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Economic Theory

Limited participation, private money, and credit in a spatial model of money*

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Summary. The purpose of this paper is to explore the implications of private money issue for the effects of monetary policy, for optimal policy, and for the role of fiat money. A locational model is constructed which gives an explicit account of the role for money and credit, and for limited financial market participation. When private money issue is prohibited, there is a liquidity effect as the result of a money injection from the central bank, but this effect goes away when private money is permitted. Private money issue changes dramatically the nature of optimal monetary policy. With private money, fiat currency is no longer used in transactions involving goods, but currency and central bank reserves play an important part in the clearing and settlement of private money returned for redemption.

Keywords and Phrases: Money, Credit, Limited participation.

JEL Classification Numbers: E4, E5.

1 Introduction

Technological advances and the relaxation of financial restrictions have opened up new opportunities for the issue of substitutes for government-issued fiat money. If financial intermediaries can issue private monies, what implications does this have for the effects of monetary policy, for optimal policy, and for the role of fiat money? To explore these issues, it is necessary to work with a model which is

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explicit about the role money plays in facilitating exchange, and in which monetary policy matters. The model constructed here, while building on the work of others, provides a novel treatment of the role of money, financial intermediation, and credit, and also constructs a more explicit foundation for the limited-participation financial frictions which can yield liquidity effects.

Monetary arrangements with privately-issued monies were common historically. In particular, in the United States before the Civil War and in Canada prior to 1935, much of the stock of currency in circulation was issued by private banks (see Gorton, 1996; Smith and Weber, 1999; Williamson, 1989). An important feature of these particular historical private money systems was that there was no central banking; the United States did not have a central bank until 1914, and Canada's central bank was established in 1935. There is virtually no historical experience to inform us about the operating characteristics of a modern central bank in the context of a monetary system with private money issue. As we will show here, the existence of privately-issued money will have important implications for the effects of central bank actions, and for the optimal behavior of a central bank. Given the current potential for private money issue, along with the increasing use of substitutes for fiat currency (such as credit and debit cards) in transactions, it is important to understand these implications.

The model constructed here is related in spirit to the locational monetary models studied by Townsend (1980). As well, it has some features in common with models in Temzelides and Williamson (2001), and Williamson (2002). In the model studied here there are wholesale and retail transactions, with some wholesale and retail transactions carried out with credit cleared through private financial intermediaries and the central bank, and some transactions requiring either fiat currency or circulating private money. The model contains two key frictions. First, one role of private financial intermediaries in the model, somewhat related to elements of the Diamond-Dybvig (1983) model or that of Champ, Smith and Williamson (1996), is to insure retail buyers against the need to make transactions when credit is not available. Intermediaries fulfil this insurance role by being able to supply retail buyers with cash at the appropriate times. If private intermediaries cannot issue money, then their ability to provide this insurance is limited. Second, private intermediaries are constrained because they cannot ship currency instantaneously to the central bank for deposit in the intermediary's reserve account with the central bank. This limits the intermediary's ability to use the currency it acquires to settle IOUs through the central bank.

The basic locational model we work with has, under some conditions, operating characteristics which are much like those of the limited participation models first developed by Grossman and Weiss (1983) and Rotemberg (1984), and later refined by Lucas (1990) and Fuerst (1992). A key feature of limited participation models is the *liquidity effect.* If there is an unanticipated increase in the stock of outside money, this alters the distribution of wealth and results in a short-run decrease in the nominal rate of interest.

In limited participation models, money is typically held to satisfy a cash-in-advance constraint. In the model here, locational frictions give rise to cash-in-advance type constraints, but these constraints are endogenous, and are affected in important ways by financial restrictions and by monetary policy, as we will show. Thus, cash-in-advance constraints, as they are typically used, are not immune to the Lucas critique (see Lucas, 1975), and that is an important point of this paper.

Using our model, we first examine a regime where private money is prohibited. Here, the nominal interest rate will in general be positive and retail buyers will pay a premium if they purchase goods with currency. An unanticipated injection of outside money redistributes wealth, alters the distribution of consumption across agents, and reduces the nominal interest rate - there is a liquidity effect. An optimal allocation can be achieved as a competitive equilibrium through the operation of a discount window by the central bank, and with an appropriate money growth rate that pegs the nominal interest rate at zero and eliminates the premium on retail cash purchases.

Next, we examine a regime where private money issue is permitted. Private money effectively undoes the liquidity effect in that it gives private financial intermediaries a device for distributing the effects of a central bank monetary injection across groups of agents making cash and credit transactions. Unanticipated money injections have no effect on the nominal interest rate and there is no distributional effect. Given private money issue, an optimal monetary arrangement is very different than in the case where private money is prohibited.

Private money issue in the model changes the roles of fiat currency and central bank reserves in important ways. As one might expect, private money displaces fiat currency in retail and wholesale transactions; in fact fiat currency will not be used to purchase goods once private money is permitted. This is because private money has the advantage of being "elastic," in that its quantity can respond to unanticipated shocks in ways that the stock of fiat currency cannot. However, currency and bank reserves are still important in a private money system, as they are needed in the clearing and settlement of private monies returned for redemption.

Previous work on models of private money includes Cavalcanti, Erosa, and Temzelides (1999), Cavalcanti and Wallace (1999), Williamson (1999), and Temzelides and Williamson (2001). Some interesting work which looks at the coexistence of government-issued money and private money is Azariadis, Bullard, and Smith (2001) and Bullard and Smith (2002). As with our model, Freeman (1996) considers an environment where credit arrangements support exchange and outside money is necessary for settlement to take place.

The model constructed here can be viewed as being consistent with the thrust of Green (1998), where it is argued that an explicit treatment of payments arrangements and credit is required to carry out sensible analyses of monetary policy. In fact, Green used standard cash-in-advance models of cash and "credit" transactions as an example of how policy analysis should not be done. Our model is a richer environment than standard cash-in-advance setups, but it nevertheless delivers cash-in-advance type constraints.

The remainder of the paper is organized as follows. In Section 2 the model is constructed, while Sections 3 and 4 contain analyses of the cases where private money is and is not prohibited, respectively. Section 5 is a conclusion.

2 The model

The environment has spatially separated locations, each inhabited by a single representative household. Locations are indexed by (i, j), where $i = -\infty, ..., -1, 0, 1, ..., \infty$, and $j = -\infty, ..., -1, 0, 1, ..., \infty$. The representative household at location (i, j) has a continuum of shoppers with unit mass, two wholesale sellers, a retail seller, a financial transactor, and a financial intermediary. During any period, a shopper from a household has one of two itineraries, which we call *itinerary 1* and *itinerary 2*. In general, these different locational itineraries imply that there will be two groups of shoppers using different means of payment and will also imply that consumption will in general be different for itinerary 1 and itinerary 2 shoppers. Let π_t denote the fraction of shoppers of the representative household at each location who follow itinerary 1, and $1-\pi_t$ the fraction who follow itinerary 2, where π_t is a random variable and $0 < \pi_t < 1$. At the beginning of period t, π_t is unknown and shoppers do not know what itinerary they will follow. For a given shopper, the probability of following itinerary 1 is π_t, conditional on π_t. Each household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t u(c_{1t}) + (1 - \pi_t) u(c_{2t})], \tag{1}$$

where $0 < \beta < 1$ and c_{it} denotes the consumption of a shopper from the household who follows itinerary i. Here, $u(\cdot)$ is twice continuously differentiable and strictly concave, with $u'(0) = \infty$.

At the beginning of each period, the household receives an endowment of y units of the perishable consumption good. Members of household (i, j) consume goods endowed to households $(k, j + 2)$, where $k = -\infty, ..., -1, 0, 1, ..., \infty$. In addition to its endowment, at the beginning of period t the household has M_t units of outside money, held as currency and reserve balances with the central bank, and B_t one-period nominal bonds maturing in period t, each of which is a claim to one unit of central bank reserves. There is a single central bank for the economy, and transactions between the central bank and households are carried out by the financial intermediaries in the households. After the household's endowment is received, the financial intermediary must decide how to divide the current money balances of the household between reserve balances held with the central bank and fiat currency. That is, the financial intermediary either makes a deposit of currency in its reserve account with the central bank, or withdraws currency. Then, agents learn π_t, following which itinerary 1 shoppers from household (i, j) travel to location $(i + 1, j + 1)$ to purchase goods, and itinerary 2 shoppers travel from location (i, j) to location $(i, j+1)$. While this is taking place, two wholesale sellers from each location travel to sell goods. Wholesale seller 1 takes $\gamma_t y$ consumption goods from location (i, j) to location $(i - 1, j - 1)$, and wholesale seller 2 takes $(1 - \gamma_t)y$ consumption goods to location $(i, j - 1)$. Here, γ_t is a random variable, with $0 < \gamma_t < 1$, that becomes known at the same time as does π_t. Goods are sold wholesale in different locations because consumption goods need a location-specific costless input in order to be consumable. In period t, a fraction γ_t of the

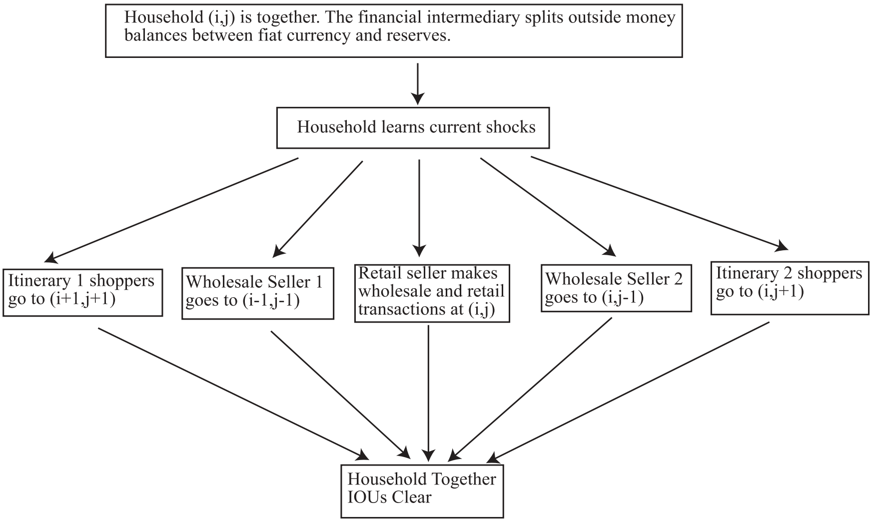

Figure 1 Timing within the period

endowment received at location (i, j) requires this input at location $(i - 1, j - 1)$ and the remaining fraction requires the input at location $(i, j - 1)$.

Once the shoppers from the household have left to purchase goods, the financial intermediary receives a lump sum transfer from the central bank, in the form of a change in the household's reserve account balance with the central bank. Then, the financial intermediary can make a withdrawal of currency from its reserve account. The financial intermediary is permitted to run an overdraft on its reserve account with the central bank during the period, but reserve balances cannot be negative at the end of the period. Next, at location (i, j) wholesale goods sellers arrive, sell their goods to the retail seller, and then leave for their home locations. Shoppers then arrive, purchase goods from the retail seller, and then return home. Following this there is communication, in a limited way, among households. In particular, central bank reserves can be transferred among some financial intermediaries in some locations. That is, the financial intermediaries in location (i, j) and location (k, l) can make transfers between their central bank reserve accounts if and only if $i = k$.

A key assumption is that currency acquired during the period cannot be deposited in the financial intermediary's central bank reserve account until the beginning of the following period. Thus, the receipts from the household's sales of retail and wholesale goods for currency cannot be used to finance the purchase of other goods within the period. This will be a critical constraint which will in general imply a positive nominal interest rate.

Currency cannot be moved between locations except by shoppers and by wholesale sellers. As well, goods cannot be transported across locations except by wholesale sellers, and must be consumed at the location of purchase by shoppers. Figure 1 shows the timing of events within the period.

Each household is endowed with $\overline{M}_0$ units of outside money at the beginning of period 0, and the money supply grows according to

$$\overline{M}_{t+1} = \theta_t \overline{M}_t, \tag{2}$$

for $t = 0, 1, 2, ...$, where θ_t is the gross money growth rate and $\overline{M}_t$ is the supply of money at the beginning of period t. The nominal lump-sum transfer received by the financial intermediary during period t is then

$$\Upsilon_t = (\theta_t - 1)\overline{M}_t. \tag{3}$$

Here, θ_t becomes known at the same time as π_t and γ_t. Assume that $(\pi_t, \gamma_t, \theta_t)$ follows a first-order Markov process.

3 A prohibition on circulating private liabilities

We will first consider how this economy functions when there is a prohibition on private liabilities that change hands, which we interpret as private monies. Thus, the kinds of transactions that are permitted in this economy are exchanges of consumption goods for fiat currency, and exchanges of consumption goods for IOUs which are then settled at the end of the period through net transfers of reserves. These IOUs can be interpreted as checks or as debit card transactions.

At the beginning of a given period t, the financial intermediary holds the household's assets, which consist of M_t units of outside money, held as central bank reserves and currency, and B_t units of one-period nominal bonds. Each nominal bond issued in period t is a promise to pay one unit of reserves at the beginning of period $t+1$. Before the household knows π_t, γ_t, and θ_t, the financial intermediary must decide how much outside money to hold as currency, and how much to hold as reserves. The financial intermediary makes this adjustment either by depositing currency with the central bank or withdrawing it. After this, the financial intermediary is able to make a cash withdrawal from the central bank only after itinerary 1 agents have departed to make retail purchases. The financial intermediary cannot make a cash deposit in its reserve account until the beginning of the following period. Given the restrictions on private liability issue, itinerary 1 shoppers, who travel from location (i, j) to location $(i+1, j+1)$ require currency in order to purchase goods, as locations (i, j) and $(i+1, j+1)$ cannot exchange reserves, and so cannot clear IOUs. Thus, letting M_{1t} denote the quantity of currency allocated to itinerary 1 shoppers, and M_{2t} the quantity of reserves allocated for wholesale goods transactions and financial transactions, the household must satisfy the constraint

$$M_{1t} + M_{2t} \leq M_t + B_t. \tag{4}$$

Itinerary 1 shoppers face the cash-in-advance constraint

$$P_{1t}\pi_t c_{1t} \leq M_{1t}, \tag{5}$$

where P_{1t} denotes the price in period t of consumption goods in terms of money in a cash transaction. Throughout we will be confining attention to symmetric competitive equilibria where prices are identical across locations.

As we will see, the friction that leads to constraint (5) is what produces a liquidity effect from a money injection by the central bank. The liquidity effect arises because the household is not able to allocate outside money between itinerary 1 and itinerary 2 shoppers after it learns the current shock $(\pi_t, \gamma_t, \theta_t)$. Thus, if a money injection occurs, this causes a redistribution of wealth between itinerary 1 and itinerary 2 shoppers, and there will be implications for asset prices, as discussed in the next subsection.

Itinerary 2 shoppers, who travel from location (i, j) to location $(i, j + 1)$, purchase consumption goods using within-period IOUs, which are promises to pay outside money when households can transfer reserves. Thus, when the transactions of itinerary 2 shoppers, wholesale transactions, and financial transactions clear, the financial intermediary has available receipts from wholesale sales of consumption goods through wholesale seller 2 (who travelled from location (i, j) to location $(i, j - 1)$), money balances M_{2t}, the money transfer from the central bank Υ_t, and receipts from retail sales of consumption goods to itinerary 2 shoppers. Note that the household does not have available the receipts from sales by wholesale seller 1, who travelled from location (i, j) to location $(i - 1, j - 1)$, and does not have available to spend the receipts of retail sales to itinerary 1 shoppers, who travelled to location (i, j) from location $(i - 1, j - 1)$. These receipts are in fiat currency, which cannot be transferred to other households to settle transactions at this point. Thus, the financial intermediary faces the constraint

$$P_{2t}(1 - \pi_t)c_{2t} + q_t B_{t+1} + W_t x_t \leq W_t(1 - \gamma_t)y + M_{2t} + \Upsilon_t + P_{2t} z_t. \quad (6)$$

In constraint (6), P_{2t} denotes the price of consumption goods purchased with IOUs, q_t is the price in units of money of a nominal bond which pays off in period $t + 1$, W_t is the price in units of money of consumption goods traded on the wholesale market, x_t is the quantity of goods purchased wholesale by the household, and z_t is the quantity of goods sold by the household to itinerary 2 shoppers. In purchasing wholesale goods, the retail seller will be indifferent between buying these goods with currency and buying with IOUs. This is because the financial intermediary can make a withdrawal from its reserve account when the current wholesale prices are known, and can thus withdraw just enough cash to make its planned wholesale goods cash purchases. On the other side of wholesale goods transactions, the wholesale goods seller arriving at location (i, j) from location $(i + 1, j + 1)$ must exchange goods for currency, and the wholesale goods buyer arriving at (i, j) from $(i, j + 1)$ exchanges goods for IOUs. Thus, given strictly positive supplies of wholesale goods sold for currency and wholesale goods sold for IOUs, and given that these goods are effectively perfect substitutes on the demand side, in equilibrium all wholesale goods must sell at the same price W_t.

At the end of the period, the financial intermediary must satisfy the budget constraint

$$\begin{aligned} &P_{1t}\pi_t c_{1t} + P_{2t}(1 - \pi_t)c_{2t} + M_{t+1} + q_t B_{t+1} + W_t x_t \\ &\leq W_t y + M_t + B_t + \Upsilon_t + P_{1t}(x_t - z_t) + P_{2t} z_t \end{aligned} \quad (7)$$

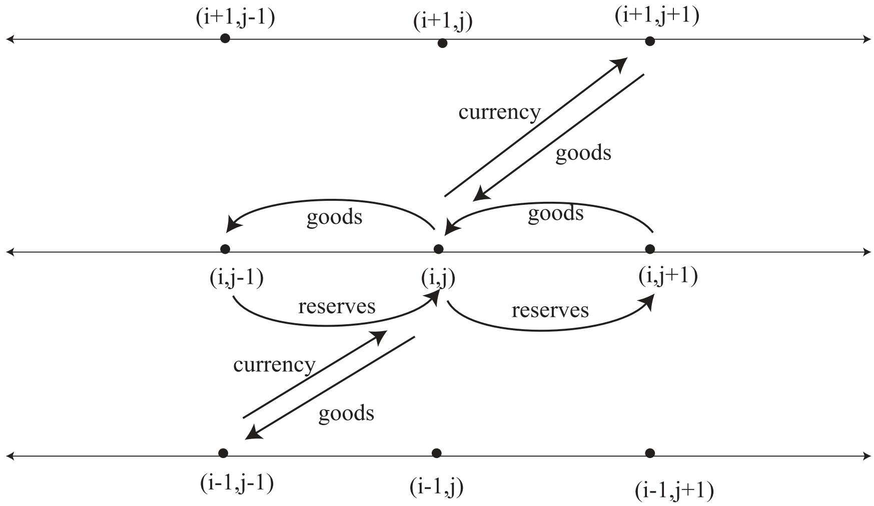

Figure 2 Transactions of household (i, j) when private money is prohibited

In constraint (7), M_{t+1} denotes the money balances that the financial intermediary takes into period $t + 1$. Figure 2 shows the patterns of exchange that take place given a prohibition on circulating private liabilities.

It will prove useful to write the constraints (4)–(7) in scaled form. That is, letting lower case letters denote the ratio of the appropriate nominal variable to the current money supply, dropping time subscripts, e.g. $p_1 \equiv \frac{P_{1t}}{M_t}$, and using primes to denote variables dated $t + 1$, we can rewrite (4)–(7) respectively, using (2) as

$$m_1 + m_2 \leq m + b \tag{8}$$

$$p_1 \pi c_1 \leq m_1 \tag{9}$$

$$p_2(1 - \pi)c_2 + q\theta b' + wx \leq w(1 - \gamma)y + m_2 + \tau + p_2 z \tag{10}$$

$$\begin{aligned} &p_1 \pi c_1 + p_2(1 - \pi)c_2 + \theta m' + q\theta b' \\ &\qquad + wx \leq wy + m + b + \tau + p_1(x - z) + p_2 z \end{aligned} \tag{11}$$

Letting "–" superscripts denote variables dated $t - 1$, $v(m, b, \pi^-, \gamma^-, \theta^-)$ will be the value function associated with the representative household's optimization problem. Then, the household's problem can be represented as

$$\begin{aligned} &v(m, b, \pi^-, \gamma^-, \theta^-) \\ &= \max_{m_1, m_2} E\left(\max_{c_1, c_2, x, z, m', b'} \{\pi u(c_1) + (1 - \pi)u(c_2) + E\left[\beta v(m', b', \pi, \gamma, \theta)\right]\} \right) \end{aligned} \tag{12}$$

subject to (8)–(11). In the optimization problem on the right-hand side of the Bellman equation (12), the inner expectation is conditional on (π, γ, θ), while the outer expectation is conditional on $(\pi^-, \gamma^-, \theta^-)$.

Now, let λ_1, λ_2, λ_3, and λ_4 denote the multipliers associated with constraints (8)–(11), respectively. In the symmetric equilibrium we study here, it must be the case that $m_1 > 0, m_2 > 0, x > 0$, and $0 < z < x$, which imply, respectively, from (12) subject to (8)–(11), that

$$-\lambda_1 + E[\lambda_2 \mid \pi^-, \gamma^-, \theta^-] = 0, \tag{13}$$

$$-\lambda_1 + E[\lambda_3 \mid \pi^-, \gamma^-, \theta^-] = 0, \tag{14}$$

$$-w(\lambda_3 + \lambda_4) + p_1\lambda_4 = 0, \tag{15}$$

$$p_2\lambda_3 + (p_2 - p_1)\lambda_4 = 0. \tag{16}$$

Assuming that the value function is differentiable and strictly concave in m and b, the first-order conditions for an optimum from (12) subject to (8)–(11) are

$$u'(c_1) - p_1(\lambda_2 + \lambda_4) = 0, \tag{17}$$

$$u'(c_2) - p_2(\lambda_3 + \lambda_4) = 0, \tag{18}$$

$$-q\theta(\lambda_3 + \lambda_4) + \beta E[v_2(m', b', \pi, \gamma, \theta) \mid \pi, \gamma, \theta] = 0, \tag{19}$$

$$-\theta\lambda_4 + \beta E[v_1(m', b', \pi, \gamma, \theta) \mid \pi, \gamma, \theta] = 0. \tag{20}$$

Next, from (12) subject to (8)–(11), envelope conditions are

$$v_1(m, b, \pi^-, \gamma^-, \theta^-) = v_2(m, b, \pi^-, \gamma^-, \theta^-) = E[\lambda_1 + \lambda_4 \mid \pi^-, \gamma^-, \theta^-]. \tag{21}$$

Finally, in a symmetric equilibrium, the quantity of goods sold on the wholesale market is equal to the household's endowment,

$$x = y, \tag{22}$$

the quantity of goods sold to itinerary 2 shoppers by the household is equal to the quantity of goods purchased by the household's itinerary 2 shoppers,

$$z = (1 - \pi)c_2, \tag{23}$$

the demand for nominal bonds is equal to the zero net supply of nominal bonds,

$$b = b' = 0, \tag{24}$$

the demand for money is equal to the supply of money,

$$m = m' = 1, \tag{25}$$

and the demand for final goods is equal to the supply,

$$\pi c_1 + (1 - \pi)c_2 = y. \tag{26}$$

Now, (15) and (16) imply that

$$w = p_2, \tag{27}$$

that is the wholesale price of goods is equal to the price of retail goods sold to itinerary 2 shoppers. As well, (15) and (16) give

$$\lambda_3 + \lambda_4 = \frac{p_1}{p_2}\lambda_4, \tag{28}$$

so that $\lambda_3 \geq 0$ implies that $p_1 \geq p_2$, and if constraint (10) binds then $p_1 > p_2$ and the price of retail goods to itinerary 1 shoppers, who buy with cash, is greater than the price charged to itinerary 2 shoppers, who make purchases with IOUs. This premium on cash retail sales arises because the proceeds from retail sales to itinerary 2 shoppers can be spent within the current period, while retail sales to itinerary 1 shoppers must be held as cash until the next period. From (19), (20), and (21) we have

$$q = \frac{p_2}{p_1}, \tag{29}$$

so that $q < 1$ and the nominal interest rate is positive if and only if constraint (10) binds. Further, (28) implies that (18) can be rewritten as

$$u'(c_2) = p_1 \lambda_4,$$

so that, if (10) binds so $\lambda_2 > 0$, then from (17) we have $c_1 < c_2$ and the competitive equilibrium allocation must be suboptimal, since the Pareto optimum has $c_1 = c_2 = y$.

There are two key frictions here for households. First, in general the household would like to have the flexibility to reallocate outside money between the "cash market" and the "credit market" after the current shocks are learned. However, this cannot be done, as currency cannot be transferred to or from the itinerary 1 shoppers after they have left their home location, as reflected in the cash-in-advance constraint (5). Basically, after they have left their home location, shoppers in the cash market do not have the ability to communicate with their own financial intermediary, either directly or indirectly through the central bank. This certainly seems a realistic friction to assume, since in practice there are circumstances where consumers wish to make transactions when they are not able to communicate with a financial intermediary in order to arrange credit. It is this friction which will produce the liquidity effect that we will examine in the next subsection.

Second, financial intermediaries cannot instantaneously ship currency to the central bank for deposit in the intermediary's reserve account. Any currency acquired during the period must be held over until the beginning of the next period. This is reflected in the finance constraint (6). Again, this is a realistic assumption, since in practice banks are constrained by the physical necessity of shipping currency to the nearest central bank branch in order to make deposits of currency in their reserve accounts. This friction is important, in that implies that the nominal interest rate is positive, in general.

The liquidity effect – an example

Given a prohibition on circulating private liabilities, there exists a liquidity effect, much like what holds in Grossman and Weiss (1983), Rotemberg (1984), or Lucas (1990). The household must allocate money balances to itinerary 1 shoppers before θ_t is known, and the household has no opportunity to reallocate money balances across the household after the money injection occurs. Just as in the limited participation literature, there is a distribution effect from monetary policy actions.

We will show the nature of the liquidity effect in this version of the model by way of example. Suppose that $\gamma_t = \gamma$ and $\pi_t = \pi$ for all t, and that θ_t is an i.i.d. random variable. For simplicity, also suppose that the distribution of θ_t is such that constraints (9) and (10) bind in all states of the world. This then implies that (8) binds, from (13) and (14). Since λ_2 and λ_3 are i.i.d. random variables in the equilibrium we study, therefore from (13) and (14) λ_1 is a constant. Then, from (20) and (21) we have, since λ_4 is an i.i.d. random variable,

$$\lambda_4 = \frac{\beta\psi}{\theta}, \tag{30}$$

where ψ is a constant. Given that (9) binds, we have

$$p_1 \pi c_1 = m_1, \tag{31}$$

where m_1 is a constant. Further, from (8), (10), and (22)–(27), we have

$$p_2 \gamma y = \theta - m_1, \tag{32}$$

that is in equilibrium there is a binding cash-in-advance constraint for the wholesale purchase of goods from wholesale seller 1. From (18), (28), and (30), we have

$$u'(c_2) - \frac{p_1 \beta\psi}{\theta} = 0. \tag{33}$$

Then, (26) and (31)–(33) solve for c_1, c_2, p_1, and p_2. It is straightforward to show that, when θ increases, c_2, p_1, and p_2 increase, and c_1 decreases. Thus, an unanticipated cash injection by the central bank increases prices, and this results in a reduction in the consumption of itinerary 1 shoppers and an increase in the consumption of itinerary 2 shoppers, as the itinerary 1 shoppers receive none of the newly-injected money. From (26) and (31)–(33), comparative statics gives

$$\frac{dq}{d\theta} = \frac{-\theta\,(c_1)^2\, u''(c_2) + m_1 u'(c_2) c_1 (1-\pi)}{\gamma y m_1 \theta[-c_1 u''(c_2) + (1-\pi) u'(c_2)]} > 0$$

so that the nominal interest rate falls when θ rises - a liquidity effect.

The liquidity effect arises from the friction which prevents the household from reallocating outside money between itinerary 1 and itinerary 2 shoppers after the current money growth shock becomes known. This friction is reflected in the cash-in-advance constraint (5). When a money injection occurs, it is only received by itinerary 2 shoppers, who consume more as a result and force up retail prices (retail sellers arbitrage between credit sales and sales for currency). The increase in the price of goods reduces the real balances of itinerary 1 shoppers, and they consume less. A money injection also relaxes the finance constraint (6), which causes the nominal interest rate to fall. If it could, the intermediary would like to borrow against receipts from cash sales of wholesale and retail goods in order to finance cash purchases of wholesale goods; the level of the nominal interest rate reflects the tightness of the finance constraint and so the nominal interest rate falls when there is a positive money injection.

Optimal monetary policy with liquidity demand shocks in the wholesale and retail markets

In this section, we want to ask whether we can find a monetary policy arrangement under which an optimal allocation can be achieved as a competitive equilibrium, in the general case where (π_t, γ_t) is some arbitrary first-order Markov process. Clearly, we cannot improve on an allocation where $c_{1t} = c_{2t} = y$ for all t. This allocation would be the one chosen by a social planner seeking to maximize the expected utility of the representative household, and with the power to confiscate endowments and move them across locations.

There are in principle many alternative monetary policy arrangements that implement the optimal allocation, some involving Friedman rules for money growth. One arrangement that works is the following. First, the central bank operates a discount window, offering zero-nominal-interest-rate one-period loans which are taken out at the time when the central bank makes money transfers to households, and are repaid at the beginning of the following period. This relaxes the finance constraint (10), which we can then eliminate. Further, constraint (8) becomes

$$m_1 \leq m + b \tag{34}$$

and the household's problem then is (12) subject to (9), (11), and (34).

In equilibrium we will then have $w = p_1 = p_2 = p$ and $q = 1$. That is, there is no premium paid on purchases of goods with cash, and the nominal interest rate is zero. Arbitrage and the first-order conditions from the household's optimization problem give

$$\lambda_1 = E[\lambda_2 \mid \pi^-, \gamma^-, \theta^-] \tag{35}$$

$$u'(c_1) - p(\lambda_2 + \lambda_3) = 0, \tag{36}$$

$$u'(c_2) - p\lambda_3 = 0, \tag{37}$$

$$\theta\lambda_4 = \beta E[\lambda_1' + \lambda_3' \mid \pi, \gamma, \theta], \tag{38}$$

where λ_1, λ_2, and λ_3 are the multipliers associated with constraints (34), (9), and (11), respectively. Now, to implement an optimal allocation as a competitive equilibrium, we must have $c_1 = c_2 = y$, which from (36) and (37) implies $\lambda_2 = 0$. Then, (35) gives $\lambda_1 = 0$. One equilibrium is then $m_1 = 1$ and $p = \frac{1}{\pi y}$, which from (38) implies that the optimal money growth factor is

$$\theta = \beta E\left[\frac{\pi'}{\pi}\right]. \tag{39}$$

There are two elements of the optimal monetary policy that correct for the two frictions we discussed in the previous subsection. First, the zero-nominal-interest rate discount window policy relaxes the financial intermediary's finance constraint, which arises because the financial intermediary cannot instantaneously make use of cash on hand to clear IOUs through the central bank. Second, the central bank manipulates the money supply growth rate so that the cash-in-advance constraint of itinerary 1 shoppers does not bind. That is, from (39), when there is a relatively large number of shoppers who need to make purchases with currency, money growth is relatively low, implying that prices are relatively low and the real quantity of currency held by itinerary 1 shoppers is relatively high. The modification to the standard Friedman rule (where the standard rule would be $\theta = \beta$) in Equation (39) results from the fact that, at the optimum, the central bank must attain the correct distribution of money balances between itinerary 1 and itinerary 2 shoppers.

4 Competitive equilibrium with private money

Now, suppose that the prohibition on private circulating liabilities is removed. In particular, we now allow households to issue private money, which is a liability that can be transferred during the period, with each unit of private money being a promise to pay one unit of outside money at the end of the period. Now, instead of issuing fiat currency to itinerary 1 shoppers, the financial intermediary can provide them with private money, and can determine the quantity of this private money to issue after learning (π_t, γ_t). The retail seller can also purchase goods from wholesale seller 1 using private money rather than fiat currency.

The retail seller and wholesale seller 1 in household (i, j) will receive some private money in exchange for goods, and this private money will have been issued by the household at location $(i-1, j-1)$. The financial transactor will then take this private money to location $(i-1, j)$ (recall the restrictions on how the financial transactor can move) and exchange it for fiat currency. Reserve balances cannot be exchanged between household (i, j) and household $(i-1, j-1)$, but they can be exchanged between household $(i-1, j)$ and household $(i-1, j-1)$. A financial intermediary that exchanges fiat currency for private money then redeems the private money for reserves through communication with the financial intermediary that issued the private money. Figure 3 shows the patterns of exchange in this case.

This arrangement shares some of the features of historically-observed private money systems, for example the Suffolk banking system in New England prior to the Civil War, and the Canadian banking system prior to 1935. In both of these systems, circulating liabilities issued by private banks were a key medium of exchange, and private money issued by one bank in the system was redeemed by other banks in the system, and then cleared through a centralized clearing mechanism. An important difference between these historical private money systems and the one in our model is that, historically, private monies were typically redeemable in commodity money (either gold or silver) rather than in outside money. Note that central banking did not exist in New England before the Civil War, or in Canada prior to 1935, when these countries had private money systems (see Champ, Smith and Williamson, 1996; Smith and Weber, 1999; Williamson, 1989).

Given the use of private money in our model, it will still be the case that retail sales of goods will take place at different prices when the goods are purchased with cash (private money) than when they are purchased with credit. If goods are sold by the household in exchange for private money, then the household will ultimately redeem the private money for fiat currency, which then must be held until the beginning of the next period. Thus, shoppers will in general pay a premium to purchase goods with private money rather than credit.

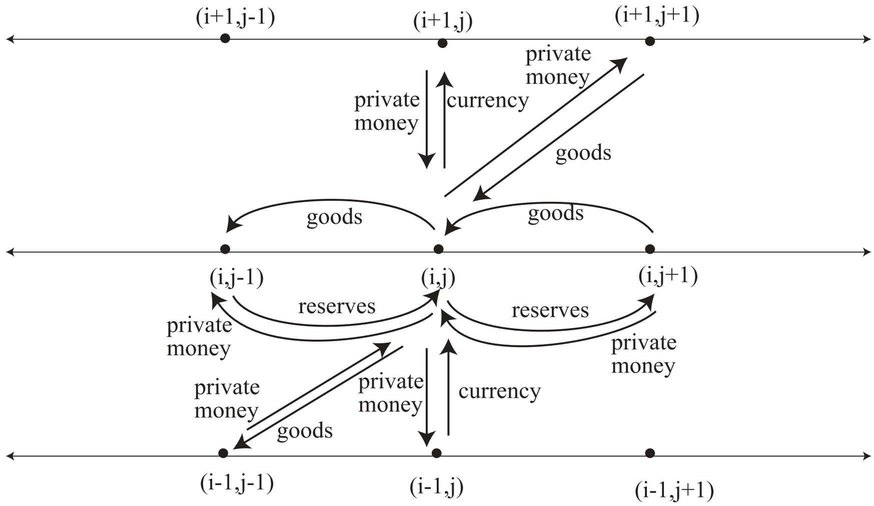

Figure 3 Transactions of household (i, j) when private money is permitted

The first modification we need to make to our approach in Section 3 is to replace constraints (8)–(10) with

$$p_1 \pi c_1 + p_2(1-\pi)c_2 + q\theta b' + wx \leq w(1-\gamma)y + m + b + \tau + p_2 z. \qquad (40)$$

Constraint (40) reflects the key effects of permitting the issue of private money. Since the household can issue private money, it need not make a commitment to the cash holdings of itinerary 1 shoppers before learning what the current aggregate shocks are. The distribution of cash between itinerary 1 and itinerary 2 shoppers is determined by how much private money the household wishes to issue, contingent on the shocks. However, the household is still not able to spend its cash receipts within the period, as indicated by constraint (40). This is because any private money the household receives in exchange for wholesale and retail goods is then redeemed for currency, which must be held until the next period. Note also that the financial intermediary can hold all of its outside money holdings as reserves at the beginning of the period, and then can make a withdrawal of fiat currency from the central bank at the time when financial transactors wish to exchange private money for fiat currency.

The effect of allowing private money issue in this model is an important example in which a cash-in-advance constraint cannot be treated as invariant to policy. In this model, cash-in-advance type constraints arise endogenously; when private money issue is allowed, there is one of these constraints (constraint (40)) rather than two (constraints (9) and (10)).

Letting $v^*(m, b, \pi, \gamma, \theta)$ denote the value function for the household in this case, we can write the household's dynamic programming problem as

$$v^*(m, b, \pi, \gamma, \theta) \qquad (41)$$
$$= \max_{c_1, c_2, x, z, m', b'} \{\pi u(c_1) + (1-\pi)u(c_2) + \beta E[v^*(m', b', \pi', \gamma', \theta')]\}$$

subject to (40) and (11). Let μ_1 and μ_2 denote the multipliers associated with constraints (40) and (11), respectively. Analogous to (13)–(16), the following arbitrage conditions hold,

$$-w(\mu_1 + \mu_2) + p_1\mu_2 = 0, \tag{42}$$
$$p_2\mu_1 + (p_2 - p_1)\mu_2 = 0, \tag{43}$$

and from the first-order conditions and envelope conditions from (42), we obtain

$$u'(c_1) - (\mu_1 + \mu_2)p_1 = 0, \tag{44}$$
$$u'(c_2) - (\mu_1 + \mu_2)p_2 = 0, \tag{45}$$
$$-q\theta(\mu_1 + \mu_2) + \beta E[\mu_1' + \mu_2'] = 0, \tag{46}$$
$$-\theta\mu_2 + \beta E[\mu_1' + \mu_2'] = 0. \tag{47}$$

As above, the equilibrium conditions are (22)–(26).

This then implies that (27) holds, just as in Section 3, and the analogue of (28) is

$$\mu_1 + \mu_2 = \frac{p_1}{p_2}\mu_2. \tag{48}$$

Note, as before, that (46) and (47) imply (29). Thus, if and only if (40) binds, the nominal interest rate is strictly positive, shoppers purchasing goods with private money pay a premium, and itinerary 1 and itinerary 2 shoppers consume different amounts.

Private money eliminates a friction, in that the household can effectively distribute money between shoppers in the "cash market" and shoppers in the "credit market." However, it still remains the case that financial intermediaries are constrained by their inability to instantaneously ship currency to the central bank for deposit in the household's reserve account. This remaining friction yields the possibility of an inefficient competitive equilibrium with a positive nominal interest rate and a premium in purchases with private money.

The absence of a liquidity effect

In this section we will show that private money serves to undo the liquidity effect that exists when private money is prohibited. We use the same example as in Section 3, where $\pi_t = \pi$ for all t and $\gamma_t = \gamma$, with θ_t an i.i.d. random variable. Then, from (42)–(47) and (22)–(26), assuming (40) is binding, c_1, c_2, p_1, and p_2 are the solution to

$$u'(c_1) - \frac{(p_1)^2\beta\Gamma}{p_2\theta} = 0, \tag{49}$$
$$u'(c_2) - \frac{p_1\beta\Gamma}{\theta} = 0, \tag{50}$$
$$\pi c_1 + (1 - \pi)c_2 = y, \tag{51}$$
$$p_1\pi c_1 + p_2\gamma y = \theta. \tag{52}$$

Here,

$$\Gamma = E[\mu_1' + \mu_2'],$$

which is a constant, since μ_1 and μ_2 are i.i.d. random variables in the equilibrium we examine.

As a result, from (49)–(52) c_1 and c_2 are invariant to θ, and p_1 and p_2 are proportional to θ. This then implies that θ has no effect on q, since $q = \frac{p_2}{p_1}$. Thus, variability in the money growth rate in this example does not imply any variability in consumption allocations, relative prices, or the nominal interest rate. There is therefore no liquidity effect here with private money. Effectively, the fact that financial intermediaries can issue private money integrates markets in such a way that there is no distributional effect from a monetary injection by the central bank. That is, the ability of the household to issue private money eliminates the cash-in-advance constraint faced by itinerary 1 households, so that the household can effectively distribute money balances between itinerary 1 and itinerary 2 households as it chooses, contingent on current aggregate shocks. It was the inability of the household to redistribute money among household members in this fashion that gave rise to the liquidity effect in the absence of private money issue.

However, the fact that the liquidity effect disappears here does not imply that there is no role for outside money or for monetary policy. Fiat currency and central bank reserves are critical elements in the mechanism by which private money is redeemed and cleared through private intermediaries and the central bank. As well, monetary policy matters for the equilibrium allocation. We will examine optimal monetary policy in the context of a private money regime in the next subsection.

Optimal monetary policy

How does optimal monetary policy differ when private money is permitted? Again, suppose that (π_t, γ_t) is a first-order Markov process. Given this, we characterize two policies that support the optimal allocation is a competitive equilibrium.

As in Section 3, an optimal allocation is one where $c_{1t} = c_{2t} = y$ for all t. To support this allocation as a competitive equilibrium implies, from (44) and (45), that $p_1 = p_2$, and so from (48) we have $\mu_1 = 0$ and constraint (40) does not bind. Further, since $p_1 = p_2 = p$ we have $q = \frac{p_2}{p_1} = 1$, and the nominal interest rate must be zero at the optimum. Thus, if an optimal money growth rule exists that supports the allocation $c_{1t} = c_{2t} = y$ for all t, it must be a Friedman rule.

As mentioned previously, there are typically many ways to support an optimal allocation in monetary models, and we will consider only two optimal policies here. The first optimal policy is directly comparable to the optimal policy considered when we studied the regime with a prohibition on private money issue. That is, suppose as in Section 3 that the central bank operates a discount window, lending at a zero nominal interest rate at the point in time during the period when transfers are made from the central bank to households, with repayment on discount window loans occurring at the beginning of the following period. This will then imply that the nominal interest rate on private bonds goes to zero, and the optimal allocation is a competitive equilibrium allocation if the money growth rate equals minus the rate of time preference, or $\theta = \beta$. Thus, private money enables the financial intermediary to correctly distribute purchasing power between shoppers buying with cash and those buying with credit, and the discount window arrangement effectively allows the household to use cash acquired during the period to finance purchases. In contrast to the case with no private money issue, the central bank does not need to exogenously manipulate the money supply so as to obtain the correct distribution of money balances across economic agents. That is, we have a standard Friedman rule $(\theta = \beta)$. This is because there is endogenous elasticity in the supply of currency through the issue of private money.

Now, consider an alternative optimal monetary policy that does not involve central bank lending. To determine the optimal money growth rule in this case, look for a monetary rule having the property that (40) holds with equality in equilibrium, so

$$p = \frac{\theta}{y(\pi + \gamma)}. \tag{53}$$

As well, (44) and (45) imply that

$$\mu_2 = \frac{u'(y)}{p} = \frac{yu'(y)(\pi + \gamma)}{\theta}, \tag{54}$$

using (53). Then, substituting for μ_2 and μ_2' in Equation (47) using (54) gives

$$-(\pi + \gamma) + \beta E\left[\frac{\pi' + \gamma'}{\theta'}\right] = 0, \tag{55}$$

and a monetary rule which satisfies (55) is then

$$\theta' = \frac{\beta(\pi' + \gamma')}{(\pi + \gamma)}.$$

That is, in this case the optimal gross money growth rate is the discount factor multiplied by the ratio of the real quantity of money necessary for settlement in the current period to the same quantity in the previous period. Essentially, monetary policy at the optimum accommodates fluctuations in the quantity of money needed in clearing and settlement. Note that this optimal money growth rule is quite different from the one in the regime with a discount window where private money was prohibited. Here, monetary growth is high (low) when the demand for cash in transactions is high (low), while the reverse was true in Section 3. The difference is due to the fact that monetary policy acts to face agents with the correct intertemporal terms of trade, whereas in the case with a discount window and a prohibition on private money issue optimal monetary policy is designed to attain the correct distribution of real money balances across the population.

5 Conclusion

The locational model constructed here provides a more explicit foundation for limited-participation models, and captures the important elements of the clearing and settlement of credit through private financial intermediaries and the central bank. Cash-in-advance type constraints arise endogenously, and are altered in important ways by financial constraints and monetary policy. When private money issue is prohibited, the nominal interest rate is in general positive, retail buyers pay a higher price for purchases made with currency rather than credit, and there exists a liquidity effect - an unanticipated positive monetary injection from the central bank reduces the nominal interest rate. When private money is permitted, the nominal interest rate will again be positive, in general, and retail buyers who make purchases with cash will pay a premium, but the liquidity effect no longer exists. This is because private money issue essentially permits private intermediaries to redistribute purchasing power to private transactors in a way that undoes the distribution effect of the central bank's money injection.

Optimal monetary policy is quite different with and without private money issue. Under either circumstance, a zero-nominal-interest-rate discount window policy allows financial intermediaries to borrow reserves in the current period against the currency that they plan to deposit with the central bank in the next period. Given this policy, if private money issue is prohibited, then it is optimal for money growth to be negatively correlated (relative to trend) with shocks to the demand for liquidity, as this achieves the correct distribution of money balances across the population. However, with private money issue, the optimal distribution of money balances is attained endogenously, and so money growth does not need to compensate optimally for fluctuations in the demand for liquidity at the optimum. With private money, an optimal allocation can also be achieved without central bank lending, in which case money growth is positively correlated with shocks to the demand for liquidity, as this achieves the correct intertemporal terms of trade.

Private money does not displace outside money in the model, though it changes the roles that fiat currency and central bank reserves play. With private money, fiat currency is no longer used in transactions, but fiat currency and central bank reserves are still important in clearing and settling private money that is returned for redemption.

This model is potentially useful for studying other issues connected with monetary policy, money, and credit. For example, with modifications it could be used to address the differences between a private money regime where private money is redeemable in commodity money (e.g. gold or silver) and the regime here where private money is redeemable in outside money. As well, it is fairly straightforward to include production, which would allow the exploration of the effects of money on output, and how these effects vary with the policy regime and financial restrictions.

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