On FDH efficiency analysis: Some methodological issues and applications to courts, and urban transit. Journal of Productivity Analysis 4, 183–210.

Malnero, A., 1994. Nonparametric approaches to the assessment of the relative ncy of bank branches, CORE Discussion Paper 9447, Belgium.

u, M., Giokas, D., 1990, A study of the relative efficiency of bank branches: An application a envelopment analysis. Journal of the Operational Research Society 41, 591–597.

hi, A., Kolari, J., 1994. Branch office economies of scale and scope: Evidence from savings in Finland, Journal of Banking and Finance 18, 421–432.

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Evolution, coordination, and banking panics

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Abstract

I study equilibrium selection by an evolutionary process in an environment with multiple equilibria, one of which involves a banking panic. The analysis is built on a repeated version of the Diamond–Dybvig (1983) model. The optimal equilibrium is uniquely selected if it is also 'risk dominant'. When there are multiple banks, the probability of observing a panic increases as the size of the banks decreases. Local interaction generates contagion effects that allow a bank run to spread first among banks in the same geographic location and then throughout the population.

Keywords: Evolution; Banking panics; Contagion

JEL classification: C73; G21

1. Introduction

Increasing the stability of the banking system has been one of the main goals of bank regulation since the Federal Reserve System was established. Correspondingly, understanding why some banking regimes have been more stable than others has been a major focus of research. Following Bryant (1980) the Diamond and Dybvig (D–D) (1983) model of suspension of convertibility

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versus public deposit insurance has widely been considered the most satisfactory model of the contrast between late nineteenth and mid-twentieth century American banking regimes. In the standard version of this model, without either deposit insurance or suspension, there are two Nash equilibria. One of them involves a Pareto-dominated banking panic. The threat of such a panic has widely been cited as a justification for a regulatory regime involving government deposit insurance.

The above argument, however, is not completely satisfactory. The welfare gain from eliminating banking panics has to be weighed against the cost of incentive problems induced by deposit insurance. While the cost of distorted incentives will be borne regardless of whether panics occur in the absence of insurance, the welfare cost of panics will be roughly proportional to their frequency in a laisez-faire regime. The simple fact that a panic equilibrium exists in the D–D model does not provide any help in thinking about the question of how frequently panics might occur or of what features of the market structure might minimize their frequency. The question, for example, of whether lack of bank-specific information will lead to less frequent panics in a system consisting of many small banks or one consisting of a few big banks cannot be answered in this framework. In the present paper, I am addressing this question focusing on the role of *learning* by depositors through previous experience with the banking system.

In recent work, Young (1993) and Kandori et al. (1993) showed that learning can lead to striking predictions about the possibility of achieving the optimal outcome in repeated coordination problems. Unlike static refinements, their approach specifies how an equilibrium emerges through explicit dynamic processes by which agents adjust their choices over time as they learn from trial and error. Here, I argue that, in addition to its use for long-run predictions, this approach might be useful for studying the transitions from one equilibrium to the other in the course of long enough histories in a stationary environment. I concentrate on the likelihood of the occurrence of a panic in the long run, after agents have gained experience from a long history of interaction with the banking system, a history potentially including several panic episodes. I study conditions on the market structure that guarantee that this experience will result in depositors learning to avoid panics.

Moreover, historically, many panics followed a particular pattern. A bank run in one bank would spread first to 'neighboring' banks and then bank runs would occur in other locations. One of the earliest references to this contagion effect appears in Gibbons (1859) in relation to the 1857 panic. Gibbons discusses how news about financial distress would travel via telegraph, the then new medium of communication, leading depositors in other parts of the country to withdraw their deposits. Sprague (1910) suggests the existence of such effects during the National Banking Era, and Friedman and Schwartz (1963) describe such effects in connection with certain episodes during the 1930s. Saunders and Wilson (1996) empirically document significant contagion effects during the 1930–32 period. They attribute it to depositors lacking bank-specific information and using news about the national or the regional banking system as a proxy. Can the relative size of banks be a determinant of the magnitude of this contagion effect?

Ellison (1993) identified the importance of 'neighborhood effects' on learning in large populations. In a spatial version of the model, where agents interact with the local banking system, I show that local learning can lead to contagion and a fast transition from a bank run in one bank to bank runs in neighboring banks and, consequently, to a panic in the entire population. The connection between the lack of bank-specific information and banking panics has also been discussed by Chari and Jagannathan (1988) and Jacklin and Bhattacharya (1988), but not in relation to learning.

I first consider a repeated version of the D–D model with a representative bank. The bank can be interpreted as many branches of the same bank or as a coalition of banks that cooperate to a large extent and, for all purposes, are viewed as one bank. Agents receive information on the number of early withdrawals throughout the banking system in each period. They play myopic best responses, following the action that worked best in the previous period. Both the optimal equilibrium and the inferior one associated with a panic are steady states of the dynamical system describing the evolution of the agents' behavior. In the presence of exogenous noise, transitions from one steady state to the other occur in each period with positive probability. I show that, over a long enough history of transitions, the optimal steady state will be observed most of the time, provided that it is also risk dominant. Then, I turn to the case where there are many banks of equal size, where the size of a bank is the number of agents that pool their resources into a collective arrangement. The banks are *ex ante* identical and agents are randomly matched in the banks within each period. Random matching is one way to model a situation where agents have information about the entire banking system but not about individual banks. As the size of the representative bank in the economy decreases, each individual bank becomes subject to increasing uncertainty and, as a result, the probability of observing the panic equilibrium increases.

Finally, I study the case where agents are locally matched in banks close to their own geographic location. Here, agents have information about the local banking system but, once again, not about individual banks. Local interaction allows for the possibility of small clusters of panic withdrawals within the population. In this case, withdrawing early pays better within the region and, therefore, will be imitated by depositors who lack information about bank-specific withdrawals. This generates contagion effects that lead a bank run in one bank to spread into a local panic in neighboring banks and then into a panic for the entire economy, a pattern consistent with some observations from the history of banking panics in the US. These effects are shown to be stronger as the size of the bank decreases.

While there is extensive literature on bank runs related to the D–D model, most papers concentrate on the problem of designing mechanisms that eliminate the run as an equilibrium outcome. One such mechanism, as discussed in D–D, is suspension of convertibility. But while suspension eliminates runs in the version of the model without aggregate risk, it is less effective when there is aggregate risk, i.e., when the fraction of early withdrawals due to liquidity needs is random.[1] On the other hand, nothing about the methodology used here depends on the no aggregate risk assumption; moreover, the results generalize in the version of the model with aggregate risk. This is because in either case the game played by patient depositors in the intermediate period is a basic game of coordination. Historically, while suspension has been relatively successful in moderating panics, it was also considered as very unpleasant by those that experienced it. The existence of the Federal Reserve System prevented banks from suspending during later episodes. This was true both because the concern of stronger banks was reduced and because of the assumption that the establishment of the System made such moves unnecessary.[2] In the light of this, one could alternatively view this model as a model of banking crisis in cases when suspensions are not extensively practiced by banks, perhaps in response to government or other pressures.

The remainder of the paper is organized as follows. Section 2 outlines the economic environment. Section 3 gives the model with a representative bank, and Section 4 gives the model with multiple banks. In Section 5, I consider the model with local matching, followed by conclusion in Section 6. The Appendix contains many of the proofs.

2. The economy

Here I give a detailed description of the environment.

2.1. Time

There is an infinite number of periods, $t=0,1,2,\ldots$. Each period is divided into three subperiods: beginning ($t0$), middle ($t1$), and end ($t2$). The economic environment is the same for all periods (index t is dropped here after).

2.2. Population and endowments

In each period, there are m agents alive. Agents are endowed with one unit of the subperiod 0 good. In subperiod 0, each individual faces a preference shock

[1] See Chari (1989) and Green (1995).

[2] See Friedman and Schwartz (1963), pp. 311–312, for a discussion of the consequences of the lack of banks' suspension during the final months of 1930.

that determines whether she wants to consume only in subperiod 1 (type 1, or *impatient agent*), or whether she is indifferent between consumption in subperiod 1 and consumption in subperiod 2 (type 2, or *patient agent*). Let n be the number of agents that are patient. I assume that n is a constant and large.[3]

2.3. Technology

A technology is available in subperiod 0 that can transform the subperiod 0 good to subperiods 1 and 2 goods. The technology set is characterized by a triple (y_0, y_1, y_2) such that: $y_1 \leq -y_0$ and $y_2 \leq (-y_0 - y_1)R$, where y_i is the amount of input (output) during subperiod i, $y_0 \leq 0$, $y_1 \geq 0$, $y_2 \geq 0$ and $R>1$ for all t. The technology is, therefore, riskless but illiquid. In addition, agents can privately store the consumption good at no cost, but no investment in the illiquid technology can start after subperiod 0.

2.4. Preferences

Everyone has the same preferences in subperiod 0. In particular, each agent alive in period t has a state-dependent utility function (with the state private information), which we assume has the form:

$$U(C) = \begin{cases} u(c_1), & \text{if impatient;} \\ u(c_1 + c_2), & \text{if patient;} \end{cases}$$

where $u:\mathscr{R}_+ \to \mathscr{R}$ is C^2 in $\mathscr{R}_{++}$, and $-(cu''(c)/u'(c)) \geq 1$.

2.5. Isolation

Agents are isolated during subperiod 1 of each period. This assumption precludes the existence of an asset market during subperiod 1. It is consistent with the notion that agents hold liquid assets because they may need to consume at times and places at which accessing asset markets is difficult.

Let c_i^j be the consumption during subperiod i of an agent of type j. Consider the problem the society faces during subperiod 0 of each period. One possibility is autarky. The autarky allocation is, for all t, $c_1^1 = 1, c_2^1 = c_1^2 = 0, c_2^2 = R$. However, everybody could be better off if they pool their resources in a collective insurance arrangement. The optimal insurance contract is the solution to the problem of maximizing $(1-(n/m))u(c_1) + (n/m)u(c_1+c_2)$ subject to $(1-(n/m))c_1 + (n/m)(c_2/R) = 1$. As D–D show, the optimal consumption levels satisfy for all t: $c_1^* > 1$, $c_2^* < R$ and $c_2^* > c_1^*$. The optimal allocation is superior

[3] This is only a simplifying assumption and the results generalize to the case where n is random, i.e., when there is aggregate risk.

to autarky since people are willing to forgo some of their consumption if they turn out to be patient in return for greater consumption if they turn out to be impatient.

2.6. Banks

A bank is a contract among agents who pull their resources together in the collective arrangement. I assume that there is a finite number ($k \geq 1$) of banks of equal size offering the optimal insurance contract as a *demand deposit contract*. The deposit contract gives each agent who withdraws in subperiod 1 $r_1 = c_1^*$ units per unit deposited in subperiod 0. The banks are liquidated in subperiod 2 of each period, so agents who do not withdraw in subperiod 1 get a pro rata share of the banks' assets in subperiod 2. The same number of banks are 're-born' at the beginning of the next period. In each period, the m agents are randomly matched in the k banks. I assume that the proportion of patient and impatient agents in each bank is the same as in the population. As in the D–D model, the demand deposit contract can achieve the full information optimal risk sharing as an equilibrium and I assume that all agents deposit initially.

3. The model with a representative bank

In each period during subperiod 0, there are m identical agents. At the beginning of subperiod 1, uncertainty is resolved and each agent learns whether she is patient or impatient. There will be n patient and $m-n$ impatient agents. Impatient agents always withdraw during subperiod 1, so in what follows, they are not viewed as being 'strategic'.

The n patient agents play a game of coordination choosing whether to withdraw in subperiod 1 or wait until subperiod 2. I assume that agents act myopically by playing best responses to the population configuration from the previous period. Therefore, what connects two consequent periods is that the strategy that gives the higher expected pay-off increases its representation in the population of patient agents, and this process continues until a 'steady state' is reached.

I first describe the stage game. Let $S = \{s_1, s_2\}$ be the set of strategies available to an agent in subperiod 1.[4] Here, s_1 stands for 'withdraw in subperiod 1' and s_2 stands for 'withdraw in subperiod 2'. Let z_t be the number of patient agents adopting strategy s_2 at time t; $t = 1, 2, \ldots, \infty$. The state space, i.e., the range of z_t is $Z = \{0, 1, \ldots, n\}$. Let $\gamma(r_1) \in \{0, 1, \ldots, n\}$ be the minimum number of patient agents not withdrawing, so that the bank will be able to pay r_1 to every agent in the waiting line during subperiod 1. Here, $\gamma(r_1)$ is a constant for fixed r_1 and $m - (m-n)r_1 - (n-z)r_1 \geq 0$ for $z \geq \gamma(r_1)$ implies that $\gamma(r_1) = (m(r_1 - 1))/r_1$.

[4] I assume that agents play pure strategies only. As is standard in this literature, mixed strategies are represented by the fractions in the population that choose the respective pure strategies.

The pay-offs of the two strategies are as follows

$$\pi(s_1, z) = \begin{cases} u(r_1), & \text{if} \quad z \geq \gamma(r_1); \\ \dfrac{m - \gamma(r_1)}{m - z} u(r_1) + \dfrac{\gamma(r_1) - z}{m - z} u(0), & \text{if} \quad z \leq \gamma(r_1); \end{cases} \tag{1}$$

$$\pi(s_2, z) = \begin{cases} u\left(\dfrac{[m - (m-n)r_1 - (n-z)r_1]R}{z} \right), & \text{if} \quad z \geq \gamma(r_1); \\ u(0), & \text{if} \quad z \leq \gamma(r_1). \end{cases} \tag{2}$$

At the beginning of subperiod 1, in each period t, uncertainty about the types is resolved. If every patient agent chooses s_2, the equilibrium providing optimal risk sharing is achieved. It might happen, however, that some patient agents will choose to withdraw early. Because of the illiquidity of the production technology, the bank in this case might not be able to pay everyone the promised amount r_1 during subperiod 1 (recall that $r_1 > 1$). If the state z is high enough, the bank can still pay r_1 to every depositor who tries to withdraw. Otherwise, the bank serves the agents who try to withdraw according to the *sequential service constraint* until it runs out of assets. The sequential service constraint (see Wallace, 1988) requires that payments to individuals depend only on the number of people who have previously withdrawn, not on the number of prospective withdrawers. Here, it is assumed that every agent contacts the bank at some random time during subperiod 1. The patient agents will then have to choose whether to 'report untruthfully', i.e., try to withdraw early. I assume that the arrival times are independent across agents and that an agent's time of arrival relative to other agents is not known to him. Let E_1 be the n-tuple $(s_1, \ldots, s_1)$ and similarly for E_2. The following lemma verifies that both the optimal allocation and the inferior one that results from a bank panic can be supported by non-cooperative equilibria of this game.

Lemma 1. *E_1 and E_2 are the only pure Nash equilibria of the stage game. Furthermore, they are both strict. There exists a mixed Nash equilibrium.*

I now turn to the repeated version of the game. I assume that the better strategy is better represented in the population in the next period. A deterministic dynamic $z_{t+1} = b(z_t)$ gives the profile of strategies that will be used at $t+1$, given that the time t profile is z_t.[5] In this game, the deterministic dynamical system has two

[5] Formally, the following weak monotonicity property and boundary conditions are assumed: $\text{sign}(b(z) - z)) = \text{sign}(\pi(s_2, z) - \pi(s_1, z))$, $b(0) = 0$, $b(n) = n$. As an example, consider the best reply dynamic:

$$B(z) = \begin{cases} n, & \text{if} \quad \pi(s_2, z) > \pi(s_1, z); \\ z, & \text{if} \quad \pi(s_2, z) = \pi(s_1, z); \\ 0, & \text{if} \quad \pi(s_2, z) < \pi(s_1, z). \end{cases}$$

pure steady states, 0 and n, and, possibly, a mixed one. The two pure steady states correspond to the optimal and the panic equilibrium of the stage game, respectively. The multiplicity is resolved if noise is introduced into the system. Assume that with probability ε, each player plays each strategy with equal probability and that these probabilities are iid across players and time. This yields a stochastic dynamical system that defines a Markov chain on the finite state space Z. It is well-known, then, that the Markov chain has a unique invariant distribution for a given rate ε. The invariant distribution satisfies global stability and ergodicity properties and is interpreted as giving the proportion of time that the society spends in each state. For expository purposes, I consider the support of the distribution μ, which results as the limit case when ε is driven to zero. Let z_* be the critical 'state' such that for all z in Z, $\text{sign}(\pi(s_2,z)-\pi(s_1,z))=\text{sign}(z-z_*)$. The following lemma is key[6] (see Fig. 1).

Lemma 2. *The two states* 0 *and* n *have basins of attraction under* $b(z)$ *given by* $\{z<z_*\}$ *and* $\{z>z_*\}$, *respectively. The limit distribution puts probability one on* n *if* $z_*<n/2$ *and on* 0 *otherwise.*

The equation $\pi(s_2,z)-\pi(s_1,z)=0$ has a unique root in Z. The sign of $z_*-(n/2)$ is a function of the risk characteristics of the game. Notice that if $z_*<n/2$, then the optimal equilibrium is also risk-dominant (see Harsanyi and Selten (1988)). The first result of this section describes conditions under which panics are very rare events.

Proposition 1. *The unique state in support of* μ *is* n *if* $(mr_1R-mR)/(nr_1R-nr_1)<\frac{1}{2}$ *and it is* 0 *if the inequality is reversed.*

Proposition 1 asserts that in environments for which the above condition is satisfied, bank panics will be very rare and the economy will spend most of the time in the optimal steady state. In the presence of noise, some patient agents withdraw early with positive probability in each period. With even lower probability, these withdrawals will be significant enough to result in a bank panic, i.e., the miscoordination is sufficiently large to shift the population into the basin of attraction of the panic steady state. In this case, a panic is inevitable and the process will spend a number of periods close to that steady state until it escapes again. In the limit, however, the process will be in the steady state with the largest basin of attraction with probability one.

[6] Kandori et al. (1993) and Young (1993) developed this result in a model with random matching in pairs, but the same result can be used for any group size up to n. For a more recent discussion on this and related dynamics see Binmore et al. (1995).

Once the economy reaches the panic steady state, it is expected to stay there for a large number of periods.[7] In this case, it is hard to justify the assumption that agents deposit in the bank at the beginning of each period. At the cost of some additional complication, I could assume that agents choose not to deposit in the bank after a panic has occurred. In that case, the resulting Pareto inferior equilibrium would correspond to the state where the banking system is not used. Provided that the 'good' equilibrium satisfies an analog of the inequality in Proposition 1, the system will spend almost all time there under this alternative specification. This suggests that banking is 'tenuous' in this model the same way as money is in monetary models.[8] The next proposition relates the size of the basin of attraction of the panic equilibrium to the underlying parameters of the model.

Proposition 2. *For some parameter values, the basin of attraction of the panic equilibrium increases as the coefficient of risk aversion, the illiquid technology parameter, and the fraction of impatient agents increase.*

The proof is given in the Appendix. An increase in b leads to an increase in the short-term interest rate r_1, which increases z_*. An increase in the fraction of impatient agents and the illiquidity of the technology has a similar effect. Hence, panics are more likely if changes in the underlying parameters make the banking system more illiquid. Some of the important observations that can discriminate among alternative models of banking systems come from the performance of the American banking industry during the period immediately before the Federal Reserve System was established, i.e., during the National Banking Era. This is a period of approximately 50 years (1863–1914), during which there were five major panics: 1873, 1884, 1890, 1893 and 1907. These panics tended to occur in the fall when demand for currency was particularly high. Proposition 1 above suggests that these observations are consistent with a stochastic steady state in which panics are relatively rare events over a long enough time horizon. In this context, the five panics during a period of approximately 50 years are interpreted as relatively infrequent transitions from the optimal to the panic equilibrium. Champ et al. (1996) study a model of banking panics in which money creation and the provision of liquidity by banks are related. They find that the probability of a panic is highest in periods of greatest seasonal demands for liquidity. In their model, private note-issue by banks can act as a mechanism that meets these demands and eliminates panics. Next, I turn to the question of how

[7] Notice that this would not necessarily be true if I assume that, in any given period, agents base their decision on a finite random sample of past histories, as in Young (1993). The main results of the paper will still hold under this alternative assumption.

[8] I owe this observation to an anonymous referee.

qualitative properties of the two static equilibria change for different specifications of the banking system.

4. The model with multiple banks

Assume that there are many banks of equal size in the economy. In each period during subperiod 0, the m agents are randomly matched in k banks. I assume that the seasonal high demands for currency are experienced the same way by all banks, i.e., that in each period and in each bank, there are $n_k = n/k$ patient and $m_k - n_k = (m-n)/k$ impatient agents. However, the relative representation of the two strategies of the patient players in each bank is random and, as we will see, subject to increasing variance as the size of the bank decreases. The fact that banks act in isolation might be the result of spatial separation and branching regulations.

Evidence from different regions in the United States as well as a comparison between Canadian and American banking suggest that during the National Banking Era, branching and cooperative interbank arrangements reduced the likelihood of panics. The different performance of Canadian and American banks is largely attributed to branching tactics and the degree of cooperation that Canadian banks managed to achieve during crises.[9] A question arises here: Suppose that the size of the banks in the economy can be influenced as a result of government policy, such as geographic restrictions on expansion for banks. If the government wants to minimize the likelihood of a banking panic, should this policy favor a centralized banking system with bigger banks or a system with many small banks?

Williamson (1989b) developed a model that incorporates the US and Canadian banking systems from 1870 to 1913 as special cases. He found that US banks under unit banking restrictions were more sensitive to idiosyncratic shocks and they experienced runs with higher probability while Canadian banks could diversify better due to the absence of unit banking restrictions and strong cooperative arrangements. He, then, used the model to study the possible implications of these banking arrangements for aggregate fluctuations in the two countries. While Williamson's analysis concentrates on asset diversification, in this paper I concentrate on liability diversification. Banks, therefore, fail in my model for reasons similar to those in the D–D static model. I reach conclusions similar to those in Williamson (1989) by concentrating on the role of *learning* as an equilibrium selection device in different banking environments with multiple equilibria. In this sense, my results can be viewed as reconciling to some extent the D–D model with Williamson's criticisms of the static version of this model.

[9] See, for example, Chari (1989), Calomiris and Gorton (1991), Sprague (1910), and Williamson (1989a).

In order to answer the posed question within my framework, I need to explore the relation of the probability of observing the panic equilibrium to the number of banks in the economy and, therefore, the size of a representative bank. Let x_{jt} be the expected number of patient agents adopting strategy s_2 at time t in bank j, $j=1,\dots,k$; $t=1,\dots,\infty$. Here, x_{jt} can be thought of as the expected representation of s_2-players in a sample of size n_k, where $k \geq 1$. Let $\gamma(r_1,k) \in \{0,1,\dots,n_k\}$ be the minimum number of patient agents not withdrawing in a given bank, so that a bank run does not occur during subperiod 1 in this bank, given that there are n_k patient depositors in this bank. As before, $\gamma(r_1,k)$ is constant among banks of the same size and equal to $(m_k(r_1-1))/r_1$. Let

$$\Omega_z^k = \left\{ \omega = (\omega_1,\dots,\omega_k) \in [0,n_k]^k :\ 0 \leq \omega_j \leq n_k \quad \text{and} \quad \sum_{j=1}^{k} \omega_j = z \right\}. \tag{3}$$

The set Ω_z^k contains all possible sample representations of z in the case where there are k banks in the economy. The pay-offs of the two strategies in bank j are as follows:

$$\pi(s_1,x_j,k) = \begin{cases} u(r_1), & \text{if} \quad x_j \geq \gamma(r_1,k); \\ \dfrac{m_k - \gamma(r_1,k)}{m_k - x_j} u(r_1) + \dfrac{\gamma(r_1,k) - x_j}{m_k - x_j} u(0), & \text{if} \quad x_j < \gamma(r_1,k); \end{cases} \tag{4}$$

$$\pi(s_2,x_j,k) = \begin{cases} u\left(\dfrac{[m_k - (m_k - n_k)r_1 - (n_k - x_j)r_1]R}{x_j} \right), & \text{if} \quad x_j \geq \gamma(r_1,k); \\ u(0), & \text{if} \quad x_j < \gamma(r_1,k). \end{cases} \tag{5}$$

Let $\pi(s_1,z_t,k)$ be the expected pay-off of a player with strategy s_1 against population configuration z_t given that there are k banks in the economy; and similarly for $\pi(s_2,z_t,k)$. Then for $s_h \in \{s_1,s_2\}$, $\omega = (\omega_1,\dots,\omega_k)$, we have:

$$\pi(s_h,z,k) = \sum_{w \in \Omega_z^k} \Pr\{x=\omega\} \cdot \left[\sum_{j=1}^{k} \frac{1}{k} \cdot \pi(s_h,x_j,k) \right].$$

The expected pay-offs reflect the uncertainty resulting from the random matching process. As before, the game is a coordination one with $E_1 = (s_1,\dots,s_1)$ and $E_2 = (s_2,\dots,s_2)$ being the only pure Nash equilibria. I assume that the evolution of the agents' strategies is subject to the same rules as in the one-bank case. For

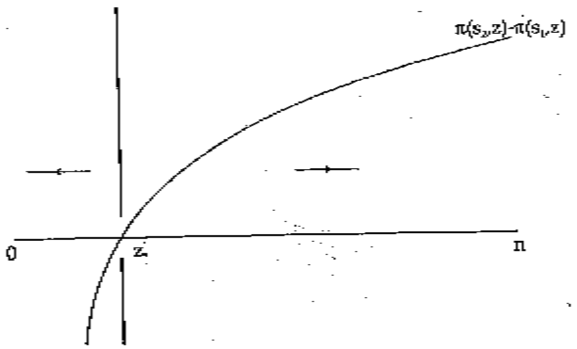

Fig. 1.

each level k, a unique z_*^k determines the basins of attraction of the two steady states. As before, the steady state with the largest basin of attraction will be observed most of the time for small levels of noise. I want to characterize the two basins of attraction as a function of k. We have the following.

Proposition 3. *Suppose that* $u(0) \leq M(m,n,b,R)$ *and let* k, k' *be integers such that:* $k' > k \geq 1$. *Then* $z_*^{k'} > z_*^k$, *i.e., the basin of attraction of the panic equilibrium increases as the size of the banks in the economy decreases.*

The proof of Proposition 3 is given in the Appendix. As the number of banks increases, each individual bank becomes subject to more uncertainty. Since the returns to s_2-players change dramatically across states, withdrawing late then becomes increasingly risky while strategy s_1, providing a constant return across most states, remains relatively 'safe'. Therefore, in the presence of increasing uncertainty, agents will shift to withdrawing early. Since z_* determines the two basins of attraction, the basin of attraction of the panic equilibrium becomes larger as the number of banks increases (see Fig. 1). It is worth mentioning that, in certain examples, this can change equilibrium selection, leading to the panic steady state having the larger basin of attraction in cases where this would not happen if there were a smaller number of banks in the economy.

Learning, therefore, results in sharp predictions about the stability of alternative banking systems over a long period of time. Small or isolated banks cannot diversify against withdrawal risk. Consequently, the entire banking system becomes riskier, in which case it is harder for depositors to coordinate. This way, in a system of small banks, panics might become frequent events.

5. The model with local interaction

The assumption of uniform matching expresses the idea that depositors have no local or other information about individual banks. In contrast, local matching rules are one way to model a situation where depositors are less likely to interact with the banking system as a whole than with local banks in the same geographic region. In addition, local matching rules allow for an overlap in the groups of depositors in neighboring banks so that a bank's neighboring bank is likely to be a neighboring bank as well. As Ellison (1993) showed, this allows for the existence of small clusters within the population, and the possibility of a new strategy's gaining a foothold within one of these clusters allows a more rapid transition to steady state than in the model with uniform matching.

Here, I assume that the m agents and the k banks are uniformly arranged around a circle. As before, each bank has the same constant fraction of patient and impatient agents as the population, and I concentrate on the behavior of patient agents. In contrast to the uniform matching model, here I assume that each depositor is matched with probability $1 - 2\tau$ with the bank in his own location, and with probability τ with each of the other two banks in the region, directly to the left and directly to the right of his location. For simplicity, I fix $\tau = \frac{1}{3}$. As before, I am interested in equilibrium selection as a function of the size of the representative bank in the economy n_k.

In the model with local interaction, I assume that depositors observe the bank failures in their own region and that the payoffs are such that depositors will switch to s_1 if at least one of the three banks in their region experienced a run in the previous period.[10] As before, this reflects a situation where agents have information on the status of the 'local banking system' but not on individual banks.[11]

First, consider the case without noise. Once again, there are two steady states with a nontrivial basin of attraction, corresponding to $E_1 = (s_1, \ldots, s_1)$ and $E_2 = (s_2, \ldots, s_2)$. First, suppose that a bank run is experienced in one bank at time t, let us say bank B (see Fig. 2). Then, under the best reply dynamic, each depositor in banks A, B and C will play s_1 in $t+1$. This way, a local panic is created in period $t+1$ that will continue to spread to neighboring banks of the neighboring

[10] Alternatively, I could assume that agents observe the number of early withdrawals in their region and that the pay-offs are such that agents will withdraw if more than one-third of the population in their region played s_1 in the previous period.

[11] As an alternative to random matching, I could assume that individuals deposit in the same bank in each period and that banks have access to a stochastic technology with a random element being unobserved by depositors and positively correlated across banks in the same region (and, therefore, across banks in neighboring regions...). In this alternative specification, agents who cannot distinguish whether or not a bank failure in their region is due to a bad technology shock, which could affect their own bank, might run in the next period to the neighboring banks as well. I owe this observation to George Mailath.

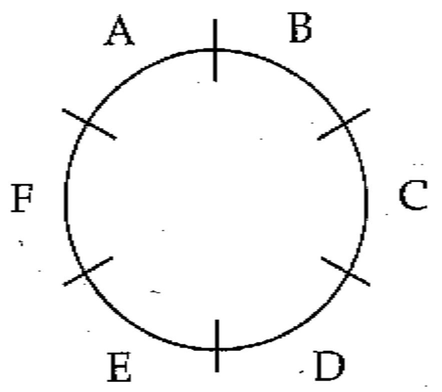

Fig. 2.

banks, until the panic steady state E_1 is reached. One important feature of this dynamic is that a small 'cluster' of depositors creating bank runs in one region is sufficient to ensure rapid convergence to the panic equilibrium. On the other hand, as I will show, another important feature of this dynamic is that a panic may become far more probable as the size of the banks in the economy decreases. More precisely, we have the following analogue of Proposition 4.

Proposition 4. *The probability of observing the panic equilibrium increases as the size of the banks in the economy decreases (the number of banks, k, increases).*

The proof of Proposition 4 can be found in the appendix. For example, suppose that $n=18$ and consider two cases: $k=3$ and $k'=6$, i.e., $n_k=6$ and $n_{k'}=3$ (see Fig. 3). Also, assume that $\gamma(1)=6$ so that $\gamma(3)=2$ and $\gamma(6)=1$, and suppose that pay-offs are such that a depositor will play s_1 if one of the three banks in the region experienced a bank run in period t. I will denote the probability of transition from E_1 to E_2, along the 'minimum noise' path, when there are k banks in the economy by p_k and the probability of transition from E_2 to E_1 when there are k banks in the economy by q_k. I define $p_{k'}$ and $q_{k'}$ similarly for the case with k' banks. We then have

$$\frac{p_k}{q_k}=\frac{\varepsilon^2\cdot\varepsilon^2\cdot\varepsilon^2}{\varepsilon^4+\varepsilon^4+\varepsilon^4}=\frac{1}{3}\varepsilon^2, \tag{6}$$

and

$$\frac{p_{k'}}{q_{k'}}=\frac{\varepsilon\cdot\varepsilon\cdot\varepsilon\cdot\varepsilon\cdot\varepsilon\cdot\varepsilon}{\varepsilon^2+\varepsilon^2+\varepsilon^2+\varepsilon^2+\varepsilon^2+\varepsilon^2}=\frac{1}{6}\varepsilon^4. \tag{7}$$

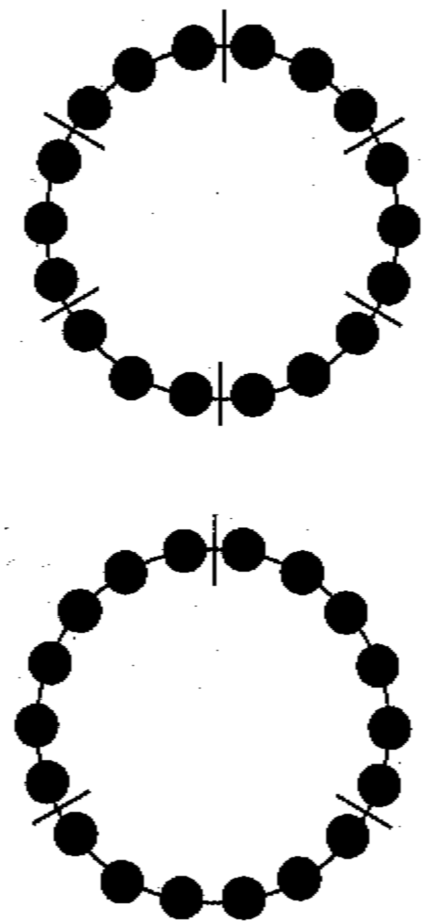

Fig. 3.

The arguments above suggest that the orders of (p_k/q_k) and $(p_{k'}/q_{k'})$ are ε^2 and ε^4, respectively, so we have

$$\frac{p_{k'}/q_{k'}}{p_k/q_k}\to 0 \quad \text{as } \varepsilon\to 0.$$

Local interaction allows for faster convergence to steady state as well as for interesting dynamics as shown in the previous example. Furthermore, the panic is more probable in a system consisting of small banks for reasons that are different from the uniform matching model and are related to 'neighborhood effects'.

Though the importance of the contagion effects is subject to some dispute, several studies have suggested that these effects were important in at least some of the banking panics experienced in the US history. According to these studies, panics did not suddenly occur at different locations simultaneously, and a typical pattern of a panic was that banks in the same geographic location experienced a run and subsequently runs spread to other locations.

6. Concluding remarks

In this paper, I have presented a model of banking panics consistent with some basic facts from the National Banking Era, the period before the Federal Reserve System was established. Panics in that period were recurrent but relatively infrequent phenomena. This pattern is consistent with a stochastic steady state in which panics occur in each period with positive probability but are relatively infrequent events over a long enough period of learning. These results challenge the view that any banking system without deposit insurance must be plagued by ever-increasing instability. The model is also consistent with observations that suggest that more centralized banking systems can better diversify against withdrawal risk and, therefore, are more likely to perform better in terms of stability than banking systems consisting of many banks that are small and isolated as a result of government regulation. Finally, local interaction generates contagion effects that allow bank runs to spread first among banks in the same geographic region and subsequently in other locations, a pattern consistent with historical observations from the same period. These effects become stronger as the size of the banks decreases.

In this version of the model, I assume that banks act in isolation. Even though, in this model, a bank can be interpreted as a *coalition of banks* that cooperate closely, liquidity risk-sharing by banks could arise if this assumption is dropped. I believe that the main conclusions of the model will still be true. This is the case since, even under perfect cooperation among banks, the banking system in this model can be at best as stable as if there were a single big bank in the economy. However, occasional panics are possible, if less likely, in this case, as shown in Section 3. On the other hand, efficiency issues and considerations about oligopoly power by banks might make such an arrangement socially undesirable.[12] Such considerations, however, are beyond the scope of this paper. The model is believed to be robust to different specifications of the dynamical system describing the learning process. However, it is not entirely clear how the results will change once the assumption that banks are not strategic is dropped. Introducing cyclical behavior in the demand for liquidity would provide a framework for strategic

[12] In addition, see McAndrews and Roberds (1995) for a study of the incentives that might undermine cooperative behavior within such a coalition of banks.

banks to adjust the interest rates over time in order to minimize the probability of a bank run. The evolutionary approach adopted here in analyzing the dynamics of financial crises can be viewed as an alternative to recent models of herd behavior.[13] Here, the emphasis is on the dynamics generated by imitating 'panic' behavior that, in the presence of noise, may become the most successful strategy locally.

Appendix: Proofs

Proof of Proposition 1. The two basins of attraction are determined by the unique solution z_* to the equation $\pi(s_2,z)-\pi(s_1,z)=0$, in Z. Clearly, $z_*>\gamma(r_1)$ and, therefore

$$\frac{[m-(m-n)r_1-(n-z)r_1]R}{z}=r_1 \quad \text{implies that } z_*=\frac{mr_1R-mR}{mr_1R-mr_1}.$$

The result then follows from Lemma 2. □

Proof of Proposition 2. Here, I find it convenient to redefine z_* as the *fraction* of patient agents that choose strategy s_2. The state space then becomes the interval $[0,1]$. Assume that $u(c)=-c^{-b}$, with $b>0$. The first order conditions for the social planner's problem give

$$u'(r_1^*)=Ru'(r_2^*).$$

The feasibility condition is

$$(m-n)r_1+\frac{nr_2}{R}=m.$$

Combining the two gives

$$r_1^*=m(m-n+nR^{\frac{-b}{b+1}})^{-1}.$$

For the risk aversion coefficient we have

$$\frac{\mathrm{d}r_1}{\mathrm{d}b}=mn\log(R)(1+b)^{-2}R^{\frac{-b}{b+1}}(m-n+nR^{\frac{-b}{b+1}})^{-2}>0,$$

and

$$\frac{\mathrm{d}z_*}{\mathrm{d}r_1}=\frac{mR}{nr_1{}^2(R-1)}>0.$$

Therefore $\mathrm{d}z_*/\mathrm{d}b>0$.

[13] See, for example, Banerjee (1992, 1993). For an alternative way to introduce learning by fully rational agents in a random environment, see Rustichini and Villamil (1996). For another application of evolutionary game theory to public economics see Langunoff and Matsui (1995).

For the technology parameter we have

$$\frac{\partial z_*}{\partial R}=\frac{1+bR-R^{\frac{b}{b+1}}-bR^{\frac{b}{b+1}}}{(1+b)(R-1)^2R^{\frac{b}{b+1}}},$$

$$\frac{\mathrm{d}r_1}{\mathrm{d}R}=bmnR^{\frac{-1-b}{b+1}}(1+b)^{-1}(m-n+nR^{\frac{-b}{b+1}})^{-2}>0,$$

and

$$\frac{\mathrm{d}z_*}{\mathrm{d}R}=\frac{\partial z_*}{\partial R}+\frac{dz_*}{dr_1}\cdot\frac{dr_1}{dR}=\frac{1-b+2bR-R^{\frac{b}{b+1}}-bR^{\frac{b}{b+1}}}{(b+1)(R-1)^2R^{\frac{b}{b+1}}}$$

Finally, for the fraction of patient agents we have:

$$\frac{\partial z_*}{\partial\left(\frac{n}{m}\right)}=-\frac{(r_1R-R)}{\left(\frac{n}{m}\right)^2(r_1R-r_1)}<0,$$

and

$$\frac{\mathrm{d}r_1}{\mathrm{d}\left(\frac{n}{m}\right)}=\frac{R^{\frac{-b}{b+1}}-1}{1-\frac{n}{m}+\frac{n}{m}R^{\frac{b}{b+1}}}.$$

For an example of a parametrization for which $\mathrm{d}z_*/\mathrm{d}(\frac{n}{m})<0$, consider $m=1000$, $n=400$, $b=5$, $R=2.5$. Then $r_1=1.27$, and $\mathrm{d}z_*/\mathrm{d}(\frac{n}{m})=-0.48$. □

Proof of Proposition 3. The proof relies on three lemmata listed below (3, 4 and 5), the proofs of which are omitted. The first lemma simplifies the analysis. It says that it is the proportion of strategies in a bank, not the size of the bank, which determines the payoffs. In other words, the payoff of playing strategy s^h when a fraction of x_j/n_k agents are playing this strategy is the same for any size of a bank n_k.

Lemma 3. *Let k,k' be integers such that $k'>k\geq 1$, and suppose that $x_j/n_k=x_{j'}/n_{k'}$. Then $\pi(s_h,x_j,k)=\pi(s_h,x_{j'},k')$, for $h=1,2$, $j=1,\ldots,k$.*

The next lemma relates the first two moments of the proportion of players playing each strategy in a bank to the size of the bank. The proof relies on standard results on the hypergeometric distribution.

Lemma 4. *Let k,k' be integers such that $k'>k\geq 1$. Then for all $j=1,\ldots,k$,*
(a) $E(\frac{x_j}{n_{k'}})=E(\frac{x_j}{n_k})=\frac{z}{n}$ *and*
(b) $\mathrm{Var}(\frac{x_j}{n_{k'}})>\mathrm{Var}(\frac{x_j}{n_k})$.

The following lemma describes the curvature of the difference of the expected payoffs of the two strategies as a function of the proportion of patient agents in the bank.

Lemma 5. *The function $\pi(s_2,z)-\pi(s_1,z)$ is a concave function of z for $z\geq\gamma(r_1)$.*

Given Lemmata 3, 4, 5, and risk aversion, I now prove the proposition for the case where $k=1$ and $k'=2$. The proof can be generalized for any integer k', $k'>1$. Let $f(z)=\pi(s_2,z)-\pi(s_1,z)$. It is sufficient to show that for any $\rho\in(0,1)$ and any $z',z'\in[0,n]$ such that $f(\rho z'+(1-\rho)z'')\geq 0$, we have: $\rho f(z')+(1-\rho)f(z')\leq f(,\rho z'+(1-\rho)z'')$. I consider three cases.

(a) First let $z',z''\in[0,\gamma(r_1))$. In this case, $f(,\rho z'+(1-\rho)z'')<0$.
(b) Next, consider the case where $z',z''\in[\gamma(r_1),n]$. In this case, concavity follows from lemma 5.
(c) Finally, consider the case where $z'\in[0,\gamma(r_1))$ and $z'\in[\gamma(r_1),n]$. Here, I define the following function

$$\tilde{f}(z)=\begin{cases}f(z), & \text{if } z\geq\gamma(r_1);\\ \bar{f}(z), & \text{if } 0\leq z\leq\gamma(r_1);\end{cases}$$

where $\bar{f}(z)=f(\gamma(r_1))'(z-\gamma(r_1))+f(\gamma(r_1))$.

Next, I define $M(m,n,b,R)=\tilde{f}(0)$. The function $\tilde{f}(z)$ is concave so $\tilde{f}(,\rho z'+(1-\rho)z')\geq\rho\tilde{f}(z')+(1-\rho)\tilde{f}(z'')$. Also, by construction of $\tilde{f}$, for all $z'\in[0,\gamma(r_1))$, we have that $f(z')<\tilde{f}(z')$, and for all $z''\in[\gamma(r_1),n]$, we have that $f(z')=\tilde{f}(z'')$. Therefore, for all $\rho\in(0,1)$ and all $z'\in[0,\gamma(r_1))$ and for all $z''\in[\gamma(r_1),n]$ such that $\rho z'+(1-\rho)z''=z$, we have: $\rho f(z')+(1-\rho)f(z'')<f(,\rho z'+(1-\rho)z')$. Since (a),(b), and (c) are the only possible cases, we conclude that $z_*^2>z_*^1$. □

Proof of Proposition 4. Here, I consider two cases. In the first case, there are k banks of size $n_k=n/k$ and in the second case, there are k' banks of size $n_{k'}=n/k'$, where $k'>k$. Since we assumed that $\gamma=n/3$ when $k=1$, a bank run occurs in the first case if the number of s_2 players in a given bank is less than $n_k/3$. Recall that $E_1=(s_1,\ldots,s_1)$ and $E_2=(s_2,\ldots,s_2)$. The weight of state E_1 in the steady state distribution is given by $\mu_{E_1}^k(\varepsilon)$ and similarly for $\mu_{E_2}^k(\varepsilon)$. First, I show that $\mu_{E_1}^{k'}(\varepsilon)>\mu_{E_1}^k(\varepsilon)$ for $k'>k$. In the case of k banks, the least costly E_1-tree is of order $\varepsilon^{\frac{1}{3}n_k}$ since a run in at least one bank is needed in order to lead the system to the E_1 state. In the case of k' banks, the similar E_1-tree is of order $\varepsilon^{\frac{1}{3}n_{k'}}$. Since $n_{k'} < n_k$, we have that $\mu_{E_1}^{k'}(\varepsilon) > \mu_{E_1}^{k}(\varepsilon)$. On the other hand, we need that no bank experiences a run in period t in order for the system to converge to E_2. So we have that $\mu_{E_2}^{k}(\varepsilon) = \varepsilon^{k\frac{1}{3}n_k}$ and $\mu_{E_2}^{k'}(\varepsilon) = \varepsilon^{k'\frac{1}{3}n_{k'}}$ and, therefore, $\mu_{E_2}^{k'}(\varepsilon) = \mu_{E_2}^{k}(\varepsilon)$. □

References

Banerjee, A.V., 1992. A simple model of herd behavior. Quarterly Journal of Economics 107, 797–817.

Banerjee, A.V., 1993. The economics of rumours. Review of Economics Studies 60, 309–327.

Bergin, J., Lipman, B., 1996. Evolution and state dependent mutations. Econometrica 64, 943–956.

Binmore, G., Samuelson, L., Vaughan, R., 1995. Musical chairs: modeling noisy evolution. Games and Economic Behavior 11, 11–35.

Bryant, J., 1980. A model of reserves, bank runs, and deposit insurance. Journal of Banking and Finance 4, 335–344.

Calomiris, C., Gorton, G., 1991. The origins of bank panics: models, facts, and bank regulation. In: Glenn Hubbard, R. (Ed.), Financial Markets and Financial Crises, University of Chicago Press, Chicago.

Champ, B., Smith, B., Williamson, S., 1996. Currency elasticity and banking panics: theory and evidence. Canadian Journal of Economics XXIX (4), 829–864.

Chari, V.V., 1989. Banking without deposit insurance or bank panics: lessons from a model of the US national banking system. Federal Reserve Bank of Minneapolis Quarterly Review 13, 3–19.

Chari, V.V., Jagannathan, R., 1988. Banking panics, information, and rational expectations equilibrium. Journal of Finance 43, 749–761.

Diamond, D., Dybvig, P., 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 91, 401–419.

Ellison, G., 1993. Learning, local interaction and coordination. Econometrica 61, 1047–1072.

Foster, D., Young, P., 1990. Stochastic evolutionary game dynamics. Journal of Theoretical Biology 38, 219–232.

Freidlin, M.I., Wentzell, A.D., 1984. Random perturbations of dynamical systems. Springer Verlag, New York.

Friedman, M., Schwartz, A.J., 1963. A monetary history of the United States 1867–1960. Princeton University Press, Princeton, N.J.

Gibbons, J.S., 1859. The banks of New York, their dealers, the clearing house, and the panic of 1857. D. Appleton & Co, New York.

Green, E.J., 1995. Implementing efficient allocations in a model of financial intermediation. Federal Reserve Bank of Minneapolis.

Harsanyi, J., Selten, R., 1988. A general theory of equilibrium selection in games. MIT Press, Cambridge.

Jacklin, C.J., Bhattacharya, S., 1988. Distinguishing panics and information-based bank runs: welfare and policy implications. Journal of Political Economy 96, 568–592.

Kandori, M., Mailath, G.J., Rob, R., 1993. Learning, mutation, and long run equilibria in games. Econometrica 61, 29–56.

Lagunoff, R.D., Matsui, A., 1995. Evolution in mechanisms for public projects. Economic Theory 6, 195–223.

McAndrews, J., Roberds, W., 1995. Banks, payments, and coordination. Journal of Financial Intermediation 4, 305–327.

Rustichini, A., Villamil, A.P., 1996. Intertemporal pricing in markets with differential information. Economic Theory 8, 211–227.

Saunders, A., Wilson, B., 1996. Contagious bank runs: evidence from the 1929–1933 period. Journal of Financial Intermediation 5, 409–423.

Sprague, O.M.W., 1910. History of crises under the national banking system. Report by the National Monetary Commission to the US Senate. Government Printing Office, Washington D.C.

Villamil, A.P., 1991. Deposit contracts, suspension of convertibility, and optimal intermediation. Economic Theory 1, 277–288.

Wallace, N., 1988. Another attempt to explain an illiquid banking system: the Diamond and Dybvig model with sequential service taken seriously. Federal Reserve Bank of Minneapolis Quarterly Review 12, 3–16.

Williamson, S.D., 1989a. Restrictions on financial intermediaries and implications for aggregate fluctuations: Canada and the United States 1870–1913, NBER Macroeconomics Annual, 303–349.

Williamson, S.D., 1989b. Bank failures, financial restrictions, and aggregate fluctuations: Canada and the United States, 1870–1913. Federal Reserve Bank of Minneapolis Quarterly Review 13, 20–40.

Young, P.H., 1993. The evolution of conventions. Econometrica 61, 57–84.