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Exposita notes

Incentives to produce quality and the liquidity of money*

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Summary. I study a version of the Williamson-Wright (1994) model that results from ruling out direct barter. Although one can no longer argue that the value of money comes exclusively from private information, one can use the simplified model to address a variety of other issues. In particular, one can characterize analytically the set of equilibria and their properties. One can also analyze the trade-off between providing liquidity to facilitate trade and providing incentives to produce high quality, and address some other issues related to policy and welfare, including the effects of inflation on the incentives to produce high quality.

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1. Introduction

Two reasons why the use of fiat money may be an equilibrium or a welfare-enhancing arrangement are that: (1) it reduces the difficulty of exchange by eliminating the "double coincidence of wants" problem with pure barter; and (2) it reduces private information problems. The former effect has been formalized using search models of the exchange process, as in Kiyotaki and Wright (1993), for example. Recently, Williamson and Wright (1994) have also formalized the latter effect in an equilibrium search model with a lemons problem along the lines of Akerlof (1970).[1] They show that fiat money can be valued in a world of qualitative uncertainty, and that its use can potentially improve welfare, simply because it is universally recognizable. These results are obtained even in circumstances where barter is not more difficult than monetary exchange.

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[1] Analogous results in Huggett and Krasa (1995) show that a fiat currency may be *essential* to implement certain Pareto optimal alllocations. Li (1996) and Cuadras-Morató (1994) look at a related issue, in a model with private information about the quality of the good used as commodity money. Koutsougeras and Yannelis (1993) show a different scenario where better informed agents can turn their information into purchasing power, playing the role of intermediaries. Li (1995) addresses a similar issue in the context of a search model. Other related literature is discussed by Williamson and Wright.

A problem with their model of money is that it is somewhat intractable. First, there are very many potential monetary equilibria.[2] Also, these equilibria are difficult to characterize analytically. It is not easy to see what conditions allow the different types of equilibria to exist, or to develop intuition as to how money works in their model. Their demonstration of a potential welfare-improving role for money consists only of some numerical examples. They do not provide general statements about existence conditions, or about the effects of the parameters or policy variables on the endogenous variables. One cannot study in their model the relationship between the two aspects of monetary exchange mentioned above.

The model presented here is much simpler, due to assumptions that rule out direct barter. Because this creates a double coincidence of wants problem, one is not able to claim that the private information is the only reason why money is privately used or socially desirable.[3] On the other hand, there are new things to be learned from this simplified version, where the lack of barter creates a trade-off between two conflicting functions of money: facilitating the frequency of exchange and providing incentives for the production of high quality. In fact, the two main results relate to that trade-off. First, a low money supply (which in the model represents a low ratio of buyers to liquidity-constrained sellers) makes it hard for sellers to find buyers, and thus reduces the attractiveness of exploiting the lemons problem. This means that in the presence of qualitative uncertainty the optimal monetary policy does not imply maximizing the frequency of consumption, since a certain amount of friction in the matching process may increase the average quality of output. Second, when a tax on money holdings (which is meant to proxy inflation in the model) is levied, more lemons are produced and sold, less potentially beneficial trading opportunities are realized, and welfare falls.

The ostensibly minor variation that simplifies the environment by ruling out direct barter also delivers much cleaner results. Only three stationary equilibria exist, and they can be ordered by the Pareto criterion. Their stability properties can be established. Closed form solutions can be found for their existence conditions and for the endogenous variables in each equilibrium. The effects of parameters that measure the money supply, the interest rate, and the extent of the private information problem on the endogenous variables can be determined. Finally, the welfare effects of fiat money can be identified more precisely.

Section 2 describes the simplified model. The potential types of equilibria, their existence conditions and main properties are studied in Section 3. Section 4 contains some concluding remarks. The Appendix follows in Section 5.

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[2] To be precise, there are eighteen qualitatively different types of potential equilibria, even after eliminating degenerate outcomes and outcomes in which money is not universally accepted. They only list nine types in their Table 1, since they do not consider monetary equilibria where no lemons are produced, even though such equilibria can be shown to exist (see below).

[3] By allowing barter, and assuming that it is no more difficult than monetary exchange, Williamson and Wright can argue that the role of fiat money in their model derives exclusively from private information and not from the fact that it reduces the double coincidence friction. That is, in fact, their main result. It is barter, however, that makes the model complicated, preventing one from looking at a variety of other issues.

2. The basic model

Time is continuous and unbounded. There is a continuum of measure one of homogeneous, infinite-lived agents. There are three kinds of tradable commodities: high quality goods, low quality goods, and fiat money. There are K varieties of the goods, and each agent is able to produce and store the high and the low quality versions of one particular variety, and unable to produce any quality of any other variety. We refer to those that produce variety k as *type-k agents*, and assume an equal number of agents of each type. For all types of agents, the cost in terms of disutility to producing one unit of output is equal to $\gamma > 0$ if it is high quality, and 0 if it is low quality, while money cannot be privately produced at any cost. Agents can store only one unit of one tradable commodity (goods or money) at a time. All objects are indivisible and disposable.

At any moment in time, an agent may want to consume some varieties of goods but not others. Consumption of one unit of the high quality good yields utility $u > \gamma$ if it is of a desired variety. Consumption of the other varieties, of any low quality good, or of money yields zero utility. The subjective rate of time preference is $r > 0$. The information structure is as follows: any agent can always recognize the *variety* of the goods produced by others, but the *quality* is recognized only with probability θ. Money is always identifiable.

At the beginning of time, a fraction M of the agents are each endowed with one unit of cash. Thereafter, agents meet pairwise and at random according to a Poisson process, and decide whether to exchange inventories. The key simplifying assumption in this paper is that preferences are ordered according to a "Wicksell triangle," meaning that a type-k agent (who is able to produce variety k) always wants to consume variety $k+1$, and never the other varieties. With $K \geq 3$, this assumption implies that strictly mutually beneficial direct barter can be ruled out (nobody who produces what I want ever wants what I produce) and guarantees there must be money on one side of every transaction.[4] We call agents holding money *buyers*, while *sellers* are agents who hold goods. Denoting by β the arrival rate of the matching process by which agents meet, the probability for any seller of type-k of meeting, in an interval of length δ, a buyer who wishes to consume good k is approximately $\beta M\delta/K$ (with the approximation becoming precise as the interval shortens) and the probability that a buyer finds, in an interval of length δ, somebody who happens to be a seller of what he wants is approximately $\beta(1-M)\delta/K$. For simplicity, and without loss of generality, we let $\beta = K$.

[4] There are other ways of describing a search environment where barter is impossible and all exchange needs to be monetary. Trejos-Wright (1995) allow for agents to want different varieties at different times, and use the fact that as K increases barter opportunities become increasingly rare relative to monetary exchange opportunities. Burdett et al. (1995) show that if buyers or sellers can chose between actively searching for others (at a cost) or staying put waiting for others to find them, equilibria exist where buyers do all the search, so sellers never meet and barter does not take place. One can also assume that sellers can only produce and store output in a given location (their "shop"), so buyers run into sellers (and into one another) but sellers never meet sellers.

3. Equilibrium

The following decisions must be made by all types of agents in this economy. First, they have to decide whether to produce high or low quality. Let π denote the probability that a given agent will choose to produce high quality. Then we can infer that, based on that decision, at each point in time there will be a proportion of the sellers, p, holding high quality output, while the other $1-p$ will hold low quality.[5] Also, buyers and sellers have to choose trading strategies for their potential meetings. A buyer has to decide whether to part with her money in exchange for a good (of the desired variety) that she either identifies as high quality, identifies as low quality, or cannot identify.[6] A seller has to decide whether he wishes to trade his production good in exchange for money. Agents make these decisions to maximize their expected intertemporal utility, taking as given p and the trading strategies of others.

Let V_J denote expected utility (*the value function*) of an agent holding object J, where $J = H, L$ or M corresponds to a high quality good, a low quality good, or money respectively. For all parameter combinations there is a degenerate (or non-monetary) equilibrium in this economy, where nobody produces high quality and money has no value. However, I am more interested here in non-degenerate, pure monetary, symmetric, steady-state equilibria. In a non-degenerate equilibrium expected utility is positive, which requires $p > 0$. In a pure monetary equilibrium money is always accepted, which means $V_M \geq V_H$ and $V_M \geq V_L$. One implication of these inequalities is that buyers always spend their money on goods they recognize as high quality, and never on goods they recognize as low quality. On the other hand, the decision of whether to buy goods whose quality cannot be recognized is not trivial. Let ω denote the probability that a buyer will decide to make a purchase in those circumstances. Further, we denote by Ω the value of ω that agents expect will be chosen by others, and of course a condition for equilibrium will be $\omega = \Omega$. Since we will only study equilibria that are stationary and symmetric across varieties, we need not specify time or type of agent in any of this notation.

The value functions satisfy the following equations:

$$rV_H = M[\theta + (1-\theta)\Omega](V_M - V_H) \tag{1}$$

$$rV_L = M(1-\theta)\Omega(V_M - V_L) \tag{2}$$

$$rV_M = (1-M)\theta p(u + V_H - \gamma - V_M) + (1-M)(1-\theta)\Omega(pu + V_H - \gamma - V_M). \tag{3}$$

Equation (1) can be interpreted as follows: the flow value of holding a high quality good (rV_H) is equal to the probability of meeting a buyer of one's variety of output ($\beta M/K = M$) times the probability she will purchase one's good (which is the probability θ that she identifies it plus the probability $(1-\theta)\Omega$ that she does not identify it but buys it anyway) times the surplus generated by trade ($V_M - V_H$). Equations (2) and (3) can be understood in a similar fashion.

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[5] Since it may not take the same average time to sell goods of different quality, in general $\pi \neq p$, except when $\pi = 0$ or $\pi = 1$.

[6] Under our assumptions, and given that we will only focus on equilibria that treat all varieties symmetrically, nobody ever accepts a good of a variety he does not wish to consume.

The best response for a buyer is to purchase something of the right variety that she cannot recognize if and only if the benefit of consuming sooner rather than later outweighs the risk of being tricked into taking a lemon. This corresponds to the condition:

$$\Omega \begin{cases} =0 & \text{if } pu+V_H-\gamma-V_M<0 \\ \in[0,1] & \text{if } pu+V_H-\gamma-V_M=0. \\ =1 & \text{if } pu+V_H-\gamma-V_M>0 \end{cases} \tag{4}$$

The decision of what quality to produce depends on whether the benefit of carrying high quality (that it can be sold faster) compensates for the production cost γ. This determines if all, some, or none of the sellers carry high-quality output, according to the condition:

$$p \begin{cases} =0 & \text{if } V_H-\gamma-V_L<0 \\ \in[0,1] & \text{if } V_H-\gamma-V_L=0. \\ =1 & \text{if } V_H-\gamma-V_L>0 \end{cases} \tag{5}$$

This means that an equilibrium of the type we are looking for (i.e. where $p>0$) satisfies $\max\{V_H-\gamma, V_L\}=V_H-\gamma$, a result which has been used, incidentally, in the contruction of (3) and (4).

An equilibrium is a combination $(V_M, V_H, V_L, p, \Omega)$ that satisfies (1) through (5). In any equilibrium where $p=1$ it has to be the case that $\Omega=1$, because when all the output is of high quality a buyer knows that what she is being offered is not a lemon, even when she cannot verify it directly. Furthermore, $\Omega=0$ rules out $p<1$, because it could not be the case that some agents find it beneficial to produce low quality (which they could never sell when $\Omega=0$). From these arguments it follows that there are only three possible types of non-degenerate monetary equilibria in this economy: A) $p=1$, $\Omega=1$; B) $p\in(0,1)$, $\Omega=1$; and C) $p\in(0,1)$, $\Omega\in(0,1)$.

Equilibrium A, in which no lemons are ever produced, is equivalent to the pure monetary equilibrium in a version of Kiyotaki-Wright (1993) with barter ruled out. In it, imperfect information is irrelevant. To verify the circumstances under which this equilibrium exists notice first that condition (4) is automatically satisfied by $\Omega=1$ when $p=1$. The only existence condition then is (5), $V_H-\gamma\geq V_L$. It holds if and only if

$$\theta\geq\theta_1\equiv\frac{(r+M)(1+r)\gamma}{M[(1-M)u+(r+M)\gamma]}. \tag{6}$$

In equilibrium B, producing either quality of the consumption good is equally profitable, and some lemons show up in the market, yet these are sufficiently scarce that no buyer walks away from a good that she cannot identify. Substituting $\Omega=1$ in the value functions, and solving for p given that equilibrium condition (5) is satisfied with equality, yields:

$$p=p_B\equiv\frac{(M+r)(1+r-\theta)\gamma}{(1-M)\theta[Mu-(M+r)\gamma]}. \tag{7}$$

For this equilibrium to exist, it has to be the case that condition (4) holds and that $p\in(0,1)$. Hence the risk of being stuck with a lemon must be so small that buyers never reject any good. This corresponds to a low θ, which makes p_B high, so buyers can expect higher utility by taking the immediate consumption opportunity rather than playing it safe. Then, condition (4) holds iff

$$\theta \leq \theta_2 \equiv \frac{(M+r)[Mu+(1-M)\gamma]}{Mu}. \tag{8}$$

The condition for $p<1$ is $\theta>\theta_1$. For $p>0$ we need $Mu>(M+r)\gamma$, which is redundant since it is also necessary for $\theta_2>\theta_1$.

In equilibrium C the number of sellers trying to trade bad output to uninformed buyers is so high that the expected surplus from the purchase of an unidentified commodity is exactly zero; conversely, Ω is so low that the expected net surplus from investing in high quality is also zero. From $V_H-\gamma=V_L$, $pu=V_M-V_L$, and the value equations, it follows that

$$p=p_C\equiv\frac{\gamma}{Mu+(1-M)\gamma} \tag{9}$$

$$\Omega=\Omega_C\equiv\frac{\theta M(1-M)(u-\gamma)-r[Mu+(1-M)\gamma]}{M(1-\theta)[Mu+(1-M)\gamma]}. \tag{10}$$

Observe that $0<p_C<1$ for any $M<1$. The only remaining condition is $\Omega_C\in(0,1)$. Ω_C is increasing in θ, and for $\Omega_C<1$ we need $\theta<\theta_2$, while for $\Omega_C>0$ we need $\theta>\theta_3$. Hence, equilibrium C exists if and only if

$$\theta_2>\theta>\theta_3\equiv\frac{r[Mu+(1-M)\gamma]}{M(1-M)(u-\gamma)}. \tag{11}$$

The analysis above can be summarized as follows:

Proposition 1. There are at most three stationary, non-degenerate, monetary equilibria. If $\theta\geq\theta_1$ then $(p,\Omega)=(1,1)$ is an equilibrium; if $\theta_2\geq\theta\geq\theta_1$ then $(p,\Omega)=(p_B,1)$ is an equilibrium; if $\theta_2\geq\theta>\theta_3$ then $(p,\Omega)=(p_C,\Omega_C)$ is an equilibrium. No non-degenerate stationary equilibrium exists if $\theta<\mathrm{Min}\{\theta_1,\theta_3\}$. A degenerate stationary equilibrium where $(p,\Omega)=(0,0)$ always exists.

Figure 1 shows the critical values θ_i as functions of M. It is drawn assuming a low value of r. (With a higher r, $\theta_1<\theta_3$ for all M, so the area where C is the unique equilibrium disappears). As seen in the figure, there is no non-degenerate equilibrium if θ is very low, because in that case all sellers will try to get away with producing low quality. If θ is not too low, non-degenerate equilibria exist. For example, if commodities are highly recognizable (θ high) selling lemons is too difficult, and the only possible out- come is equilibrium A. For intermediate levels of θ all three equilibria exist. For even lower levels of θ, equilibrium C is unique. To understand why, consider that the incentives to produce lemons are the strongest when θ is low, since finding an uninformed buyer is easier. So, if an equilibrium where some producers provide high quality is to exist, one needs to add difficulty to selling a lemon. Equilibrium C, unlike the others, does that, since in it low quality sellers have not only to find an uninformed buyer, but one that is willing to purchase unidentified goods.

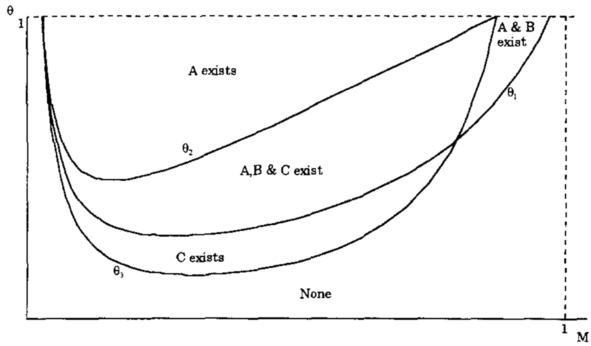

Figure 1. Existence of equilibria in (M, θ) space.

Even for $\theta = 1$, no equilibria exist when M is too close to 0 or to 1. To understand this, consider the expected time spent searching for trading partners for both buyers and sellers. If M is too big, the seller's wait will be too long, so the investment in quality would not be worthwhile. If M is too small, the buyer's wait will be too long, so money would not be very valuable and monetary equilibria would not exist. Also, equilibrium C becomes impossible at a lower critical M than the one that rules out the other equilibria. This is because the harder it is to find a seller, the less likely it is to observe an equilibrium where any buyer declines a trading opportunity. There are values of θ for which the only possible non-degenerate outcome is equilibrium C.

Since there may be multiple equilibria, one may ask which of these equilibria are robust to small perturbations in p (the only true state variable of this economy). Starting in equilibrium I, $I \in \{A, B, C\}$, would an exogenous, one time perturbation in p lead to an permanent departure away from (p_I, Ω_I)? Here, we will say that equilibrium I is *stable* if, given $p = p_I + \varepsilon$, for small enough $|\varepsilon|$ there is a set of strategies $(\omega(p), \pi(p))$ that satisfy the best response conditions and, given those strategies, the economy converges to (p_I, Ω_I). Hence the following proposition:

Proposition 2. The non-degenerate equilibria A and C, and the degenerate equilibrium $(p, \Omega) = (0, 0)$ are stable. Equilibrium B is unstable.

Proof: In the Appendix.

We now also ask how Ω and p depend on parameters in the different equilibria. In equilibria A and B, Ω is fixed, while $\partial\Omega_C/\partial r < 0$ and $\partial\Omega_C/\partial\theta > 0$. The sign of $\partial\Omega_C/\partial M$ is ambiguous; while more money reduces the probability of getting high quality when buying something unknown, it also reduces the premium from holding money rather than a good, and these effects go in different directions. Also, p is fixed in equilibrium A, while $\partial p_B/\partial M > 0$ and $\partial p_C/\partial M < 0$. Nevertheless, the stock of high quality goods in circulation, $Y \equiv p(1 - M)$, is decreasing in M in all three equilibria. The number of producers is less when M is higher, and that effect is enough to compensate the positive effect of M on p in equilibrium B, and of course reinforces the negative effect in equilibrium C. Also, p_B is increasing in r and decreasing in θ while, perhaps surprisingly, p_C is independent of both.

Welfare can be measured from the perspective of a representative agent who does not know his initial endowment: $Z = MV_M + (1-M)pV_H + (1-M)(1-p)V_L$. Solving for Z_J with the value functions from equilibrium I, I = A, B, C, we obtain:

$$Z_A = \frac{M(1-M)(u-\gamma)}{r} \tag{12}$$

$$Z_B = \frac{M(M+r)(1+r-\theta)\gamma(u-\gamma)}{r\theta[Mu-(M+r)\gamma]} \tag{13}$$

$$Z_C = \frac{(1-M)\gamma(u-\gamma)[M(\theta-r)u-r(1-M)\gamma]}{r[Mu+(1-M)\gamma]^2}. \tag{14}$$

An analysis of what determines Z must include two aspects. First, M, θ and the endogenous variable Ω determine how frequently agents trade, by pinning down the frequency of meetings between buyers and sellers and the proportion of those encounters that lead to an exchange. Second, the endogenous variable p determines what proportion of those exchanges involve goods of high quality that generate utility. The equilibria can be Pareto ranked along those lines, since $\Omega_C \leq 1$ and, if both equilibria B and C exist, $p_B \geq p_C$. Hence, the next proposition follows from (12)–(14) and the definitions of θ_1, θ_2 and θ_3.

Proposition 3. If equilibria A and B exist, $Z_B \leq Z_A$. If equilibria B and C exist, $Z_C \leq Z_B$.

Provided that condition (6) is satisfied, so that equilibrium A exists, the optimal money supply is $M = 1/2$, which maximizes the frequency of consumption. Since this equilibrium dominates the other two, $M = 1/2$ is then globally optimal, reminiscent of similar models with full information ($\theta = 1$) like Kiyotaki-Wright (1993). However, there are parameter values where equilibrium A does not exist for $M = 1/2$, only for some lower values of M. Then, Z_A is maximized at the highest M that would still satisfy the existence condition (6). Furthermore, there are parameter values for which the only non-degenerate outcome is equilibrium C. In this case, the best we can do by controlling M is maximize Z_C subject to the existence condition (11). This yields an optimal money supply $M < 1/2$, because $\partial p_C/\partial M < 0$, so the value of M that maximizes welfare is less than the value that maximizes the frequency of meetings between buyers and sellers. In other words, incentives to the production of high quality goods increase the difficulty of selling lemons, and this may increase welfare. The cost of such a policy is a reduction in the volume of transactions and the frequency of consumption.

Figure 2 shows the optimal money supply as a function of θ. It is drawn so that the dashed lines correspond to a level of the money supply M_I that maximizes one of the expressions Z_I in equations (12)–(14), while the solid lines show the level of M that maximizes welfare across the three equilibria. For low values of θ there are no non-degenerate equilibria. As θ increases, equilibrium C is within reach for some values of M, and the optimal money supply is the one that maximizes Z_C conditional on it being a sustainable outcome. For higher values of θ, equilibria A and B become

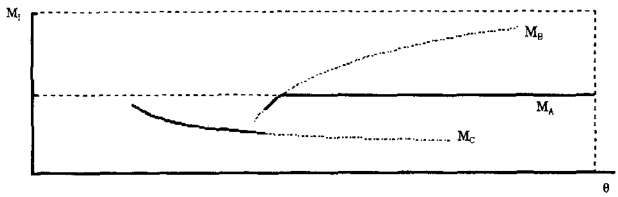

Figure 2. Optimal money supply.

possible for some $M < 1/2$, and above a critical level of θ the former outcome dominates equilibrium C.[7] For still higher values of θ equilibrium A becomes sustainable at $M = 1/2$, which then is globally optimal.

Lastly, we consider the taxation of money holdings, which is meant to proxy for inflation.[8] We will focus now on equilibrium C: the effects on equilibrium A are trivial, and equilibrium B is perhaps uninteresting since it is unstable. Assume that there is a process by which, occasionally, some buyers have their money holdings confiscated by the government, who then uses the proceeds to acquire output from randomly selected sellers. Let η be the rate of arrival of government purchases for the sellers (so the arrival rate of confiscation for buyers is $\eta(1-M)/M$). Then, equilibrium values for p_C and Ω_C can be rewritten as

$$p_C \equiv \frac{\gamma[M\theta - \eta(1-M)]}{M\theta[Mu + (1-M)\gamma]} \tag{9a}$$

$$\Omega_C \equiv \frac{\theta M(1-M)(u-\gamma) - r[Mu + (1-M)\gamma] - \eta[u + (1-M)\gamma]}{M(1-\theta)[Mu + (1-M)\gamma]}. \tag{10a}$$

Notice that both p_C and Ω_C are decreasing in η. In other words, the higher is η, the lower is average quality, but at the same time, the more selective are buyers. The latter occurs because the effect on quality dominates the urge to spend money faster caused by possible confiscation. Welfare Z_C is unambiguously reduced by increases in η.

The precise way in which this policy is introduced matters; it needs to be a tax whose burden increases with the length of time the money is held (again, proxy for inflation). To illustrate this, assume instead that in a fraction τ of monetary transactions, upon agreement of trade, the purchased goods are confiscated from the

[7] Although $Z_A > Z_C$ for the same θ and M, it is still possible that, even when θ allows for equilibrium A to exist, equilibrium C can generate higher welfare for a value of M under which A is not a sustainable outcome. Consequently, the possibility of Pareto ranking the different equilibria is not contradictory with the result shown in Figure 2.

[8] Since there are no prices in this model, inflation cannot be introduced in the usual way as a loss of purchasing power of currency, but must instead be thought of simply as a tax on money holdings. Trejos and Wright (1995) use bargaining theory to endogenize prices in a search theoretic model of money where goods are divisible and hence the terms of trade are not predetermined by nature. That model contains a continuum of inflationary equilibria, but it also assumes perfect information ($\theta = 1$) making the only equilibrium analogous to equilibrium A.

buyer, and transferred to the government. Formally, this is equivalent to reducing the utility of consumption from u to $(1-\tau)u$ in the basic model. Then, p_C is increasing and Ω_C is decreasing in τ. A higher tax on purchases in this case makes the prospect of a purchase less attractive, so in equilibrium buyers are more selective of what they buy and there are more incentives for sellers to produce quality. Indeed, it can even be shown that Z_C is increasing in τ for some parameter values consistent with the existence of equilibrium C. A tax on purchases can increase average quality as well as enhance welfare.

4. Conclusions

This note has presented a simplified version of the model in Williamson and Wright (1994) concerning bilateral exchange in the presence of private information. The simplifying assumptions that rule out barter made the model significantly more tractable. This made it possible to completely characterize the set of equilibria, and to better study how the role of fiat money is changed by qualitative uncertainty. Three non-degenerate equilibria exist and can be Pareto ranked. The optimal money supply may be lower than the value that other search models of money suggest (the value that maximizes the frequency of consumption), precisely because when selling is too easy the extra cost of producing high quality is harder to justify. Consequently, in the choice of the optimal ratio of buyers to sellers there is a trade-off between providing liquidity to facilitate exchange and providing incentives to produce output of high quality. A tax on money holdings reduces the average quality of supply as well as welfare; other forms of taxes, including taxes on purchases, may have the opposite effect.

5. Appendix

We prove now Proposition 2. Notice first that given parameter values that satisfy the existence conditions for equilibrium I, $I \in \{A, B, C\}$, with strict inequality, the expressions in the right hand side of (4) and (5) are continuous on p around $p = p_I$, $\Omega = \Omega_I$. Consequently, given (6), for $p \approx 1$ the strategies $(w(p), \pi(p)) = (1, 1)$ are incentive compatible, and clearly lead the economy to converge asymptotically to equilibrium A, so that it is a stable equilibrium.

Look now at equilibrium B. Define $\Gamma(p) \equiv pu + V_H - \gamma - V_M$. We know that, for parameters that allow the existence of equilibrium B, $\Gamma(p_B + \varepsilon) = f(p_B) + (\dot{V}_H - \dot{V}_M) + o(\varepsilon)$, where $f(p_B) > 0$ and the whole expression is continuous in p. Since the last two terms are continuous in ε, and 0 at $\varepsilon = 0$, necessarily $\Gamma(p_B + \varepsilon) > 0$ for ε small enough, and thus incentive compatibility requires $\omega(p_B + \varepsilon) = 1$. (This is true as long as we only look at strategies where money is believed to be accepted by all sellers always. Other strategies could not possibly work to demonstrate stability, so the results would not be modified by looking at them.) Furthermore, under $\omega(p) = 1$, we know that $V_H - \gamma - V_l$ is continuous and strictly increasing in p, and that $V_H - \gamma - V_L = 0$ under $p = p_B$. Then, if $\varepsilon < 0$, incentive compatibility requires $(\omega(p_B + \varepsilon), \pi(p_B + \varepsilon)) = (1, 0)$, implying divergence away from $p = p_B$. This proves that equilibrium B is unstable.

We focus now on equilibrium C. To show that it is stable, consider for instance the strategies

$$(\hat{\omega}(p), \hat{\pi}(p)) = \begin{cases} (1,0) & \text{if } p > p_C \\ (\Omega_C, p_C) & \text{if } p = p_C \\ (0,1) & \text{if } p < p_C \end{cases}$$

under which p converges to p_C in finite time, where the wait until convergence is denoted $\Delta(\varepsilon)$. Define $\hat{V}_J(p)$ the value function for $J \in \{M, H, L\}$ given p and the strategies $(\hat{\omega}, \hat{\pi})$, while $\lambda(p) = \hat{V}_H(p) - \gamma - \hat{V}_L(p)$ and $\eta(p) = pu + \hat{V}_H(p) - \gamma - \hat{V}_M(p)$. By the nature of the Poisson process, for values of p close enough to p_C, one can approximate the value functions $\hat{V}_J(p)$ by the value V_J at equilibrium C, plus the value of the option of making exactly one match during the wait $\Delta(p - p_C)$, since the probability of making more than one match during that wait, which we can denote $o(\Delta(p - p_C))$, satisfies that $o(\Delta(p - p_C))/\Delta(p - p_C) \to 0$ as $p \to p_C$. Expanding those expressions and substituting into $\lambda(p)$ and $\eta(p)$ above, these take the same values as if deriving them from solving (1)–(3), after substituting $\Omega = \hat{\omega}(p)$ and $p = p_C$. Then, provided $\theta_3 < \theta < \theta_2$, it follows that, for p close enough to p_C, $\eta(p) > 0 > \lambda(p)$ for $p > p_C$ and $\eta(p) < 0 < \lambda(p)$ for $p < p_C$. Hence, the strategies $(\hat{\omega}, \hat{\pi})$ are incentive compatible and consequently equilibrium C is stable.

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