

## Synoptic Meteorology I: Map Projections and Coordinate Systems

### For Further Reading

Section 3-1 and Appendix G of the Djurić *Weather Analysis* textbook provide useful information regarding map projections and common vertical coordinate systems. Section 3.2 of *Numerical Weather and Climate Prediction* provides useful information regarding map projections with an emphasis on applying them to numerical models. Chapter 2 of *An Introduction to Dynamic Meteorology* provides useful information regarding Eulerian and Lagrangian reference frames.

### An Introduction to Atmospheric Scales of Motion

Atmospheric phenomena may be found across a wide range of both time, or temporal, and space, or spatial, scales. Consequently, it is common to classify meteorological phenomena by their typical duration and horizontal extent. The most-commonly used classifications for doing so are:

- **Microscale:**  $\leq 10$  km, minutes
- **Mesoscale:** 10-1000 km, hours to approximately one day
- **Synoptic-Scale:** 1000-10000 km, one to seven days
- **Planetary-Scale:**  $\geq 10000$  km, beyond one week

Note, however, that the distinctions between individual classifications are arbitrary in nature. Furthermore, many atmospheric phenomena – and the physical and dynamical processes that govern their formation and evolution – straddle the dividing lines between different classifications.

In this class, we are primarily interested in phenomena that inhabit and physical processes that occur on the synoptic-scale and, to lesser extent, the larger end of the mesoscale. These phenomena form and evolve within the environment established by the planetary-scale, and in turn these phenomena influence the formation and evolution of microscale and mesoscale features. Thus, though our focus may lie on the synoptic-scale, and in specific upon the middle latitudes, what we learn here is of importance to weather analysis and forecasting across scales.

### Map Projections

As we know, the Earth is spherical – or, to be precise, nearly so. However, weather data is almost always displayed, interpreted, and analyzed in two dimensions. As a result, we need some means of accurately representing the three-dimensional Earth upon a two-dimensional sheet of paper or computer screen. For this, we utilize a *map projection*.

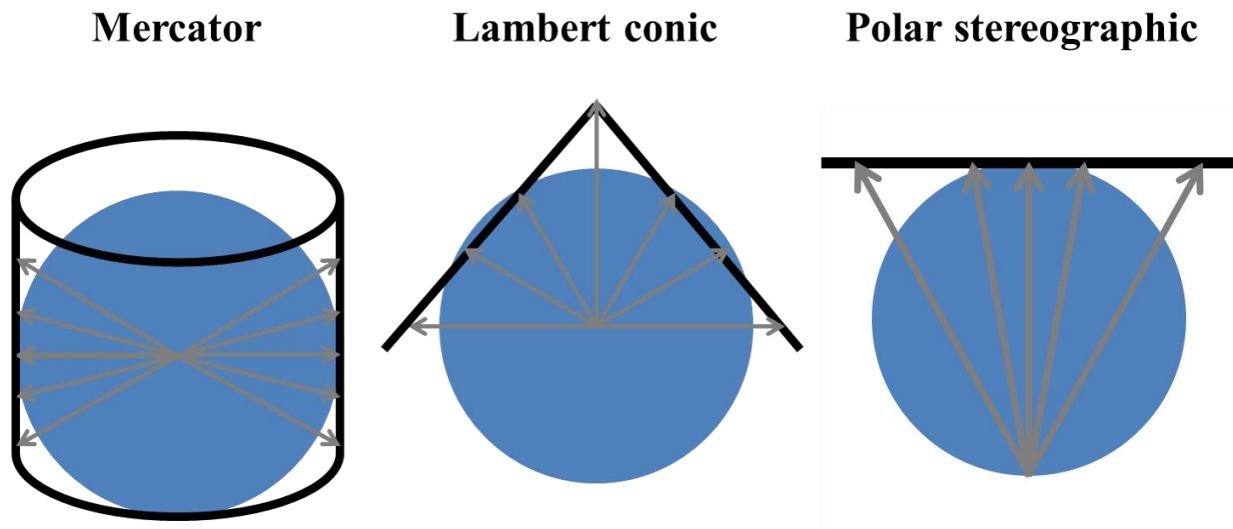
Map projections are geometric, or mathematical, relationships that translate information from a sphere to a flat surface. There exist several desirable traits for any map projection. These include:

- **Preservation of angles.** Angles on the flat surface and sphere should be equivalent.

- **Preservation of areas.** Areas on the flat surface and sphere should be equivalent.
- **Preservation of shapes.** Shapes on the flat surface and sphere should be equivalent.
- **Correct directions.** Cardinal directions on the flat surface and sphere should be equal.
- **Shortest distance between two lines should be a great circle.**

No individual map projection can satisfy all five of these considerations; a map projection is merely an approximation to the spherical Earth. In the atmospheric sciences, the most important consideration is that *angles* be preserved. This is particularly important for the accurate representation of kinematic properties such as wind, vorticity, and divergence that we will discuss in more detail later this semester.

The type of map projection that enables angles to be preserved is known as a *conformal* map projection. Note that the latitude and longitude lines on a conformal map projection are perpendicular to each other everywhere. The most commonly used types of conformal map projections in synoptic meteorology are the *Lambert conic*, *Mercator*, and *polar stereographic* map projections. These map projections do not preserve areas or shapes, but their distortion is small over specific latitudes that vary between projections. As a result, we typically apply these projections only over the limited ranges of latitudes for which area and shape distortion are small.



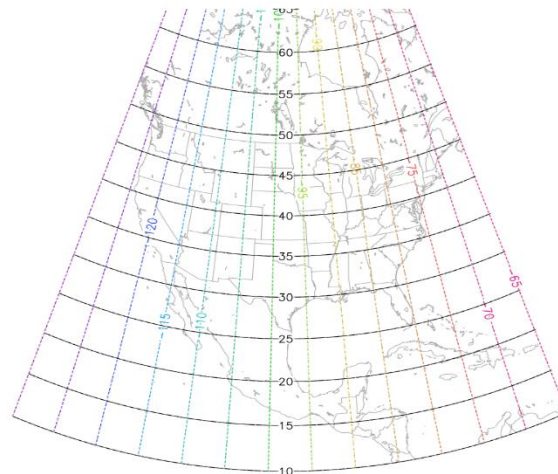
**Figure 1.** Graphical depiction of how the spherical Earth is projected onto the Mercator (left), Lambert conic (middle), and polar stereographic (right) conformal map projections. Each projection is created by projecting outward from the Equator to the flat map surface. Note the tangent character to the sphere and plane for the Mercator and polar stereographic projections, respectively, but the secant character to the cone for the Lambert conic projection.

The Lambert conic map projection is obtained by fitting a cone with tip directly above a pole secant to the Earth. The secant points, or where this cone cuts through the Earth, are typically taken

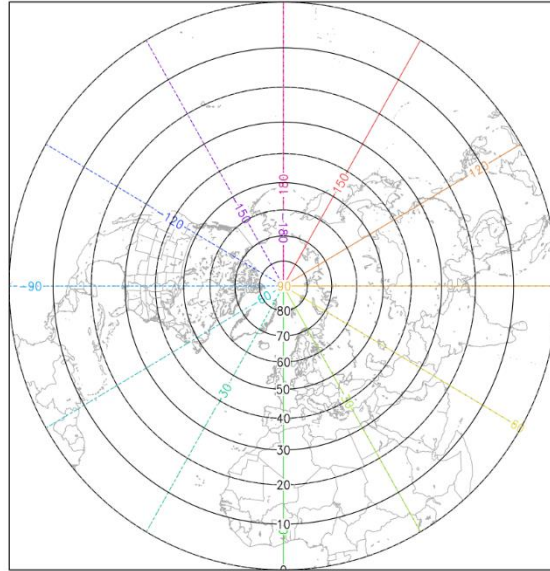
to be at 30° and 60° latitude. The Mercator map projection is obtained by fitting a cylinder tangent to the Earth at the Equator. Finally, the polar stereographic map projection is obtained by fitting a plane tangent to the Earth at the North or South Pole. Graphical examples of how each projection is obtained are in Fig. 1, while sample maps utilizing each projection are in Figs. 2-4.



**Figure 2.** Example Mercator map projection. Note how Greenland is depicted as being much larger than Australia, when in fact the surface of Australia is over 3.5 times larger than Greenland. This is emblematic of the higher-latitude distortion of the Mercator projection. Image obtained from Google Maps.



**Figure 3.** Example Lambert conic map projection encompassing the continental United States. Note the orientation of the latitude (solid black) and longitude (dashed color) lines, as well as the “pinching” of the map at its top (e.g., higher latitudes) compared to its bottom.



**Figure 4.** Example polar stereographic map projection encompassing the Northern Hemisphere. Note the circular orientation of the latitude lines (solid black) as well as the convergence of the longitude lines (dashed color) at the North Pole.

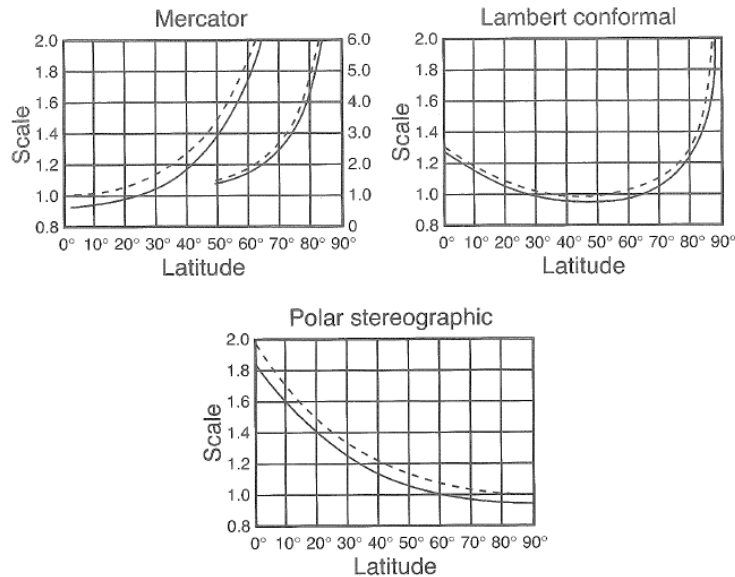
An important note for us to consider moving forward is the representation of the cardinal directions on the maps associated with each projection. Although the latitude and longitude lines are always *locally* perpendicular on all three of the conformal map projections we have considered, they are not universally perpendicular for the Lambert conic and polar stereographic projections. Thus, true north is only always to the top of the map for the Mercator projection; for the other projections, it is parallel to a line of constant longitude. Likewise, true east is only always to the right side of the map for the Mercator projection; for the other projections, it is parallel to a line of constant latitude.

The degree to which distances, and thus by extension areas and shapes, are distorted in conformal map projections can be expressed by what is known as the *map-scale factor*, given by:

$$m = \frac{\Delta x_{map}}{\Delta x_{earth}} \quad (1)$$

In the above,  $\Delta x_{map}$  refers to the horizontal distance between two points on the conformal map and  $\Delta x_{earth}$  refers to the horizontal distance between the same two points on the Earth. On Fig. 1, the distance on the perimeter of the blue circle between two adjacent grey arrows is  $\Delta x_{earth}$ , whereas the corresponding distance on the black line to which the grey arrows extend is  $\Delta x_{map}$ . The case of  $m = 1$  represents no distortion; i.e., the distance between two points on the map and Earth are equal. For each of our conformal map projections, the distortion is smallest at and near the latitudes where the cone, cylinder, or plane intersect or lie tangent to the Earth. It grows larger, sometimes rapidly so, as you move away from this/these latitude(s).

We desire that the map-scale factor be as close to 1 as possible, or roughly between 0.9 and 1.1. In Fig. 5, curves of the map-scale factor as a function of latitude for each of our conformal map projections are presented. We find that the map-scale factor falls within this range from the Equator to about 30°N for the Mercator, 20°N to 70°N for the Lambert conic, and 50°N to 90°N for the polar stereographic map projection. Given our interest in studying mid-latitude synoptic-scale meteorology, the Lambert conic projection will generally be our projection of choice this semester.



**Figure 5.** Map-scale factor as a function of latitude for the Mercator (upper left), Lambert conic (upper right), and polar stereographic (bottom) map projections. For the Mercator and polar stereographic projections, please analyze the dashed curves; for the Lambert conic projection, please analyze the solid curve. Figure obtained from *Numerical Weather and Climate Prediction* by T. T. Warner, their Fig. 3.5.

### Horizontal Coordinate Systems

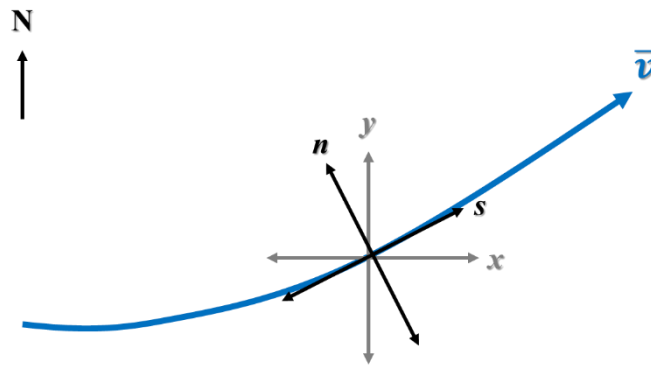
Most generally, our horizontal coordinates are given by  $x$ , referring to the east-west (or zonal) direction, and  $y$ , referring to the north-south (or meridional) direction. These horizontal coordinates are known as *Cartesian coordinates* and are geometric in nature. The positive  $x$ -axis always points to the east, while the positive  $y$ -axis always points to the north. As one might expect, care must be taken to correctly define the  $x$ - and  $y$ -axes on Lambert conic, polar stereographic, and other related conformal projections where latitude and longitude lines are not universally parallel.

Because the Earth is spherical, or very nearly so, *spherical coordinates* are often utilized. A *spherical coordinate system* is one in which the horizontal coordinates are not  $x$  and  $y$  but, rather, are latitude ( $\phi$ ) and longitude ( $\theta$ ). When an isolated, cylindrically-shaped feature (e.g., a cyclone)

is being analyzed, *cylindrical coordinates* may be used. A *cylindrical coordinate system* is one in which the horizontal coordinates are radius ( $r$ ), or distance, and azimuth ( $\lambda$ ), or angle.

It is possible to transform equations between Cartesian, spherical, and cylindrical coordinates. Except where otherwise stated, however, our focus in this class will be on the analysis and interpretation of data presented in Cartesian coordinates. Many Calculus textbooks provide basic transform relationships for each coordinate system, while many dynamic meteorology textbooks provide applications of such relationships to the equations governing atmospheric motions.

It is also possible to define what is known as a *natural coordinate system*. A natural coordinate system is one defined not based upon the geography and/or geometry of the Earth but, rather, based upon the local air flow (e.g., wind). In a natural coordinate system, there exist two horizontal coordinates:  $s$ , following the flow (**streamwise**), and  $n$ , perpendicular (**normal**) to the flow. The positive  $s$ -axis is defined in the direction to which the wind is blowing, while the positive  $n$  axis is defined perpendicular and to the left of the positive  $s$ -axis, as depicted in Fig. 6 below. We will discuss natural coordinates in more detail at times throughout this semester.



**Figure 6.** Graphical example of Cartesian ( $x, y$ ; in grey) and natural ( $s, n$ ; in black) coordinate systems. Note how the  $y$ -axis points to the north and the  $x$ -axis points to the east, whereas the  $s$ -axis points along the wind (depicted by a *streamline* in blue) and the  $n$ -axis points perpendicular and to the left of the  $s$ -axis. At every location, the  $x$ - and  $y$ -axes are of identical construction, whereas the  $s$ - and  $n$ -axes change orientation depending upon the local wind direction.

### Vertical Coordinate Systems

Most generally, a vertical coordinate depicts what we know to be “up” and “down.” Vertical coordinates enable us to represent the atmosphere by a series of distinct levels, each of which is characterized as having the same value of some quantity that defines the vertical coordinate. In the atmospheric sciences, there exist four widely-used vertical coordinates: height (or  $z$ ), pressure (or  $p$ ), potential temperature (or  $\theta$ ), and sigma (or  $\sigma$ ).

Since it is derived based upon height above sea level, the height (or  $z$ ) coordinate is the most straightforward to both visualize and conceptualize. Here, we divide the atmosphere into a series of distinct levels each having the same height above sea level at every horizontal location. In this coordinate, the positive axis (known as the  $z$ -axis) points upward. The  $x$ - $y$ - $z$  coordinate system is commonly referred to as the Cartesian coordinate system. Constant height surfaces at low altitudes above sea level often intersect the Earth's surface, particularly in mountainous regions.

The pressure, or  $p$ , vertical coordinate is typically referred to as the *isobaric* vertical coordinate. Here, the prefix *iso-* implies “equal,” while the suffix *-baric* has the pressure unit of measurement *bar* at its heart. Thus, on an isobaric surface, pressure is constant at every horizontal location. A given isobaric surface is generally not found at the same height above sea level at every horizontal location, however, because of the relationship between pressure and mass. Recall that pressure is related to the mass of the air present within the atmosphere above a given point. Consequently, pressure is highest at the Earth's surface, where it has an average sea level value of approximately 1013.25 hPa, and decreases going upward. Thus, in this coordinate, the positive  $p$ -axis points downward. Isobaric surfaces at relatively high pressures ( $\geq 850$  hPa) can intersect Earth's surface, particularly near higher terrain. Meteorological data collected above the Earth's surface, including temperature, relative humidity, wind speed and direction, and geopotential height, are typically reported on isobaric surfaces. This makes the isobaric coordinate the most frequently used vertical coordinate in synoptic meteorology, particularly for map analysis. We will make frequent use of data plotted on isobaric surfaces such as 850, 700, 500, and 300 hPa during this semester.

The potential temperature, or  $\theta$ , vertical coordinate is typically referred to as the *isentropic* vertical coordinate. Here, the prefix *is-* implies “equal,” while the suffix *-entropic* refers to entropy, which is closely related to potential temperature. Thus, on an isentropic surface, potential temperature is constant at every horizontal location. As a result, a given isentropic surface is generally not found at the same height above sea level, or pressure, at every horizontal location. The isentropic vertical coordinate is appealing because many physical processes in the atmosphere conserve (i.e., keep constant) potential temperature as they occur, even if air moves vertically upward or downward with respect to height. We will consider isentropic analysis in more detail early next semester.

The sigma coordinate is a terrain-following coordinate that is commonly used in numerical weather prediction models, primarily for computational reasons, but that is not traditionally used elsewhere. We will not utilize the sigma coordinate in this class.

## Reference Frames

There exist three primary reference frames: *Eulerian*, *quasi-Lagrangian*, and *Lagrangian*. An Eulerian reference frame is a fixed reference frame. It is established over a fixed geographic area. Because it does not move, it also does not change shape. Air can move into and out of an Eulerian reference frame. Most conventional weather analyses use an Eulerian reference frame.

By contrast, a Lagrangian reference frame is a moving reference frame. It is not fixed in space, but rather moves with the flow, which consists of an infinitesimal set of air parcels. The simplest example of a Lagrangian reference frame is given by a single *trajectory* following the motion of an air parcel. Air does not move into or out of a Lagrangian reference frame; rather, the reference frame changes shape following the flow when following multiple air parcels.

A quasi-Lagrangian reference frame combines elements of both Eulerian and Lagrangian reference frames. A quasi-Lagrangian reference frame is a moving reference frame but with a fixed shape. A simple example of a quasi-Lagrangian reference frame is a fixed box that follows the motion of some atmospheric feature such as an area of low pressure. Air can move into and out of a quasi-Lagrangian reference frame, but the feature(s) being tracked cannot.