Liquidity, Money Creation and Destruction, and the Returns to Banking*

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We build on our earlier model of money in which bank liabilities circulate as medium of exchange. We investigate optimal bank behavior and the resulting provision of liquidity under a range of central-bank regulations. In our model, banks issue inside money under fractional reserves, facing the possibility of excess redemptions. Banks consider the float resulting from money creation and make reserve-management decisions which affect aggregate liquidity conditions. Numerical examples demonstrate positive bank failure rates when returns to banking are low. Central-bank regulations may improve banks' returns and welfare through a reduction in bank failure.

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Running Head: Money Creation and Banking

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1. Introduction

In this paper, we build on Cavalcanti, Erosa, and Temzelides (CET, 1999). There, we developed a model of private money issue and redemption in the random-matching environment of Kiyotaki and Wright (1989). This was accomplished by assuming that a fraction of the population, the banks, are no longer anonymous. They are subject to monitoring and are able to issue substitutes to government currency, or outside money. Our earlier work demonstrated that such a private monetary system can be self-sustaining. In particular, we showed that, under certain conditions, a "law of reflux" can discipline note issue, thus, preventing inflationary overissue by banks. While our earlier work demonstrated the existence of a stationary equilibrium implying a stable private monetary system, we did not investigate the precise properties of the banks' policy rules, nor how such policies would respond to regulation imposed, for example, by a central bank. This is a necessary step towards investigating the implications of various government policies for equilibrium bank failure and welfare.

Our current study is influenced by two models of private money that built on extensions of CET. Cavalcanti and Wallace (1999b) study a model in which banks are perfectly monitored, and there is no concern about how bank returns are related to liquidity conditions.[2] Moreover, their focus on optimal allocations prevents them from studying bank failure and its consequences for the supply of money. Wallace and Zhu (WZ, 2003) present a model in which banks are again prevented from failing by assumption. They call attention to the fact that banks may choose to avoid note issue in trading opportunities with the government, or the central bank, if the expected float of their notes in these "markets" is too low. Their result is derived under

[2] A form of imperfect monitoring appears in Cavalcanti and Wallace (1999a), but bank problems there are not sufficiently severe so as to affect the supply of liquidity.

general money holdings that would prevent the computation of steady states with illiquid banks, a topic that we pursue here. Moreover, banks never meet with others banks in WZ, and note issue in low-float opportunities never occurs in equilibrium. In CET, by contrast, banks meet other banks without knowing their identity, thus, they are not able to distinguish between diverse trade opportunities. Here, we demonstrate that this assumption in CET can be easily dropped. As a result, our simulations allow us to concentrate on the general equilibrium implications of note issue.[3] If a bank meets another bank and issues a note, it is common knowledge that this note will be redeemed instantly. Thus, notes enter circulation only if they are issued to non-bank producers. Indeed, in some computed examples we find that illiquid banks do not issue to other banks, but issue to non-banks in order to maximize the payoff from float. Our general approach differs from existing papers in many ways.[4] Perhaps the most important difference is that random float opportunities, together with random opportunities to build reserves, give rise to a non-trivial reserve management problem for banks. We concentrate in studying the effects of various regulations on reserve management and, thus, on banks' optimal policy rules.

Another extension that we borrow from WZ is the payment of interests on bank reserves in units of a common good with linear utility. Since the extension of a common good affecting bank's payoffs also appears in other recent papers in this literature (see also He, Huang, and Wright, 2003), we find it useful to explore some consequences of this linearity in our setup. In particular, we show that, by preventing banks from becoming illiquid, WZ also eliminate the need to track down the float of notes. We demonstrate this by duplicating the same bank behavior as in WZ within an economy in which the central bank pays an interest proportional to money holdings in

[3] In CET the decision to issue a note did depend on the risk of failure associated with the issuer's net worth.

[4] See also Cavalcanti (2004) and Williansom (1999).

accordance with the expected return from float in WZ.

Our analysis proceeds as follows. In Section 2 we describe the general economic environment. In Section 3 we impose bank liquidity (100 percent reserves) and demonstrate the irrelevance of float. In Section 4, we present the existence argument and a description of banks' optimal policy rules. We emphasize that this result extends the main theorem in CET (1999) in that here we demonstrate the existence of a stationary equilibrium with illiquid banking, which implies that a fraction of banks fail in each period. We characterize the steady state policy rules for banks and demonstrate that banks that are "sufficiently liquid" will always issue a note in order to consume, and might decline possibilities to build costly reserves. Somewhat surprisingly, the same behavior is shown to be optimal for banks that are sufficiently "illiquid." In other words, if a bank's notes in circulation far exceed its reserves, the bank will have to exit with high probability in the near future. We demonstrate that building costly reserves will not benefit such banks; they will only issue notes up to the point when they exit. We term such banks "wildcats."

Our theoretical results guide several computational experiments that we perform, in Section 5, in order to investigate the effects of various regulating polices on bank optimal behavior, probability of failure, and welfare. A variety of regulations result in different levels of money creation by banks. Overall, we find support for the view that a system of privately created liquidity can be self-stabilizing. For example, in one configuration of our model, we find that banks may stop creating money despite its private benefits in the short run. We also verify in simulations that in certain, if special, circumstances banks may give up trying to reverse their illiquid position. Such banks focus on short-run returns only and eventually fail. Finally, we show that, in a case of a liquidity shortage, some infusion of reserves by a central bank may lead to an increase in trade and a reduction of bank failure. Finally, Section 6 concludes. The Appendix contains many of the proofs.

2. The Model

Time is discrete and there are k perishable goods per date. Goods are indivisible. A measure-one continuum of individuals inhabit the economy, and they produce and consume either 0 or 1 unit of a good per date. The common discount factor is $\beta \in (0,1)$. People specialize in consumption and production of goods. Individuals of type s consume only good s and produce only good $s+1$, modulo k. People cannot commit to future actions, and their histories are to some extent, to be made precise below, private. Individuals meet randomly in pairs once per period, and the probability of meeting with a relevant consumer is the same as meeting with a relevant producer: $\frac{1}{k}$, independent of s. As is standard, we assume that $k > 2$, so that barter is not possible. We study only steady states with symmetric allocations with respect to s. Consumption gives an instantaneous utility u, and production gives a disutility e, with $u > e$.

The above environment can be used to derive a role for outside money. That is done by endowing a fraction (of each type) of the population with one unit of fiat money, and by assuming that money is durable but indivisible, and that holdings of money are restricted to either 0 or 1. Here, we divide the population of each type into two sets: the banks and the non-banks. We assume that banks can hold money in the form of reserves, r, taking values in the set of integers $\{0, 1, 2, ..., N\}$. We assume that banks can issue money (notes) identified by the name of the issuer. The assumption that notes are indexed by the identity of the issuer gives rise to a reserve-management problem, as a stochastic process resulting from the random trades governs the float of such notes. As in CET, banks build reserves by receiving in trade a note issued by another bank. When they do so, the reserve balance of the issuer is reduced by one unit, and the reserve balance of the receiving bank, who deposits the note with a fictitious central bank, is increased by one unit. We assume that all notes are treated the same way by the non-bank population. The state of a bank at the start of a period is (r, m), where r is total reserves and m is the total number of notes in the hands of non-banks.

We assume that there is constant inflow of new individuals in the population, both banks and non-banks, in addition to a constant outflow. That is accomplished by the simple assumption that, with probability $\delta \in (0,1)$, an individual dies and is replaced by another individual of the same type. Notes in circulation issued by a bank that dies remain valid for deposits with the central bank. A note held by a non-bank that dies is cleared by the central bank, so that δ imposes a lower bound on the probability of redemption π. The effective discount factor, after death rates are taken into account, is $\beta = (1-\delta)\tilde{\beta} \in (0,1)$, where $\tilde{\beta}$ is the original discount rate. Unlike CET, we assume that banks know when their trade partner is a bank (who deposits all notes immediately).

We let p_0^n denote the measure of non-banks holding 0 notes, divided by k, and let p_1^n denote the measure of non-banks holding 1 note of some bank, also divided by k. Also, we let $p_{r,m}^b$ denote the measure of banks holding r units of reserves and having m notes in circulation in the beginning of the first sub-period, divided by k. A monetary steady state requires that these measures are time invariant and that non-banks trade for notes. For the moment, we let $\phi_{r,m}^n$ denote the probability that banks choose, as a contingent strategy, to issue a note to a non-bank in exchange for consumption, and let $\gamma_{r,m}$ denote that probability of acquiring a note in exchange for production.[5] The non-bank values are given by

[5] In contrast to WZ, we allow banks to issue notes to other banks. in addition, in contrast to CET, we allow banks to condition the decision of whether to issue a note on the type of the producer in the meeting (either a non-bank, superscript n, or a bank, superscript b).

$$v_1^n = \beta v_1^n + (p_0^n + \sum_{r,m} p_{r,m}^b \phi_{r,m}^n)[u + \beta(v_0^n - v_1^n)] \tag{2.1}$$

and

$$v_0^n = \beta v_0^n + (p_1^n + \sum_{r,m} p_{r,m}^b \gamma_{r,m})[-e + \beta(v_1^n - v_0^n)], \tag{2.2}$$

where v_1^n is the discounted expected utility of starting with one note, and v_0^n is that of starting without a note. In contrast to CET, we allow banks with unfavorable reserves to avoid note issue in meetings with other banks. We use numerical methods and phase diagrams to illustrate bank behavior in the (r, m)-space. In the computed examples, some banks with $r < m$ do not issue to other banks, but issue to non-banks in order to maximize the payoff from float. Banks in CET are allowed to issue as many notes as they want, thus, they can become illiquid. Since the float of notes is affected by bank policies, general equilibrium effects link bank behavior to float averages and back to bank behavior.[6] Here we explore these possibilities and we investigate how the parameters of the model, including policies injecting money into private-banks' reserves, interact through such general equilibrium effects.

Following WZ, we introduce a general good used to pay interest on reserves at rate R. This good is also used in tax collection in order to finance these payments. We assume that the utility of consuming general goods (earning interest) and the disutility of producing general goods (paying taxes) are linear (with slope one). Hence, with lump-sum taxation, it suffices to subtract from the equilibrium value functions, when $R > 0$, a constant that equals the total interest paid to

[6] Later we consider a central-bank policy of paying interests on bank reserves. Float, however, need not result from interest payments on reserves. It could also result from the attempt to reach a balance between a fractional-reserve policy, with good short-run consumption opportunities, and a tolerable risk of bank failure.

the bank sector. For simplicity of the presentation, we ignore this constant (the tax rate) from our Bellman equations, although we make the necessary adjustments when computing welfare measures.

There exists a "central bank" that shuts down banks that experience a number of redemptions that exceeds their reserves. Any notes issued by a failed bank that are still in circulation remain valid and are accepted in trade. The central bank "honors" these liabilities by accepting them as deposits of reserves from any other bank. A failed bank is, however, prohibited from issuing new notes. Hence all notes in circulation are valid notes, and they never pose a risk to non-banks. In order to produce the simplifying result that banks always deposit notes with the central bank, even if $R = 0$, we assume that a failed bank exits the banking industry with a valid note and continues to trade as a non-bank. Hence, depositing notes is a dominant strategy for active banks, even in the absence of interest payments on reserves.

We denote the bank expected discounted utility, before taxes, by $v^b_{r,m}$ ($w^j_{r,m}$ just before redemptions are computed). Their present value is zero when dead. The sequence of events for within a period is displayed below.

$t \rightarrow$ possibility of death $\rightarrow v^n_i$, $v^b_{r,m} \rightarrow$ meetings $\rightarrow$ trade and deposits $\rightarrow w^j_{r',m'} \rightarrow$ reserves updated and interest paid $t+1$

Sequence of Events

We let $p^b_{r,m}$ denote the measure of banks in state (r, m) before the trade meetings occur. Due to discounting, spending money is a dominant strategy for non-bank consumers. Thus, the only relevant decision for non-banks refers to the acceptance of money in exchange for production. Clearly, a non-bank producer in state $i = 0$ engages in trade whenever $\beta(v^n_1 - v^n_0) > e$. We shall restrict parameters so that this is indeed the case in monetary steady states.

When banks in state (r, m) are in a meeting where they can produce, they improve their reserve position by accepting money in exchange for production with probability $\gamma_{r,m} \in [0, 1]$.

Notice that while non-banks always agree to exchange money for consumption, that decision is nontrivial for banks. We shall see that a concern about building reserves can induce banks to behave differently. When deciding whether to issue a note, in exchange for consumption, their behavior may depend on their reserve position and also on whether the recipient of the note is a bank or a non-bank. We let $\phi^n_{r,m}$ and $\phi^b_{r,m}$ denote the probabilities that a bank in state (r, m) issues a note to a non-bank and to a bank, respectively.

If a bank issues a new note to a recipient that is also a bank, the fact that the new note is deposited and does not enter circulation is marked by adding to the bank state, before redemptions are computed, the indicator $j = 0$. If the new note does enter circulation, because it has been issued to a non-bank, the indicator is $j = 1$. After reserve balances are updated, banks know the quantity of notes that remain in circulation. Taking into account that banks that reach a negative reserve balance exit the industry with one note, the current $w^j_{r,m}$ depends on next period values according to

(2.3)

$$w^j_{r,m} = rR + \beta \sum_{i \leq r} \binom{m-j}{i} \pi^i (1-\pi)^{m-i-j} v^b_{r-i,m-i} + \beta \sum_{i > r} \binom{m-j}{i} \pi^i (1-\pi)^{m-i-j} v^n_1,$$

with the understanding that the sum is taken with respect to i, with redemption values ranging from 0 to $m - j$. Since the number of banks willing to create money in meetings with other banks is given by the inner product $\phi^b \cdot p^b$, it follows that the fraction of the population willing to purchase from a bank is $\phi^b \cdot p^b + p^n_1$. Thus, a bank's value is given by

$$\begin{aligned} v^b_{r,m} &= w^0_{r,m} + (\phi^b \cdot p^b + p^n_1) \max_{\gamma_{r,m}} \gamma_{r,m}(-e + w^0_{r+1,m} - w^0_{r,m}) + \\ &\quad p^n_0 \max_{\phi^n_{r,m}} \phi^n_{r,m}(u + w^1_{r,m+1} - w^0_{r,m}) + \\ &\quad \gamma \cdot p^b \max_{\phi^b_{r,n}} \phi^b_{r,m}(u + w^0_{r-1,m} - w^0_{r,m}). \end{aligned}$$

We now turn to the supply of money. There are two sources of money injections. The first source is exogenous. We assume that, in every period, a fraction, μ_n, of the newborns are non-banks receiving one monetary unit. Hence, $\mu_n \delta$ equals the inflow measure of outside assets. The second source is endogenous. We assume that a fraction μ_b of the newborns, with $\mu_b < 1 - \mu_n$, are each endowed with a banking technology. The stationary measure of potential banks is μ_b, and that of non-banks is $1 - \mu_b$. Besides the newborns starting with outside money or with the banking technology, there exists a fraction of non-bank newborns, $1 - \mu_n - \mu_b$, starting with zero monetary units. We shall restrict attention to outcomes in which private money is treated as a perfect substitute to outside money in all trades, and we use our notation accordingly. In particular, there is no need to decompose the state of a non-bank, i, into holdings of either outside or inside money.

The final aspect of bank regulation is the assignment of initial states to newborn banks. Let us suppose that all newborn banks are assigned $(r, m) = (\bar{r}, 0)$ as an initial state. Consider for the moment an arbitrary cut-off reserve level, r^*, for determining failure. If $R = 0$, it is clear that the same banking behavior should follow from assigning $(\bar{r} - r^*, 0)$ as an initial state and changing the cut-off level to zero. Hence, with respect to policies that keep $R = 0$, we can, without loss of generality, fix the lower bound for reserves as $r^* = 0$ and consider different choices of initial balances $\bar{r}$. We do so assuming that the state of a newborn bank is $(\bar{r}, 0)$. The distributions (p^n, p^b) must remain invariant, given the interest-rate policy, the money-injection parameters, the bank strategies, and the fact that non-banks accept money in trade. The restrictions imposed by stationarity are described in the Appendix by means of an operator mapping a current distribution into one for the next period. An invariant distribution is a fixed point of that operator. Statistics of interest include the probability of redemption, π, and the measure of bank failure per period, z. We compute these later in several numerical examples.

In summary, the money supply is endogenous and depends on banking regulation and monetary policy. Monetary policy is characterized in the model by μ_n, describing the distribution of outside money given to non-bank newborns, by $(\mu_b, \bar{r})$, describing the entry rate and reserve position of new banks, and by R, describing the interest rate on reserves.

In a steady state, banks and non-banks make their consumption and savings decisions taking as given the policy parameters and the variables p^n, p^b and π, as well as the decisions of others. Here, we discuss how these variables are determined in equilibrium. We let T_{γ,ϕ^n,ϕ^b} denote the continuous mapping, implied by any given list of strategies (γ, ϕ^n, ϕ^b), which maps (p^n, p^b) into the next period's distribution of individuals across states (see the Appendix). We conclude this section by defining steady states as follows.

Definition 2.1. *A monetary steady-state equilibrium is an array* $(v^n, v^b, w, \gamma, \phi^n, \phi^b, p^n, p^b, \pi)$ *satisfying the Bellman equations (2.1)-(2.2), with* $\phi = \phi^n$*, as well as (2.3)–(2.4), and such that* $\beta(v_1^n - v_0^n) > e$*,* (γ, ϕ^n, ϕ^b) *attains the maximum in (2.4),* (p^n, p^b) *is a fixed point of* T_{γ,ϕ^n,ϕ^b}*, and* $\pi = \delta + (1-\delta)\gamma \cdot p^b$.

Regarding the last equilibrium condition, we note the following. Inside money held by non-banks is redeemed upon death. It is also redeemed when non-banks survive and trade in meetings with bank producers, an event occurring with probability $(1-\delta)\gamma \cdot p^b$. As a result, $\pi = \delta + (1-\delta)\gamma \cdot p^b$.

3. The Irrelevance of Float under Liquid Banking

Before we discuss the existence of steady states in the general case, we consider briefly a special case of our model where banks are not allowed to issue more notes than their current level of reserves. We find it instructive to demonstrate that this economy, in which banks cannot become illiquid, is equivalent to an economy in which the duration of note circulation is ignored. This result is a consequence of the linearity of bank payoffs in the interest rate, R. The alternative economy that we construct looks very similar to one under a gold standard. We proceed by restricting banks' states to the set $Z = \{(r, m) \in \mathbb{N} \times \mathbb{N} : r \geq m, 0 \leq r \leq N\}$ (banks cannot become illiquid, say due to the existence of a sufficiently high penalty if they are caught with negative reserves).

Proposition 3.1. *There exists a unique value function v^b solving the Bellman equation for banks. Moreover, for i such that $v^b_{r+i,m+i}$ is well defined, v^b satisfies the following linearity condition: $v^b_{r+i,m+i} = v^b_{r,m} + Ai$, where $A = \frac{R}{1-\beta(1-\pi)}$ is a positive constant.*

Proof. Let V^b be the set of functions $v^b : Z \to [0, (u+RN)/(1-\beta)]$ with $v^b_{r+i,m+i} = v^b_{r,m} + Ai$. We want to show that the Bellman equations defining the value of banks define an operator that maps functions in V^b into V^b. Start with $v^b \in V^b$. From the linearity on v^b, it follows that

$$w^j_{r,m} = rR + \beta v^b_{r,m} - \beta A f(m - j), \tag{3.1}$$

where $f(m)$ denotes the mean of the binomial distribution over $m - j$ possible events with probability of success π (so that $f(m - j) = \pi \times (m - j)$). Now consider the operator T^b defined by the Bellman Equation for banks. We want to show that $T^b v^b_{r+i,m+i} - T^b v^b_{r,m} = Ai$. Since T^b is the sum of four terms, we can write $T^b v^b_{r+i,m+i} - T^b v^b_{r,m}$ as the sum of four terms. It suffices to show that each of these terms is a linear function of i. Adding up the four terms, and after some algebra, we obtain $T^b v^b_{r+i,m+i} - T^b v^b_{r,m} = i\{R + \beta A(1 - \pi)\} = iA$, with $A = \frac{R}{1-\beta(1-\pi)}$. We conclude that $T^b v^b$ satisfies the linearity property. The uniqueness of v^b follows from the fact that the Bellman equation for banks satisfies the monotonicity and discounting properties, Blackwell's sufficient conditions for a contraction. QED

Next, we demonstrate that an economy with float but in which banks are not allowed to become illiquid is equivalent to an outside money economy with no float of notes. This finding illustrates that float does not imply the presence of inside money. Instead, float is a way of increasing banks' returns due to an implied interest on reserves. Returns on float are decreasing on the redemption probability and increasing on the interest paid on reserves.

We proceed as follows. Instead of using a pair (r, m) in order to denote the state of a bank, let us now consider a regulation or central-bank policy that attaches to each bank a single state variable, a natural number g. The state g represents net holdings (or wealth) and is equivalent to the difference $r - m$ in the original formulation. In order to induce the same behavior in random meetings as before, we assume that in the second sub-period R units of the common good are paid per unit of holdings, with gR being paid in total. We assume that when money is spent in the first sub-period the balance g is reduced by one unit, and when money is accepted the balance g is increased by one unit. It is clear that the proposed simplification treats all money use as if it generated no float, that is, as if all notes issued are redeemed in the same period. We compensate this absence of float with a payment of $\bar{R}$ units of the second sub-period good whenever money is paid out to a non-bank (whenever notes are put in circulation). When money is paid to a bank, however, the issuing bank is not paid $\bar{R}$ because, in this case, money is not put into circulation in the corresponding float economy. By choosing $\bar{R}$ according to the expected forgone interest payments on float,

$$\bar{R} = R \sum_{i=0}^{\infty} \beta^{i+1} (1 - \pi)^i, \tag{3.2}$$

we are able to generate the same bank decisions.

We formalize this outside-money economy as follows. With a certain abuse of notation, the Bellman equation for non-banks remain the same, except that the pair (r, m) is replaced by the scalar g as the bank state. The Bellman equation for banks is now

$$\begin{aligned} v_g^b &= \beta v_g^b + gR + p_0^n \max_{\phi_g^n} \phi_g^n [\bar{R} + u + \beta(v_{g-1}^b - v_g^b)] + \\ &(\phi^b p^b + p_1^n) \max_{\gamma_g} \gamma_g [R - e + \beta(v_{g+1}^b - v_g^b)] + \\ &\gamma p^b \max_{\phi_g^b} \phi_g^b [u - R + \beta(v_{g-1}^b - v_g^b)], \end{aligned} \tag{3.3}$$

with the understanding that $\phi_0 = 0$.

Thus, in this outside-money version, there is no float. The proposition relating this no-float, outside-money economy to the float economy, provided that $(p_0^n, p_1^n, \phi^b.p^b, \gamma.p^b)$ does not change, is as follows.

Proposition 3.2. *If $(\phi_{r,m}, \gamma_{r,m})$ is an optimal strategy in the economy with float, then $(\phi_g, \gamma_g) \equiv (\phi_{g,0}, \gamma_{g,0})$ is an optimal strategy in the economy without float.*

Proof. The Bellman equation for banks with $m = 0$ in the float economy is given by

$$\begin{aligned} v_{r,0}^b &= \beta v_{r,0}^b + rR + (\phi^b.p^b + p_1^n) \max_{\gamma_{r,0}} \gamma_{r,0} [R - e + \beta(v_{r+1,0}^b - v_{r,0}^b)] + \\ &p_0^n \max_{\phi_{r,0}^n} \phi_{r,0}^n [u + \beta(v_{r,1}^b - v_{r,0}^b)] + \gamma.p^b \max_{\phi_{r,0}^b} \phi_{r,0}^b [u - R + \beta(v_{r-1,0}^b - v_{r,0}^b)]. \end{aligned} \tag{3.4}$$

By Proposition 3.1, $v_{r,1}^b - v_{r-1,0}^b$ is equal to $A = \frac{R}{1-\beta(1-\pi)}$ (for all r). Note that $\bar{R} = R \sum_{i=0}^{\infty} \beta^{i+1} (1-\pi)^i = \beta \frac{R}{1-\beta(1-\pi)} = \beta A = \beta(v_{r,1}^b - v_{r-1,0}^b)$. The result then follows by substituting $v_{r,1}^b = v_{r-1,0}^b + \bar{R}/\beta$ in the above expression, and by setting $v_g^b = v_{g,0}^b$ to obtain the Bellman equation of the economy without float. QED

To summarize, the main implication of this section is that a float economy, with an interest-rate on reserves, R, can be duplicated by a no-float economy with a corresponding interest rate on money holdings, R, which increases to $R + \bar{R}$ in the last holding period if money is paid to a non-bank. Thus, the no-float economy loses some of its inside-money interpretation, except that for computing $\bar{R}$ it is necessary to know π, the exogenous redemption probability. The duplication requires setting $p_g^b = \sum_m p_{m+g,m}^b$, since p_g^b must be an invariant distribution implied by (ϕ_g, γ_g) if $p_{r,m}^b$ is an invariant distribution implied by $(\phi_{r,m}, \gamma_{r,m})$. With this construction, (p_0^n, p_1^n) is the same across the two economies. Also, since banks decisions do not change, $(\phi^b.p^b, \gamma.p^b)$ are the same across the float and non-float economies.

4. Existence and Steady-State Bank Policies

In every period there is a constant inflow of newborn non-banks without money, of measure $\delta(1-\mu_n-\mu_b) > 0$. This fact can be used to derive a condition (see Lemma 4.1 below) relating β, u/e, k and $1-\mu_b-\mu_n$, so that $\beta(v_1^n - v_0^n) > e$ holds for any fixed point (p^n, p^b). This condition suffices to produce a degree of scarcity of money so that money has value independently of bank behavior, provided that a stationary distribution of banks exists. we have the following.

Lemma 4.1. *If* $\left(\frac{\delta}{\delta+\mu_b}\right)\left(\frac{1-\mu_n-\mu_b}{k}\right) \geq \left(\frac{1}{\beta}-1\right)\frac{1}{\frac{u}{e}-1}$, *then* $\beta(v_1^n - v_0^n) \geq e$.

Proof. Non-banks accept money if $\beta(v_1^n - v_0^n) > e$. Using (2.1) and(2.2) we obtain

$$\beta(v_1^n - v_0^n) = \beta\left(\frac{pu+qe}{1-\beta(1-p-q)}\right) > e \text{ if } p > \left(\frac{1}{\beta}-1\right)\frac{1}{\frac{u}{e}-1}, \tag{4.1}$$

where $p = p_0^n + \sum_{r,m} p_{r,m}^b \gamma_{r,m}$, and $q = p_1^n + \sum_{r,m} p_{r,m}^b \phi_{r,m}$. In steady state, $p_0^n = p_0^n + (p_0^n + p^b\cdot\gamma^b)p_1^n - (p_1^n + p^b\cdot\phi^b)p_0^n + \frac{\delta}{k}(1-\mu_b-\mu_c)$. Solving for p_0^n we obtain $p_0^n = \frac{p^b\cdot\gamma^b p_1^n}{\delta+p^b\cdot\phi^b} + \left(\frac{\delta}{\delta+p^b\cdot\phi^b}\right)\left(\frac{1-\mu_n-\mu_b}{k}\right) > \left(\frac{\delta}{\delta+\mu_b}\right)\left(\frac{1-\mu_n-\mu_b}{k}\right)$, where the last inequality is obtained using that $\mu_b > p^b\cdot\phi^b$. As a result, we can write $p = p_0^n + p^b\cdot\gamma^b > p_0^n > \left(\frac{\delta}{\delta+\mu_b}\right)\left(\frac{1-\mu_n-\mu_b}{k}\right)$. Combining this expression with (4.1), we obtain that a sufficient condition for non-banks to accept money is $\left(\frac{\delta}{\delta+\mu_b}\right)\left(\frac{1-\mu_n-\mu_b}{k}\right) > \left(\frac{1}{\beta}-1\right)\frac{1}{\frac{u}{e}-1}$, which holds true for β close to 1. QED

Our argument on the existence of a monetary equilibrium with banking proceeds by following similar steps as in CET. We restrict attention to the case in which every newborn bank starts with

$\bar{r} = 0$ reserves.[7] We first restrict non-banks to accept money (as stated in the Bellman equations for non-banks). We then construct a correspondence, ψ, producing new steady-state candidates from any given array $(v^n, v^b, w, \gamma, \phi^n, \phi^b, p^n, p^b, \pi)$ as follows. We determine new lists for (γ, ϕ^n, ϕ^b) according to the correspondence of strategies that attains the maximum in the right-hand side of Bellman equations, given the old list $(v^n, v^b, w, \gamma, \phi^n, \phi^b, p^n, p^b, \pi)$. The maximizations on the Bellman equation also give a new triple (v^n, v^b, w) as the left-hand side. Next, the new measures over states are given by T_{γ,ϕ^n,ϕ^b} applied on (p^n, p^b). Finally, π is computed according to the formula $\pi = \delta + (1-\delta)\gamma \cdot p^b$. The existence of a restricted steady-state equilibrium ($\alpha = 1$) then follows since ψ satisfies the conditions of Kakutani's fixed-point theorem.[8]

Next, we argue that the restriction $\alpha = 1$ is nonbinding when β is sufficiently large. To an individual non-bank, the fact that non-banks without money, as well as newborn banks, are willing to accept money, implies a lower bound on $(p_0^n + p^b \cdot \gamma)$. Thus, if β is sufficiently high, the restriction $\beta(v_1^n - v_0^n) > e$ is non-binding (see Lemma 4.1).This completes the argument for the existence of a monetary steady state.

Proposition 4.2. *If β is sufficiently close to one, there exists a monetary steady state for the economy with banking.*

We now argue that the monetary steady state features bank production in equilibrium (banks accumulate reserves). To this end, we restrict banks to back note issue by 100 percent reserves on a range of the state space indexed by a constant S (banks in this range are also restricted to

[7] Some numerical experiments use different specifications. we specify these alternatives later.

[8] We assume an upper bound (N, N) on the state space for banks, so that this space becomes compact. Since $\delta > 0$, the measure of banks who survive until they reach a state outside this upper bound can be made arbitrarily small by the choice of a sufficiently large N. Hence, this compactness restriction is innocuous.

accept money). More precisely, the restrictions on banks require: $\gamma_{r,r} = 1$, and $\phi^b_{r,r} = \phi^n_{r,r} = 0$, for $r \leq S$. Non-restricted banks choose their strategies optimally. We can then follow the argument in the proof of Proposition 4.2 to construct a correspondence ψ that satisfies the conditions of Kakutani's fixed-point theorem. The existence of a restricted steady-state equilibrium follows. Next, we argue that the restrictions are nonbinding when β is sufficiently large. To an individual non-bank, the fact that other non-banks without money, as well as newborn banks, are willing to accept money with probability one implies a lower bound on $(p^n_0 + p^b \cdot \gamma)$. Thus, if β is sufficiently high, the restriction that $\beta(v^n_1 - v^n_0) > e$ is nonbinding (see also Lemma 4.1). For β sufficiently large, the restrictions for banks are also nonbinding. Notice that if $\phi^b_{r,r} = 0$ and $\phi^n_{r,r} = 0$ are nonbinding for $r \leq S$, then $\gamma_{r,r} = 1$ is also nonbinding for $r \leq S$; otherwise, banks would be choosing a form of autarky after reaching a level of reserves $r \leq S$. We now show that $\phi^b_{r,r} = \phi^n_{r,r} = 0$ is indeed nonbinding for $r \leq S$. It suffices to show that $v^b_{0,0} - v^n_1$ can be made arbitrarily large, by picking β sufficiently close to one in a restricted steady state. If $\phi^b_{r,r} = 1$, a bank faces, with probability 1, the possibility of losing $v^b_{0,0} - v^n_1$. The relevant case, thus, concerns the case where a bank consumer meets a non-bank producer.

If $\phi_{r,r} = 1$, a bank faces, with positive probability, the possibility of losing $v^b_{0,0} - v^n_1$ after one period in the event of excess redemptions. From the Bellman equation for banks, we know that $\phi^n_{r,r} = 0$ is implied if $u + w^1_{r,r+1} < w^0_{r,r}$ holds or, equivalently, if $u < w^0_{r,r} - w^0_{r,r+1}$. Now, $w^0_{r,r} - w^0_{r,r+1}$ is bounded below by $\beta\pi^r(1 - q_b)(v^b_{0,0} - v^n_1)$, where π^r equals the probability that all notes are redeemed and $(1 - q_b)$ represents the probability that the bank cannot increase reserves in the next period ($q^b = p^n_1 + \phi^b \cdot p^b$). Thus, $\phi^b_{r,r} = \phi^n_{r,r} = 0$ if $u < \beta\, \pi^r(1 - q_b)(v^b_{0,0} - v^n_1)$, for all $r \leq S$. Since $\pi^r \geq \delta^r > 0$ and $1 - q_b > \frac{(k-1)}{k}(1 - \mu_n)\delta > 0$, the conclusion follows if $v^b_{0,0} - v^n_1$ is sufficiently large. The same argument as in CET (p. 941) can now be applied to show that

$v^b_{0,0} - v^n_1 \to \infty$ as $\beta \to 1$ (notice that if $R > 0$ the argument in CET still applies since the penalty for being caught with negative reserves is even larger). This completes the existence argument with positive bank production. Thus, we have the following:

Proposition 4.3. *If β is sufficiently close to one, there exists a monetary steady state with positive bank production.*

We emphasize that because banks earn interest on reserves it is not true in general that they will be willing to issue notes in a monetary equilibrium. We therefore need a condition for existence of a monetary equilibrium with positive note issue.

Corollary 4.4. *If $u > \frac{R}{1-\beta}$ banks issue a positive amount of notes in equilibrium.*

Proof. Assume, by way of contradiction, that banks do not issue notes in equilibrium. Thus, banks never become illiquid and the only payoff resulting from holding reserves is the discounted flow of interest rates. The value functions must satisfy: $v^b_{r+i,m} = v^b_{r,m} + Bi$, where $B = \frac{R}{1-\beta}$, for $i \geq 0$. Now consider a bank in state $(r, 0)$, for some $r > 0$. The bank is willing to issue a note to another bank (that is, $\phi^b_{r,0} = 1$) if $u + w^0_{r+1,0} - w^0_{r,0} > 0$. Now, $u + w^0_{r+1,0} - w^0_{r,0} = u - R + \beta(v^b_{r-1,0} - v^b_{r,0}) = u - R - \beta R/(1-\beta) > 0$ iff $u > \frac{R}{1-\beta}$. It follows that $\phi^b_{r,0} = 1$ when $u > \frac{R}{1-\beta}$, contradicting that banks do not issue notes in equilibrium (notice that $\phi^b_{r,0} = 1$ also implies that $\phi^n_{r,0} = 1$, since banks always prefer to issue to a non-bank). QED.

To show existence of a monetary steady state with note issue and illiquid banking, fix β so that the condition in Lemma 4.1 holds and assume that $u > \frac{R}{1-\beta}$. Now consider economies that differ in N. For fixed β, if N is sufficiently large, there are some banks with sufficiently high reserves which choose to issue notes when $m = r$, thus becoming illiquid. This is because, for m sufficiently high, the probability that all m notes are cleared at the same time becomes arbitrarily low. Notice that since the condition stated in Lemma 4.1 does not depend on N, we can safely conclude that there exists a monetary equilibrium with illiquid banks provided N is sufficiently large. We have proved the following.

Proposition 4.5. *For a fixed β satisfying the condition in Lemma 4.1 and $u > \frac{R}{1-\beta}$, if the upper bound (N, N) on banks is sufficiently large (N goes to infinity), then the monetary equilibrium features a positive mass of illiquid banks.*

Next, we describe optimal strategies in any given monetary steady-state equilibrium with $R = 0$. We first establish some properties of the value functions and of the banks' strategies. These properties are of independent interest but they are also used in guiding the computational experiments that we perform in the next Section. We start with a sequence of preliminary Lemmas. The proofs appear in the Appendix. The next Lemma asserts that the value function of a bank is increasing in the amount of reserves and decreasing in the amount of liabilities in circulation.

Lemma 4.6. *The function $v^b_{r,m}$ is weakly increasing in r and weakly decreasing in m.*

Our next Lemma asserts that the value of a bank is always greater than the value of a non-bank holding one unit of money which, in turn, is greater than the value of a non-bank with no money holdings. Banks can do everything that non-banks can and, in addition, they can build reserves and "borrow" by costlessly issuing new money. This additional value of being a bank, which partly relies on assumptions that guarantee limited entry into the banking sector, plays an important role in our numerical results when we study the effects of different policies on note issue.

Lemma 4.7. *For all r and m, $v_0^n < v_1^n < v_{r,m}^b$.*

Our remaining goal in this subsection is to describe key policy rules for banks throughout the state space. We divide our analysis into three Propositions regarding meetings with other banks and with non-banks. First, we describe the behavior of *liquid* banks. Such banks have reserves that exceed their total amount of money in circulation. Since future payoffs are discounted, and $R = 0$, a liquid bank will always find it optimal to issue an additional unit of money to a non-bank in order to consume now.[9] In addition, if a bank's reserves sufficiently exceed the amount of its money in circulation, a bank will find it optimal to stop building additional reserves. This is because the benefit from extra reserves will come in the distant future and is, thus, discounted, while the cost of production is incurred today.

Proposition 4.8. *For all r and m, with $r > m$, $\phi_{r,m}^n = 1$. Moreover, for every m, there exists an r_m such that, if $r \geq r_m$, then $\gamma_{r,m} = 0$.*

Next, we describe the behavior of *illiquid* banks. These are banks whose total amount of money in circulation exceeds their reserves. When r is relatively low, such banks are facing a

[9] That behavior is not necessarily optimal in meetings between banks. In the next section we study simulations in which some banks may direct new issue towards non-banks only, avoiding the short-term cost resulting from immediate redemption.

positive probability of closure. Note that, as r grows, a bank issuing money at state (r, r) faces a diminishing probability of being caught with negative reserves. In the limit, this probability becomes arbitrarily small. In this case, the short-term gains dominate and the bank issues money, even to other banks, in order to consume. Of course, some illiquid banks are caught with negative reserves and have to exit the sector. To avoid exit, such banks typically find it optimal to suffer the disutility of production in order to improve their reserves.

Proposition 4.9. *There exists* $\hat{r} > 0$ *such that, if* $r \geq \hat{r}$ *then* $\phi^n_{r,r} = 1$.

The next Proposition describes the behavior of banks that are "too illiquid" in the sense that $m - r$ is positive and large. Banks that have very low reserves compared to the amount of their money in circulation will concentrate on issuing more notes instead of building additional reserves, since they expect to exit the banking sector with high probability in the near future. We call such banks *wildcats*.[10]

Proposition 4.10. *For every* r *there exists* m_r *such that, if* $m \geq m_r$ *then* $\phi^n_{r,m} = 1$ *and* $\gamma_{r,m} = 0$.

{Insert Figure .1}

The findings in Propositions 4.8 - 4.10 are summarized in Figure .1. As the values of r and m vary, the optimal strategies give rise to four regions in the (r, m)-space. In region I reserves are high compared to money in circulation. In that case, a bank finds it optimal to issue money. At

[10] The term "wildcat bank" originates in the 1800s, when it was applied to banks that were established in inaccessible locations in order to avoid redemptions of their notes. The term has become synonymous with unsound banking practices regarding note issue and the obligations of banks regarding redemptions of their notes.

the same time, such a bank rejects opportunities to increase reserves. Banks in region II still find it optimal to issue money, but being less liquid, they also accept trades that increase reserves. Banks in region III are becoming alarmingly illiquid and find it optimal to both improve their reserves and stop issuing new notes. In other words, concerned about the possibility of having to exit, these banks concentrate on improving their reserve position.[11] Finally, banks in region IV have too few reserves compared to their notes in circulation and will thus have to exit the banking sector with high probability in the near future. These banks would not benefit from a single-unit increase in their reserves. They issue money until they are caught with a negative balance. Notice that the redemption process reduces both reserves and the notes in circulation by the same number and thus always moves a bank southwest, in parallel to the 45-degree line. We remark that banks might enter region IV either accidently, when the redemption process brings them there from another region, or intentionally, after issuing too many notes. In both cases, they never leave this region before exiting the banking sector.[12]

5. Numerical Findings

In this Section we present the outcomes of numerical simulations, including some policy experiments, for several different parameterizations. We emphasize that these simulations are not meant to generate quantitative predictions. Rather, they serve as an additional device for exploring the qualitative effects of varying the economic environment on optimal bank behavior. We start by describing the equilibrium strategies of banks throughout the state space. Then, we study the effects of paying interest on reserves as well as the effects of varying the stock of outside money.

[11] Region III might be a subset of the 45-degree line.

[12] It can be shown that $\phi^n_{r,m} = 0 \Rightarrow \phi^b_{r,m} = 0$, and that $\phi^b_{r,m} = 1 \Rightarrow \phi^n_{r,m} = 1$. In general, even when $R = 0$, the possibility that $\phi^b_{r,m} = 0$ for $r > m$ cannot be ruled out.

Finally, we study the effects of different regulations, such as changing the value of initial reserves and prohibiting money creation, followed by increasing entry into the banking sector.

Before presenting the simulation results, we shall briefly comment on certain assumptions that simplified the numerical task. We have assumed that goods are indivisible and money holdings are restricted to $\{0, 1\}$, so that there is no need to compute the *intensive margin*; that is, how much output is traded in each monetary transaction. When we allow for bank failure the state space for banks must include both reserves and notes in circulation. Moreover, predicting higher moments of the redemption process becomes complicated if non-banks can hold multiple units. Therefore, restricting non-bank holdings to $\{0, 1\}$ greatly simplifies each relevant history to a simple pair, (r, m). The assumption of indivisible production further simplifies the analysis. Thus, whether we can extend our experiments with bank failure to the case of general money holdings is an open question for future research. With that in mind, our results are only suggestive and should be interpreted with some caution.

The numerical examples below are based on the following parameter values: we set the number of types, k, to 3; the exogenous probability of death, δ, is fixed at .2; the fraction of newborn agents holding government currency, μ_n, and the fraction of agents born as banks, μ_b, are set at .2 and .1, respectively; utility associated to consumption, u, is fixed at .5 and the disutility from production, e, is set at .2. These choices imply that the sufficient condition in Lemma 4.1 is satisfied as long as $\beta > .741$. We set the effective discount factor at $\beta = .995$. We also force banks to have a maximum of nine units of reserves and ten units of money in circulation ($N = 9$).

We assume initially that banks are born with two units of reserves ($\bar{r} = 2$), and that no interest is paid on reserves ($R = 0$). Unless stated otherwise, the parameter values remain constant in the examples below.

5.1. An Inside-Money Economy

The parameters and policy settings of this subsection define our benchmark for what follows. Under this parametrization we find in our simulations that banks become illiquid. That is, we find that some banks cross the 45-degree line in the (r, m)-plane. However, banks still exhibit conservative behavior regarding issue: there are four states on the diagonal $((0, 0), (1, 1), (2, 2), (3, 3))$ for which banks do not issue to non-banks. Moreover, there are states in which a bank, concerned about redemptions, would issue to a non-bank but not to a bank (states $(4, 4)$ and $(5, 5)$). Note also that there are states where banks exhibit wildcat behavior. In particular, Figures .2 and .3 indicate that there are states in the northwest corner in which illiquid banks do not produce while they choose to issue money. Thus, banks in that region exit the banking sector in the near future. To document wildcat behavior by some banks in equilibrium, it remains to be shown that a positive fraction of banks will enter that region. Figure .3 demonstrates that this is indeed the case. Interestingly, banks do not enter into the wildcat region unwillingly. Rather, banks enter that region because of their decision to print an extra note, and not due to the redemption process. In fact, the redemption process alone will not bring a bank into that region since it moves a bank southwest on a negative 45-degree line (see Figures .2 and .3). Notice that it is possible for a bank to enter the wildcat region even if none of its notes has been redeemed up to that point. This is exhibited, for example, by the behavior of a bank that has four units of reserves and keeps issuing notes to non-banks with no notes being redeemed. Eventually the bank reaches state $(r, m) = (4, 8)$, thus, entering the ergodic wildcat region.

{Insert Figures .2 and .3}

5.2. Interest on Reserves

Illiquidity has non-trivial implications for aggregate welfare in the economy of subsection 5.1. Illiquid banks that produce, and even wildcats that do not, may play a positive role in providing scarce liquidity. However, there is also a social cost associated to wildcats since these banks stop building reserves and eventually fail. Thus, additional policies might be required in order to limit financial fragility. In order to assess the role of bank returns, we now consider the case where a positive interest is paid on reserves; that is, $R > 0$. The interest payments are financed by imposing a lump sum tax on both banks and non-banks. This tax is deduced from utility in our welfare measures. We consider three values for the interest rate on reserves: 0, .002, and .02 (the economy with $R = 0$ corresponds to that of subsection 5.1).

Table 1 demonstrates that a policy of "high" interest rates on reserves ($R = .02$) can reduce non-bank welfare. We find that, as R increases from 0 to .02, average non-bank welfare decreases from 4.54 to 2.911, while the average welfare of banks, not surprisingly, increases from 6.9 to 28.09. The interest makes banks more concerned about losing reserves. They, thus, choose not to issue to non-banks even when they are liquid (see Figures 3 and 4 below). Banks, thus, create no liquidity in the economy. The mass of consumers holding notes decreases significantly, relative to subsection 4.1, which explains the decline in non-bank welfare.

Table 1 illustrates some interesting general-equilibrium effects. The policy of increasing interest rates from 0 to .002 raises average non-bank welfare but decreases average bank welfare (recall that the welfare effects of increasing R from 0 to .02 had the opposite implication). As R is raised to .002, the fraction of the population per type that is willing to spend a note, q, increases from .1083 to .1159. Non-bank consumers, thus, have a higher ability to spend in the economy with $R = .002$, which leads to an increase in average consumption and welfare (despite the existence of lump sum taxes). The reason why non-banks consume more frequently when $R = .002$ is that private note issue to non-banks is relatively high in this economy ($p^b \cdot \phi^n$ equals .0318 when $R = .002$, instead of .0279 in subsection 5.1). Thus, banks are more willing to issue notes to non-banks in the economy with $R = .002$ than in the economy with $R = 0$.

When the central bank pays interest on reserves, one would expect banks to be more conservative regarding note issue and to work harder in accumulating reserves. General-equilibrium effects, however, can reverse the sign of these effects. When a low interest rate is used ($R = .002$), banks compete harder for reserves, as illustrated by the observation that in states $(1, 0), (2, 0)$ and $(2, 1)$ in Figure .7 banks do not issue notes to other banks. Competition for reserves decreases the consumption frequency of banks without improving aggregate reserves of the banking sector. More precisely, we find that banks are less willing to consume from other banks when $R = .002$, since $p^b \cdot \phi^b$ is now .0064 instead of .0279. This effect translates into a reduction in the welfare of the average bank. A reduction in the average value of banks, in turn, makes banks more willing to issue notes as the penalty for exiting the industry is reduced. These effects lead to an increase of liquidity and an increase in average non-bank welfare, which, in turn, reduces the penalty from exiting the industry and reinforces the incentives to issue more notes. As a result, as R is raised from 0 to .002, the fraction of illiquid banks increases from .003 percent to 3.6 percent. In addition, banks produce less (when R is set to .002 banks stop producing in states $(0, 3), (1, 4)$ and $(2, 5)$).

Table 1: Effects of paying interest on reserves			
R (interest on reserves)	.0	.002	.02
v_0^n	4.423	4.619	2.824
v_1^n	4.835	5.020	3.267
Avg. non-bank welfare	4.54	4.731	2.911
Avg. bank welfare	6.90	6.814	28.09
p_0^n	.2196	.2167	.2412
p_1^n	.0804	.0840	.0588
$p = p_0^n + p^b \cdot \gamma^b$	.2529	.2492	.2745
$q = p_1^n + p^b \cdot \phi^n$	.1083	.1159	.0588
$p \times q$	.0274	.0289	.0161
$q^b = p_1^n + p^b \cdot \phi^b$	.1083	.0905	.0588
$p^b \cdot \phi^n$	.0279	.0318	.0000
$p^b \cdot \phi^b$	.0279	.0064	.0000
$p^b \cdot \gamma^b$	.0333	.0326	.0333

The following table compares the two environments discussed previously. An interest rate of .02 results to no illiquid banks, while an interest rate of .002 results in a larger mass of illiquid banks than in the economy with $R = 0$.

Table 2: Bank behavior and the interest rate			
	$R = 0$	$R = .002$	$R = .02$
No. states with no consumption	10	7	136
when meeting a bank	6	6	68
when meeting a non-bank	4	1	68
No. states with no production	25	28	15
Fraction of illiquid banks	.003%	3.6%	0%
Fraction of wildcats	.00004%	.17%	0%
No. of banks exiting	.0002%	.44%	0%

5.3. Eliminating Failure Through Injections

We now return to the case where $R = 0$ and investigate the effects of endowing banks with additional reserves. As mentioned earlier, injecting additional initial reserves is equivalent to allowing a less strict shut-down rule. This can also be interpreted as a policy of lending of last resort to banks with difficulties. We emphasize that in our model the central bank (or clearing authority) needs to know only the reserve balances of banks and not the number of their notes in circulation.

We compare the equilibrium under $\bar{r} = 2$, our benchmark, to the case where $\bar{r} = 4$. The corresponding decision rules are represented graphically in Figures .8 and .9. We find that banks behave more conservatively when they can borrow 4 units of reserves rather than 2. In fact, banks, in the computed range of $r, m \leq 10$, do not become illiquid when $\bar{r} = 4$ (they only issue notes in states in which $r > m$, both in meetings with banks and with non-banks). Thus, in this simulation we find that banks do not risk failure. Moreover, they are concerned about building reserves. Notice that the number of states in which banks decide to produce increases. Thus, by injecting money into the banking sector, the central bank increases both trade and the value of banks. Consequently, the banks' policies become more conservative.

{Insert Figures .8 and .9}

Some additional findings are summarized in the following table. They reveal that all liquidity injected by the central bank is absorbed by the banks, so that the mass of non-banks holding money does not increase.

Table 3: Effects of an injection of reserves

	$\bar{r} = 2$	$\bar{r} = 4$
v_0^n	4.423	4.621
v_1^n	4.835	5.023
Avg. non-bank welfare	4.54	4.733
Avg. bank welfare	6.90	8.147
p_0^n	.2196	.2165
p_1^n	.0844	.0835

As Table 3 demonstrates, both banks' and non-banks' utility increases when banks receive additional initial reserves. Notice that the increase in utility is larger for banks than for non-banks. Thus, the penalty for banks exiting the industry is larger in this case.[13] As a result, banks behave conservatively even in this case of less stringent reserve requirements. This experiment suggests that there is excessive bank failure in the economy of subsection 5.1.

[13] We also have simulations (not included) in which setting β to a high value produces a similar effect: the value of a bank increases relative to that of a non-bank, virtually eliminating bank illiquidity.

5.4. Inside Money as a Source of Liquidity

In this experiment we modify our benchmark so that outside money is more scarce. To this end, we decrease the fraction of newborn individuals to $\delta = .1$ (instead of $\delta = .2$) and we decrease the fraction of newborn non-banks with money to $\mu_n = .1$ (instead of $\mu_n = .2$). As a result, the measure of newborn non-banks with money is reduced by one fourth relative to the benchmark (the product $\delta\mu_n$ is .01 instead of .04). We consider four banking environments, keeping $R = 0$. We vary $\bar{r}$ in $\{1, 4\}$, and consider two possible regulations: one imposing a 100 percent reserve requirement and the other allowing banks to become illiquid.

We first study the case where inside money is allowed. Since liquidity is potentially scarce with $\delta = .1$ and $\mu_n = .1$, banks have a hard time building reserves to support issue and, as a result, they are quite willing to become illiquid. The decisions of banks regarding issue are represented graphically in Figure .10, with $\bar{r} = 1$ and $\bar{r} = 4$ represented on the left and right panels, respectively. Figure .10 reveals that issue is indeed less conservative now than in the benchmark case. There are now fewer states in the diagonal for which banks do not issue notes. Surprisingly, there are states in which banks, despite being liquid, forgo a consumption opportunity when meeting another bank . In such states banks avoid issuing money that will be cleared immediately. instead, they prefer to use the extra reserves to support multiple money issue in the future.

{Insert Figure .10}

In order to study the role of inside money as a source of liquidity, we now evaluate how welfare would be affected if banks were not allowed to become illiquid. Such regulation could be implemented by imposing an arbitrarily large penalty to banks caught with negative reserves, or by a central-bank policy that prevents banks from printing their own currency, allowing them to put in circulation only government currency (which is earned as they accumulate reserves). A comparison of columns 1 and 2 in Table 4 reveals that allowing banks to become illiquid, when $\bar{r} = 1$, can provide a significant boost to the average welfare of both banks and non-banks. Inside money responds to liquidity scarcity, as indicated by the fact that 18.6 percent of banks in this economy are illiquid. Moreover, private money, as a fraction of the aggregate money stock, increases from 48.7 percent to 64 percent as banks are allowed to become illiquid. The increase in liquidity comes at a cost since 1.7 percent of banks are shut down each period due to a negative reserve position.[14]

In evaluating the role of banks in providing liquidity, it is essential to distinguish between inside money and private money. In our model, inside money takes the form of note issue not backed by reserves. That is, banks create liquidity when they issue non-backed liabilities. When banks issue notes that are backed by accumulated reserves, they are changing the physical aspects of the means of payments without necessarily adding liquidity in the economy. The degree to which extra liquidity is generated by inside money is a difficult question as the choice of an appropriate measure is not trivial.

Table 4 reveals that, when $\bar{r} = 4$, average welfare for both banks and non-banks improves when banks are allowed to print their own money (compare columns 3 and 4). We also notice that inside money increases welfare by a larger amount in the economy with low $\bar{r}$. Inside money plays a less prominent role in the economy with $\bar{r} = 4$ since liquidity is less scarce. When banks are born with four units of reserves they have an easier time printing money. This, in turn, has positive external effects on other banks, making it easier for them to accumulate reserves. Since the value of being a bank is quite large in this economy, banks behave more conservatively. The

[14] The fraction of failed banks that become wildcats intentionally is .9993 for $\bar{r} = 1$, and .995 for $\bar{r} = 4$.

fraction of illiquid banks is only 3.5 percent (instead of 18.6 percent in the case where $\bar{r} = 1$). Our findings illustrate that inside money plays an important role when liquidity is scarce, an observation that we believe is consistent with several historical episodes.

Table 4: Inside money under scarce liquidity ($\delta = .1$, $\mu_n = .1$)

	$\bar{r} = 1$		$\bar{r} = 4$	
Reserve Regulation	*Outside M.*	*InsideM.*	*Outside M.*	*Inside M.*
Avg. non-bank welfare	3.13	4.31	4.35	4.51
Avg. bank welfare	3.95	6.10	7.08	7.61
p_1^n/p_0^n	.200	.319	.314	.335
Number of banks	.10	0.085	.10	.0978
Fraction illiquid banks	0%	18.6%	0%	3.5%
Fraction exiting banks	0%	1.71%	0%	.22%
Share of private money	48.7%	64.0%	64.2%	65.8%
Share of inside money	0%	28.2%	0%	5.5%

5.5. Increasing Entry into the Banking Sector

An important question is to what extent redemptions discipline the issue of money by the banking sector. We investigate this question by increasing competition for reserves through an increase in entry. To this end, we fix $\mu_n = .1$ and compare three economies that differ in the fraction of the newborn agents that are banks ($\mu_b = .1, .5, .90$). Table 5 shows that as the fraction of banks in the population increases from .1 to .90, the probability that a unit of money in circulation is redeemed increases from .126 to .367. It should be noted that the speed at which notes are redeemed is determined by several factors, not just the number of banks. In our economy, the rule of closing down banks with negative reserves is quite effective in disciplining banks. Since banks strive to accumulate reserves, the redemption probability increases with the number of banks. This, in turn, reinforces conservative behavior. Figures .11 and .12 compare banks' decision rules when $\mu_b = .1$ and $\mu_b = .5$. When $\mu_b = .5$, banks prefer not to issue money whenever the prospects of excessive redemptions put them at risk. In this economy there is no illiquid banking.

Table 5 demonstrates that the welfare of both banks and non-banks improves in the number of banks in all three examples considered. A larger fraction of banks leads to a higher level of liquidity and makes both banks and non-banks better off. Welfare is the largest in the economy with the smallest aggregate stock of money, which suggests that choosing an appropriate notion of liquidity in our economy is not straightforward. Total welfare depends on both the measure of people willing to produce, p, as well as that willing to consume, q, in a multiplicative way (the product is a measure of the frequency of successful trades). In equilibrium, $p = p_0^n + p^b \cdot \gamma$ represents how easily a consumer can spend a note, and $q = p_1^n + p^b \cdot \phi^n$ represents how easily a producer can earn a note. The dependence on the product demonstrates that liquidity is not sufficiently summarized by the money stock. The amount of money that is created and destroyed is also relevant.

While the number of potential banks is our model is exogenous, one could model free entry in the following way. Suppose, in the spirit of He, Huang, and Wright (2003), that in each period there is an afternoon market where a general good can be traded. Moreover, suppose that there is a fixed cost of setting up a communication network with the central bank. Then, we could consider free entry by allowing individuals to pay an amount that would make them indifferent between being a banker or not. We conjecture that a stable monetary system will arise in which banks make zero ex-ante profits.

Table 5: Chartering more banks

Fraction of newborn banks	.10	.50	.90
Redemption probability	.126	.25	.367
Avg. non-bank welfare	4.31	5.86	7.50
Avg. bank welfare	6.10	8.90	11.92
p_1^n/p_0^n	.319	.468	1.0
Number of banks	0.085	.50	.90
Fraction of illiquid banks	18.6%	0%	0%
Fraction of exiting banks	1.71%	0%	0%
Share of private money	64.0%	75%	
Share of inside money	28.2%	0%	
Aggregate money stock	.221	.159	.0499

6. Final Remarks

We extended our earlier work in order to study the consequences of central-bank policies for liquidity and the risk of bank failure. In our model, banks can be profitable without earning interests on reserves, since the capability to create liquidity has private benefits. We have studied whether a stable monetary system emerges, as well as whether there is under-issue or overissue of notes. In addition, we studied whether some infusion of reserves or an interest rate paid by the central bank can reduce instability. The answers proved to be non-trivial as the model displayed important non-linearities and general-equilibrium effects. We have showed that banks may under-issue if trading with other banks results in excessive redemptions. Small increases on the interest rate paid on reserves may reduce banks' returns and increase risk-taking. High interest rates may produce an outside-money economy, reducing non-bank welfare. We also found cases in which banks willingly cross a threshold and start overissuing until failure. In summary, monetary stability depends on central-bank polices as well as general liquidity conditions. Allowing banks to run negative reserves, a policy that resembles lending by a central-bank, may facilitate strict reserve management, increase returns, and promote trade and financial stability and a lower rate of bank failure.

As indicated earlier, limiting non-bank money holdings to $\{0, 1\}$ is important for our computational experiments. With general holdings, the individual history of note issue would have to be carried for each bank. This is because many notes held by a single non-bank would give rise to different risk for the issuing bank compared to an equal amount of notes spread across several holders. In that case, beliefs about the distribution of note holdings of each bank would have to be updated each period. When holdings are restricted to $\{0, 1\}$, these complications are not present. It is however important to consider which results are likely to be sensitive to this assumption. The effects of large interest payments on bank behavior do not seem to depend of how many notes a bank can issue in a period. The fact that large interest rates may shift the distribution of money towards bank reserves, with likely disturbances on non-bank welfare, also seems not to rely on the restricted money holdings. Moreover, as Cavalcanti and Wallace (1999b) demonstrate, inside money beneficial to society under general conditions, and the higher the number of active banks the better. Our paper involves the additional complication that bank illiquidity is likely beneficial but there is also a cost associated with bank failure. We do not believe that allowing for general money holdings will change this picture. We recognize, however, that our findings on the effects of small changes in policy parameters, such as the interest rates on reserves, may change under general money holdings. In summary, our paper demonstrates the effects of general equilibrium interactions on the returns to banking. We believe that, at least qualitatively, these effects will remain present under more general specifications of the basic model.

Appendix

Here we formally describe the mapping T_{γ,ϕ^b,ϕ^n} governing the law of motion of (p^n, p^b). For states such that $\sum_{r,m} p^b_{r,m} + p^n_0 + p^n_1 = \frac{1}{k}$, the mapping is given by the right hand side of the equations below, together with the expressions from the equilibrium definition relating p^b, p^n, and π to (γ, ϕ^n, ϕ^b). For non-banks, we have

$$p^n_1 = (1-\delta)(p^n_1 + q\ p^n_0 - p\ p^n_1) + \frac{1}{k}(z + \delta\mu_c) \tag{.1}$$

and

$$p^n_0 = (1-\delta)(p^n_0 - q\ p^n_0 + p\ p^n_1) + \frac{\delta}{k}(1 - \mu_c - \mu_b), \tag{.2}$$

where $p = p_0^n + p^b \cdot \gamma$ and $q = p_1^n + p^b \cdot \phi^n$. For banks, we have

$$
\begin{aligned}
z &= k\{\sum_{m\geq 1}\sum_{r=0}^{m}\sum_{i=r+1}^{m} \binom{m}{i}\pi^i(1-\pi)^{m-i}p_{r,m}^b + \sum_{m\geq 0} p_{m,m}^b\ \phi_{m,m}^b \binom{m}{m}\pi^m(1-\pi)^{m-m} \\
&\quad - \sum_{m\geq 1}\gamma_{m-1,m}\binom{m}{m-1}\pi^{m-1}(1-\pi)^1 p_{m-1,m}^b\}
\end{aligned} \tag{.3}
$$

and

$$
\begin{aligned}
p_{r',m'}^b &= (1-\delta)\sum_{m\geq m'}\binom{m}{m-m'}\pi^{m-m'}(1-\pi)^{m'} \\
&\quad \times\{q^b\left[\gamma_{r'+m-m'-1,m}\ \ p_{r'+m-m'-1,m}^b + (1-\gamma_{r'+m-m'-1,m})\ \ p_{r'+m-m',m}^b\right] \\
&\quad +p_0^n\ \left[\phi_{r'+m-m',m-1}^n p_{r'+m-m',m-1}^b + (1-\phi_{r'+m-m',m}^n)\ p_{r'+m-m',m}^b\right] \\
&\quad +\gamma\cdot p^b\ \left[\phi_{r'+m-m'+1,m}^b\ p_{r'+m-m'+1,m}^b + (1-\phi_{r'+m-m',m}^b)\ p_{r'+m-m',m}^b\right] \\
&\quad +(\frac{1}{k} - q^b - p_0^n - \gamma\cdot p^b)\ p_{r'+m-m',m}^b\} + I_{r',m'}\frac{\delta\mu_b}{k},
\end{aligned} \tag{.4}
$$

for all $r', m' \geq 0$, where $q^b = p_1^n + p^b \cdot \phi^b$, and $I_{r',m'}$ is an indicator function that assumes value one if $(r', m') = (\bar{r}, 0)$ and value zero otherwise.

Proof of Lemma 4.6: We fix $R = 0$ and first demonstrate that v^b is weakly increasing in r. Let $G = \{f : \mathbb{N}\times\mathbb{N} \to \mathbb{R}\}$. Define the operator $T : G \to G$ by the right hand side of the functional equation for a bank. We know that T has a unique fixed point, v^b, in the space of bounded continuous functions. If T also maps the space of weakly increasing continuous functions into itself, and since this space is complete, this fixed point will also be a weakly increasing continuous function. We need to show that T preserves monotonicity, i.e., for any fixed m, if $r_1 \leq r_2$, then $Tv^b(r_1, m) \leq Tv^b(r_2, m)$. We first show that, for all m and any j, $w_{r_1,m}^j \leq w_{r_2,m}^j$. First, consider the case where $r_1 \geq m$. Then $w_{r_h,m}^j = \sum_{0\leq i\leq m}\binom{m}{i}\pi^i(1-\pi)^{m-i}v_{r_h-i,m+j-i}^b$, where $h = 1, 2$. Notice that the terms multiplying v^b are the same for $h = 1, 2$. Since v^b is monotone, it follows that $w_{r,m}^j$ is monotone. Next, consider the case where $r_2 < m$. Then

$$w^j_{r_h,m} = \sum_{0\le i\le m} \binom{m}{i}\pi^i(1-\pi)^{m-i}v^b_{r_h-i,m+j-i} + \sum_{r_h\le i\le m} \binom{m}{i}\pi^i(1-\pi)^{m-i}v^n_1.$$

The first r_1 terms in the first sum are weakly greater when $r_h = r_2$ than when $r_h = r_1$. The result then follows since, for all (r, m), $v^b_{r,m} > v^n_1$. By the same reasoning, the desired inequality follows for the case where $r_1 < m < r_2$, since v^b is monotonically increasing in r and, for all (r, m), $v^b_{r,m} > v^n_1$.

Next, we demonstrate that v^b is weakly decreasing in m. Applying the same reasoning, we need to show that for any fixed r, $m_1 \le m_2$ implies that $Tv^b(r, m_1) \ge Tv^b(r, m_2)$. Like before, we first show that for all r and any j, $w^j_{r,m_1} \ge w^j_{r,m_2}$. We consider the case where $m_2 < r$ first. In that case, $w^j_{r,m_h} = \sum_{0\le i\le m_h} \binom{m_h}{i}\pi^i(1-\pi)^{m_h-i}v^b_{r-i,m_h+j-i}$. Define $p(i, m)$ by $p(i, m) = \binom{m}{i}\pi^i(1-\pi)^{m-i}$. We then have $\frac{p(i,m)}{p(i,m+1)} = \frac{\frac{m!}{(m-i)!i!}}{\frac{(m+1)!}{(m+1-i)!i!}}\frac{\pi^i}{\pi^i}\frac{(1-p)^{m-i}}{(1-p)^{m+1-i}} = \frac{m+1-i}{m+1}\frac{1}{1-p}$. This expression is less than one if and only if $i > p(m + 1)$. Therefore, $p(i, m)$ is greater than $p(i, m + 1)$ for low values of i and is lower than $p(i, m + 1)$ for high values of i. The result then follows since $w^j_{r,m}$ is a convex combination of decreasing functions of m. Therefore, $w^j_{r,m_1} \ge w^j_{r,m_2}$. Since $v^b_{r,m} > v^n_1$, the same argument can be used to demonstrate the result both when $r < m_1$ and when $m_1 < r < m_2$. ■

Proof of Lemma 4.7: The first inequality clearly holds. Regarding the second inequality, since v^b is increasing in r and decreasing in m, it is sufficient to show that $v^b_{0,m} > v^n_1$, for m large. In this case, the bank will have a negative balance with probability 1 at the end of the period and will, thus, exit the sector. If it issues one more note this period, it will have the value of a non-bank with one unit of money, v^n_1, in the next period. Therefore, $v^b_{0,m} > v^n_1$, for arbitrarily high m. ■

Proof of Proposition 4.8: (First part). The proof follows by contradiction. Assume that there is an optimal policy (ϕ^n, ϕ^b, γ) such that $\phi^n_{r_0,m_0} = 0$, for some state (r_0, m_0) with $r_0 > m_0$. Consider a bank that is in state (r_0, m_0). Notice that this bank decides not to issue a note in the current period. Also, notice that this decision would only be optimal if the bank could reach with positive probability in the future a state where it will issue a note. Denote this state by (r_1, m_1).[15] We shall show that there is a policy $(\hat{\phi}^n, \hat{\phi}^b, \hat{\gamma})$ that gives a strictly higher expected discounted utility to the bank than policy (ϕ^n, ϕ^b, γ), contradicting the optimality of (ϕ^n, ϕ^b, γ). In order to define the new policy, it is convenient to expand the state space to include two new artificial state variables, representing artificial reserves, r_a, and notes, m_a. Initialize the current (expanded) state as $(r, m, r_a, m_a) = (r_0, m_0, r_0, m_0)$. Set $\hat{\phi}_{r_0,m_0,r_0,m_0} = 1$ and, immediately after the note is issued, let the expanded state variable become $(r, m, r_a, m_a) = (r_0, m_0 + 1, r_0, m_0)$. From now on, update r_a and m_a in the same way as r and m, but with the following exception. If the note just issued in state (r_0, m_0, r_0, m_0) is redeemed, do not count this redemption in the state for artificial reserves and notes (that is, do not decrease r_a and m_a by 1). Set $\hat{\phi}^n_{r,m,r_a,m_a} = \phi^n_{r_a,m_a} \hat{\phi}^b_{r,m,r_a,m_a} = \phi^b_{r_a,m_a} \hat{\gamma}_{r,m,r_a,m_a} = \gamma_{r_a,m_a}$ for all states (r, m, r_a, m_a) but with the following exception. Recall that there exists a state (r_1, m_1) that can be reached with positive probability and where $\phi^n_{r_1,m_1} = 1$. When such a state is reached for the first time, set $\hat{\phi}^n_{r_1,m_1,r_a,m_a} = 0$ and change the law of motion for the artificial state to allow the note that would have just been issued under the original policy in state (r_1, m_1) (but in fact was issued some periods ago under the alternative policy) to be redeemed with probability π (this is an artificial redemption since the original note could be redeemed earlier). From then on, let decisions by the bank be as indicated

[15] Clearly, since no new notes were issued before reaching state (r_1, m_1), it is the case that $r_1 > m_1$ for all possible histories of redemptions and production decisions.

by $(\hat{\phi}^n, \hat{\phi}^b, \hat{\gamma})$. The optimality of (ϕ^n, ϕ^b, γ) is then contradicted by noting the following.

1. Under the alternative policy, the bank consumes today (in state (r_0, m_0)), rather than at some point in the future when the state (r_1, m_1) is reached.

2. Consumption and production decisions are the same while the bank transits from (r_0, m_0) to (r_1, m_1). Since $r_0 > m_0$ and $r_1 > m_1$, the bank does not risk failure while transiting from (r_0, m_0) to (r_1, m_1) under policy $(\hat{\phi}^n, \hat{\phi}^b, \hat{\gamma})$.

3. Once state (r_1, m_1) is reached, the stochastic process for (r, m) induced by the redemption process and by (ϕ^n, ϕ^b, γ) is the same for (r_a, m_a), as induced by the modified redemption process. Therefore, consumption and production decisions implied by the two policies coincide.

Since banks are impatient, it follows that the policy $(\hat{\phi}^n, \hat{\phi}^b, \hat{\gamma})$ gives higher utility, contradicting that (ϕ^n, ϕ^b, γ) is optimal.

(Second part). Consider a bank facing an opportunity to increase reserves. We have that $\gamma_{r,m} = 0$ if and only if $-e + w^0_{r+1,m} \leq w^0_{r,m}$. Note that $w^0_{r,m}$ is bounded, since it belongs to the interval $(0, \frac{u}{1-\beta})$. It is also an increasing function of r. Therefore, for fixed m, we have that $\lim_{r\to\infty} w^0_{r,m} = \lim_{r\to\infty} w^0_{r-1,m} = K_m$. The result then follows for r large. ■

Proof of Proposition 4.9: Consider the case where $m = r$, where r is large. We have that $\phi^n_{r,r} = 1$ if and only if $u + w^1_{r,r+1} > w^0_{r,r}$. Notice that $w^0_{r,r}$ is bounded, since it belongs to the interval $(0, \frac{u}{1-\beta})$. It is also an increasing function of r. Therefore, $\lim_{r\to\infty} w^0_{r,r} = \lim_{r\to\infty} w^0_{r,r+1} = K$, for some finite constant K. Also, $w^0_{r,r+1} < w^1_{r,r+1} < w^0_{r,r}$, which implies that $\lim_{r\to\infty} w^0_{r,r} = \lim_{r\to\infty} w^0_{r,r+1} = \lim_{r\to\infty} w^1_{r,r+1} = K$. Thus, the above inequality follows for r large. ■

Proof of Proposition 4.10: A bank that is given the opportunity to issue a note to a non-bank faces the following problem.

$$\max_{\phi^n_{r,m}} \left\{ \phi^n_{r,m}[u + w^1_{r,m+1}] + (1-\phi^n_{r,m}) w^0_{r,m} \right\}. \tag{.5}$$

We have that, for any fixed r,

$\lim_{m\to\infty} w^j_{r,m} = \lim_{m\to\infty} \beta \left\{ \sum_{i=0}^{r} p(m,i) v^b_{r-i,m+j-i} + \sum_{i=r+1}^{m} p(m,i) v^n_1 \right\}$.

The first sum in the bracket involves a finite number of terms, so we have $\lim_{m\to\infty} p(m,i) = \lim_{m\to\infty} \left\{ \frac{m!}{(m-i)!i!} \pi^i (1-\pi)^{m-i} \right\} = \lim_{m\to\infty} \pi^i (1-\pi)^{m-i} = 0$.

Thus, $\lim_{m\to\infty} w^j_{r,m} = \lim_{m\to\infty} \left\{ 0 + v^n_1 \sum_{i=r+1}^{m} p(m,i) \right\} = v^n_1$, and, for any fixed r, there exists an m_r large enough such that $u + \beta v^n_1 > \beta v^n_1$ and, therefore, $\phi^n_{r,m} = 1$. We know that $\gamma_{r,m} = 0$ if and only if $-e + w^0_{r+1,m} \leq w^0_{r,m}$. Now, fix $r \geq 0$. We have that $\lim_{m\to\infty} w^j_{r+1,m} = \lim_{m\to\infty} w^j_{r,m} = v^n_1$. Therefore, given r, there exists a large enough m_r such that $-e + \beta w^0_{r+1,m} = -e + \beta v^n_1 < \beta v^n_1 = \beta w^0_{r,m}$, for $m \geq m_r$. Thus, $\gamma_{r,m} = 0$ for all $m \geq m_r$. ■

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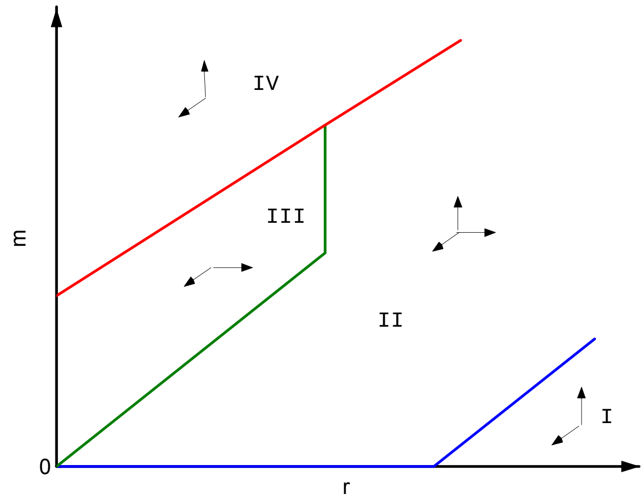

Figure .1: Phase Diagram

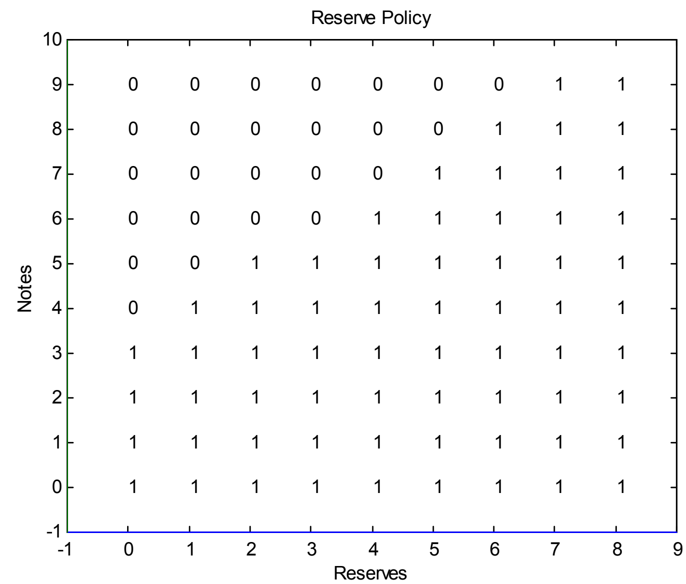

Figure .2: Values of $\gamma_{r,m}$ in the Benchmark Economy

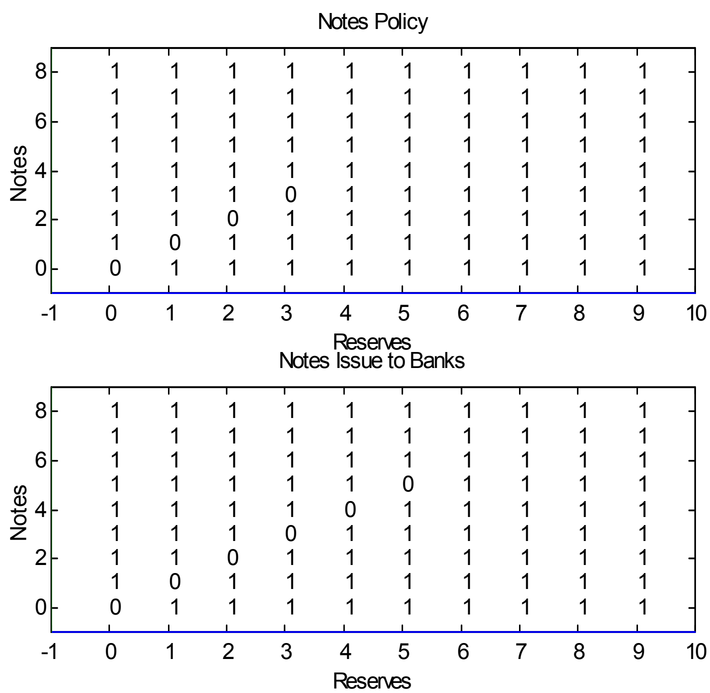

Figure .3: Values of $\phi^{n}_{r,m}$ and $\phi^{b}_{r,m}$ in the Benchmark Economy

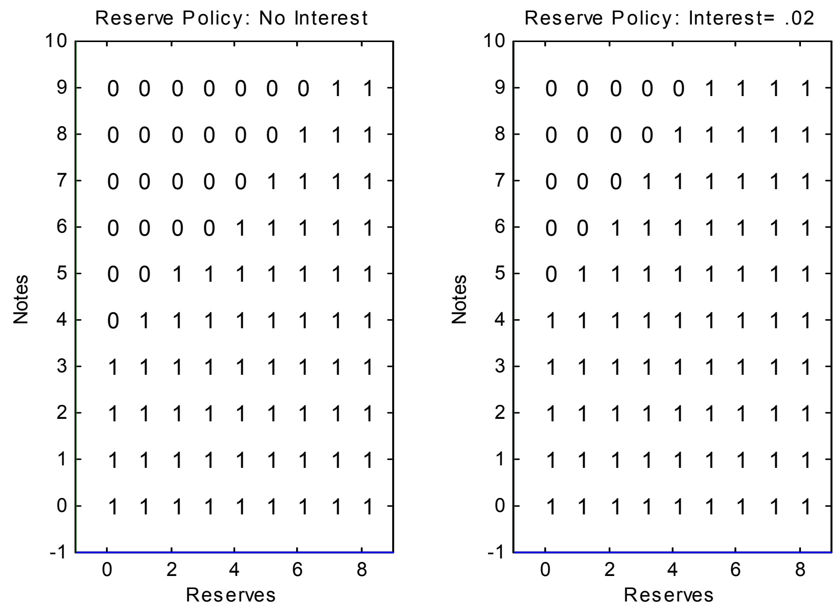

Figure .4: Values of $\gamma_{r,m}$ in the Economies with $R = 0$ and $R = .02$

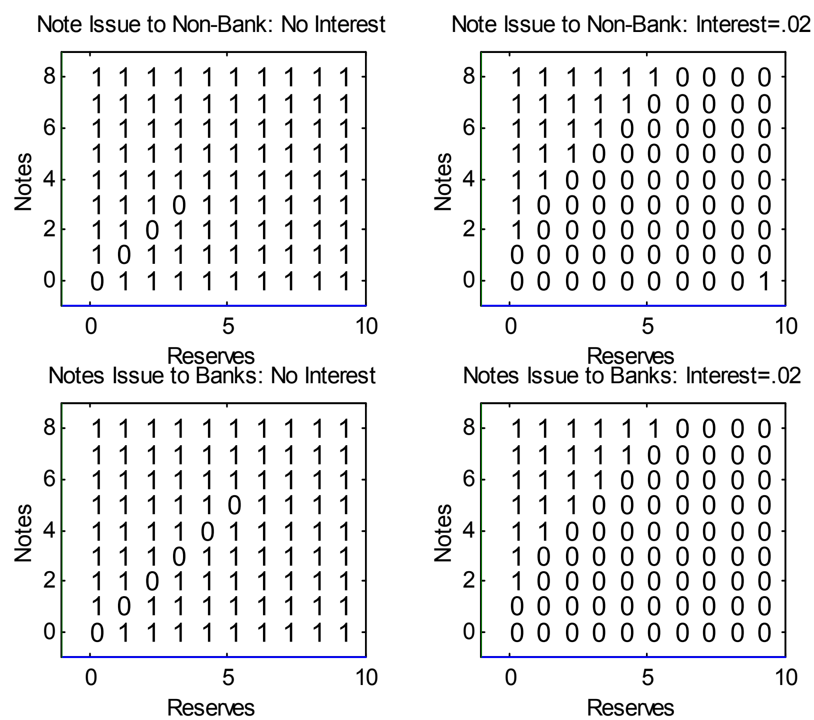

Figure .5: Values of $\phi^n_{r,m}$ and $\phi^b_{r,m}$ in the Economies with $R = 0$ and $R = .02$

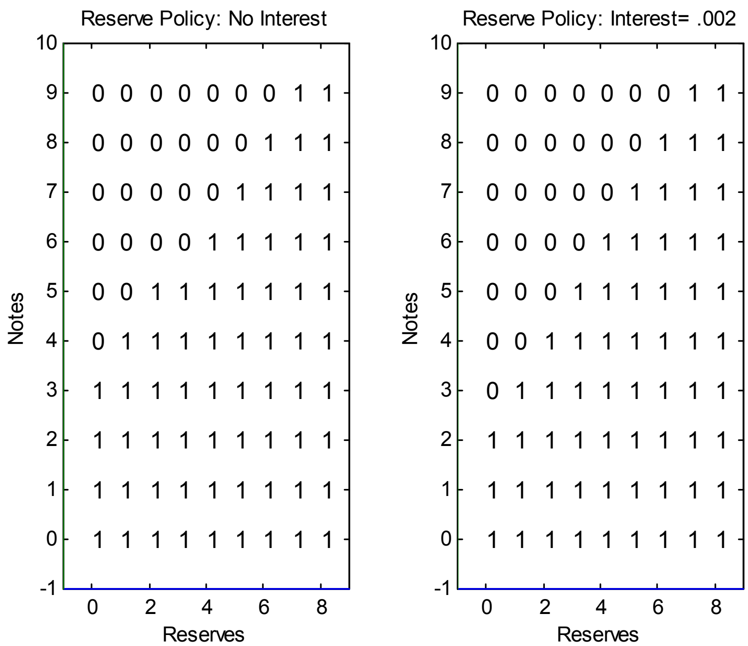

Figure .6: Values of $\gamma_{r,m}$ in the Economies with $R = 0$ and $R = .002$

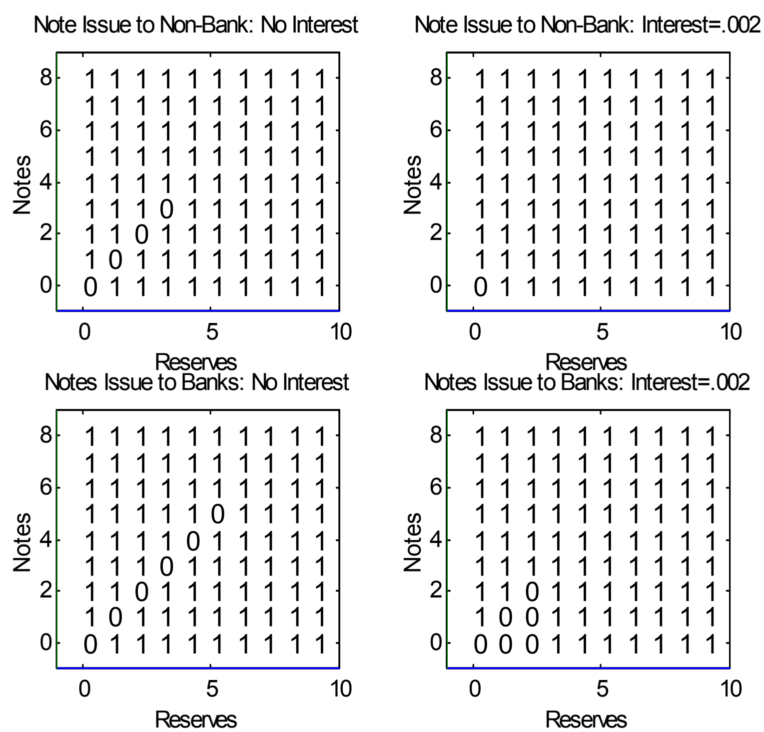

Figure .7: Values of $\phi^n_{r,m}$ and $\phi^b_{r,m}$ in the Economies with $R = 0$ and $R = .002$

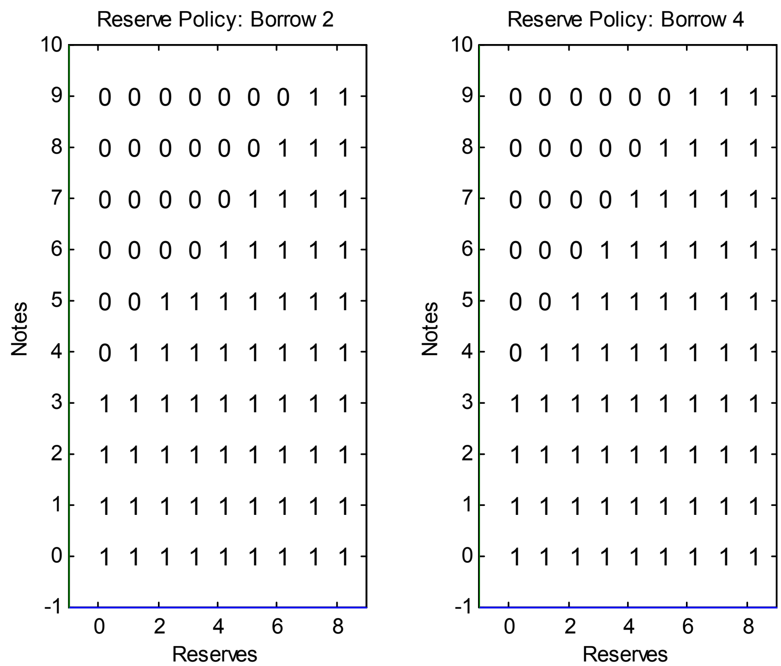

Figure .8: Values of $\gamma_{r,m}$ in the Economies with $\bar{r} = 2$ and $\bar{r} = 4$

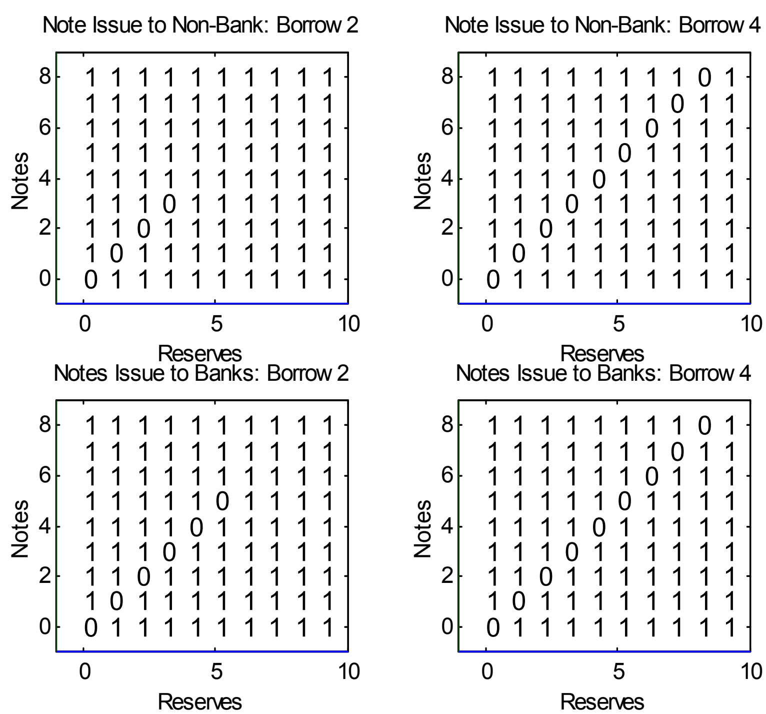

Figure .9: Values of $\phi^n_{r,m}$ and $\phi^b_{r,m}$ in the Economies with $\bar{r} = 2$ and $\bar{r} = 4$

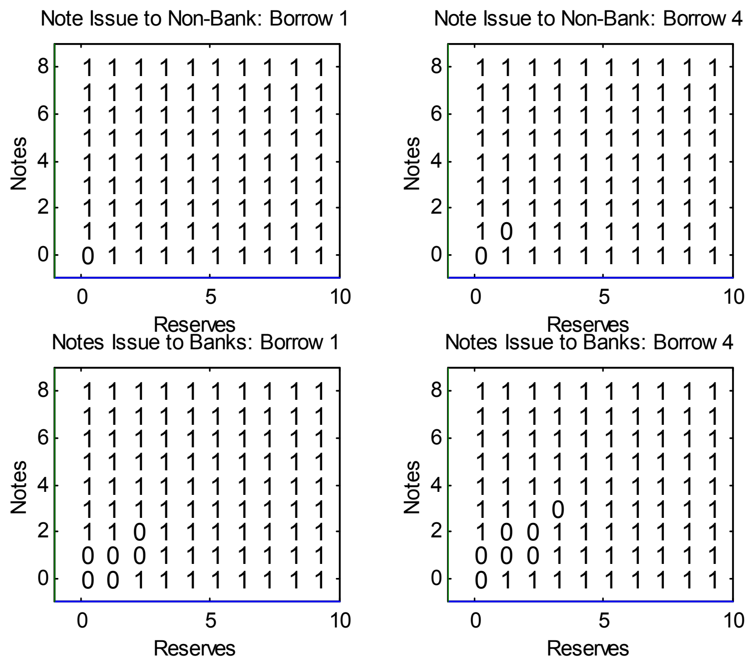

Figure .10: Values of $\phi^n_{r,m}$ and $\phi^b_{r,m}$ under scarce liquidity, with $\bar{r} = 2$ and $\bar{r} = 4$

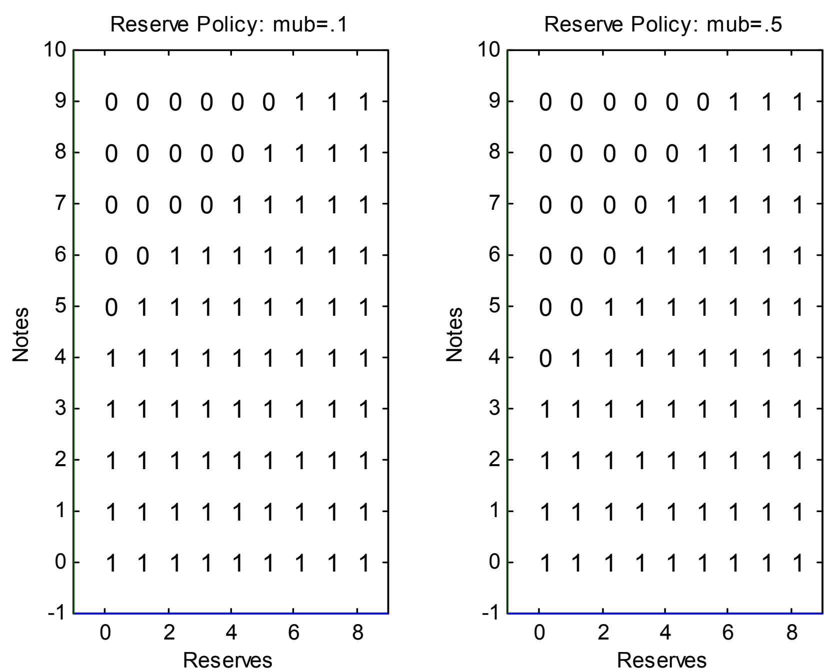

Figure .11: Values of $\gamma_{r,m}$ in the Economies with $\mu_b = .1$ and $\mu_b = .5$

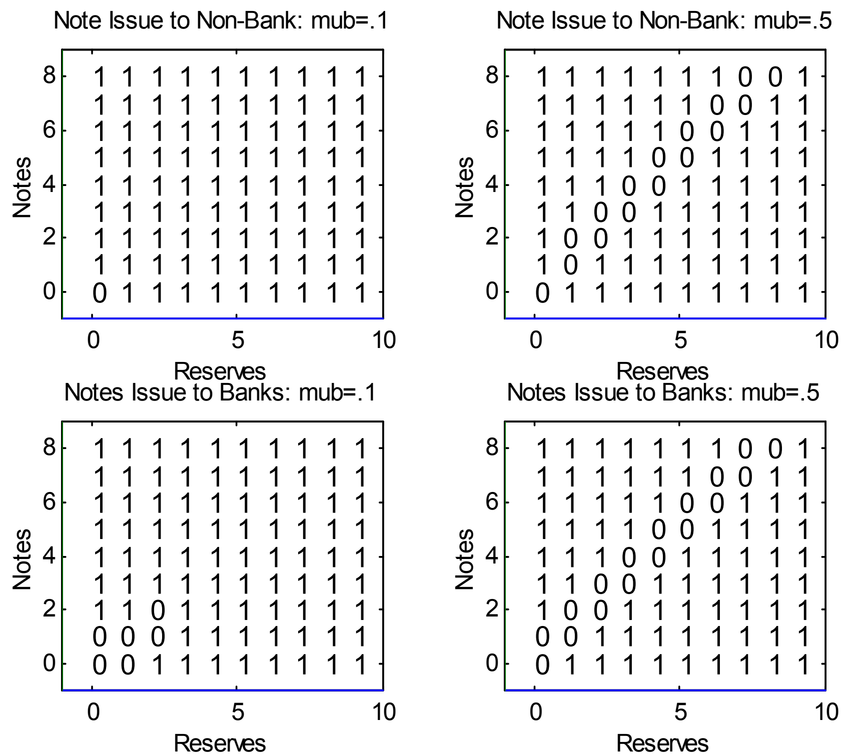

Figure .12: Values of $\phi^n_{r,m}$ and $\phi^b_{r,m}$ in the Economies with $\mu_b = .1$ and $\mu_b = .5$