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International Currency

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Abstract

We develop a two-country, two-currency, search-theoretic model of monetary exchange, extending previous such models by endogenizing prices using bargaining theory. We analyze features of the environment that make it more likely that a given money circulates internationally. We show the value of a given currency rises if it circulates abroad, and falls if foreign money circulates locally. Also, we show that international monies have more value at home than abroad. These results help to explain the otherwise anomalous observation that the US dollar is an outlier in the empirical relationship between income and PPP deflators.

KEYWORDS: money, search, international

Introduction

The US dollar is an *international currency* in the sense that it is used for transactions in many foreign countries, including transactions between locals. The same was true of other currencies in the past (see, e.g., Lopez 1951 and Cippolla 1956), and time will tell if it happens in the future with other monies, such as the new Euro. The goal of this paper is to analyze the factors that make it more or less likely that a given currency will circulate abroad, and to study the implications for the values of different monies when it does. To this end we construct a two-country, two-currency, search-theoretic model where money serves as a medium of exchange, as in Matsuyama, Kiyotaki and Matsui (1993), but rather than assuming prices are fixed as in that paper, we determine them here using bargaining theory. Hence, in addition to studying their realms of circulation we can also study the values of the two currencies, and ask how these values depend on which money circulates where.[1]

In terms of results, we show that there can be 3 distinct types of equilibria, where in every case monies circulate locally, and either one, both, or neither circulates internationally. We analyze how the existence of each type of equilibrium depends on parameters. For example, we find international currencies tend to be issued by larger or more efficient economies, and that a higher tax on a given currency (a proxy for inflation, perhaps) reduces the likelihood that it will circulate abroad and increases the likelihood that foreign currency will circulate locally. However, given fundamentals, multiple equilibria often coexist, so whether a money circulates abroad may also depend on beliefs. In terms of some new results that follow from having endogenous prices, we find that the domestic value of a money rises if it circulates abroad and falls if the foreign money circulates locally. Also, we find that an international currency tends to have more value at home than abroad.

One issue we wish to emphasize that the model addresses quite well is the following. There is much discussion of the failure of the purchasing power parity (PPP) hypotheses, and in particular, of the finding that when market

[1] Other papers with two-country search models include Zhou (1997) and Soller-Curtis and Waller (2000), who also assume prices are exogenous. For a review of the general literature on international currencies, and on the topic of currency substitution more generally, see Giovannini and Turtelboom (1992). The bargaining approach used here follows Shi (1995) and Trejos and Wright (1995).

prices are translated to a common currency one tends to find higher prices in richer countries. Some theories can account for the overall relationship between income and the price level, but none can explain why the US is a consistent outlier – i.e., why US prices are much lower than income-price regressions predict.[2] The model here accounts for this observation by predicting that when a currency is used internationally as a medium of exchange its purchasing power at home rises. Intuitively, in an environment with explicit trade frictions, it is liquidity (acceptability) that gives money value, and being liquid abroad enhances a money's purchasing power at home. Hence, the model implies that prices should be lower in a country issuing an international currency than they would be if this money did not circulate abroad.

The General Model

There are two countries, $i = 1, 2$. Each starts with a continuum of agents and both populations grow at the same rate $\gamma \geq 0$. Once born, agents live forever. The fraction of individuals from country i is N_i, with $N_1 + N_2 = 1$. Agents are specialized: for any pair picked at random, regardless of nationality, given that the first agent is able to produce something desired by the second, the probability that the second can also produce something desired by the first is $y \in [0, 1]$. Thus, y relates to the frequency of a double coincidence of wants. If an agent produces q units of output for a consumer of that good, the latter enjoys utility $u(q)$ while the former suffers disutility $c(q)$. We assume $u'(q) > 0$, $c'(q) > 0$, $u''(q) < 0$ and $c''(q) \geq$ for all $q > 0$. Also, $u(0) = 0$, $c(0) \geq 0$, $u'(0) = \infty$, $c'(0) = 0$ and there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q})$. Consumption goods are nonstorable (so they cannot be used as commodity money).

Agents meet bilaterally according to an anonymous matching process described by four Poisson parameters, α_{ij}, $i, j = 1, 2$, where α_{ij} is the rate at which an individual from country i meets a producer of his consumption good from country j. One can interpret α_{ii} as a measure of the size or efficiency of economic activity of country i, and α_{ij} as the degree of economic integration between countries. We assume $\alpha_{ii} \geq \alpha_{ji}$, which means, e.g., that it is at least as easy for a Mexican buyer to meet Mexican sellers as it is for an American

[2]Some of the best known articles on the correlation between national income levels and the pattern of deviations from purchasing power parity are Balassa (1964), Samuelson (1964), Kravis and Lipsey (1983), and Bhagwati (1984). For a comprehensive discussion, see Rogoff (1996).

buyer to meet Mexican sellers. The matching technology in Matsuyama et al. (1993) is a special case of this specification.

With anonymous matching all trade must be quid pro quo (i.e., there is no credit). Hence, agents who do not barter directly must use some form of fiat money, but we do not impose *which* money is used in which transactions – that is endogenous. The government of country j issues one unit of currency j to some fraction $M_j \in (0, 1)$ of its newborn citizens at each point in time. We assume that an agent with money spends it all at once (for instance, because it is indivisible), and cannot acquire a second unit before spending what he has (for instance, because one cannot produce twice in a row without consuming). This implies that the population can always be partitioned into a group of *buyers* each with one unit of money, and a group of *sellers* with no money. Let the fraction of buyers from country i with currency j be m_{ij}, and let $m_{i0} = 1 - m_{i1} - m_{i2}$ be the fraction of agents from country i with no money.[3]

Two types of trades are possible, direct barter and monetary exchange. When an individual without money meets a producer of his consumption good without money they can barter if there is double coincidence of wants between them, which occurs with probability y. If an individual with money meets a producer of his consumption good without money, they choose if they want to enter a bargaining process or alternatively continue searching for other potential trading partners. For reasons that will be important later, we assume that first the seller and then the buyer decides if they want to bargain. We also assume agents choose to bargain iff they assign a positive probability to the event that agreement will occur, which can be interpreted as a consequence of an infinitesimal but positive transaction cost to bargaining.

When a buyer from country j with money k meets a seller from country i who produces the right good, let $\sigma^j_{ik} = 1$ if the seller chooses to bargain and 0 otherwise, and let $\beta^j_{ik} = 1$ if the buyer chooses to bargain and 0 otherwise. When both parties agree they enter a bargaining game. As is well known, various bargaining games generate outcomes that can be represented by the

[3] The assumptions that make agents hold either one or zero units of money make things much simpler than some search models (e.g. Green and Zhou 1998 or Molico 1996) where solving for the distribution of money holdings is complex even with one country and one currency. Of course, these assumptions have a cost: it is hard to interpret M_i as a money supply parameter. Since we do not analyze the impact of changes M_i on any variables here, this is less of an issue than in some papers.

Nash solution with particular bargaining powers and threat points, so here we adopt the Nash representation without explicit reference to the underlying game. Given a buyer from country j with money k and a seller from country i agree to trade, we call q^j_{ik} the amount of output that the latter produces for the former´s money, so the nominal price is $p^j_{ik} = 1/q^j_{ik}$. Notice prices can vary both with the nationalities of the two parties and with the currency. We also use bargaining when agents barter, and in this case we adopt the symmetric Nash solution, which can be shown to imply they exchange q^* where $u'(q^*) = c'(q^*)$.

We distinguish different regimes, or types of equilibria, by specifying whether there are 0, 1, or 2 international currencies. To be precise, define a money to be a national currency if it is only used in transactions where the buyer, seller and money all come form the same country (e.g., pesos are a national currency if they only change hands when both the buyer and seller are Mexican), and define a money to be international if it is used in every possible kind of transaction (e.g., dollars are an international currency if we observe both American and Mexican buyers using dollars to trade with both American and Mexican sellers). Thus, currency j is an international currency when $\beta^h_{kj}\sigma^h_{kj} = 1\ \forall h, k$ and a national currency when $\beta^j_{jj}\sigma^j_{jj} = 1$ and $\beta^j_{ij}\sigma^j_{ij} = 0$ for $j \neq i$.[4]

Let V_{ij} be the value function for a buyer from country i with currency j and V_{i0} the value function for a seller from country i. If r is the discount rate, the value functions satisfy standard dynamic programming equations:

$$\begin{aligned}
rV_{ii} &= \alpha_{ii}m_{i0}\beta^i_{ii}\sigma^i_{ii}[u(q^i_{ii}) + V_{i0} - V_{ii}] + \alpha_{ij}m_{j0}\beta^i_{ji}\sigma^i_{ji}[u(q^i_{ji}) + V_{i0} - V_{ii}] \\
rV_{ij} &= \alpha_{ii}m_{i0}\beta^i_{ij}\sigma^i_{ij}[u(q^i_{ij}) + V_{i0} - V_{ij}] + \alpha_{ij}m_{j0}\beta^i_{jj}\sigma^i_{jj}[u(q^i_{jj}) + V_{i0} - V_{ij}] \\
rV_{i0} &= \alpha_{ii}m_{i0}y[u(q^*) - c(q^*)] + \alpha_{ij}m_{j0}y[u(q^*) - c(q^*)] \qquad (1) \\
&\quad + \alpha_{ii}m_{ii}\beta^i_{ii}\sigma^i_{ii}[V_{ii} - V_{i0} - c(q^i_{ii})] + \alpha_{ii}m_{ij}\beta^i_{ij}\sigma^i_{ij}[V_{ij} - V_{i0} - c(q^i_{ij})] \\
&\quad + \alpha_{ij}m_{ji}\beta^j_{ii}\sigma^j_{ii}[V_{ii} - V_{i0} - c(q^j_{ii})] + \alpha_{ij}m_{jj}\beta^j_{ij}\sigma^j_{ij}[V_{ij} - V_{i0} - c(q^j_{ij})]
\end{aligned}$$

For example, the first equation in (1) sets the flow payoff for a buyer from country i holding money i, rV_{ii}, equal to the sum of two terms. The first term is the rate at which he meets sellers from his own country, $\alpha_{ii}m_{i0}$, times the probability both want to trade, $\beta^i_{ii}\sigma^i_{ii}$, times his surplus from trading,

[4] Other regimes are possible, including cases where one money does not circulate at all or circulates abroad but not at home. We focus on these regimes since they seem the most interesting, and they are the analogues of those emphasized by Matsuyama et al.

consuming and switching from buyer to seller, $u(q^i_{ii}) + V_{i0} - V_{ii}$. The second term is the rate at which he meets foreign sellers, times the probability both want to trade, times the surplus.

If a seller from country h and a buyer from country i with money j both choose to bargain, the outcome is given by the generalized Nash solution,

$$q^i_{hj} = \arg\max_q \, [-c(q) + V_{hj} - V_{h0}]^\theta [u(q) + V_{i0} - V_{ij}]^{1-\theta} \tag{2}$$

where θ is the bargaining power of the seller. Of course, agents will not bargain if there is no surplus to be had. As the most a buyer can get from a seller is the q that satisfies $c(q) = V_{hj} - V_{h0}$, they bargain iff the utility of this q exceeds $V_{ij} - V_{i0}$:

$$\beta^i_{hj} = \sigma^i_{hj} = 1 \Longleftrightarrow u[c^{-1}(V_{hj} - V_{h0})] > V_{ij} - V_{i0}. \tag{3}$$

We now discuss the determination of m_{ij}. Standard reasoning yields

$$\begin{aligned} \dot{m}_{ii} &= \alpha_{ij} m_{i0} m_{ji} \beta^j_{ii} \sigma^j_{ii} - \alpha_{ij} m_{ii} m_{j0} \beta^i_{ji} \sigma^i_{ji} + \gamma(M_i - m_{ii}) = 0 \\ \dot{m}_{ij} &= \alpha_{ij} m_{i0} m_{jj} \beta^j_{ij} \sigma^j_{ij} - \alpha_{ij} m_{ij} m_{j0} \beta^i_{jj} \sigma^i_{jj} - \gamma m_{ij} = 0 \end{aligned} \tag{4}$$

as the steady state conditions.[5] One can easily solve for m_{ij} in each regime. In the regime with two national and no international monies, $m_{ii} = M_i$ and $m_{12} = m_{21} = 0$. In the regime with two international monies,

$$m_{10} = 1 - \tfrac{(\gamma+\alpha_{21})M_1 + \alpha_{12} M_2}{\gamma+\alpha_{12}+\alpha_{21}}, \; m_{11} = \tfrac{\gamma(\gamma+\alpha_{12}+\alpha_{21}) + \alpha_{21}[(\gamma+\alpha_{21})(1-M_1)+\alpha_{12}(1-M_2)]}{(\gamma+\alpha_{12}+\alpha_{21})[\gamma+\alpha_{21}(1-M_1)+\alpha_{12}(1-M_2)]}$$

for citizens of country 1, while for citizens of country 2 we simply reverse the subscripts. And for the regime where money 2 is international and money 1 national, we have $m_{11} = M_1$, $m_{21} = 0$,

$$m_{12} = \tfrac{\alpha_{12}(1-M_1)M_2}{\gamma+\alpha_{12}+\alpha_{21}(1-M_1)}, \; m_{22} = \tfrac{(\gamma+\alpha_{12})M_2}{\gamma+\alpha_{12}+\alpha_{21}(1-M_1)}.$$

[5] To see where this comes from, first note that trades between a buyer and seller of the same nationality cannot alter m_{ij}, since the two agents simply switch states and leave the aggregate distribution unchanged. Now consider the first equation in (4). The first term says that m_{ii} increases when a seller from country i meets a buyer from country j with currency i and they trade. The second term says that m_{ii} decreases when a buyer from country i with currency i meets a seller from country j and they trade. The final term says that m_{ii} increases when the fraction of the newborn in country i who receive currency from their government, M_i, exceeds m_{ii}.

We now define equilibrium. Let $q = (q_{ij})$, $V = (V_{ij})$, $\beta = (\beta_{ij}^k)$, $\sigma = (\sigma_{ij}^k)$ and $m = (m_{ij})$. Since (3) implies we do not have to keep track of both β and σ, an equilibrium can be defined as a list (q, V, β, m) satisfying (1)-(4). The method for analyzing equilibria is to choose a regime (that is, a value for β), solve for V, q and m, and then determine if (3) is satisfied given parameter values.

Deriving analytical results is difficult in this general version of the model. In the next section we provide a set of simplifying assumptions under which the model can be solved analytically, but we first discuss some numerical results. For this purpose, we set $u(q) = \sqrt{q}$ and $c(q) = q$, and choose parameter values so that country 1 is small and open compared to country 2. Figure 1 shows the regions of $(\alpha_{11}, \alpha_{12})$ space where each of the regimes is an equilibrium for a typical specification; for example, if (n, i) appears in a region then for those parameters there is an equilibrium where the currency 1 is national and currency 2 international. One salient feature is the possibility of multiplicity: for many parameters more than one equilibrium exists, and for some parameters all equilibria exist.[6] However, fundamentals are also important: it is not the case that all equilibria always exist, and for some parameter values the equilibrium is unique.

In terms of when the different regimes arise, the figure – which is typical of the examples we studied – shows that the equilibrium with two national monies and no international money exists as long as the economies are not too open (that is, α_{12} and α_{21} are low) and not too different in terms of local arrival rates (that is, α_{11} and α_{22} are similar). The intuition is that, even if one expects agents to only trade using their local money, one may find foreign money valuable if one has an opportunity to trade with foreigners often enough, or if foreign prices are low enough because the foreign market is very efficient. Hence, we require α_{12} and α_{21} to be relatively low and α_{11} and α_{22} to be similar for this regime to be an equilibrium.

The equilibrium with two international currencies exists as long as the differences in meeting rates among the two countries are not too big. The intuition is that, if a money is to be international, it must move back and forth between the two countries, which requires that the price levels across countries are not so different that one would prefer to wait for a chance

[6] The multiplicity of equilibria is not confined to multiple regimes: although it cannot be seen in the figure, within a given regime there may be more than one (q, V) satisfying all of the equilibrium conditions.

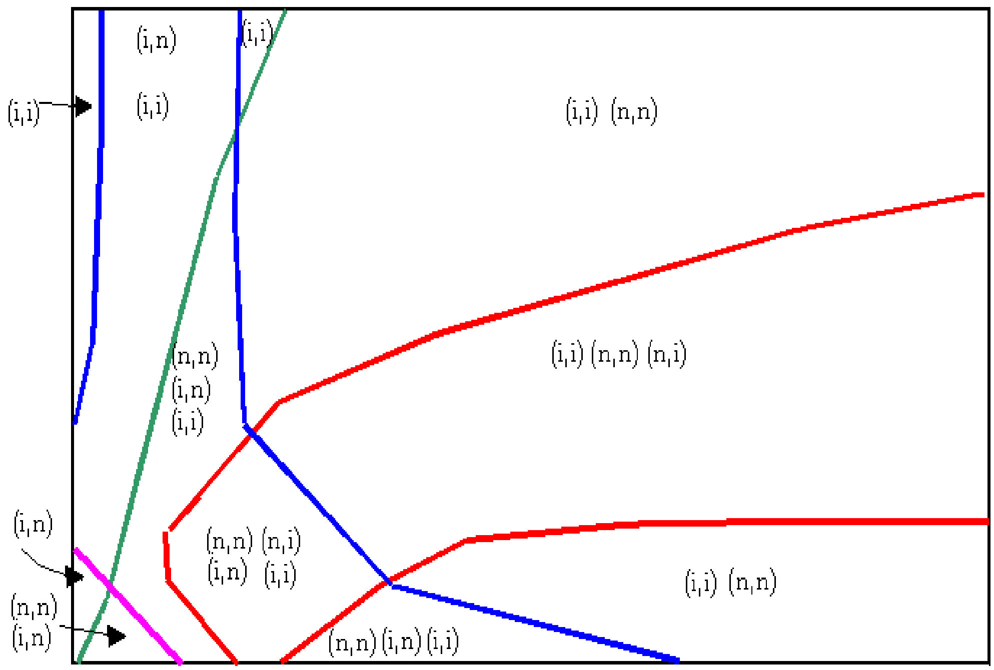

Figure 1: Areas where different regimes are equilibria

to spend money in the low-price country rather than spend it right away in the high-price country. Also note that in this regime the equilibrium often involves the monies being perfect substitutes (prices vary with the nationalities of the traders but not with the type of the money). For some parameters, however, there are equilibria with two international monies that are not perfect substitutes.

Consider now the regime with one national and one international money. As the figure shows, one condition for this equilibrium to exist is that the economy absorbing foreign money cannot be too closed, and that the economy not using foreign money can be neither too open nor too efficient (too cheap) relative to the other. For a currency to be international, the purchasing power it commands at home must not differ too much the one it has abroad, or else the money will only flow in one direction.

In terms of the values of the monies, for typical parameter values we found that across regimes, a money has higher value if it circulates internationally than if it does not, and a money has lower value if the other circulates internationally. Also, within a regime, an international currency tends to purchase more at home than abroad. Within the regime with one national and one international money, the former tends to buy more than the latter – although this last result relies on identifying a single unit of each currency as the basis for comparison (note that the other findings do not). In the next section we will be able to give some analytic results, at the cost of specializing some aspects of the specification.

A Simplified Model

In this section, we normalize $c(q) = q$, and also impose three assumptions that make the model very tractable. First, we set $\theta = 0$, which means that buyers hold all the bargaining power against sellers, and make take-it-or-leave-it offers. Second, we set $y = 0$, which means double coincidence of wants is impossible, and there are no barter transactions. Third, we assume that sellers cannot recognize the nationality of a buyer, but only the nationality of the currency the buyer carries, so $\sigma^1_{ik} = \sigma^2_{ik} \equiv \sigma_{ik}$.

The first two assumptions make the model simple enough to solve analytically. Notice that under $c(q) = q$ and $\theta = 0$, it follows that $q^h_{ji} = V_{ji} - V_{j0} \equiv q_{ji}$. Also, with no barter opportunities and no surplus in monetary transactions, (1) implies $V_{i0} = 0$, and thus

$$q_{ij} = V_{ij}. \tag{5}$$

Then (1) simplifies to

$$\begin{aligned} rq_{ii} &= \alpha_{ii} m_{i0} \beta^i_{ii} \sigma^i_{ii} [u(q_{ii}) - q_{ii}] + \alpha_{ij} m_{j0} \beta^i_{ji} \sigma^i_{ji} [u(q_{ji}) - q_{ii}] \\ rq_{ij} &= \alpha_{ii} m_{i0} \beta^i_{ij} \sigma^i_{ij} [u(q_{ij}) - q_{ij}] + \alpha_{ij} m_{j0} \beta^i_{jj} \sigma^i_{jj} [u(q_{jj}) - q_{ij}], \end{aligned} \tag{6}$$

and (3) simplifies to

$$\beta^i_{hj} = 1 \Leftrightarrow u(q_{hj}) \geq q_{ij}. \tag{7}$$

These two assumptions make the model much more tractable. However, we have the problem that some of the regimes that have been analyzed so far may not be equilibria under $y = \theta = 0$ for any combination of the other parameters. To be precise, the regimes with national currencies analyzed in the previous section cannot be subgame perfect Nash equilibria if $V_{i0} = 0$. That is the reason why we need the third simplifying assumption.

Why can´t equilibria with national currencies exist when $V_{i0} = 0$? Suppose, for instance, that we want dollars to be an exclusively national currency (e.g., to circulate in the US but not in Mexico). This must be because Americans expect that Mexicans do not value dollars very much, and so an American with a dollar prefers waiting to meet a fellow American rather than trading with a Mexican, who gives very little for dollars. But why would Mexicans not value dollars very much? It must be because in this regime they believe that other Mexicans do not value dollars very much either, and so they themselves must wait until they meet an American to be able to spend dollars at an acceptable price. But such a belief cannot be an equilibrium belief, since the less valuable is a dollar to a Mexican buyer the more willing he is to trade, and the seller is always willing to trade if he holds no bargaining power in a monetary exchange, and no outside value in waiting for a barter opportunity. It would only be if $V_{i0} > 0$ that Mexican sellers could prefer to pass on the dollars and wait for a better trade.[7]

This is not a problem under the third assumption, that sellers cannot recognize a buyer´s nationality. If the seller must make the same decision whenever he encounters any buyer with a given money ($\sigma^1_{ik} = \sigma^2_{ik} \equiv \sigma_{ik}$), the equilibrium condition for σ is no longer (3), but rather

$$\sigma_{ik} = 1 \Leftrightarrow \beta^1_{ik} m_{1k} + \beta^2_{ik} m_{2k} > 0. \tag{8}$$

This says that a seller is willing to bargain iff he assigns positive probability to the event that the buyer will want to do so.[8] Then, the regimes with

[7] To illustrate the problem more formally, consider the regime where both monies are national. Currency 1 does not circulate in country 2, so $\beta^1_{11} = 1$ and $\beta^1_{21} = 0$. By (7), this implies $u(q_{11}) \geq u(q_{21})$, so $q_{11} \geq q_{21}$, and given the concavity of $u(q)$, we conclude $u(q_{12}) > q_{12}$ and hence $\beta^2_{21} = 1$. This makes it difficult that $q_{11} > u(q_{21})$, as required by $\beta^1_{21} = 0$.

For citizens of country 1, (6) becomes

$$\begin{aligned} rq_{11} &= \alpha_{11} m_{10}[u(q_{11}) - q_{11}] \\ rq_{12} &= \alpha_{12} m_{20}[u(q_{22}) - q_{12}] + \alpha_{11} m_{10}[u(q_{12}) - q_{12}]. \end{aligned}$$

These equations imply that $q_{12} > q_{11}$, and as $q_{22} > q_{12}$, we conclude that $q_{22} > q_{11}$. However, by symmetry, the same argument regarding the fact that money 2 does not circulate in country 1 would imply $q_{11} > q_{22}$. This is a contradiction, so there cannot be an equilibrium with two national monies and no international currency.

[8] Recall that the seller decides first, and chooses to bargain iff he believes an agreement will be reached with positive probability. If $\beta^1_{ik} m_{1k} + \beta^2_{ik} m_{2k} = 0$ then there are no buyers with money k who want to bargain with him, so his best response is $\sigma_{ik} = 0$.

national currencies may constitute subgame perfect Nash equilibriua, even if $V_{i0} = 0$. This allows us to tap on the tractability brought by $y = \theta = 0$, while still being able to make cross-regime comparisons as in the last section.

Inserting $\sigma^j_{ik} \equiv \sigma_{ik}$ into (6), we have

$$\begin{aligned} rq_{ii} &= \alpha_{ii} m_{i0} \beta^i_{ii} \sigma_{ii} [u(q_{ii}) - q_{ii}] + \alpha_{ij} m_{j0} \beta^i_{ji} \sigma_{ji} [u(q_{ji}) - q_{ii}] \\ rq_{ij} &= \alpha_{ii} m_{i0} \beta^i_{ij} \sigma_{ij} [u(q_{ij}) - q_{ij}] + \alpha_{ij} m_{j0} \beta^i_{jj} \sigma_{jj} [u(q_{jj}) - q_{ij}]. \end{aligned} \tag{9}$$

An equilibrium for the modified version of the model can now be defined as a list (q, β, σ, m) satisfying (4), (7), (8) and (9). In the rest of this section, we consider in turn each of the three types of equilibria and discuss the conditions under which they exist, as well as some of their properties.

Consider first the regime with no international money, $\sigma_{12} = \sigma_{21} = 0$ and $\sigma_{11} = \sigma_{22} = \beta^1_{11} = \beta^2_{22} = 1$ (we do not need to specify the other values of β^i_{jk}). Equilibrium requires

$$u(q_{ii}) \geq q_{ii} \geq u(q_{ji}), \tag{10}$$

for $i = 1, 2$, $i \neq j$, which says that country i buyers with money i spend it on country i sellers, but not on country j sellers. In this case, (9) becomes

$$\begin{aligned} rq_{ii} &= \alpha_{ii} m_{i0} [u(q_{ii}) - q_{ii}] \\ rq_{ij} &= \alpha_{ij} m_{i0} [u(q_{jj}) - q_{ij}] \end{aligned} \tag{11}$$

which implies unique values for q_{ii} and q_{ij}.[9] Given this, the first inequality in (10) holds for all parameter values, while the second reduces to

$$u \left[\frac{\alpha_{ji}}{\alpha_{ii}} \, \frac{r + \alpha_{ii}(1 - M_i)}{r + \alpha_{ji}(1 - M_i)} \, q_{ii} \right] \leq q_{ii},$$

which holds iff α_{ji} is small compared to α_{ii}. Intuitively, under the belief that foreign money does not circulate at home, foreigners hold on to their money rather spend it on locals iff locals interact sufficiently infrequently with foreigners, which means the economies must be relatively isolated.

[9] Note that the second equation in (11) is different from the second equation in (6), which was derived without the assumption that sellers cannot identify a buyer's nationality. In that model, a local seller and a local buyer with foreign money necessarily trade. Here they do not because the seller believes that the buyer is a foreigner who will not trade, and hence he does not even enter negotiations. That is, we still have $\beta^i_{ij} = 1$, but now $\sigma_{ij} = 0$.

Consider now the regime with two international currencies. Existence requires $u(q_{ij}) \geq V_{kj} \quad \forall i, j, k$. Before proceeding to say when this equilibrium exists, we first claim that now we must have $q_{ii} = q_{ij} = Q_i$; that is, the two monies are perfect substitutes in that they purchase the same amount from a given seller. To prove this, first note in this regime (9) implies the subsystem

$$\begin{aligned} rq_{11} &= \alpha_{11}m_{10}[u(q_{11}) - q_{11}] + \alpha_{12}m_{20}[u(q_{21}) - q_{11}] \\ rq_{21} &= \alpha_{21}m_{10}[u(q_{11}) - q_{21}] + \alpha_{22}m_{20}[u(q_{21}) - q_{21}], \end{aligned} \tag{12}$$

which can be solved for q_{11} and q_{21} independently of q_{12} and q_{22}. It is easy to show that (12) has a unique solution $(q_{11}, q_{21}) = (Q_1, Q_2)$. Similarly, (9) also implies a subsystem that can be solved for q_{12} and q_{22} independent of q_{11} and q_{21}. Inspection reveals that it is identical to the above system, and so $(q_{12}, q_{22}) = (Q_1, Q_2)$.

Since q_{ij} does not depend on which currency the buyer is using, if there were a market in which agents can trade the two monies the market clearing price would be 1. But one can also compute the PPP exchange rate, given by $e = Q_1/Q_2$.[10] In general e differs from unity because, even though the terms of trade do not depend on which currency the buyer is using, they do depend on the nationality of the seller. In any case, we do not pursue the implications of changes in parameters (say, M_i) on exchange rates in this paper.

To see when this regime constitutes an equilibrium, we need to check the conditions $u(Q_1) \geq Q_2$ and $u(Q_2) \geq Q_1$ (the other conditions hold for all parameter values). If the countries are symmetric, in the sense that $M_1 = M_2$, $\alpha_{ii} = \alpha_{jj}$ and $\alpha_{ij} = \alpha_{ji}$, it follows that $Q_1 = Q_2$ and the relevant conditions hold with strict inequalities. By continuity these conditions hold and the equilibrium with two international currencies exists as long as the two countries are not too different. However, if the countries are sufficiently dissimilar then the relevant conditions will be violated, and having two international currencies cannot be an equilibrium. In the close-to-symmetric case, with

[10] Again, this is not a market exchange rate, but the rate that would be implied by PPP. Thus, suppose 1 peso buys q_{11} units of output in Mexico and 1 dollar buys q_{22} units of output in America, and suppose as a thought experiment that there is a market in which currencies can be traded at the nominal exchange rate e (i.e., one peso buys e dollars), even though no such market actually exists in the model. Then one peso could be used to buy eq_{22} units of output in America using dollars. Purchasing power parity holds if a peso buys the same amount directly at home and using dollars abroad: $q_{11} = eq_{22}$.

α_{ij} small, this equilibrium necessarily coexists with the equilibrium that has two national currencies.

Finally, consider the regime where citizens of country 1 accept money 2 but not vice-versa. We require $u(q_{12}) \geq q_{12}, q_{22}$ (so that all buyers spend money 2 on country 1 sellers); $u(q_{22}) \geq q_{12}, q_{22}$ (so that all buyers spend money 2 on country 2 sellers); $u(q_{11}) \geq q_{11}$(so that country 1 buyers spend money 1 on country 1 sellers); and $u(q_{21}) < q_{11}$ (so that country 1 buyers do not spend money 1 on country 2 sellers). The equations in (9) can be written

$$\begin{aligned} rq_{11} &= \alpha_{11}m_{10}[u(q_{11}) - q_{11}] \\ rq_{21} &= \alpha_{21}m_{10}[u(q_{11}) - q_{21}] \\ rq_{12} &= \alpha_{11}m_{10}[u(q_{12}) - q_{12}] + \alpha_{12}m_{20}[u(q_{22}) - q_{12}] \\ rq_{22} &= \alpha_{21}m_{10}[u(q_{12}) - q_{22}] + \alpha_{22}m_{20}[u(q_{22}) - q_{22}]. \end{aligned} \tag{13}$$

The first and second determine q_{11} and q_{21} independent of q_{12} and q_{22}. In fact, these two equations are qualitatively identical to the equations in (11) for q_{ii} and q_{ij} in the regime with two national monies. However, since m_{10} is lower in this regime, q_{11} and q_{21} are lower in this regime than when both monies are national. Intuitively, the influx of foreign money inflates prices denominated in the domestic currency.

Now the third and fourth equations in (13) determine the purchasing power of the international currency, q_{12} and q_{22}, independent of q_{11} and q_{21}. In fact, these two equations are qualitatively identical to the equations in (12) from the regime with two international monies, and the same analysis implies that there exists a unique solution. Moreover, $q_{12} > q_{11}$. Hence, citizens of country 1 value the international currency more than their own domestic currency. In general, q_{12} can be greater or less than q_{22} depending on M_i and α_{ij}; in the close-to-symmetric case, however, one can show unambiguously that $q_{22} > q_{12}$. Hence, *the same amount* of international currency tends to purchase more at home than abroad. The model therefore predicts that a country that issues an international currency will tend to be an outlier in the relationship between purchasing power and its fundamental determinants, because local prices are relatively low. As discussed in the introduction, this is what one sees in the data.

We still need to check when the equilibrium with one national and one international money exists. It turns out that the binding conditions are $u(q_{22}) \geq V_{12}$, $u(q_{12}) \geq V_{22}$ and $u(q_{21}) < V_{11}$. Each of these conditions looks qualitatively like one that has been encountered earlier and holds under similar circumstances: we need country 2 to be isolated enough for money

1 to be national, and the two countries similar enough for money 2 to be international, consistent with the numerical findings in the previous section for the more general model.

An Inflation-Like Tax

In this section, following Li (1995), we introduce a tax on currency holdings as a proxy for inflation. The motivation is that it has been observed in many countries that the use of foreign money can be triggered by an episode of inflation in the local money. We then consider the equilibrium with two national monies, and ask if differences in our inflation proxy can preclude the existence of such an equilibrium, by either forcing locals to want to trade the foreign currency or to abandon the national currency. Our taxation scheme is a simple but hopefully reasonable way to get at this issue without actually having inflation (which would be very complicated in this context).

Assume that at rate π_i a buyer with money i has his money taxed away by the government, so that he becomes a seller without consuming. Also, with arrival rate π_i^*, a country i seller gets a chance to sell to the same government at the going market price. If we set $\pi_i^* = \pi_i M_i/(1-M_i)$ the per capita supply of money i does not change over time. One can think of π_i as a proxy for the money i inflation rate in several senses. First, it is a tax on money holders, and, unlike a tax on purchases, the expected cost of the tax is proportional to the time the money is held. Also, the size of the tax is proportional to the monetary authorities' purchases with new money. And, finally, we shall see that the tax erodes the real value of a given nominal amount of money.

We analyzed this model numerically (note that we set $\theta = 1/2$, so this is *not* a version of the simple model in the previous section with take-it-or-leave it offers, and hence it is analytically difficult to analyze). Rather than giving details, we simply report the flavor of the results from all the examples we studied. First, high π_1 – say, high Mexican inflation – always reduces the pesos' purchasing power. This reduces the value of holding pesos for Americans, and increases the value of holding dollars for Mexicans. Hence, a higher π_1 increases the potential gains from trade between an American buyer and a Mexican seller, and thus it is more likely that dollars begin to circulate in Mexico. In fact, at a high enough value of π_1, only equilibria in which dollars circulate in Mexico exist, and if π_1 is even higher, pesos do not circulate, not even in country Mexico.

To the extent to which π_j is a proxy for inflation, this kind of a model therefore can be used to articulate the observed relationships between inflation and the degree of dollarization or currency substitution. Due to the multiplicity of equilibria, it can also be used to account for the hysteresis apparently observed in the determination of monetary regimes, since a high inflation may instigate a transition away from the regime with two national currencies, but inflation is not necessary to keep an economy in the new regime once it has emerged. Thus, continuing peso inflation is not necessary to maintain the foreign circulation of dollars, because the very belief that dollars circulate in Mexico has an important effect on whether they do.

Conclusion

We extended the two-country, two-currency, model in Matsuyama et al. (1993) by endogenizing prices. This gives us a framework for addressing a variety of issues relating to international currencies in a new light. Some findings are that, other things being equal, a currency has a higher purchasing power if it circulates internationally than if it does not; the value of a local currency is diminished by the influx of a foreign currency; and an international money purchases more at home than abroad. As discussed in the introduction, these results are consistent with the fact that US prices are much lower than income-price regressions predict.

Although the model is obviously abstract and contains many simplifying assumptions, we think that it also has some virtues. Namely, it allows one to determine of the realms of circulation of the different monies as well as their values endogenously. Moreover, the basic economic ideas underlying the key results seem robust. In particular, when the value of a money is determined at least in part by its liquidity, or acceptability, an international currency will have higher value because it is more liquid. Although this may seem very intuitive, to make the point in the context of a formal model it is evident that one would like both acceptability and prices to be endogenous, as is the case here.

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