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Search, evolution, and money

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Abstract

This paper describes a search-theoretic model that can be used to determine which objects serve as media of exchange, or money. Existing versions of the model are generalized to allow arbitrary distributions of agents who specialize in different consumption–production activities. I characterize the way the numbers of consumers and producers of the various goods help determine which goods serve as money. The distribution is then endogenized, so that agents can choose their type. This generates a unique equilibrium outcome. Ideas from evolutionary dynamics are employed as a way to interpret the model, and to compute equilibria.

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1. Introduction

Two of the classic questions in economics are: (1) why do transactions so frequently occur using a medium of exchange, or money, and (2) what determines which objects come to serve as media of exchange? Extended

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discussions of these questions, and some interesting answers, appear in the literature dating back at least to Smith (1776), Jevons (1875), Menger (1892), Wicksell (1911), and so on. Nevertheless, my view is that it is useful to formalize the insights in this traditional literature using the techniques of modern, dynamic, economic analysis. This not only provides a check on the intuition of the classical economists, it often generates new qualitative insights into monetary economics, and it provides the first step towards quantitative analysis.[1]

Many attempts have been made to provide a theoretical foundation for monetary economics, of course; see, for example, the survey by Ostroy and Starr (1990). One thing that is clear is that in order to make any progress along these lines one has to depart at least somewhat from the classical general equilibrium model, as epitomized by Debreu (1959), for example. The purpose of that model is to determine the properties of final (equilibrium) allocations consistent with individual maximization at given prices and given initial resources; it is silent on the process by which the economy gets from an initial allocation to a final allocation. Consequently, it has nothing to say about the role of money, or any of the related institutions that seem to be so important for allocating resources in actual economies, such as credit, middlemen, stores, banks, and so on.[2]

Recently, monetary economics has progressed by using search theory to formalize frictions in the process of exchange, frictions that are crucial for generating a role for money and related institutions, but difficult to build into standard economic models.[3] This paper describes one version of the

[1] Not only are the above questions fascinating and challenging from a purely theoretical point of view, it is also possible that coming up with answers may be important in terms of practical policy considerations. I will not pursue this, however, so that I may focus on more technical issues in this paper.

[2] I will avoid the temptation to discuss this point at length. I will also avoid critiquing the standard perturbations of the classical model, such as imposing a cash-in-advance constraint or assuming money enters utility or production functions, except to say that whatever the merits of such a reduced-form approach, it has no chance of answering the two questions given in the opening paragraph. Extended discussions of these methodological issues can be found in many places, including Kareken and Wallace (1980), Starr (1989), and Hellwig (1993). A classic reference that is still relevant in this regard is Hicks (1935).

[3] Research in this vein that is explicitly concerned with determining which objects circulate as commodity money includes Kiyotaki and Wright (1989), Marimon, McGrattan, and Sargent (1990), Aiyagari and Wallace (1991), Kehoe, Kiyotaki, and Wright (1993), Cuadras-Morateo (1993), and Li (1993). Versions of the model that are more suitable for analyzing fiat money are contained in Kiyotaki and Wright (1991, 1993), Aiyagari and Wallace (1992, 1993), Li (1992, 1993), Burdett et al. (1993), Matsuyama, Kiyotaki, and Matsui (1993), Kultti (1995), Trejos (1993), Williamson and Wright (1994), Wright (1994), and Zhou (1993). A contribution in the spirit of the search-theoretic approach is the model of Jones (1976) and the extensions by Iwai (1988) and Oh (1989). Models that use the search framework but get money into the system via a cash-in-advance constraint are contained in Diamond (1984), Gale (1986), and Diamond and Yellen (1987, 1990). One deficiency in all of these papers (except some of those with cash-in-advance) is that, in order to focus on the exchange process, they neglect the determination of exchange rates by assuming that prices are more or less fixed exogenously (say, by assuming that objects are indivisible, and so must trade one-for-one). Shi (1993) and Trejos and Wright (1993a, 1993b) determine prices endogenously in a search-based model of fiat money using bilateral bargaining theory.

search-theoretic approach to monetary economics and uses it to address the above questions. Along the way, some new economic and technical issues arise. The goal is to demonstrate how this class of models can be employed both to study some substantive questions in monetary economics – in this particular case, how do the relative numbers of agents producing and consuming various objects interact with the equilibrium choice of which objects serve as media of exchange – and as a laboratory within which one can experiment with new techniques – in this case, the evolutionary approach to noncooperative game theory.

The starting point is the commodity money model described in Kiyotaki and Wright (1989). In that framework, agents specialize in both production and consumption, and meet randomly over time in a way that implies exchange must be bilateral and quid pro quo. Each individual chooses a trading strategy to maximize his expected discounted utility from consumption net of production and transaction (storage) costs, given the strategies of others and rational expectations concerning the endogenous process of meetings. Equilibrium determines the trades that occur and, therefore, the good or goods that serve as commodity money. (Fiat money can also serve as a medium of exchange in this model, but for simplicity I concentrate on commodity money here.) Part of what determines which objects serve as money is their intrinsic properties, such as storage costs, although this does not tell the whole story. For example, a belief by agents that a good will be widely accepted as a medium of exchange can be self-fulfilling even if it has intrinsic properties that are relatively inferior to other potential monies, but not if they are too inferior to other potential monies.

The first thing I do is to generalize the model to allow arbitrary distributions of types across the population, where types are indexed by their consumption–production specialties (existing specifications assume equal numbers of each type). This is interesting because it expands the set of considerations that help determine which objects can or cannot serve as media of exchange, and there is a long-held notion that a consideration that ought to be important is the number of people who consume or produce different commodities [see Menger (1892) or Jones (1976), for example]. The set of equilibria is characterized for any fixed population distribution, and it is shown how the equilibrium trading strategies, and therefore media of exchange, depend on both the storage costs and the relative numbers of agents who produce and consume the different goods. Equilibria studied in previous analyses of this model appear as special cases, and new equilibria can also emerge for certain population distributions.

One might argue that the distribution of consumption–production types ought to be endogenized. For instance, not only does the number of gold producers or gold consumers have a potential impact on whether or not gold serves as a medium of exchange, but whether or not gold serves as a medium of exchange presumably can have an impact on the number of gold producers and consumers. I therefore also consider the possibility that agents can choose their specialization. Several results emerge from this exercise. First, the outcome called the 'fundamental' equilibrium in Kiyotaki and Wright (1989), and often discussed as though it is the most natural equilibrium [see, for example, Marimon, McGrattan, and Sargent (1990)], does not even exist when types are endogenous. In fact, there is a unique equilibrium and it involves what were called the 'speculative' trading strategies in Kiyotaki and Wright (1989). In this unique equilibrium, except for the degenerate case in which storage costs are identical for all goods, the distribution of types will not be uniform, as has been assumed exogenously in previous analyses of the model.

There are some technical and conceptual issues involved in making the distribution of types endogenous. Technically, it becomes difficult to solve for the equilibrium. Conceptually, to the extent that specialization in consumption is determined by agents' preferences in this model, it may be difficult to motivate one's type as a choice variable. I address both of these issues by introducing evolutionary dynamics into the analysis [see Mailath (1992), and the other papers in that volume, for an introduction to evolutionary game theory]. For a fixed population distribution, depending on parameter values and, also, on which equilibrium the economy selects if more than one exists, some types enjoy greater payoffs than others. For example, types that specialize in the production of low storage cost output or in the production of output that serves as a medium of exchange in a particular equilibrium would seem to be in position to derive a larger payoff.

I then consider alternative specifications by which the distribution of types responds over time, including the 'evolutionary dynamic' whereby types reproduce in proportion to their average payoff, and the 'best response dynamic' whereby all new entrants into the population emulate the type that is currently enjoying the greatest payoff. It is then possible to analyze the dynamic behavior of the population either analytically or numerically. For example, if the distribution of types settles down to a steady state where all types derive the same payoff, then the limiting distribution constitutes an equilibrium of the larger game in which agents choose their type. Hence, in particular, one can use the evolutionary process as a computational method for constructing equilibria numerically. For some specifications, the outcome can depend on which dynamic process is assumed, and generates cycles rather than convergence to a steady state.

The rest of the paper is organized as follows. Section 2 presents the assumptions underlying the basic model. Section 3 characterizes the set of steady state equilibria for a fixed population distribution, and discusses how commodity money in equilibrium depends on storage costs and the numbers of agents producing or consuming the various objects. Section 4 endogenizes the distribution of types, and analyzes the evolutionary dynamics. Some concluding remarks are contained in section 5.

2. The basic model

Time is discrete and continues forever (a similar analysis applies in continuous time when the meeting technology described below is specified in terms of Poisson arrivals). There are three indivisible commodities, labeled $j = 1, 2, 3$, and a continuum of agents, each of whom can be one of three types, labeled $i = 1, 2, 3$. [The basic model can be generalized to N commodities and N types, as in Aiyagari and Wallace (1991); $N = 3$ is the minimum number that makes the analysis interesting.] What distinguishes types is that type i consumes only good i and produces only good $i + 1$, modulo 3: that is, type 1 consumes good 1 and produces good 2, type 2 consumes good 2 and produces good 3, type 3 consumes good 3 and produces good 1. Unlike earlier analyses of this model, the distribution of types is not necessarily uniform; it is described by $\theta = (\theta_1, \theta_2, \theta_3)$, where θ_i is the fraction of type i, with $\theta_i > 0$ and $\theta_1 + \theta_2 + \theta_3 = 1$.

For any agent i, the common utility of consuming one unit of good i is $u > 0$, the disutility cost of producing one unit of good $i + 1$ is normalized to zero, and $r > 0$ is the discount rate. Any commodity can be stored by any agent, one unit at a time. If commodity j is stored between t and $t + 1$, there is a cost in terms of disutility c_j, paid at $t + 1$, where $c_1 < c_2 < c_3 < u$. (If $c_j < 0$, it can be interpreted as a rate of return rather than a storage cost.) This is the consumption–production–storage specification called Model A in Kiyotaki and Wright (1989). There is another version, called Model B, that reorders production so that i produces $i - 1$ modulo 3 or, equivalently, reorders storage costs so that $c_1 > c_2 > c_3$.[4] For most of the discussion in this paper I will focus exclusively on Model A, although a similar analysis can be performed on Model B.

Agents meet bilaterally at each date, which implies that the probability of meeting an agent of type i is θ_i. When two agents meet, they trade if and only if mutually agreeable. All trades are quid pro quo, and there are no credit arrangements, since two agents who meet at data t will meet again at date $t' > t$ with probability 0. All trades involve a one-for-one swap of inventories, since commodities are indivisible.[5] When type i acquires good i, he immediately consumes it, produces a new unit of good $i + 1$, and stores it until the next date when he meets a new trading partner. Hence, in equilibrium type i always enters a trading period with either good $i + 1$ or good $i + 2$, and never good i, and therefore $p(t) = [p_1(t), p_2(t), p_3(t)]$ completely describes the distribution of inventories at a point in time, where $p_i(t)$ is the proportion of type i agents holding

[4] Note that there are only two different versions of the model possible, assuming that no type produces his own consumption good; everything else is just a relabeling.

[5] This means that relative prices all equal unity. Shi (1993) and Trejos and Wright (1993a, b) assume divisible commodities, so that prices can be determined endogenously, in a version of the model with fiat money; I prefer to avoid these complications in this paper.

their production good $i+1$ at the beginning of period t. A steady state inventory distribution satisfies $p(t) = p$ for all t.

Agents choose strategies for deciding when to trade, given the strategies of others and the inventory distribution. As agents are always willing to trade for their own consumption goods, all that needs to be determined is if they are willing to trade their production goods for goods that are not their consumption goods. For now, I consider only symmetric, time-invariant, pure strategies (mixed or nonsymmetric strategies are also considered below). Such a strategy profile is denoted $s = (s_1, s_2, s_3)$, where $s_i = 1$ if type i wants to trade his production good $i+1$ for good $i+2$, and $s_i = 0$ otherwise; by assumption, if i trades good j for good k, then he does not trade good k back for good j. If $s_i = 1$, then type i is willing to make an indirect trade of good $i+1$ for $i+2$ and then use good $i+2$ to acquire good i, which makes good $i+2$ a medium of exchange, or a commodity money. If $s_i = 0$, on the other hand, then type i holds on to his production good $i+1$ until he can trade directly for good i.

I begin the analysis by solving for the steady state inventory distribution, given a strategy vector s and population distribution θ. Consider, for example, type 1, who always has either good 2 or good 3. The only way for type 1 to go from good 2 to good 3 is to meet a type 2 agent with good 3 and trade, which occurs with probability $\theta_2 p_2 s_1$ (notice that the probability of trading is s_1 since type 2 always accepts good 2). In steady state he will never switch from good 3 to good 2 directly, since if he preferred good 2, his production good, then he would not have traded it for good 3 in the first place; however, he can switch by trading good 3 for good 1, consuming, and producing a new unit of good 2. This happens when he meets type 3 with good 1, which occurs with probability $\theta_3 p_3$, or type 2 with good 1 who wants to trade, which occurs with probability $\theta_2(1-p_2)(1-s_2)$ (notice that $1-s_2$ is the probability that type 2 trades good 1 for good 3, since s_2 is the probability that type 2 trades good 3 for good 1).

Equating the flow of type 1 agents from good 2 to good 3 with the flow of type 1 agents from good 3 to good 2, a steady state requires

$$p_1\theta_2 p_2 s_1 = (1-p_1)[\theta_3 p_3 + \theta_2(1-p_2)(1-s_2)].$$

A symmetric equation holds for each i. Therefore, a steady state inventory distribution p is a solution to

$$\left.\begin{aligned} p_1\theta_2 p_2 s_1 &= (1-p_1)[\theta_3 p_3 + \theta_2(1-p_2)(1-s_2)] \\ p_2\theta_3 p_3 s_2 &= (1-p_2)[\theta_1 p_1 + \theta_3(1-p_3)(1-s_3)] \\ p_3\theta_1 p_1 s_3 &= (1-p_3)[\theta_2 p_2 + \theta_1(1-p_1)(1-s_1)] \end{aligned}\right\} . \tag{1}$$

These equations can actually be simplified slightly further, by using the result that $s_i = 0$ implies $p_i = 1$, but the above expressions suffice for present purposes.

I now describe the individual decision problem. Let V_{ij} be the expected discounted utility at the end of period t for type i with good j (the payoff, or value, function). A maximizing strategy for type i then satisfies $s_i = 1$ if $V_{i,i+1} < V_{i,i+2}$ and $s_i = 0$ if $V_{i,i+1} > V_{i,i+2}$. Thus, if the value of holding the production good $i + i$ is less than the value of holding good $i + 2$, then i will set $s_1 = 1$ and opt for an indirect trade whenever the opportunity presents itself (although, of course, he may still acquire his consumption good i directly if it is offered in exchange for $i + 1$ before he has the chance to trade for $i + 2$). On the other hand, if the value of holding the production good $i + 1$ exceeds the value of holding good $i + 2$, then i will not accept good $i + 2$ and will hold on to his production good until he can trade it directly for good i. If $V_{i,i+1} = V_{i,i+2}$, then type i is indifferent between holding goods $i + 1$ and $i + 2$, and he could randomize between $s_i = 0$ and $s_i = 1$. In the analysis of pure strategy equilibria, however, I simply assume that agents do not trade if they are indifferent.

To illustrate the workings of the model, consider the payoffs of type 1. Standard techniques deliver the following expressions for the *flow payoffs* (the flow payoff is simply the rate of time preference multiplied by the value function):

$$rV_{12} = -c_2 + \theta_2 p_2 s_1 (V_{13} - V_{12}) + [\theta_2(1 - p_2) + \theta_3 p_3 s_3] u,$$

$$rV_{13} = -c_3 + \theta_3 p_3 (u + V_{12} - V_{13}).$$

The flow payoff from holding good 2 is the disutility of storage, plus the probability of meeting a type 2 with good 3 and trading if desirable (i.e., if $s_1 = 1$), plus the expected utility from consumption, which occurs when the agent meets either a type 2 with good 1 or a type 3 with good 1 who is willing to accept good 2. The flow payoff from holding good 3 is the disutility of storage plus the probability of meeting a type 3 with good 1 and trading, consuming, and producing a new unit of good 2.[6]

Manipulation of the above expressions yields

$$\Delta_1 = c_3 - c_2 + [\theta_2(1 - p_2) - \theta_3 p_3 (1 - s_3)] u,$$

[6] I have used the result that, in steady state, type 1 never switches from good 3 to good 2 without consuming, and the result that type 1 can never get good 1 from type 2 in exchange for good 3 (since if type 2 preferred good 3 to good 1 he never would have acquired good 1 in the first place). These expressions can be derived from the perhaps more intuitive expressions for the value functions in Kiyotaki and Wright (1989), although there would be a slight difference because that paper assumes the cost of storing a good between t and $t + 1$ is incurred at t rather than at $t + 1$. The reason the expressions are so much more compact here than in that paper is because they are written in flow terms – that is, because I have solved for rV_{ij}.

where $\Delta_1 \equiv (r + \theta_2 p_2 s_1 + \theta_3 p_3)(V_{12} - V_{13})$ takes the same sign as $V_{12} - V_{13}$, and therefore the sign of Δ_1 is sufficient to determine s_1. By symmetry, the sign of Δ_i is sufficient to determine s_i, where

$$\Delta_i = c_{i+2} - c_{i+1} + \theta_{i+1}(1 - p_{i+1}) - \theta_{i+2} p_{i+2}(1 - s_{i+2}), \tag{2}$$

given a normalization $u = 1$ (and, as always, addition with respect to types is understood to be modulo 3). Maximization for type i requires $s_i = 1$ if $\Delta_i < 0$ and $s_i = 0$ if $\Delta_i > 0$.

Intuitively, on the right-hand side of (2), $c_{i+2} - c_{i+1}$ represents the cost to acquiring a medium of exchange in terms of relative storabilities, while the remaining terms represent the change in *liquidity* that results from trading good $i + 1$ for good $i + 2$, where liquidity reflects the rate at which type i expects to acquire his consumption good. If $\Delta_i > 0$, the net storability plus liquidity cost is positive, so type i sets $s_i = 0$ and never uses indirect trade. If $\Delta_i < 0$, then the net storability plus liquidity cost is negative, so type i sets $s_i = 1$ and acquires a medium of exchange whenever he can.

The above considerations lead to the following definition of an equilibrium for the model with θ fixed.

Definition 1. Given the storage costs and the population distribution, $c = (c_1, c_2, c_3)$ and $\theta = (\theta_1, \theta_2, \theta_3)$, a (symmetric, steady state, pure strategy) equilibrium is defined to be a vector of inventories $p = (p_1, p_2, p_3)$ and a vector of strategies $s = (s_1, s_2, s_3)$ satisfying:

(a) the steady state condition (1),

(b) for each agent of type i, given the strategies of others and p, s_i satisfies the maximization condition: $s_i = 1$ if $\Delta_i < 0$ and $s_i = 0$ if $\Delta_i > 0$.

3. Equilibria with a fixed population

There are exactly $2^3 = 8$ candidate strategy vectors that could potentially constitute (symmetric, steady state, pure strategy) equilibria, corresponding to each s_i taking on either a value of 1 or 0. Several cases can be ruled out immediately.

Case 1: $s = (1, 1, 1)$. This is not an equilibrium because, given these strategies, (2) reduces to $\Delta_1 = c_3 - c_2 + \theta_2(1 - p_2) > 0$ for type 1, and therefore $s_1 = 1$ is not a maximizing strategy.

Case 2: $s = (1, 0, 1)$. This is not an equilibrium because, given these strategies, (2) reduces to $\Delta_1 = c_3 - c_2 > 0$ for type 1, and again $s_1 = 1$ is not a maximizing strategy.

Case 3: $s = (1, 0, 0)$. This is not an equilibrium because, given these strategies, (2) reduces to $\Delta_2 = c_1 - c_3 < 0$ for type 2, and therefore $s_2 = 0$ is not a maximizing strategy.

Case 4: $s = (0, 1, 1)$. This is not an equilibrium because, given these strategies, (2) reduces to $\Delta_3 = c_2 - c_1 > 0$ for type 3, and therefore $s_3 = 1$ is not a maximizing strategy.

Case 5: $s = (0, 0, 0)$. This is not an equilibrium because, given these strategies, (2) reduces to $\Delta_2 = c_1 - c_3 - \theta_1 p_1 < 0$ for type 2, and therefore $s_2 = 0$ is not a maximizing strategy for type 2.

This leaves three cases, $s = (0, 1, 0)$, $s = (1, 1, 0)$, and $s = (0, 0, 1)$. The first two possibilities were found to be equilibria for some specifications of storage costs in Kiyotaki and Wright (1989) when $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$, and were referred to there as the fundamental and speculative equilibria, respectively. The third possibility cannot be an equilibrium when $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$, but it remains to be seen what happens for other θ. The steady state inventory distributions for each of the three cases, which can be found by solving (1), is given by

$$s = (0, 1, 0) \quad \text{implies} \quad p = [1, \theta_1/(1 - \theta_2), 1];$$

$$s = (1, 1, 0) \quad \text{implies} \quad p = [p_1, p_2, 1],$$

where

$$p_1 = \frac{\theta_3}{2\theta_1(1 - \theta_1)}\left[\theta_1 - \theta_3 + \sqrt{(1 - \theta_2)^2 + 4\theta_1\theta_2}\right],$$

$$p_2 = \frac{1}{2\theta_2}\left[\theta_2 - 1 + \sqrt{(1 - \theta_2)^2 + 4\theta_1\theta_2}\right];$$

and

$$s = (0, 0, 1) \quad \text{implies} \quad p = [1, 1, \theta_2/(1 - \theta_3)].$$

Consider first $s = (0, 1, 0)$. In this case, type 2 uses good 1 as a medium of exchange, while types 1 and 3 opt for direct exchange. Type 2 agents therefore act as middlemen, acquiring good 1 from type 3 and trading it to type 1, and good 1 becomes commodity money. Under what circumstances will this be an equilibrium? Given s and the implied distribution for p, condition (2) implies:

$$\Delta_1 = c_3 - c_2 - \theta_3(1 - 2\theta_2)/(1 - \theta_2),$$

$$\Delta_2 = c_1 - c_3 - \theta_1,$$

$$\Delta_3 = c_2 - c_1.$$

Hence, $\Delta_2 < 0$ and $\Delta_3 > 0$, and so $s_2 = 1$ and $s_3 = 0$ are maximizing strategies for types 2 and 3, while $\Delta_1 \geq 0$ and $s_1 = 0$ is a maximizing strategy for type 1 if and only if the following condition holds: $\theta_2 \geq \frac{1}{2}$, or $\theta_2 < \frac{1}{2}$ and

$$\theta_3 \leq (c_3 - c_2)(1 - \theta_2)/(1 - 2\theta_2) \equiv e(\theta_2). \tag{3}$$

A graphical illustration of the region of parameter space in which these conditions are satisfied – and,therefore, in which $s = (0, 1, 0)$ constitutes an equilibrium – is provided below.

In the special case where $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$, (3) implies that $s = (0, 1, 0)$ is an equilibrium if and only if $c_3 - c_2 \geq \frac{1}{6}$. This is the same as the condition in Kiyotaki and Wright (1989, Theorem 1(a)) – although the discount rate does not appear because it is assumed here that storage costs between t and $t + 1$ are incurred at $t + 1$, rather than at t. Another special case of interest is the one where $c_j \to 0$ for all j, which implies that $s = (0, 1, 0)$ is an equilibrium if and only if $\theta_2 \geq \frac{1}{2}$. As the storability factor vanishes, all that matters is liquidity, and type 1 will accept good 3 as a medium of exchange if and only if it allows him to acquire his consumption good more quickly. He can acquire his consumption good more quickly in exchange for good 2 than in exchange for good 3 if and only if $\theta_2(1 - p_2) \geq \theta_3 p_3$, which reduces to $\theta_2 \geq \frac{1}{2}$, because this equilibrium implies $p_2 = \theta_1/(1 - \theta_2)$ and $p_3 = 1$. Therefore, in the absence of storage costs, $s = (0, 1, 0)$ is an equilibrium if and only if the majority of the population are type 2 agents – that is, consumers of type 1's output.

Consider now $s = (1, 1, 0)$. In this case, types 2 and 3 are using the same strategies they used in the previous case, but now type 1 is accepting good 3 as a medium of exchange; hence, both goods 1 and 3 serve as commodity money. Given s and the implied distribution for p,

$$\Delta_1 = c_3 - c_2 + \tfrac{1}{2}\left\{1 + \theta_2 - 2\theta_3 - \sqrt{(1 - \theta_2)^2 + 4\theta_2(1 - \theta_2 - \theta_3)}\right\},$$

$$\Delta_2 = c_1 - c_3,$$

$$\Delta_3 = c_2 - c_1 + \theta_1(1 - p_1).$$

Hence, $\Delta_2 < 0$ and $\Delta_3 > 0$, and so $s_2 = 1$ and $s_3 = 0$ are maximizing strategies for types 2 and 3, while $\Delta_1 < 0$ and $s_1 = 1$ is a maximizing strategy for type 1 if and only if

$$\theta_3 > c_3 - c_2 + \tfrac{1}{2} - \tfrac{1}{2}\sqrt{1 - 4\theta_2(\theta_2 + c_3 - c_2)} \equiv f(\theta_2). \tag{4}$$

A graphical depiction of the region of parameter space in which this is satisfied will also be provided below.

In the special case in which $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$, (4) is satisfied and therefore $s = (1, 1, 0)$ is an equilibrium if and only if $c_3 - c_2 < (\sqrt{2} - 1)/3$. This is the same as the condition in Kiyotaki and Wright (1989, Theorem 1(b)) – again, except that the discount rate does not appear. In general, (4) indicates that good 3 has to be not too much more costly to store than good 2, and there has to be a sufficiently large number of type 3 agents relative to type 2 agents in order for type 1 to wish to use indirect trade. This is reasonable, since the advantage to type 1 of using good 3 as a medium of exchange is that he can always trade it to type 3 and type 3 always carries good 1. If the probability of meeting a type 3 agent is great – in other words, if there is a large number of agents who demand good 3 for consumption purposes – then acquiring good 3 reduces the time it takes type 1 to acquire his consumption good. In this case, type 1 opts for liquidity over storability.

Finally, consider $s = (0, 0, 1)$. In this case, type 3 accepts good 2 as a medium of exchange, while types 1 and 2 hold on to their production goods, and good 2 becomes the unique commodity money – a complete reversal of the situation in the previous case, where goods 1 and 3 are commodity monies but good 2 was not. With these strategies and the implied distribution for p,

$$\Delta_1 = c_3 - c_2 ,$$

$$\Delta_2 = c_1 - c_3 + \theta_1(2\theta_3 - 1)/(1 - \theta_3) ,$$

$$\Delta_3 = c_2 - c_1 - \theta_2 .$$

Hence, $s_1 = 0$ is a maximizing strategy for type 1, $s_2 = 0$ is a maximizing strategy for type 2 if and only if $\Delta_2 \geq 0$, which holds if and only if $\theta_3 \geq \frac{1}{2}$ and

$$\theta_2 \leq (1 - \theta_3)(1 + c_3 - c_1 - 2\theta_3)/(1 - 2\theta_3) \equiv g(\theta_3) , \tag{5}$$

and $s_3 = 1$ is a maximizing strategy for type 3 if and only if $\Delta_3 < 0$, which holds if and only if

$$\theta_2 > c_2 - c_1 . \tag{6}$$

In the limit as $c_j \to 0$ for all j, this equilibrium exists if and only if $\theta_3 \geq \frac{1}{2}$.

Figs. 1a–d depict regions in the (θ_2, θ_3) plane in which the various equilibria exist, for various values of the storage costs. There are several things to notice about the results. First, because conditions (3) and (4) can never be satisfied simultaneously, the two equilibria $s = (0, 1, 0)$ and $s = (1, 1, 0)$ never coexist. Furthermore, there always exist values of θ such that neither (3) nor (4) is

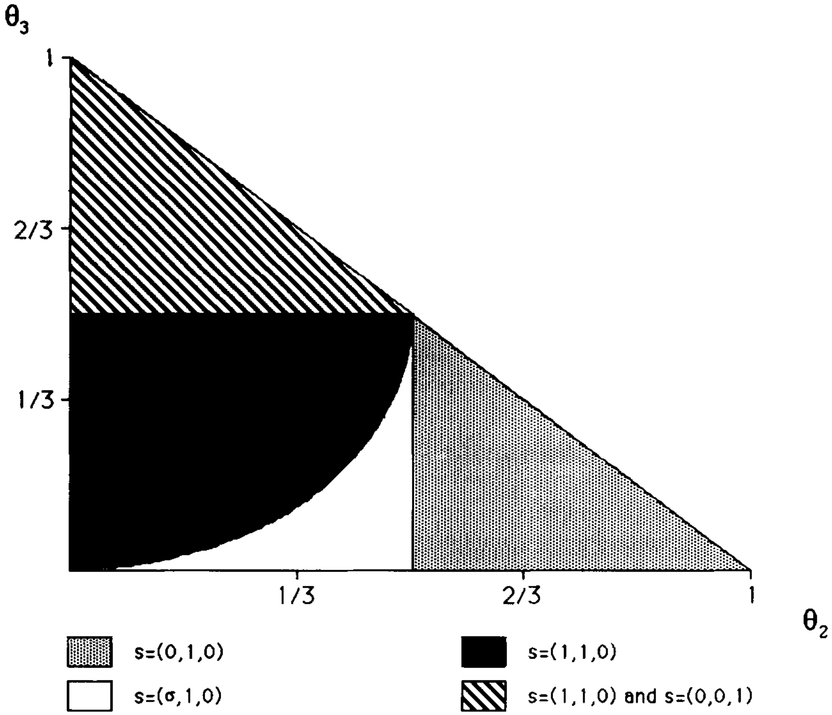

Fig. 1a. Existence of equilibria, $c = (0, 0, 0)$.

satisfied, and so neither the $s = (0, 1, 0)$ nor the $s = (1, 1, 0)$ equilibrium exists (a mixed strategy equilibrium will be constructed below for these parameter values). These findings generalize known results for the case $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$. The new equilibrium $s = (0, 0, 1)$ can exist for some values of c and θ, although not for any c when $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$, which is why it did not come up in Kiyotaki and Wright (1989). The region in the (θ_2, θ_3) plane in which the new equilibrium exists shrinks and eventually vanishes as the storage costs become bigger.

An interesting finding is that whenever the new equilibrium $s = (0, 0, 1)$ exists, so does $s = (1, 1, 0)$. These equilibria are mirror images, in the sense that agents use opposite strategies in the two cases, and the commodities monies are goods 1 and 3 in the latter case and good 2 in the former case. This demonstrates that which objects arise endogenously as media of exchange depends not only on fundamentals, like storage costs and the numbers of agents that consume or produce the various goods, but also on which equilibrium the economy selects. This multiplicity of equilibrium outcomes does not arise in the special case

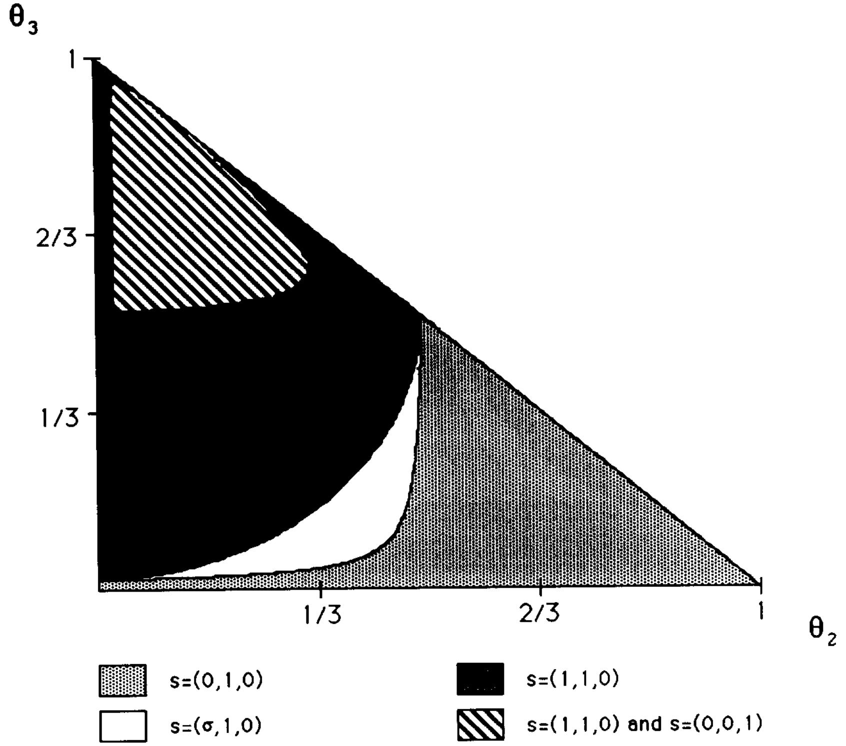

Fig. 1b. Existence of equilibria, $c = (0, 0.01, 0.02)$.

where $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$, where there is never more than one equilibrium, and the commodity monies can therefore depend only on storage costs.[7]

As remarked above, there is always a region of parameter space in which no pure strategy equilibrium exists, in between the $\theta_3 = e(\theta_2)$ and $\theta_3 = f(\theta_2)$ curves, shown as the white region in fig. 1. Although I am mainly concerned with symmetric pure strategies in this paper, for completeness I now demonstrate how to fill in this region by constructing a symmetric mixed strategy equilibrium where $s = (\sigma, 1, 0)$, for some σ in the open interval $(0, 1)$. That is, type 1 agents randomize in their decision to accept good 3.[8] For this to

[7] The alternative version of the model called Model B does admit multiple equilibria with different commodity monies, for some storage costs, even if $\theta_1 = \theta_2 = \theta_3 = 1/3$; see Kiyotaki and Wright (1989, Theorem 2).

[8] The construction follows Kehoe, Kiyotaki, and Wright (1993). Notice that the symmetric mixed strategy equilibrium can always be reinterpreted as a nonsymmetric pure strategy equilibrium, where a fraction σ of the type 1's uses $s_1 = 1$ and a fraction $1 - \sigma$ uses $s_i = 0$.

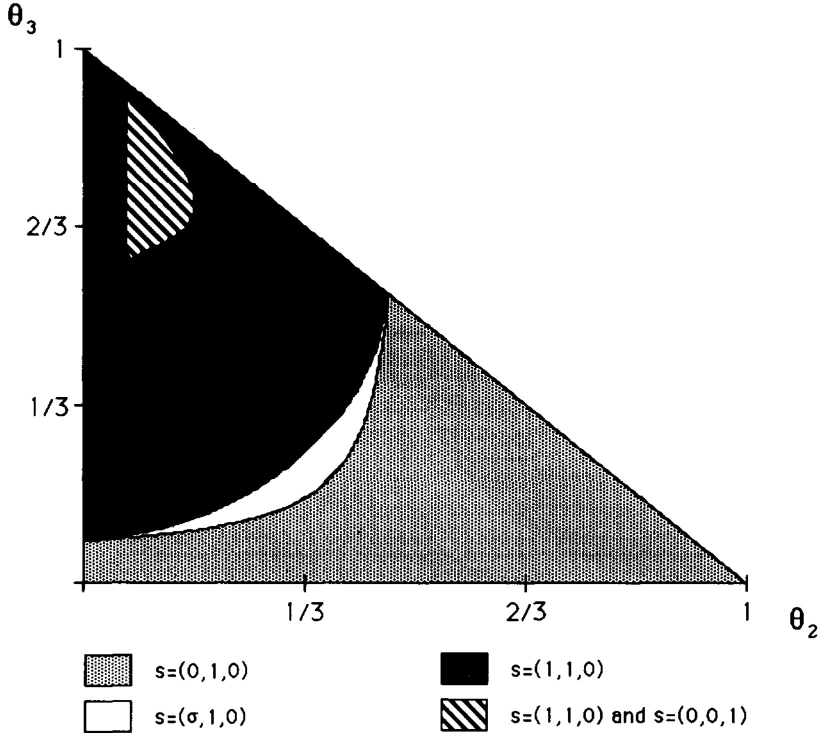

Fig. 1c. Existence of equilibria, $c = (0, 0.1, 0.2)$.

be consistent with individual maximization, Δ_1 must be zero. Also, given $s = (\sigma, 1, 0)$, the steady state distribution is $p = [p_1, p_2, 1]$ where

$$p_1 = \frac{\theta_3}{2\theta_1(\sigma\theta_2 + \theta_3)}\left\{1 - \theta_2 - 2\theta_3 + \sqrt{(1 - \theta_2 - 2\theta_3)^2 + 4\theta_1(\sigma\theta_2 + \theta_3)}\right\}.$$

$$p_2 = \frac{1}{2\sigma\theta_2}\left\{\theta_2 - 1 + \sqrt{(1 - \theta_2)^2 + 4\sigma\theta_1\theta_2}\right\}.$$

Given s and p, it is easy to check that $\Delta_2 < 0$ and $\Delta_3 > 0$, so that $s_2 = 1$ and $s_3 = 0$ are maximizing strategies for types 2 and 3, and that

$$\Delta_1 = c_3 - c_2 + \theta_2(1 - p_2) - \theta_3\,,$$

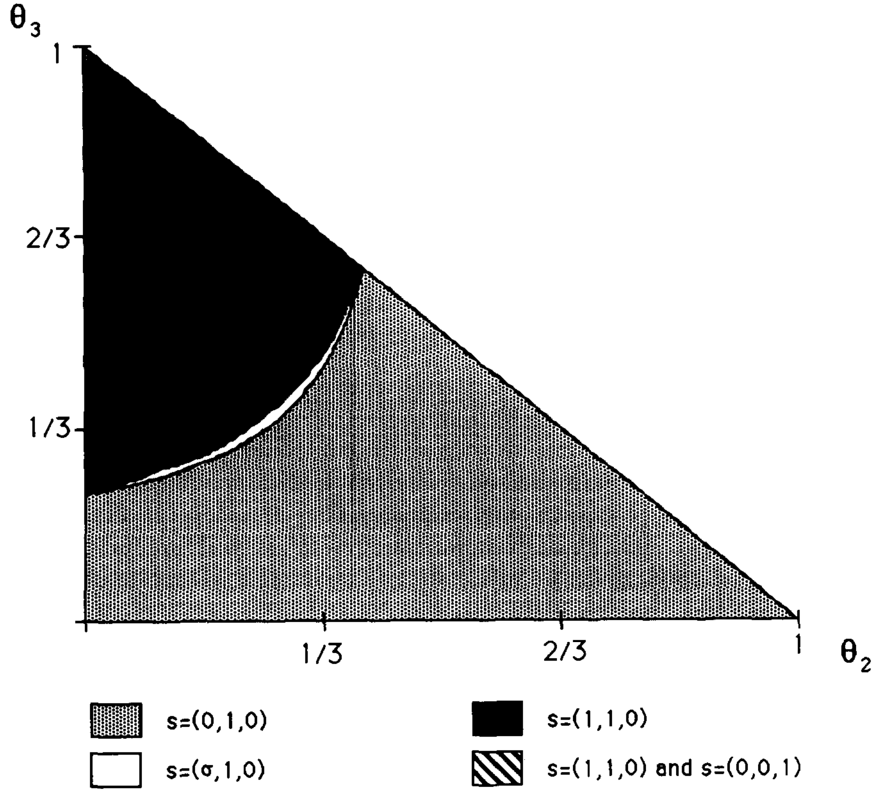

Fig. 1d. Existence of equilibria, $c = (0, 0.2, 0.4)$.

so that $\Delta_1 = 0$ if and only if (after inserting p_2 from above and solving)

$$\sigma = \frac{\theta_1\theta_2 + (1-\theta_2)(c_3 - c_2 + \theta_2 - \theta_3)}{(c_3 - c_2 + \theta_2 - \theta_3)^2}.$$

Notice that $0 < \sigma < 1$ if and only if $e(\theta_2) < \theta_3 < f(\theta_2)$. Therefore, in the region between the $\theta_3 = e(\theta_2)$ and $\theta_3 = f(\theta_2)$ curves, this mixed strategy equilibrium exists and ties together the two equilibria $s = (0, 1, 0)$ and $s = (1, 1, 0)$. This establishes the existence of equilibrium for all θ.

4. Endogenous types

Until now, the distribution of types has been taken to be exogenous. What if agents are allowed to choose their type? How will the distribution of types be determined by the intrinsic properties of the objects, and by the choice as to which objects get used as money? Instead of studying how the number of producers or consumers of each good affects the equilibrium commodity money, the idea in this section is to study how the population distribution itself is determined.

To study this issue, one needs to know the equilibrium payoffs. The payoff to type i, *not* conditioned on the particular good he is holding at a point in time, is given by

$$W_i = p_i V_{i,i+1} + (1 - p_i) V_{i,i+2} \, .$$

Since this payoff is not conditioned on the current inventory, it measures type i's steady state utility – or, equivalently, average utility at a point in time across type i agents. One could alternatively measure payoffs according to some other criteria, such as utility given an inventory of i's production good as an initial condition, perhaps. I work with W_i as defined above because it is tractable, and also because it seems reasonable under the evolutionary interpretation described below.

As it turns out, these equilibrium payoffs can also be represented in the following way:

$$\begin{aligned} rW_i = & -p_i c_{i+1} - (1 - p_i) c_{i+2} + \theta_{i+1}(1 - p_{i+1})[p_i + (1 - p_i) s_{i+1}] \\ & + \theta_{i+2} p_{i+2} [p_i s_{i+2} + (1 - p_i)] \, . \end{aligned}$$

On the left-hand side is the flow payoff, or expected utility per period, rW_i. On the right-hand side, the first two terms give the average storage costs incurred by type i. The final two terms give the rate of consumption per period for type i: the probability of acquiring good i from type $i+1$ plus the probability of acquiring good i from type $i+2$. This expression is simple and intuitive, in principle, although it becomes unwieldy as soon as one inserts the steady state p distribution.

In any case, if agents are allowed to choose their type, equilibrium requires $W_1 = W_2 = W_3$ (it may seem more reasonable to require equal payoffs only for types that exist in positive numbers, but it is easily seen that there exist no nondegenerate equilibria where $\theta_i = 0$ for some i). This leads to the following generalization of Definition 1:

Definition 2. Given storage costs, c, a (symmetric, steady state, pure strategy) equilibrium with an endogenous distribution of types is a list (s, p, θ) satisfying:

(a) the steady state conditions (1),

(b) for each agent of type i, given the strategies of others and p, s_i satisfies the maximization condition: $s_i = 1$ if $\Delta_i < 0$ and $s_i = 0$ if $\Delta_i > 0$,

(c) equalization of expected utility: $W_1 = W_2 = W_3$.

It will be shown that endogenizing the distribution of types reduces the set of equilibria considerably: the only possible nondegenerate steady state equilibrium involves $s = (1, 1, 0)$, and an equilibrium with this s exists as long as the storage costs are not too high. (Clearly, if the storage costs are too high, then the only equilibrium is degenerate.) The idea behind the result is as follows: if θ is such that one of the other possible equilibria exists, and if the economy is in one of these equilibria, then the W_i are not equated across agents, and as θ changes to equate W_i, the existence conditions for these equilibria no longer hold. In particular, other things being equal, type 3 agents get a high payoff and type 2 get a low payoff, given the storage costs of their production goods. Hence, equilibrium must entail a relatively large θ_3 and low θ_2, and this allows only the $s = (1,1,0)$ equilibrium.

To verify this result, consider the strategy profile $s = (0, 1, 0)$. Explicit calculation of the W_i, given this s, implies

$$W_2 \gtreqless W_3 \quad \text{if and only if} \quad \theta_3 \gtreqless \theta_2 + c_3 - c_1 \,.$$

One can easily show that the curve along which $W_2 = W_3$ does not pass through the region in the (θ_2, θ_3) plane within which $s = (0, 1, 0)$ is an equilibrium. Hence, there is no point at which $s = (0, 1, 0)$ is an equilibrium strategy profile and the W_i are equated. Now consider $s = (0, 0, 1)$. Given this s,

$$W_1 \gtreqless W_3 \quad \text{if and only if} \quad 2\theta_3 \gtreqless 1 - \theta_2 + c_2 - c_1 \,.$$

One can easily show that the curve along which $W_1 = W_3$ does not pass through the region in which $s = (0, 0, 1)$ is an equilibrium. Therefore, there is no point at which $s = (0, 0, 1)$ is an equilibrium strategy profile and the W_i are equated. A similar argument applies to the mixed strategy equilibria discussed in the previous section.

These results establish that the only possible equilibrium involves the strategy profile $s = (1, 1, 0)$. Unfortunately, for general storage costs, an explicit expression for the locus of points in θ space in which $W_1 = W_2$, $W_2 = W_3$, or $W_3 = W_1$ is too complicated to convey much information when $s = (1, 1, 0)$. However, in the limiting case in which $c_j \to 0$ for all j, each locus turns out to be linear. In this case, there is a unique point such that $W_1 = W_2 = W_3$ in the interior of the region in which $s = (1, 1, 0)$ is an equilibrium – that is, the region

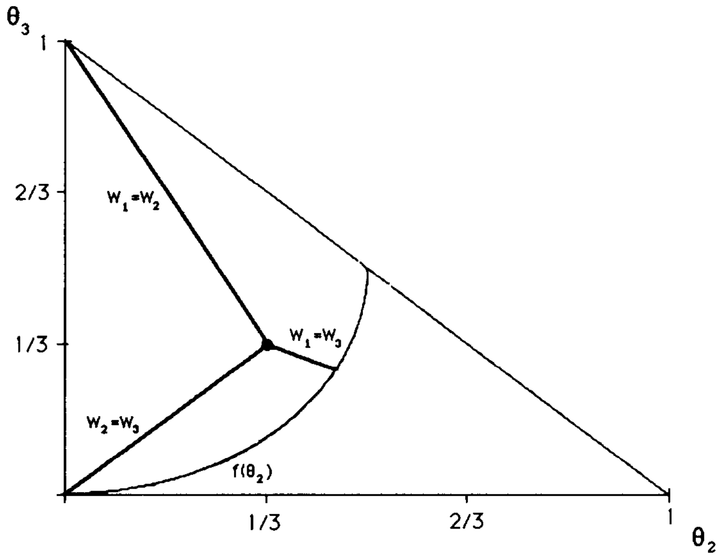

Fig. 2a. Equilibrium with endogenous types, $c = (0, 0, 0)$.

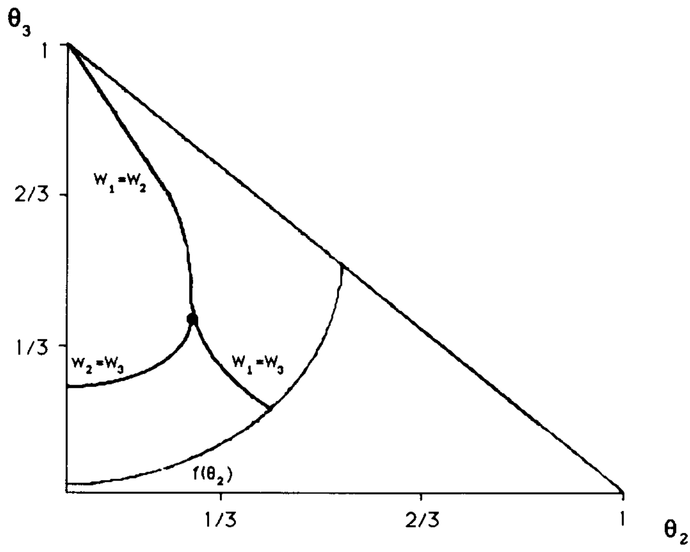

Fig. 2b. Equilibrium with endogenous types, $c = (0, 0.01, 0.02)$.

where $\theta_3 \geq f(\theta_2)$. By continuity, there exists a unique equilibrium in this region whenever the c_j are not too big. Figs. 2a and 2b depict the equilibrium in the (θ_2, θ_3) plane for two values of the storage costs. Notice that, when $c_j \to 0$ for all j, the equilibrium is the uniform distribution: $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$. Whenever $c_3 > c_2 > c_1$, on the other hand, $\theta_3 > \frac{1}{3}$ and $\theta_2 < \frac{1}{3}$.

Several aspects of these results are worth emphasizing. First, the equilibrium $s = (0, 1, 0)$, which was called the fundamental equilibrium in Kiyotaki and Wright (1989) and often discussed as though it is the most natural outcome [see, e.g., Marimon, McGrattan, and Sargent (1990)], does not even exist when θ is endogenized. Rather, the unique outcome is what was called a speculative equilibrium (since type 1 'speculates' by trading the relatively low storage cost good 2 for the high storage cost good 3, because he rationally believes the latter conveys greater liquidity). Also, the uniform distribution for θ, which is the specification exogenously imposed in previous analyses of this model, is never an equilibrium once θ is endogenized, excluding the degenerate case in which the storage costs are all identical. In general, there will be a more (fewer) agents who produce the good with the lowest (highest) storage cost.

There are two features of endogenizing θ that raise difficulties. First, technically, it is difficult to compute the equilibrium; although we can deduce its qualitative properties for small storage costs, in general it is not easy to find the solution to the equations $W_1 = W_2 = W_3$. Second, conceptually, it may be hard to understand how one's type can be a choice variable, to the extent that type in this model determines one's taste in consumption goods, and tastes are typically not open to choice in economic analysis.[9] I address both of these issues by introducing evolutionary dynamics into the model.

Although there are a variety of interpretations to the evolutionary approach, one is that over time new agents are born, and they inherit the characteristics of their parents [see Mailath (1992) for a discussion of some other interpretations, including learning and bounded rationality]. Conceptually, one does not choose one's type at all – it is given at birth; but as long as types reproduce at a faster rate when they enjoy a higher payoff (a greater 'fitness' in the language of evolutionary biology), the system can evolve in such a way that the economy

[9]The assumption that one's type determines both one's consumption good and production good was made partly for simplicity and partly to facilitate comparison with the previous literature, but it is not really necessary. An alternative is to assume that every agent at each point in time desires to consume one particular good, which varies stochastically (say, after consuming one thing he realizes a taste for something else drawn at random from the set of goods). This makes the agents generalists in consumption even though they are specialists in production; this may be more natural, but it is also somewhat more complicated and therefore will not be pursued here. Another interpretation of the assumption that one's type determines both one's demand and supply is that agents are in fact choosing production processes that take some goods as inputs and turn them into other goods as outputs. Then one's choice of which output to produce will simultaneously determine which goods one needs as inputs in the production chain.

behaves as if agents were choosing their types. Technically, studying the dynamic process may not only be interesting in its own right, but if it converges then its limiting distribution can be regarded as an equilibrium of the game in which types are chosen endogenously. As discussed by Sargent (1993), one can thereby regard the evolutionary approach as providing a method for computing equilibria.

The idea is to first fix $\theta(t)$ at a point in 'evolutionary time'. Then choose an equilibrium in trading strategies for this $\theta(t)$, which can be nontrivial when multiple equilibria exist, as they do in this model for some θ. Then compute the equilibrium payoffs, $W_i(t)$. Then allow the population to evolve (with evolutionary time) in such a way that types with greater payoffs increase their relative numbers. If $\theta(t) \to \theta^*$, then θ^* constitutes an equilibrium distribution of types according to Definition 2. One has to be careful, however, because whether or not a particular equilibrium exists depends on θ; therefore, as θ evolves, the economy may have to jump from one type of equilibrium to another.[10]

The next question is, how should the population evolve? A common specification in the literature is the so-called *replicator dynamic* [see Mailath (1992) and the references contained therein]:

$$\theta_i(t+1) = \theta_i(t)(1-\beta) + \theta_i(t)\beta W_i(t)/\mathrm{E}\,W(t)\,, \tag{7}$$

where $\mathrm{E}\,W(t) = \sum_i \theta_i(t) W_i(t)$ is the average payoff and $\beta \in (0, 1)$. This assumes some fraction of the population $1-\beta$ does not change at all, while a fraction of the population β (the newborn) evolves in such a way that types with a high payoff relative to the average payoff increase their relative numbers. The parameter β merely affects the speed of adjustment. At alternative that has been studied by Gilboa and Matsui (1991) and Matsui (1992), for example, is the so-called *best response dynamic*:

$$\theta_i(t+1) = \begin{cases} (1-\beta)\theta_i(t) + \beta & \text{if} \quad W_i(t) = \max\limits_j W_j(t)\,, \\ (1-\beta)\theta_i(t), & \text{otherwise.} \end{cases} \tag{8}$$

This specification assumes that all newborn agents next period emulate the type of agents who are currently enjoying the maximum payoff.[11]

Figs. 3a and 3b depict the evolution of the population in the (θ_2, θ_3) plane according to the replicator dynamic for two values of c. The arrows indicate the

[10] By evoking the notion of evolutionary time in this discussion, I mean that the population evolves much more slowly than the rate at which the agents interact in the trading process. This is what makes steady state utility W_i a reasonable indicator of reproductive fitness.

[11] If one interprets the model as saying that newborn agents *choose* their type, then both specifications imply agents are to some extent myopic or unsophisticated [Mailath (1992)]. In particular, the population evolves according to current payoffs and not rationally anticipated future payoffs, which is why evolutionary theory is often interpreted in terms of bounded rationality. By contrast, the agents in this paper are fully rational and sophisticated in choosing their trading strategies.

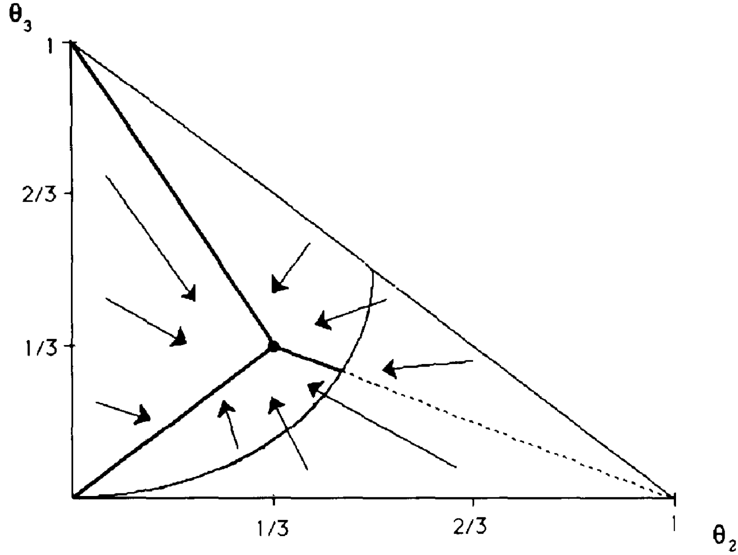

Fig. 3a. Replicator dynamic, $c = (0, 0, 0)$.

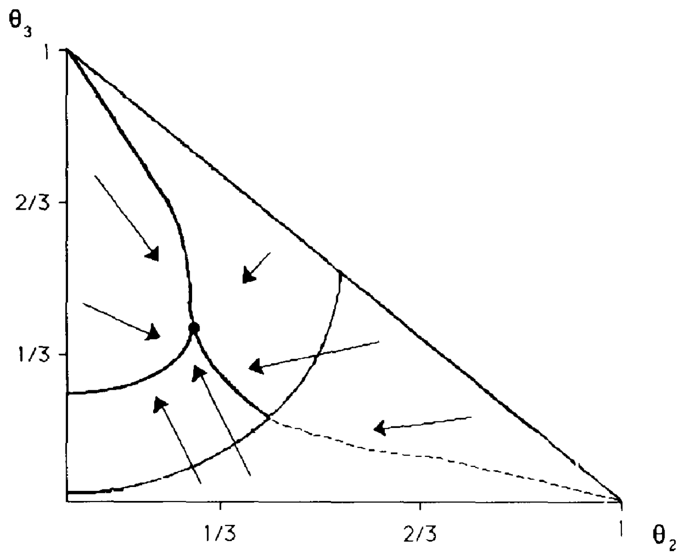

Fig. 3b. Replicator dynamic, $c = (0, 0.01, 0.02)$.

direction of motion, derived by calculating the W_i and then the change in θ_i as implied by (7) in each region of the plane, given the type of equilibria that exist in the various regions. In the region where the unique equilibrium involves $s = (0, 1, 0)$, type 3 are doing well and type 2 poorly. Hence, the system moves to the northwest, first entering the region where the mixed strategy $s = (\sigma, 1, 0)$ is used, and then leaving this region and entering the region where $s = (1, 1, 0)$ is used. In the region where both $s = (1, 1, 0)$ and $s = (0, 0, 1)$ are equilibria, in either equilibrium type 2 are doing well and type 3 poorly. In this region, the motion of the system is qualitatively the same in either equilibrium, and it heads southeast out of that region, into the region where $s = (1, 1, 0)$ is the unique equilibrium trading strategy profile. Once in the region where $s = (1, 1, 0)$ is the unique equilibrium, the system never escapes.

It is apparent from the above discussion that, starting from any initial θ, the population tends to move into the region where $s = (1, 1, 0)$ is the unique equilibrium trading strategy profile. In a large number of numerical experiments with the replicator dynamic, from any initial conditions, the system always converged to a unique steady state [as is the case in many applications in economics and biology; see Mailath (1992)]. Of course, this steady state of the dynamical system is the unique equilibrium population distribution of the economy where agents are allowed to choose their type. One reason that this is useful is that it provides a simple way of calculating an equilibrium. For general storage costs it is not possible to say very much analytically about the equilibrium value of θ; when the evolutionary process converges to a steady state, it provides a very simple way of calculating the equilibrium θ numerically.

Figs. 4a and 4b depict the evolution of the population according to the best response dynamic (8) for two values of c. The system with this specification for the evolutionary process can actually display some fairly complicated dynamical behavior, depending on storage costs and on the speed of adjustment parameter β; but a common occurrence is a three-cycle [reminiscent of the results of Gilboa and Matsui (1991)]. In fact, with the best response dynamic, the direction of adjustment depends on the payoffs but the size of the adjustment depends only on the parameter β. This implies that if a three-cycle exists, it must be one of the following two candidates:

A. (θ_2, θ_3) cycles between $(D\beta^2, D)$, $(D, D\beta)$, and $(D\beta, D\beta^2)$,

B. (θ_2, θ_3) cycles between $(D\beta^2, D\beta)$, $(D, D\beta^2)$, and $(D\beta, D)$,

where $D = 1/(1 + \beta + \beta^2)$.

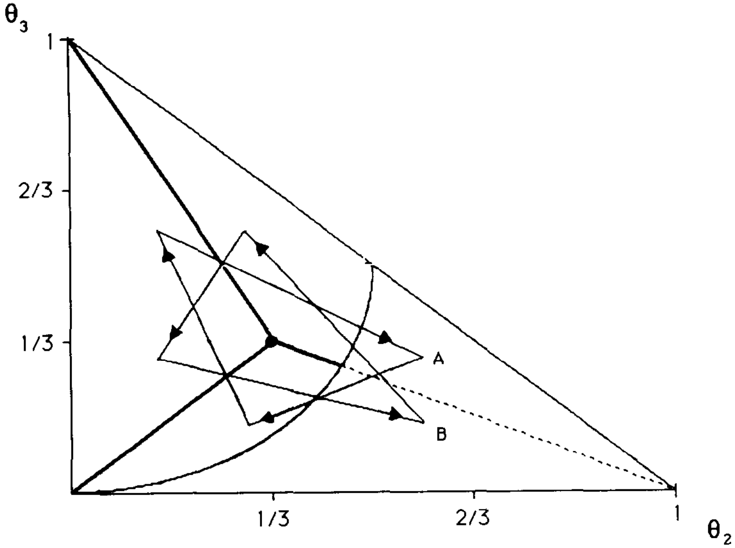

Fig. 4a. Best-response dynamic, $c = (0, 0, 0)$.

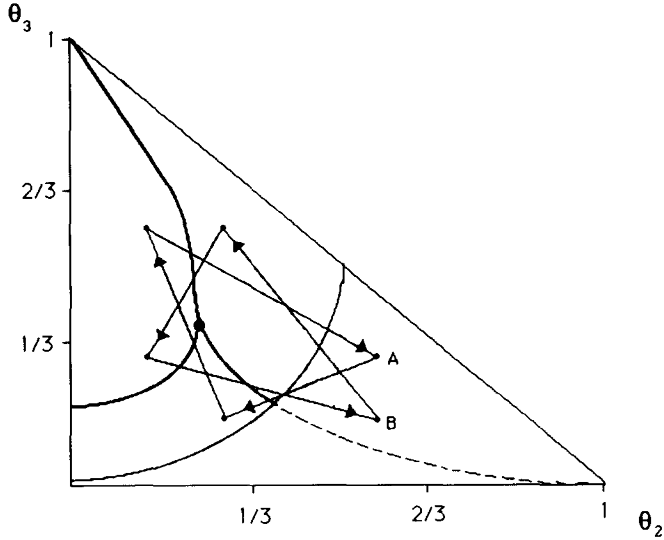

Fig. 4b. Best-response dynamic, $c = (0, 0.01, 0.02)$.

In case A, when $(\theta_2, \theta_3) = (D\beta^2, D)$, *if* W_2 is the maximum payoff, then next period the β newborn agents are all of type 2. This makes $(\theta_2, \theta_3) = (D, D\beta)$, and *if* W_1 is the maximum payoff, then next period the β newborn agents are all of type 1. This makes $(\theta_2, \theta_3) = (D\beta, D\beta^2)$, and *if* W_3 is the maximum payoff, then next period the β newborn agents are all of type 3. This takes the system back to where it started, $(\theta_2, \theta_3) = (D\beta^2, D)$, and it continues to cycle clockwise in the (θ_2, θ_3) plane. Whether this outcome is actually generated by the best response dynamic depends on whether parameter values are such that the correct W_i is the maximum payoff at each point on the cycle so as to move the system in the right direction. Case B is similar, except that the system cycles counterclockwise. The cycles trace triangles in the (θ_2, θ_3) plane around the steady state, as shown in fig. 4, where the size of the triangles depends only on β.[12]

In the limit when $c_j = 0$ for all j (and therefore, by continuity, when the c_j are close to 0), one can show that both three-cycles are implied by the best response dynamic, as shown in fig. 4. When the c_j are large, this depends on β. In numerical experiments, the following results were observed. When only one of the candidate three-cycles is implied by the best response dynamic, it is stable. When both candidate three-cycles are implied by the best response dynamic, both are locally stable: depending on where the system starts, it converges to one or the other. When neither of the candidate three-cycles is consistent with the best response dynamic, the system converges to a cycle of higher periodicity. It never settles down to the steady state. Of course, these features of the best response dynamic are due to its discontinuous nature, where *all* of the newborn agents copy the type that earned the maximum payoff last period, even if the maximum payoff was very close to the payoffs of the other types.

5. Summary and conclusion

In this paper, I have described a version of a framework that uses search theory to model the exchange process in such a way that it is possible to determine endogenously which objects circulate as commodity money. Previous analyses of related models were generalized to allow arbitrary distributions of types. This allows one to discuss how the outcome is influenced by not only the intrinsic properties of goods, like their storability, but also by the relative numbers of agents who supply and demand the different commodities. I then endogenized the distribution of types, which was shown to lead to a unique equilibrium. I also considered an evolutionary dynamic approach to the

[12] Notice that the cycles can actually take the system across regions, as shown in the figure; hence, the regime can switch – between $s = (1, 1, 0)$ and $s = (0, 1, 0)$, in this case. For β closer to 0, the cycles shrink, and the system remains in the region where $s = (1, 1, 0)$.

determination of the population distribution. The outcome of the evolutionary process sometimes depended on the specification, and some fairly complicated outcomes were possible. When convergent, the evolutionary approach can be thought of as a way to compute the equilibrium population distribution.

Both the class of models encompassing the specific example studied and the techniques employed are more general than it might appear. Many varieties of search-based monetary theory can be constructed and applied (see the references in the introduction). The evolutionary approach can be used in these models as a method of computing equilibria (as I have done here), of selecting equilibria when multiplicities exist [as emphasized in Matsuyama, Kiyotaki, and Matsui (1993)], and of studying learning, bounded rationality, and related phenomena [as Marimon, McGrattan, and Sargent (1990) do using an artificial intelligence approach]. I adopted a particular formulation to illustrate the basic ideas, but there are many places where alternative modeling choices could have been made. For instance, it may be more natural or interesting to apply the evolutionary approach to trading strategies rather than (or in addition to) the choice of types. The goals of this paper were simply to demonstrate some possibilities and to suggest potential future research topics.

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