been considered the most satisfactory model of the contrast between late nineteenth and mid-twentieth century American banking regimes. In the standard version of this model without either deposit insurance or suspension, there are two Nash equilibria. One of them involves a Pareto-dominated banking panic. The threat of such a panic has widely been cited as a justification for a regulatory regime involving government deposit insurance. However, this argument is not completely satisfactory. The welfare gain from eliminating banking panics has to be weighed against the cost of incentive problems that deposit insurance places in the way of banks. While the cost of these distortions will be borne regardless of whether panics would occur in the absence of insurance, the welfare cost of panics will be roughly proportional to their frequency in a laisez-faire regime. The simple fact that a panic equilibrium exists in the D-D model does not provide any help in thinking about the question of how frequently panics might occur or of what features of the market structure might minimize their frequency. The goal of the present paper is to investigate these questions.

I first consider a repeated version of the model with a single bank. The single bank can be interpreted as many branches of the same bank or as a coalition of banks that cooperate to a large extent and for all purposes are viewed as one bank. Agents receive information on the number of early withdrawals throughout the banking system in each period. They are bounded rational and take myopic best responses, imitating the action that worked best in the previous period. Both the optimal equilibrium and the inferior one associated with a bank run are steady states of the dynamical system describing the evolution of the agents' behavior. In the presence of noise, I show that the optimal steady state will be observed most of the time, provided that it is not "too risky." Then, I turn to the case where there are many banks of equal size, where the size of a bank is the number of agents that pool their resources into a collective arrangement. The banks are ex ante identical and agents are randomly matched in the banks within each period. Agents, therefore, have information about the entire banking system but not about individual banks. I show that as the size of the representative bank in the economy decreases, each individual bank becomes subject to increasing uncertainty, and as a result, the probability of observing the panic equilibrium increases.

Finally, I turn to the case where agents are locally matched in banks close to their own geographic location. Here, agents have information about the local banking system but, once again, not about individual banks. Local interaction allows for the possibility of small clusters of panic withdrawals within the population. This generates contagion effects that lead a bank run in one bank to spread into a local panic in neighboring banks and then into a panic for the entire economy, a pattern consistent with observations from the history of banking panics in the U.S.

II. The Economy

Here I give a more detailed description of the environment.

Time

There is an infinite number of periods, labeled $t = 0, 1, 2, \ldots$. Each period is divided into three subperiods: beginning ($t0$), middle ($t1$), and end ($t2$). The economic environment is the same for each period and I will drop the index t.

Population and Endowments

In each period, there are m agents alive. Agents are endowed with one unit of the subperiod 0 good. In subperiod 0 each individual faces a preference shock that determines whether she wants to consume only in subperiod 1 (type 1, or *impatient agent*), or whether she is indifferent between consumption in subperiod 1 and consumption in subperiod 2 (type 2, or *patient agent*). Let n be the number of agents that are patient. I assume that n is a constant and large.[2]

Technology

A technology is available in subperiod 0 that can transform the subperiod 0 good to subperiod 1 and 2 goods. The technology set is characterized by a triple (y_0, y_1, y_2) such that: $y_1 \leq -y_0$, and $y_2 \leq (-y_0 - y_1)R$, where y_i is the amount of input (output) during subperiod i, $y_0 \leq 0$, $y_1 \geq 0$, $y_2 \geq 0$ and $R > 1$ for all t. The technology is, therefore, riskless but illiquid. In addition, agents can privately store the consumption good at no cost, but no investment in the illiquid technology can start after subperiod 0.

Preferences

Everyone has the same preferences in subperiod 0. In particular, each agent alive in period t has a state-dependent utility function (with the state private information), which we assume has the form: $U(C) = \begin{cases} u(c_1), & if \quad impatient; \\ u(c_1 + c_2), & if \quad patient; \end{cases}$ where: $u : \Re_+ \rightarrow \Re$ is C^2 in $\Re_{++}$, and $-\frac{cu''(c)}{u'(c)} \geq 1$.

Isolation

Agents are isolated during subperiod 1 of each period. This assumption precludes the existence of an asset market during subperiod 1. It is consistent with the notion that agents hold liquid assets because they may need to consume at times and places at which accessing asset markets is

[2]This is only a simplifying assumption and the results generalize to the case where n is random, i.e., when there is aggregate risk.

difficult.

Let c_i^j be the consumption during subperiod i of an agent of type j. Consider the problem the society faces during subperiod 0 of each period. One possibility is autarky. The autarky allocation is, for all t, $c_1^1 = 1, c_2^1 = c_1^2 = 0, c_2^2 = R$. However, everybody will be better off if they pool their resources in a collective insurance arrangement. The optimal insurance contract is the solution to the following problem:

$$\max \left(1 - \frac{n}{m}\right) u(c_1) + \frac{n}{m} u(c_1 + c_2)$$

$$s.t. \left(1 - \frac{n}{m}\right) c_1 + \frac{n}{m}\frac{c_2}{R} = 1. \tag{1}$$

As D-D show, the optimal consumption levels satisfy for all t: $c_1^* > 1, c_2^* < R$ and $c_2^* > c_1^*$. The optimal allocation is superior to autarky since people are willing to forgo some of their consumption if they turn out to be patient in return for greater consumption if they turn out to be impatient.

Banks

A bank is a contract among agents who pull their resources together in the collective arrangement. I assume that there is a finite number ($k \geq 1$) of banks of equal size offering the optimal insurance contract as a d*emand deposit contract*. The deposit contract gives each agent who withdraws in subperiod 1 $r_1 = c_1^*$ units per unit deposited in subperiod 0.

The banks are liquidated in subperiod 2 of each period so agents who do not withdraw in subperiod 1 get a pro rata share of the banks' assets in subperiod 2. The same number of banks are "re-born" at the beginning of the next period. In each period the m agents are randomly matched in the k banks. I assume that the proportion of patient and impatient agents in each bank is the same as in the population. As in the D-D model, the demand deposit contract can achieve the full information optimal risk sharing as an equilibrium and I will assume that all agents deposit initially.

III. The Game With One Bank

In each period during subperiod 0, there are m identical agents. At the beginning of subperiod 1, uncertainty is resolved and each agent learns whether she is patient or impatient. There will be n patient and $m-n$ impatient agents. Impatient agents always withdraw during subperiod 1, so in what follows they are not viewed as "strategic" players.

The n patient agents play a game of coordination choosing whether to withdraw in subperiod 1 or wait until subperiod 2. I assume that agents act myopically by playing best responses to the population configuration from the previous period. What connects two consequent periods is that the strategy that gives the higher expected payoff increases its representation in the population of patient agents, and this process continues until a "steady state" is reached.

I first describe the stage game. Let $S = \{s_1, s_2\}$ be the set of strategies available to an agent in subperiod 1.[3] Here s_1 stands for "withdraw in subperiod 1" and s_2 stands for "withdraw in subperiod 2." Let z_t be the number of patient agents adopting strategy s_2 at time t; $t = 1, 2, ..., \infty$. The state space, i.e., the range of z_t is $Z = \{0, 1, ..., n\}$. Let $\gamma(r_1) \in \{0, 1, ..., n\}$ be the minimum number of patient agents not withdrawing, so that the bank will be able to pay r_1 to every agent in the waiting line during subperiod 1. Here, $\gamma(r_1)$ is a constant for fixed r_1 and $m - (m-n)r_1 - (n-z)r_1 \geq 0$ for $z \geq \gamma(r_1)$ implies that $\gamma(r_1) = \frac{m(r_1-1)}{r_1}$. The payoffs of the two strategies are as follows:

$$\pi(s_1, z) = \begin{cases} u(r_1), & if \quad z \geq \gamma(r_1); \\ \frac{m-\gamma(r_1)}{m-z}u(r_1) + \frac{\gamma(r_1)-z}{m-z}u(0), & if \quad z \leq \gamma(r_1); \end{cases} \tag{2}$$

[3] I assume that agents play pure strategies only.

$$\pi(s_2, z) = \begin{cases} u\left(\frac{[m-(m-n)r_1-(n-z)r_1]R}{z}\right), & if \quad z \geq \gamma(r_1); \\ u(0), & if \quad z \leq \gamma(r_1). \end{cases} \tag{3}$$

At the beginning of subperiod 1, in each period t, uncertainty about types is resolved. If every patient agent chooses s_2, the equilibrium providing optimal risk sharing is achieved. It might be, however, that some patient agents will choose to withdraw early. Because of the illiquidity of the production technology, the bank in this case might not be able to pay everyone the promised amount r_1 during subperiod 1 (recall that $r_1 > 1$). If the state z is high enough, the bank can still pay r_1 to every depositor who tries to withdraw. Otherwise, the bank serves the agents who try to withdraw according to the *sequential service constraint* until it runs out of assets.[4] Let E_1 be the $n-$tuple $(s_1, ..., s_1)$ and similarly for E_2. The following lemma verifies that both the optimal allocation and the inferior one that results from a bank panic can be supported by noncooperative equilibria of this game. The proof is given in the appendix.

Lemma 1: *E_1 and E_2 are the only pure equilibria of the stage game. Furthermore, they are both strict. There exists a mixed equilibrium.*

I now turn to the repeated version of the game. I assume that the better strategy is imitated and, therefore, better represented in the population in the next period. The deterministic dynamic $z_{t+1} = b(z_t)$ gives the profile of strategies that will be used at $t+1$, given that the time t profile is z_t.[5] In this game the deterministic dynamical system has either two or three steady states, 0, n

[4]The sequential service constraint (see Wallace, 1988) requires that payments to individuals depend only on the number of people who have previously withdrawn, not on the number of prospective withdrawers. Here it is assumed that every agent contacts the bank at some random time during subperiod 1. The patient agents will then have to choose whether to "report untruthfully," i.e., try to withdraw early. I assume that the arrival times are independent across agents and that an agent's time of arrival relative to other agents is not known to him.

[5]Formally, the following weak monotonicity property and boundary conditions are assumed: $sign\,(b(z)-z)) = sign\,(\pi(s_2, z) - \pi(s_1, z))\,,\, b(0) = 0,\, b(n) = n$. As an example consider the best reply dynamic:

and a mixed one.[6] The two pure steady states correspond to the optimal and the panic equilibrium of the stage game, respectively. The multiplicity is resolved if noise is introduced into the system. Assume that with probability ϵ each player plays each strategy with equal probability.[7] This yields a stochastic dynamical system that defines a Markov chain on the finite state space Z. It is well known, then, that the Markov chain has a unique invariant distribution for a given rate ϵ. The invariant distribution satisfies global stability and ergodicity properties and is interpreted as giving the proportion of time that the society spends in each state. For expository purposes, I consider the support of the distribution μ, which results as the limit case when ϵ is driven to zero. Let $z_* \in [0, n]$ be the critical state such that[8] for all z in Z, $sign\,(\pi(s_2, z) - \pi(s_1, z)) = sign(z - z_*)$. The following lemma is key[9] (see Figure 1).

Lemma 2: *The two states* 0 *and* n *have basins of attraction under* b *given by* $\{z < z_*\}$ *and* $\{z > z_*\}$*, respectively. The limit distribution puts probability one on* n *if* $z_* < \frac{n}{2}$ *and on* 0 *otherwise.*

The equation $\pi(s_2, z) - \pi(s_1, z) = 0$ has a unique root in Z. This is a result of the coordination structure of the game and of the fact that $\pi(s_2, z) - \pi(s_1, z)$ is a continuous function of z in the neighborhood of z_*. The sign of $z_* - \frac{n}{2}$ is a function of the risk characteristics of the game. Notice that if $z_* < \frac{n}{2}$, then the optimal equilibrium is also risk dominant. The first result of this section

$$B(z) = \begin{cases} n, & if \quad \pi(s_2, z) > \pi(s_1, z); \\ z, & if \quad \pi(s_2, z) = \pi(s_1, z); \\ 0, & if \quad \pi(s_2, z) < \pi(s_1, z). \end{cases}$$

[6]For reasons that will become clear later, I ignore the mixed steady state.

[7]I assume that these probabilities are *iid* across players and time.

[8]Here z_* need not be an element of Z.

[9]Kandori, Mailath, and Rob (1993) and Young (1993) use this result in a model with random matching in pairs but the same result can be used for any group size up to n. For a more recent discussion on this and related dynamics see Binmore, Samuelson, and Vaughan (1995).

describes conditions under which panics are very rare events. See the appendix for a proof.

Proposition 1: *The unique state in the support of μ is n if $\frac{mr_1R-mR}{nr_1R-nr_1} < \frac{1}{2}$ and it is 0 if the inequality is reversed.*

Proposition 1 asserts that in environments for which the above condition is satisfied, bank panics will be very rare and the economy will spend most of the time in the optimal steady state. In the presence of noise, some agents withdraw early with positive probability in each period. With even lower probability, these withdrawals will be significant enough to result in a bank panic, i.e., the miscoordination is sufficiently large to shift the population into the basin of attraction of the panic steady state. In this case a panic is inevitable and the process will spend a number of periods close to that steady state until it escapes again. In the limit, however, the process will be in the steady state with the largest basin of attraction with probability one.[10]

As the following proposition suggests, the condition of the above proposition will tend to be satisfied if the fraction of agents who will need to withdraw early (impatient) is low, the technology is not too illiquid, and the coefficient of relative risk aversion is not too high. Otherwise, the optimal equilibrium becomes "too risky" and the agents will decide to withdraw early "most of the time."[11]

Proposition 2: *Suppose that $u(c) = -e^{-bc}, b > 1$[12]. Then:*

(a) z_* increases as the fraction of impatient agents $\left(1 - \frac{n}{m}\right)$ increases.

(b) z_* increases as the illiquid technology parameter R increases.

[10] The continuity of μ on ϵ implies that for any small ϵ the system will be in the optimal steady state with probability close to one.

[11] On the other hand, if this is the case, the assumption that they deposit in the bank at the beginning of each period might not be convincing.

[12] For this utility function we have that $1 < c_1^* < c_2^* < R$ if $-\frac{cu''(c)}{u'(c)} \geq 1$ for $c > 1$.

(c) z_* increases as the coefficient of risk aversion b increases.

The proof is given in the appendix. An increase in the fraction of impatient agents increases the level of illiquidity in the economy. An increase in b leads to an increase in the short-term interest rate r_1, which will have the same effect. An increase in R has two effects. It decreases z_* directly but increases it indirectly through an increase in r_1. However, the second effect dominates the first and z_* becomes larger as R increases.

Some of the important observations that can discriminate among alternative models of banking systems come from the performance of the American banking industry before the Federal Reserve System was established, i.e., during the National Banking Era.[13] This is a period of approximately 50 years (1863-1914), during which there were five major panics: 1873, 1884, 1890, 1893, and 1907. These panics were of relatively short duration, and they tended to occur in the fall when, in a predominantly agricultural economy, demand for currency was particularly high. The two propositions above suggest that these observations are consistent with a stochastic steady state in which panics are rare events over a long enough time horizon. In this context the five panics during a period of approximately 50 years are interpreted as relatively infrequent transitions from the optimal to the panic equilibrium. In addition, the first part of proposition 2 suggests that these transitions are more probable during periods when the demand for early withdrawals increases exogenously. Next, I turn to the question of how qualitative properties of the two static equilibria change for different specifications of the banking system.

[13] Sprague (1910) provides a detailed discussion of the banking history of this period. Sprague wrote a history of the panics under the National Banking System for the National Monetary Commission. His work has been extensively used by Friedman and Schwartz (1963).

IV. The Game with Many Banks

Now assume that there are many banks of equal size in the economy. In each period during subperiod 0, the m agents are randomly matched in k banks. I assume that the seasonal high demands for currency are experienced the same way by all banks, i.e., that in each period and in each bank there are $n_k = \frac{n}{k}$ patient and $m_k - n_k = \frac{m-n}{k}$ impatient agents. However, the relative representation of the two strategies of the patient players in each bank is random and, as we will see, subject to increasing variance as the size of the bank decreases. The fact that banks act in isolation might be the result of spatial separation and branching regulations.

During the National Banking Era evidence from different regions in the United States as well as a comparison between Canadian and American banking suggests that branching and cooperative interbank arrangements reduced the likelihood of panics. The failure rate for national banks in the U.S. during the period 1870-1909 was 0.36, while the one in Canada, based on branches, was less than 0.1. The different performance of Canadian and American banks is largely attributed to branching tactics and the degree of cooperation that Canadian banks managed to achieve during crises.[14] The question that I am asking here is the following: Suppose that the size of the banks in the economy can be influenced as a result of government policy, such as geographic restrictions on expansion for banks. If the government wants to minimize the likelihood of a banking panic, should this policy favor a centralized banking system with bigger banks or a system with many small banks?

To answer this question we need to explore the relation of the probability of observing the panic equilibrium to the number of banks in the economy and, therefore, the size of a representative bank. Let x_{jt} be the expected number of patient agents adopting strategy s_2 at time t in bank j, $j = 1, ..., k$; $t = 1, ..., \infty$. Here, x_{jt} can be thought of as the expected representation of s_2-players in a sample of size n_k, where $k \geq 1$. Let $\gamma(r_1, k) \in \{0, 1, ..., n_k\}$ be the minimum number of patient agents not withdrawing in a given bank, so that a bank run does not occur during subperiod 1 in

[14] See, for example, Chari (1989), Calomiris and Gorton (1991), and Sprague (1910).

this bank, given that there are n_k patient depositors in this bank. As before, $\gamma(r_1, k)$ is constant among banks of the same size and equal to $\frac{m_k(r_1-1)}{r_1}$. Let:

$$\Omega_z^k = \left\{\omega = (\omega_1, ..., \omega_k) \in [0, n_k]^k : 0 \leq \omega_j \leq n_k \ and \ \sum_{j=1}^k \omega_j = z\right\}, \tag{4}$$

.

i.e., Ω_z^k is the set of all possible sample representations of z in the case where there are k banks in the economy. The payoffs of the two strategies in bank j are as follows:

$$\pi(s_1, x_j, k) = \begin{cases} u(r_1), & if \quad x_j \geq \gamma(r_1, k); \\ \frac{m_k - \gamma(r_1,k)}{m_k - x_j} u(r_1) + \frac{\gamma(r_1,k) - x_j}{m_k - x_j} u(0), & if \quad x_j < \gamma(r_1, k); \end{cases} \tag{5}$$

$$\pi(s_2, x_j, k) = \begin{cases} u\left(\frac{[m_k - (m_k - n_k) r_1 - (n_k - x_j) r_1] R}{x_j}\right), & if \quad x_j \geq \gamma(r_1, k); \\ u(0), & if \quad x_j < \gamma(r_1, k). \end{cases} \tag{6}$$

Let $\pi(s_1, z_t, k)$ be the expected payoff of a player with strategy s_1 against population configuration z_t given that there are k banks in the economy; and similarly for $\pi(s_2, z_t, k)$. Then for $s_h \in \{s_1, s_2\}$, $\omega = (\omega_1, ..., \omega_k)$ we have:

$$\pi(s_h, z, k) = \Pr\{x = \omega\} \cdot \left[\sum_{j=1}^k \frac{1}{k} \cdot \pi(s_h, x_j, k)\right]. \tag{7}$$

The expected payoffs reflect the uncertainty resulting from the random matching process. Like before, the game is a coordination one with $E_1 = (s_1, ..., s_1)$ and $E_2 = (s_2, ..., s_2)$ being the only pure equilibria. I assume that the evolution of the agents' strategies is subject to the same rules as in the one-bank case. For each level k, a unique z_*^k determines the basins of attraction of the two steady states. As before, the steady state with the largest basin of attraction will be observed most of the time for small levels of noise. I want to characterize the two basins of attraction as a function of k. The following lemma simplifies the analysis. It says that it is the proportion of strategies in a bank, not the size of the bank, which determines the payoffs. In other words, the payoff of playing strategy s^h when a fraction of $\frac{x_j}{n_k}$ agents are playing this strategy is the same for any size of a bank n_k.

Lemma 3: *Suppose that* $\lambda = ak$, $a \geq 1$ *and* $\frac{x_j}{n_k} = \frac{x_j}{n_\lambda}$. *Then* $\pi(s_h, x_j, k) = \pi(s_h, x_j, \lambda)$, *for h=1,2, j=1,...,k.*

The next lemma relates the first two moments of the proportion of players playing each strategy in a bank to the size of the bank. The proof relies on standard results on the hypergeometric distribution and is given in the appendix.

Lemma 4: *For all* $k \geq 1$, $a > 1$, *j=1,...,k, suppose* $\lambda = ak$. *Then:*
(a) $E\left(\frac{x_j}{n_\lambda}\right) = E\left(\frac{x_j}{n_k}\right) = \frac{z}{n}$; *for all* $j = 1, ..., k$, and
(b) $Var\left(\frac{x_j}{n_\lambda}\right) > Var\left(\frac{x_j}{n_k}\right)$.

The following lemma describes the curvature of the difference of the expected payoffs of the two strategies as a function of the proportion of patient agents in the bank. Its proof is given in the appendix.

Lemma 5: *The function* $\pi(s_2, z) - \pi(s_1, z)$ *is a concave function of* z *for* $z \geq \gamma(r_1)$.

Given lemmas 3, 4, 5, and risk aversion, we have the following proposition:

Proposition 3: *Suppose that* $u(0) \leq M(m, n, b, R)$ and $k \geq 1, a > 1, \lambda = ak$. *Then* $z_*^\lambda > z_*^k$, *i.e.,* *the basin of attraction of the panic equilibrium increases as the size of the banks in the economy decreases.*

The proof of proposition 3 is given in the appendix. As the number of banks increases, each individual bank becomes subject to more uncertainty. Since the returns to s_2-players change dramatically across states, withdrawing late then becomes increasingly risky while strategy s_1, providing a relatively constant return across states, remains relatively "safe." Therefore, in the presence of increasing uncertainty, agents will shift to withdrawing early. Since z_* determines the two basins of attraction, the basin of attraction of the panic equilibrium becomes larger as the number of banks increases (see Figure 1). It is worth mentioning that this can change equilibrium selection, leading to the panic steady state having the larger basin of attraction in cases where this would not happen, should there be a smaller number of banks in the economy.

V. The Game with Local Interaction

The assumption of uniform matching expresses the idea that depositors have no local or other information about individual banks. In contrast, local matching rules describe situations where depositors are less likely to interact with the banking system as a whole than with local banks in the same geographic region. In addition, local matching rules allow for an overlap in the groups of depositors in neighboring banks so that a bank's neighboring bank is likely to be a neighboring bank as well. As Ellison (1993) showed, this allows for the existence of small clusters within the population, and the possibility of a new strategy's gaining a foothold within one of these clusters allows a more rapid transition to steady state than in the model with uniform matching.

Here I assume that the m agents and the k banks are uniformly arranged around a circle. As before, each bank has the same constant fraction of patient and impatient agents as the population, and I concentrate on the behavior of patient agents. In contrast to the uniform matching model, here I assume that each depositor is matched with probability $1-a$ with the bank in his own location, and with probability a with each of the other two banks in the region, directly to the east and directly to the west of his location. For simplicity, I fix $a = \frac{1}{3}$. As before, I am interested in equilibrium selection as a function of the size of the representative bank in the economy n_k.

In the model with local interaction, I denote the possible states by $n-$tuples $(s^1, ..., s^n) \in \{s_1, s_2\}^n$ in order to keep track of the locations of the patient depositors using each strategy. I also assume that the payoffs are such that depositors will switch to s_1 if at least one of the three banks in their region experienced a run in the previous period.[15] As before, this reflects a situation where agents have information on the status of the "local banking system" but not on individual banks.

First, consider the case without noise. Once again there are two steady states, $E_1 = (s_1, ..., s_1)$ and $E_2 = (s_2, ..., s_2)$ with a nontrivial basin of attraction. First, suppose that a bank run is experienced in one bank at time t, let us say bank B (see Figure 2). Then each depositor in banks A, B, and C will play s_1 in $t+1$. This way a local panic is created in period $t+1$ that will continue to spread to neighboring banks of the neighboring banks... until the panic steady state E_1 is reached. One important feature of this dynamic is that a small "cluster" of depositors creating bank runs in one region is sufficient to ensure rapid convergence to the panic equilibrium. On the other hand, as I will show, another important feature of this dynamic is that a panic becomes far more probable as the size of the banks in the economy decreases. More precisely, we have the following analogue of proposition 4:

Proposition 4: *Under the conditions of proposition 3 the probability of observing the panic equilibrium increases as k increases.*

The proof of proposition 4 can be found in the appendix. For an example suppose that $n = 18$

[15] Alternatively, I could assume that the payoffs are such that agents will withdraw if more than $\frac{1}{3}$ of the population in their region played s_1 in the previous period.

and consider two cases: $k = 3$ and $k^{'} = 6$, i.e., $n_k = 6$ and $n_{k'} = 3$. Also assume that $\gamma = 6$ when $k = 1$ so that $\gamma(3) = 2$ and $\gamma(6) = 1$, and suppose that payoffs are such that a depositor will play s_1 if one of the three banks in the region experienced a bank run in period t. I will denote the probability of transition from E_1 to E_2, along the minimum mutation path, when there are k banks in the economy by p_k and the probability of transition from E_2 to E_1 when there are k banks in the economy by q_k. I define $p_{k'}$ and $q_{k'}$ similarly for the case with $k^{'}$ banks. We then have:

$$\frac{p_k}{q_k} = \frac{\epsilon^2 \cdot \epsilon^2 \cdot \epsilon^2}{\epsilon^4 + \epsilon^4 + \epsilon^4} = \frac{1}{3}\epsilon^{-2}, \ and \tag{8}$$

$$\frac{p_{k'}}{q_{k'}} = \frac{\epsilon \cdot \epsilon \cdot \epsilon \cdot \epsilon \cdot \epsilon \cdot \epsilon}{\epsilon^2 + \epsilon^2 + \epsilon^2 + \epsilon^2 + \epsilon^2 + \epsilon^2} = \frac{1}{6}\epsilon^{-1}. \tag{9}$$

The arguments above suggest that the orders of $\frac{p_k}{q_k}$ and $\frac{p_{k'}}{q_{k'}}$ are $\epsilon^{-2}$and ϵ^{-1}, respectively, so we have: $\frac{\frac{p_{k'}}{q_{k'}}}{\frac{p_k}{q_k}} \to 0 \ as \ \epsilon \to 0$.

Local interaction allows for faster convergence to steady state as well as for interesting dynamics as the previous example shows. Furthermore, the panic is more probable in a system consisting of small banks for reasons that are different from the uniform matching model and are related to “neighborhood effects.” The study by Calomiris and Gorton (1991) suggests that these effects were important in the banking panics experienced in the U.S. during the National Banking Era. According to this study panics did not suddenly occur at different locations simultaneously, and the typical pattern of a panic was that banks in the same geographic location simultaneously experienced a run and subsequently runs spread to other locations.

VI. Concluding Remarks

In this paper I have presented a model of banking panics consistent with some basic facts from the National Banking Era, the period before the Federal Reserve System was established. Panics in that period were recurrent but relatively infrequent phenomena. This pattern is consistent with a stochastic steady state in which panics occur in each period with positive probability but are relatively infrequent events over a long time horizon. These results challenge the view that any banking system without deposit insurance must be plagued by ever-increasing instability. The model is also consistent with observations that suggest that more centralized banking systems can better diversify against withdrawal risk and, therefore, are more likely to perform better in terms of stability than banking systems consisting of many banks that are small and isolated as a result of government regulation. Finally, local interaction generates contagion effects that allow bank runs to spread first among banks in the same geographic region and subsequently in other locations, a pattern consistent with historical observations from the same period. These effects become stronger as the size of the banks decreases.

In this version of the model, I assume that banks act in isolation. Even though here a bank can be interpreted as a coalition of banks that cooperate closely, by dropping this assumption, liquidity risk-sharing by banks and interbank lending could arise. I believe that the main conclusions of the model will still be true in this case. The model is also believed to be robust to different specifications of the dynamical system describing the learning process. However, it is not entirely clear how the results will change once the assumption that banks are not strategic is dropped. Introducing cyclical behavior in the demand for liquidity would provide a framework for strategic banks to adjust the interest rates over time in order to minimize the probability of a bank run.

VII. References

Adao B., and Temzelides T. (1994): Beliefs, Competition, and Bank Runs. University of Minnesota.

Bergin J., and Lipman B. (1994): Evolution and State Dependent Mutations. Queen's University.

Binmore G., Samuelson L., and Vaughan R. (1995): Musical Chairs: Modeling Noisy Evolution. *Games and Economic Behavior* **11**, 11-35.

Calomiris C., and Gorton G. (1991): The Origins of Bank Panics : Models, Facts, and Bank Regulation, in "Financial Markets and Financial Crises" (R. Glenn Hubbard, Ed.), University of Chicago Press, Chicago.

Chari V.V. (1989): Banking without Deposit Insurance or Bank Panics : Lessons from a Model of the U.S. National Banking System. *Federal Reserve Bank of Minneapolis Quarterly Review* **13** (Summer), 3-19.

Diamond D., and Dybvig P. (1983): Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy* **91**, 401-419.

Ellison G. (1993): Learning, Local Interaction and Coordination. *Econometrica* **61**, 1047-1072.

Ellison G. (1994): Basins of Attraction and Long Run Equilibria. Harvard Institute of Economic Research Working Paper.

Foster D., and Young P. (1990): Stochastic Evolutionary Game Dynamics. *Journal of Theoretical Biology* **38**, 219-232.

Freidlin M.I., and Wentzell A.D. (1984): Random Perturbations of Dynamical Systems. New York: Springer Verlag.

Friedman M., and Schwartz A.J. (1963): A Monetary History of the United States 1867-1960. Princeton, N.J.: Princeton University Press.

Green E.J. (1995): Implementing Efficient Allocations in a Model of Financial Intermediation. Federal Reserve Bank of Minneapolis.

Harsanyi J., and Selten R. (1988): A General Theory of Equilibrium Selection in Games. Cambridge: MIT Press.

Kandori M., Mailath G.J., Rob R. (1993): Learning, Mutation, and Long Run Equilibria in Games. *Econometrica* **61**, 29-56.

Kandori M., and Rob R. (1992): Evolution of Equilibria in the Long Run: A General Theory and Applications. CARESS Working Paper 92-06R.

Lagunoff R. (1994): On the Evolution of Pareto Optimal Behavior in Repeated Coordination Problems. University of Pennsylvania.

Morris S., Rob R., and Shin H. (1993): Risk Dominance and Stochastic Potential. *Econometrica* **63**, 145-157.

Postlewaite A., and Vives X. (1987): Bank Runs as an Equilibrium Phenomenon. *Journal of Political Economy* **95**, 485-491.

Smith B.D. (1986): Bank Panics, Suspensions, and Geography: Some Notes on the "Contagion of Fear" in Banking. University of Western Ontario.

Sprague O.M.W. (1910): History of Crises Under the National Banking System. Report by the National Monetary Comission to the U.S. Senate. Washington D.C.: Government Printing Office.

Villamil J. (1991): Deposit Contracts, Suspension of Convertibility, and Optimal Intermediation. *Economic Theory* **1**.

Wallace N. (1988): Another Attempt to Explain an Illiquid Banking System : The Diamond and Dybvig Model with Sequential Service Taken Seriously. *Federal Reserve Bank of Minneapolis Quarterly Review* **12** (Summer), 3-16.

Young P.H. (1993): The Evolution of Conventions. *Econometrica* **61**, 57-84.

VIII. Appendix: Proofs

Proof of Lemma 1: Since the payoffs of the game satisfy the inequalities listed before the statement of the lemma, the unique best response to $(s_1, ..., s^i, ..., s_1)$ is s_1^i and the unique best response to $(s_2, ..., s^i, ..., s_2)$ is s_2^i . So E_1 and E_2 are pure strategy strict Nash equilibria. The mixed equilibrium comes from the unique solution of the equation $\pi(s_2, z) - \pi(s_1, z) = 0$, in Z. ■

Proof of Proposition 1: The two basins of attraction are determined by the unique solution z_* to the equation $\pi(s_2, z) - \pi(s_1, z) = 0$, in Z. Clearly $z_* > \gamma(r_1)$ and, therefore, $\frac{[m-(m-n)r_1-(n-z)r_1]R}{z} = r_1$ implies that $z_* = \frac{mr_1R-mR}{r_1R-r_1}$. ■

Proof of Proposition 2: First, recall that the coefficient of relative risk aversion is bc, which increases as b increases. Also, by proposition 3, $z_* = \frac{mr_1R-mR}{r_1R-r_1}$. The first order conditions for the social planner's problem give: $u^{'}(r_1^*) = Ru^{'}(r_2^*)$ or $r_2^* = r_1^* + \frac{\ln(R)}{b}$. The feasibility condition is: $(m-n)r_1 + \frac{nr_2}{R} = m$. Combining the two, gives: $r_1^* = \frac{mbR-n\ln(R)}{b(Rm-Rn+n)}$.

(a) Let $\bar{z} \in [0,1]$ be the fraction of s_2-players in the population of patient agents. Then $\bar{z}_* = \frac{mr_1R-mR}{nr_1R-nR}$ and we have[16]:

$$\frac{\partial \bar{z}_*}{\partial \left(\frac{n}{m}\right)} = -\frac{(r_1R-R)}{\left(\frac{n}{m}\right)^2(r_1R-r_1)} < 0, \tag{10}$$

$$\frac{d\bar{z}_*}{dr_1} = \frac{mnR(R-1)}{(nr_1R-nr_1)^2} > 0, \tag{11}$$

$$\frac{dr_1}{d\left(\frac{n}{m}\right)} = \frac{1-R^{\frac{1-a}{a}}}{\left(1-\frac{n}{m}+\frac{n}{m}R^{\frac{1-a}{a}}\right)^2} > 0. \tag{12}$$

$$So: \frac{d\bar{z}_*}{d\left(\frac{n}{m}\right)} = \frac{\partial \bar{z}_*}{\partial \left(\frac{n}{m}\right)} + \frac{d\bar{z}_*}{dr_1} \cdot \frac{dr_1}{d\left(\frac{n}{m}\right)} = \tag{13}$$

[16] Here the state space is considered to be the interval $[0,1]$.

$$\frac{m^2R(b-bR+\ln(R))(-bmR+b^2mR+n\ln(R))}{n(1-R)(-bmR+n\ln(R))^2}<0. \tag{14}$$

(b) We have:

$$\frac{d\bar{z}_*}{dr_1}=\frac{mnR(R-1)}{(nr_1R-nr_1)^2}>0, \tag{15}$$

$$\frac{\partial\bar{z}_*}{\partial R}=\frac{mnr_1(-r_1+1)}{(nr_1R-nr_1)^2}<0, \tag{16}$$

$$\frac{dr_1}{dR}=\frac{mnb^2-mnb+n^2b-(\frac{n^2b}{R})+bmn\ln(R)-bn^2\ln(R)}{(bRm-brn+nb)^2}>0, \tag{17}$$

and, finally,

$$\frac{d\bar{z}_*}{dR}=\frac{\partial\bar{z}_*}{\partial R}+\frac{dr_1}{dR}\cdot\frac{d\bar{z}_*}{dr_1}>0^{17}. \tag{18}$$

(c) We have:

$$\frac{dr_1}{db}=\frac{Rmn\ln(R)-Rn^2\ln(R)+n^2\ln(R)}{(bRm-bRn+nb)^2}>0.$$

Therefore,

[17]This is true since: $\frac{\partial\bar{z}_*}{\partial R}+\frac{dr_1}{dR}\cdot\frac{d\bar{z}_*}{dr_1}=$
$\frac{[m(bn+bmR-2bnR-bmR^2+bnR^2-bn\ln(R)+3bnR\ln(R)+bmR^2\ln(R)-bnR^2\ln(R)-n\ln(R)^2)]}{(-1+R)^2(-bmR+n\ln(R))^2}>0.$

$$\frac{d\bar{z}_*}{db} > 0. \blacksquare$$

Proof of Lemma 3: Without loss of generality consider the case with k banks and the case with λ banks where $\lambda = ak$, $a > 1$ and, therefore, $n_k = an_\lambda$ and $m_k = am_\lambda$. Then:

$$\gamma(r_1, \lambda) = \frac{\frac{m}{\lambda}(r_1 - 1)}{r_1} = \frac{\frac{m}{ak}(r_1 - 1)}{r_1} = \frac{1}{a}\frac{\frac{m}{k}(r_1 - 1)}{r_1} = \frac{1}{a}\gamma(r_1, k).$$

First suppose $\frac{x_j}{n_\lambda} < \frac{\gamma(r_1,\lambda)}{n_\lambda}$ and, therefore, $\frac{x_j}{n_k} < \frac{\gamma(r_1,k)}{n_k}$. Then:

$$\pi(s_2, x_j, \lambda) - \pi(s_1, x_j, \lambda)$$

$$= u(0) - \frac{m_\lambda - \gamma(r_1, \lambda)}{m_\lambda - x_j}u(r_1) - \frac{\gamma(r_1, \lambda) - x_j}{m_\lambda - x_j}u(0)$$

$$= u(0) - \frac{\frac{m_k}{a} - \frac{\gamma(r_1,k)}{a}}{\frac{m_k}{a} - \frac{x_j}{a}}u(r_1) - \frac{\frac{\gamma(r_1,k)}{a} - \frac{x_j}{a}}{\frac{m_k}{a} - \frac{x_j}{a}}u(0)$$

$$= u(0) - \frac{m_k - \gamma(r_1, k)}{mk - x_j}u(r_1) - \frac{\gamma(r_1, k) - x_j}{m_k - x_j}u(0)$$

$$= \pi(s_2, x_j, k) - \pi(s_1, x_j, k).$$

Next suppose $\frac{x_j}{n_\lambda} > \frac{\gamma(r_1,\lambda)}{n_\lambda}$ and, therefore, $\frac{x_j}{n_k} > \frac{\gamma(r_1,k)}{n_k}$. Then:

$$\pi(s_2, x_j, \lambda) - \pi(s_1, x_j, \lambda) = u\left(\frac{[m_\lambda - (m_\lambda - n_\lambda)r_1 - (n_\lambda - x_j)r_1]R}{x_j}\right) - u(r_1)$$

$$= u\left(\frac{\left[\frac{m_k}{a} - \left(\frac{m_k}{a} - \frac{n_k}{a}\right)r_1 - \left(\frac{n_k}{a} - \frac{x_j}{a}\right)r_1\right]R}{\frac{x_j}{a}}\right) - u(r_1)$$

$$= u\left(\frac{[m_k - (m_k - n_k)r_1 - (n_k - x_j)r_1]R}{x_j}\right) - u(r_1)$$

$$= \pi(s_2, x_j, k) - \pi(s_1, x_j, k). \blacksquare$$

Proof of Lemma 4: (a) Let n be the population of patient agents. Suppose that z agents play strategy s_2 and $n-z$ agents play strategy s_1 in the population and that a sample of $n_k = \frac{n}{k}$ agents is taken from the population. Let 1 stand for a draw of an s_2 strategist and 0 stand for a draw of an s_1 strategist. Let $\{I_1, ..., I_{n_k}\}$ be the sequence of indicator functions describing the outcome of the draws, where:

$$I_i = \begin{cases} 1, & if \quad s_2 \quad player \quad in \quad draw \quad i; \\ 0, & if \quad s_1 \quad player \quad in \quad draw \quad i. \end{cases} \tag{19}$$

Define $x_{n_k} = I_1 + ... + I_{n_k}$, the number of s_2 players drawn from n_k trials. The Is are identically distributed. We then have: $\Pr\{I_i = 1\} = \frac{z}{n}$, for all i. With regard to the number of 1s among the n_k objects drawn, it does not matter whether we draw one at a time or we simultaneously draw n_k objects. So[18]:

$$\Pr\{x_{n_k} = \omega\} = f(\omega; n, z, n_k) = \frac{\binom{z}{\omega}\binom{n-z}{n_k-\omega}}{\binom{n}{n_k}}. \tag{20}$$

[18] I assume that the agents do not exclude themselves in this calculation. This is consistent with a large population of agents.

Now, $E(x_{n_k}) = E(I_1, ..., I_{n_k}) = \frac{z}{n} + ... + \frac{z}{n} = \frac{n}{k}\frac{z}{n} = \frac{z}{k}$. So $E\left(\frac{x_{n_k}}{n_k}\right) = \frac{z}{n}$, i.e., the expected proportion of 1s among the n_k objects is the same regardless of the sample size.

(b) Here, the variance of x_{n_k} is not the sum of the variances of the Is, since the latter are not independent. Following the convention that $\binom{\alpha}{\beta} = 0$, when $0 < \alpha < \beta$ we obtain that:

$$Var(x_{n_k}) = \frac{n_k \frac{z}{n} \frac{n-z}{n}(n - n_k)}{n-1}, \text{ therefore,}$$

$$Var\left(\frac{x_{n_k}}{n_k}\right) = \frac{1}{(n_k)^2} Var\left(x_{n_k}\right) = \frac{1}{n_k} \frac{\frac{z}{n}\frac{n-z}{n}(n - n_k)}{n-1}.$$

Now suppose $n_\lambda = a n_k$, $a > 1$. Let $p = \frac{z}{n}$ and $1 - p = \frac{n-z}{n}$. Then:

$$\frac{Var\left(\frac{x_{n_k}}{n_k}\right)}{Var\left(\frac{x_{n_\lambda}}{n_\lambda}\right)} = \frac{\frac{p(1-p)(n-n_k)}{n_k(n-1)}}{\frac{p(1-p)(n-n_\lambda)}{n_\lambda(n-1)}} = \frac{\frac{p(1-p)(n-n_k)}{n_k(n-1)}}{\frac{p(1-p)(n-an_k)}{an_k(n-1)}}$$

$$= \frac{an - a\frac{n}{k}}{n - a\frac{n}{k}} > 1.$$

$$i.e.\ Var\left(\frac{x_{n_k}}{n_k}\right) > Var\left(\frac{x_{n_\lambda}}{n_\lambda}\right). \blacksquare$$

Proof of Lemma 5: We have: $\pi(s_2, z) = u\left(\frac{[m-(m-n)r_1-(n-z)r_1]R}{z}\right)$.

$$\text{Now: } \frac{d\left(\frac{[m-(m-n)r_1-(n-z)r_1]R}{z}\right)}{dz} = \frac{mr_1R - mR}{z^2} > 0,$$

$$\text{and } \frac{d^2\left(\frac{[m-(m-n)r_1-(n-z)r_1]R}{z}\right)}{dz^2} = \frac{-2mR(r_1-1)}{z^3} < 0.$$

Therefore, $\pi(s_2, z)$ is a concave function of z as a composition of two concave functions. Next, $\pi(s_1, z)$ is also concave since it is a composition of a linear (constant) function and a concave function. So $\pi(s_2, z) - \pi(s_1, z)$ is concave in z. ■

Proof of Proposition 3: I will prove this proposition for the case where $k = 1$ and $a = 2$. The proof generalizes for any $a > 1$. By lemma 5, it is sufficient to show that for any $\rho \in (0, 1)$ and any $z^{'}, z^{''} \in [0, n]$ such that $\pi(s_2, \rho z^{'} + (1-\rho)z^{''}) - \pi(s_1, \rho z^{'} + (1-\rho)z^{''}) \geq 0$. We have:

$$\rho\left(\pi(s_2, z^{'}) - \pi(s_1, z^{'})\right) + (1-\rho)\left(\pi(s_2, z^{'}) - \pi(s_1, z^{'})\right)$$

$$\leq \pi(s_2, \rho z^{'} + (1-\rho)z^{''}) - \pi(s_1, \rho z^{'} + (1-\rho)z^{''}).$$

I consider three cases: (a) First let $z^{'}, z^{''} \in [0, \gamma(r_1))$. In this case:

$$\pi(s_2, \rho z^{'} + (1-\rho)z^{''}) - \pi(s_1, \rho z^{'} + (1-\rho)z^{''}) < 0.$$

(b) Next, consider the case where $z^{'}, z^{''} \in [\gamma(r_1), 1]$. In this case concavity follows from lemma 6. (c) Finally, consider the case where $z^{'} \in [0, \gamma(r_1))$ and $z^{''} \in [\gamma(r_1), 1]$. Here I first need to define the following function:

$$\tilde{\pi}(s_2, z) - \tilde{\pi}(s_1, z) = \begin{cases} \pi(s_2, z) - \pi(s_1, z), & if \quad z \geq \gamma(r_1); \\ \bar{f}(z), & if \quad 0 \leq z \leq \gamma(r_1); \end{cases} \tag{21}$$

where:

$$\bar{f}(z) = \left(\pi(s_2, \gamma(r_1)) - \pi(s_1, \gamma(r_1))\right)^{'}(z - \gamma(r_1)) + \left(\pi(s_2, \gamma(r_1)) - \pi(s_1, \gamma(r_1))\right). \tag{22}$$

Next, define $M(m,n,r_1)=\bar{f}(0)$. The function ar $\bar{f}(z)$ is concave and for all $\rho \in (0,1)$ and all $z^{'} \in [0,\gamma(r_1))$ and for all $z^{''} \in [\gamma(r_1),1]$ such that $\rho z^{'}+(1-\rho)z^{''}=z$, we have:

$$\pi(s_2,z)-\pi(s_1,z) \geq \tilde{\pi}(s_2,\rho z^{'}+(1-\rho)z^{''})-\tilde{\pi}(s_1,\rho z^{'}+(1-\rho)z^{''})$$

$$\geq \pi(s_2,\rho z^{'}+(1-\rho)z^{''})-\pi(s_1,\rho z^{'}+(1-\rho)z^{''}),$$

where the first inequality is strict if $z^{'}, z^{''}$ or both belong to the interval $[\gamma(r_1),1]$. Since (a),(b), and (c) are the only possible cases and since $\pi(s_2,z)-\pi(s_1,z)$ is strictly concave in the neighborhood of z_*, we have that $z_*^2 > z_*^1$. ■

Proof of Proposition 4: Here I consider two cases. In the first case there are k banks of size $n_k=\frac{n}{k}$ and in the second case there are $k^{'}$ banks of size $n_{k'}=\frac{n}{k'}$. Since $\gamma=\frac{n}{3}$ when $k=1$, a bank run occurs in the first case if the number of s_2 players in a given bank is less than $\frac{n_k}{3}$. Recall that $E_1=(s_1,...,s_1)$ and $E_2=(s_2,...,s_2)$. The weight of state E_1 in the steady state distribution $\mu_{E_1}^k(\epsilon)$ can be characterized by[19]: $\mu_{E_1}^k(\epsilon)=c(\epsilon)\sum_{h\in H_{E_1}}\prod_{i\neq E_1}p_{h(i)}^k i^{\epsilon}$, where c is a constant, h is any E_1-tree, and H_{E_1} is the set of E_1-trees on $\{s_1,s_2\}^n$ and similarly for $\mu_{E_2}^k(\epsilon)$. First, I show that $\mu_{E_1}^{k^{'}}(\epsilon)>\mu_{E_1}^k(\epsilon)$ for $k^{'}>k$. In the case of k banks the least costly E_1-tree is of order $\epsilon^{\frac{2}{3}n_k}$ since a run in at least one bank is needed in order to lead the system to the E_1 state. In the case of $k^{'}$ banks the similar E_1-tree is of order $\epsilon^{\frac{2}{3}n_{k'}}$. Since $n_{k'}<n_k$, we have that $\mu_{E_1}^{k^{'}}(\epsilon)>\mu_{E_1}^k(\epsilon)$. On the other hand, we need that no bank experiences a run in period t in order for the system to converge to E_2 in period $t+1$. So we have that $\mu_{E_2}^k(\epsilon)=\epsilon^{k\frac{1}{3}n_k}$ and $\mu_{E_2}^{k^{'}}(\epsilon)=\epsilon^{k^{'}\frac{1}{3}n_{k'}}$ and, therefore, $\mu_{E_2}^{k^{'}}(\epsilon)=\mu_{E_2}^k(\epsilon)$. ■

---

[19] See Ellison (1993).