Synoptic Meteorology I: Applying Finite Differences Through Advection

One of the most important attributes of the wind is its ability to transport. The transport of some quantity by the wind is known as *advection*. We are most often interested in its horizontal transport, or *horizontal advection*, where the horizontal surface can be taken to be Earth's surface, a constant height surface, an isobaric surface, or even an isentropic surface. For convenience, we sometimes refer to horizontal advection simply as advection.

In synoptic meteorology, we are particularly interested in *temperature advection*, referring to the horizontal transport of energy (recall that temperature is simply a measure of the average kinetic energy of the air) by the wind. Patterns of cold air advection and warm air advection reflect the (horizontal) motion of air masses and, as we will see next semester, play a crucial role in forcing vertical motions, can bring about changes in the amplitude of troughs and ridges, and can influence cyclone and anticyclone development.

Mathematically, temperature advection is expressed in vector notation as:

$$advection = -\vec{\mathbf{v}} \cdot \nabla T \tag{1}$$

There are three important elements to (1):

- $\vec{\boldsymbol{v}}$, representing the wind vector. In two dimensions, this includes the zonal wind u (the east-west wind component) and the meridional wind v (the north-south wind component). Thus, $\vec{\boldsymbol{v}}$ contains information about both *wind speed* and *wind direction*.
- ∇T, representing the gradient ∇ of temperature T. The gradient of any quantity refers to its change over the dimensions represented by the gradient, such as x and y for the horizontal dimensions in a Eulerian reference frame. The gradient has a large magnitude when either its numerator, representing the change in the given quantity, is large and/or its denominator, representing the distance over which the change is calculated, is small. The gradient itself, however, is a vector that points in the perpendicular direction from low to high values.
- The dot product between the two vectors described above. Mathematically, the dot product is zero when two vectors are perpendicular and is maximized when two vectors are parallel or antiparallel. For instance, temperature advection is maximized when the wind is directed from high toward low temperatures (i.e., along the temperature gradient) and is zero when the wind is directed parallel to the isotherms (i.e., along constant temperature perpendicular to the temperature gradient).

In only the horizontal Cartesian directions, (1) is expanded into its component form as:

$$advection = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} \tag{2}$$

In this sense, temperature advection is expressed as the product of the appropriate component of the wind – whether east-west (*u*; also known as the zonal wind component) or north-south (*v*; also known as the meridional wind component) – and the local change of temperature in some direction – east-west (*x*) or north-south (*y*).

The units of temperature advection are the units of wind – m s^{-1} – multiplied by the units of temperature – generally either °C or K – divided by distance units – m. As a result, temperature advection has units of °C s^{-1} or K s^{-1}; in other words, how temperature is changing *locally* over some finite amount of time Δt. We can evaluate (2) from charts of weather data using our centered finite difference approximation developed above.

Consider the hypothetical analysis presented in Fig. 1. We are interested in computing the horizontal temperature advection at the point marked by the closed circle and wind observation. We have already completed an isotherm analysis using temperature data from this point as well as the other locations that surround it. We thus have everything we need for our calculation.

To compute horizontal temperature advection, we must first set up our *x*- and *y*-axes. Fortunately, since we are told in the Fig. 1 caption that the data are plotted on a Mercator map projection, the positive *x*-axis points to the right, or due east, while the positive *y*-axis points up, or due north. Since our centered finite difference approximation is only valid over finite distances – here, Δx and Δy – we must set up a *small*, uniform grid centered on the wind observation location. This is done so that we can estimate the temperature at points *x+1*, *x-1*, *y+1*, and *y-1* – in other words, the terms that enter the numerator of our centered finite differences. The result of doing so is given in Fig. 2.

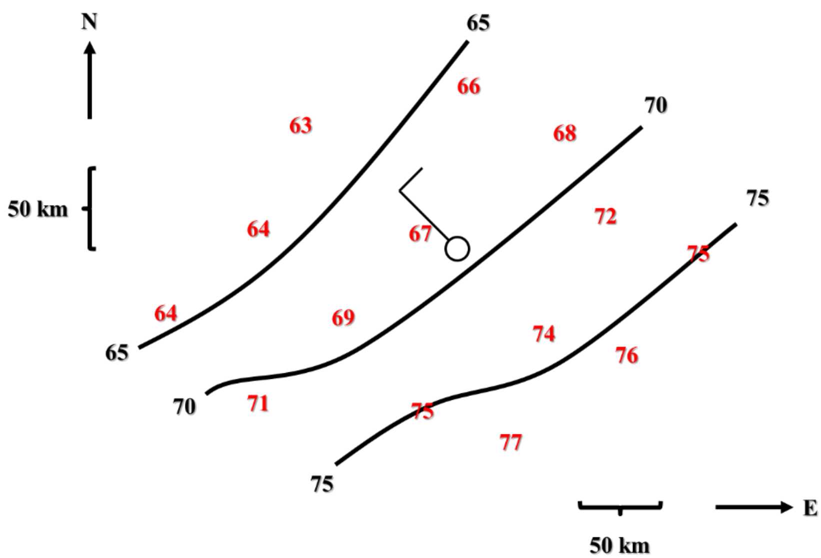

Figure 1. Hypothetical surface temperature observations (°F, red numbers), isotherm analysis (every 5°F, black lines), and a single wind observation (10 kt = 5.15 m s^{-1} out of the northwest).

Depicted for reference are horizontal scales and the north and east cardinal directions. Data are plotted on a map constructed using the Mercator map projection.

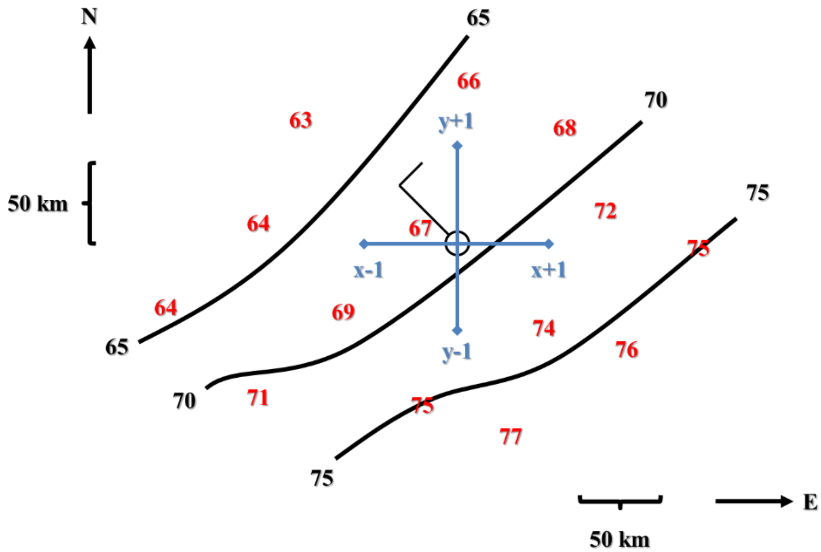

Figure 2. As in Fig. 1, except with a finite grid drawn in centered on our wind observation. Both in this example and in practice, the distance Δx is taken to be equal to the distance Δy. In this case, using the distance references on the edges of the map, both Δx and Δy are 50 km (or 50,000 m).

Next, we use our isotherm analysis to estimate the temperature at points *x+1*, *x-1*, *y+1*, and *y-1*. We must do so because we do not have an exact temperature observation at any of these locations. Visually doing so, we estimate that the temperature at *x+1* is 72°F, at *x-1* is 67°F, at *y+1* is 67°F, and at *y-1* is 73°F. This enables us to compute the finite difference approximations to our partial derivatives, where:

$$advection = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} = -u\frac{T_{x+1} - T_{x-1}}{2\Delta x} - v\frac{T_{y+1} - T_{y-1}}{2\Delta y} = -u\frac{72°F - 67°F}{100000m} - v\frac{67°F - 73°F}{100000m} \quad (3)$$

Note that, per Fig. 2's caption, we know that $\Delta x = \Delta y = 50{,}000$ m, such that $2\Delta x = 2\Delta y = 100{,}000$ m. Now, we need to know the values of *u* and *v*, the zonal (east-west) and meridional (north-south) wind components, respectively. To obtain these values, we need to use a bit of trigonometry. Recall that in meteorological convention, from the north = 0°/360°, from the east = 90°, from the south = 180°, and from the west = 270°. If the wind direction (in degrees) is known, then the *u* and *v* components of the wind can be obtained using the following equations:

$$u = -\|\mathbf{v}\|\sin\left(\frac{\pi}{180} * wdir\right) \quad (4)$$

$$v = -\|\mathbf{v}\|\cos\left(\frac{\pi}{180} * wdir\right) \tag{5}$$

In both (4) and (5), ||**v**|| is the magnitude of the wind vector **v**. In applied terms, ||**v**|| is simply equal to the wind speed. The *wdir* variable is the wind direction in degrees, and the π/180 factor in both statements converts wind direction from degrees to radians for trigonometric calculations.

Returning to our example in Fig. 1, we know that the wind speed is equal to 10 kt = 5.15 m s^{-1}. We also know that the wind is from the northwest. Expressed in degrees, from the northwest = 315° (halfway between 270°/west and 360°/north). If we substitute these values into (4) and (5), we obtain:

$$u = -5.15ms^{-1}\sin\left(\frac{\pi}{180} * 315\right) = 3.64ms^{-1} \tag{6}$$

$$v = -5.15ms^{-1}\cos\left(\frac{\pi}{180} * 315\right) = -3.64ms^{-1} \tag{7}$$

A bit of a sanity check is in order before proceeding. The positive *x*-axis is to the east, while the positive *y*-axis is to the north. The wind is blowing ***from*** the north and west and, thus, ***to*** the south and east. Our wind thus blows in the positive *x* but negative *y* directions. Since *u* is along the *x*-axis (east-west) and *v* is along the *y*-axis (north-south), we would expect that *u* should be positive and *v* should be negative for a northwest wind – and, indeed, we find that this is true.

If we plug (6) and (7) into (3) and run through the calculations, we obtain:

$$advection = -3.64ms^{-1}\frac{72°F - 67°F}{100000m} - \left(-3.64ms^{-1}\right)\frac{67°F - 73°F}{100000m} = -0.0004°Fs^{-1} \tag{8}$$

In other words, due solely to horizontal advection, the temperature at the location of our wind observation is cooling by 0.0004°F every second. If we multiply this by 3,600 (the number of seconds in one hour) or 84,600 (the number of seconds in one day), we can convert this to °F h^{-1} or °F day^{-1}, respectively. Doing so, we obtain values of -1.44°F h^{-1} and -34.56°F day^{-1}. In other words, due solely to horizontal advection, the temperature at the location of our wind observation is cooling by 1.44°F every hour and 34.56°F every day. Of course, horizontal advection is far from the only thing controlling the temperature at a given location – for instance, changes in cloud cover or time of day exert a significant influence on temperature – such that the estimated temperature change due to advection is usually much larger than the actual temperature change.

Before we proceed further, it is again time for another sanity check. In Fig. 2, we see that the wind is blowing toward the station from where it is colder. As a result, we would expect the wind to be advecting (or transporting) colder air toward the observation station. Our calculation suggests that this is true – due to advection, the temperature at the observation station is cooling.

The above calculation process is a complex means of evaluating horizontal temperature advection. By contrast, our sanity check hints at another, far less complex means of doing so. Instead of using Cartesian (*x*,*y*) coordinates, as we did before, we may use a *natural coordinate system* to assess horizontal temperature advection.

Recall that in the natural coordinate system, the appropriate coordinates become *s*, or along (**streamwise**) the wind, and *n*, or **normal** to the wind. For the example given in Fig. 1, the positive *s*-axis points to the southeast, in the direction that the wind is blowing, and the positive *n*-axis points to the northeast, or 90° to the left of the positive *s*-axis. Fig. 3 provides a graphical depiction of the natural coordinate system applied to the example from Figs. 1 and 2.

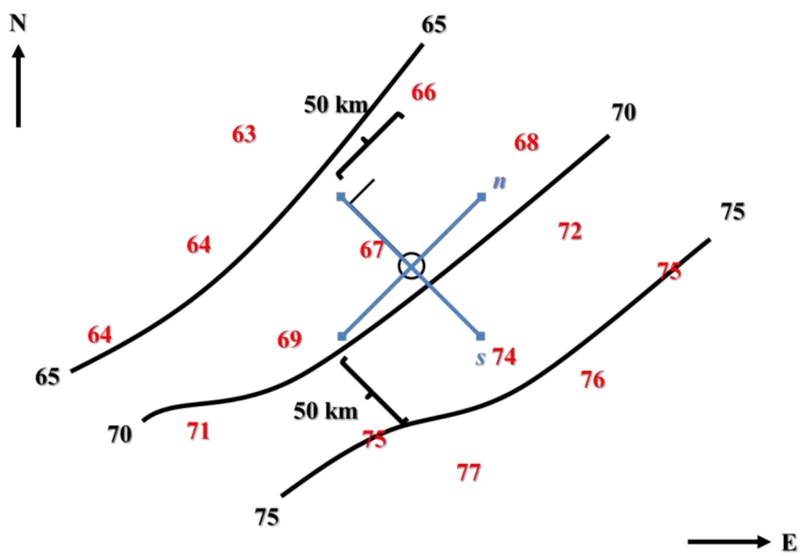

Figure 3. As in Fig. 2, except with the finite grid drawn in the natural (rather than Cartesian) coordinate system. Both in this example and in practice, the distance Δs is taken to be equal to the distance Δn. In this case, using the distance references on the finite grid, both Δs and Δn are 50 km (or 50,000 m).

In the natural coordinate system, advection is expressed mathematically as:

$$advection = -\|\mathbf{v}\|\frac{\partial T}{\partial s} = -V\frac{\partial T}{\partial s} = -V\frac{T_{s+1} - T_{s-1}}{2\Delta s} \quad (9)$$

In the above, *V* is the *wind speed*, equivalent to the magnitude of the velocity vector ($\|\mathbf{v}\|$). There is no term in the normal direction since the wind is only directed (by definition) in the streamwise direction; this also allows for the wind speed to be used directly rather than breaking it into zonal and meridional components. Thus, (9) is a second-order–accurate finite difference approximation for horizontal temperature advection. We only need to know the wind speed – 10 kt or 5.15 m s^{-1} in this example – and the change in temperature along the *s*-axis to compute the advection.

Evaluating from Fig. 3, we estimate that T_{s+1}, or the temperature at the grid point along the positive *s*-axis, is 73.5°F and that T_{s-1}, or the temperature at the grid point along the negative *s*-axis, is 66°F. Plugging these values into (9), we obtain:

$$advection = -\left(5.15ms^{-1}\right)\frac{73.5°F - 66°F}{2\left(50000m\right)} = -0.000386°Fs^{-1} \tag{10}$$

Note that this result is very nearly identical to that in Equation (8), as we expect using the same data. That this is true provides a sanity check upon our result. The two are not exactly equal because of the inherent approximate nature to each of our two analyses, namely in obtaining the values of *T* at each of our grid points.

Though this example only demonstrates two ways of computing horizontal temperature advection for a single set of data, we can nevertheless build on the concepts developed through the example to state several general rules for temperature advection in specific and advection more generally:

- Cold air advection occurs when the wind blows from cold toward warm air. Conversely, warm air advection occurs when the wind blows from warm toward cold air. In the general case, negative advection occurs when the wind blows from smaller toward larger values, whereas positive advection occurs when the wind blows from larger toward smaller values.

- When the change in advected quantity over a fixed distance is large, the magnitude of the advection is large. When the change in advected quantity over a fixed distance is small, the magnitude of the advection is small. For example, if the temperature at the *s+1* and *s-1* points in Fig. 3 were 77.5°F and 62°F, respectively, instead of 73.5°F and 66°F, then the change in temperature over the $2\Delta s$ interval would be 15.5°F and the advection would be -0.000798°F s^{-1}.

- When the wind is *parallel* to the isolines (and thus lies perpendicular to the gradient), the horizontal advection is zero. For example, if the wind in Fig. 3 were from the southwest instead of from the northwest, the temperature at the *s+1* and *s-1* points would both be 68°F, and the advection would be 0°F s^{-1}.

- Conversely, when the wind is *perpendicular* to the isolines as in Figs. 2 and 3 (such that it lies parallel to the gradient), horizontal advection is maximized.

- When the wind component blowing perpendicular to the isolines is larger, horizontal advection is larger. For example, if the wind in Fig. 3 were 20 kt (10.3 m s^{-1}) instead of 10 kt (5.15 m s^{-1}), the advection would be doubled (-0.000773°F s^{-1}). Conversely, when the wind component blowing perpendicular to the isolines is smaller, horizontal advection is smaller.

Let us now demonstrate a qualitative application of these rules to horizontal temperature advection, colloquially known as “cherries and berries.” Consider the 700 hPa observations, including wind and temperature, given in Fig. 4. We can use our principles of isoplething to create a temperature (isotherm) and geopotential height (isohypse) analysis, as is depicted in Fig. 5. In Fig. 5, we can see that the wind generally blows parallel to the isohypses, with lower geopotential heights to the left of the wind. This information allows us to conduct our “cherries and berries” analysis at each intersection of an isohypse with an isotherm, and the result of doing so is given by the blue (for cold air advection) and red (for warm air advection) dots on Fig. 5.

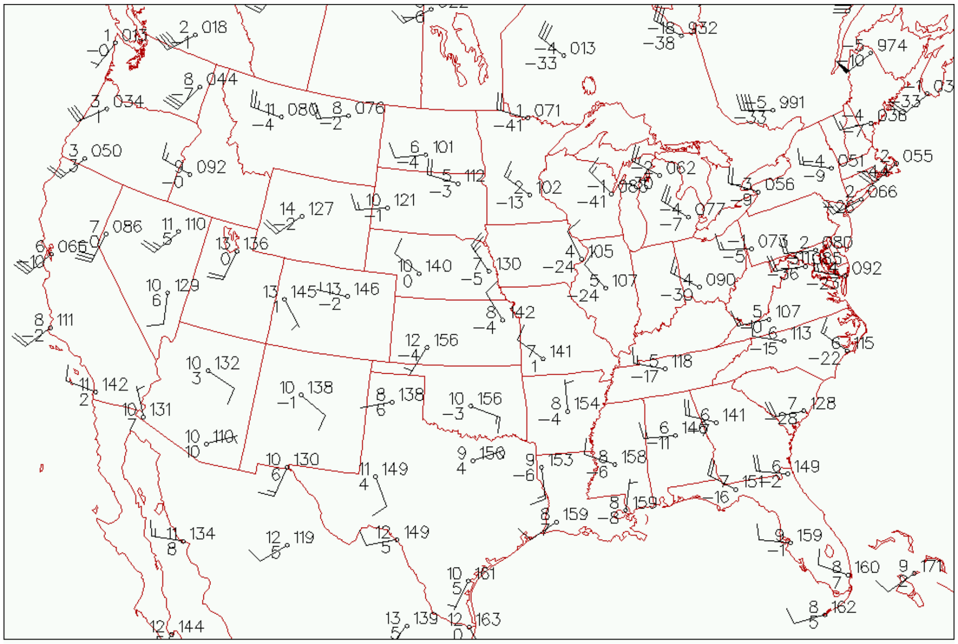

Figure 4. 700 hPa temperature (upper left), dew point temperature (lower left), geopotential height (upper right; leading 3 omitted), and wind (half-barb: 5 kt, barb: 10 kt, pennant: 50 kt) observations valid 1200 UTC 18 September 2014. Image obtained from Plymouth State University.

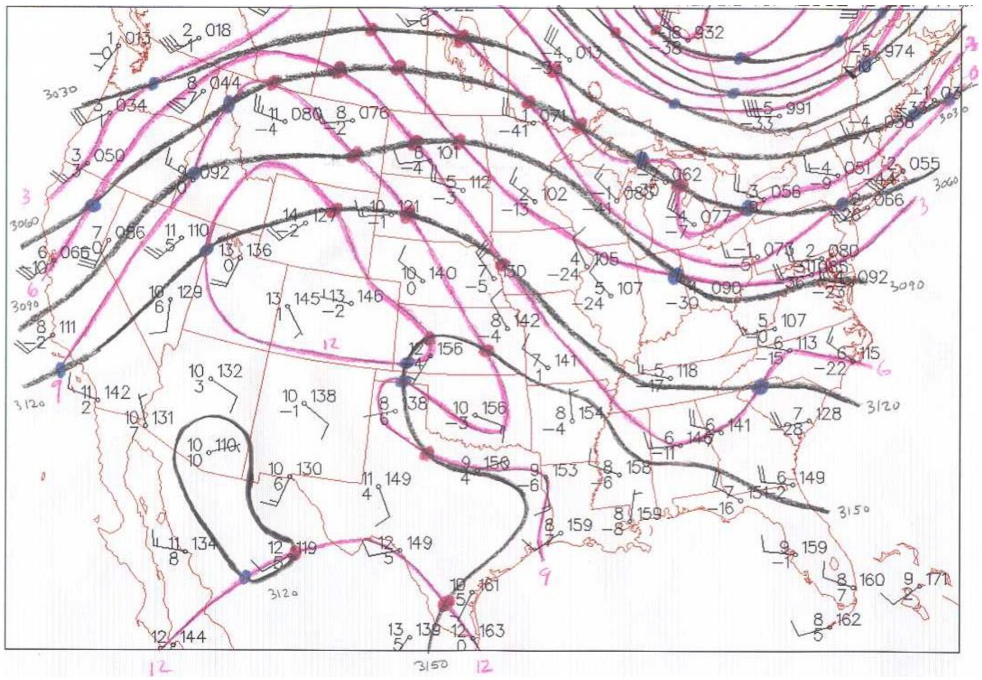

Figure 5. As in Fig. 4, except with isotherms (red lines every 3°C), isohypses (black lines every 30 m), and cherries (red dots; warm air advection) and berries (blue dots; cold air advection). Refer to the text for further details.

In Fig. 5, note that the cherries and berries are not random but instead are organized in a distinct pattern: primarily cold air advection along the west and east coasts, primarily warm air advection in the central United States. Dots are more densely packed where the isotherms and/or isohypses are more densely packed, representing regions in which temperature changes are large over a small horizontal distance and where the wind speed is large, respectively; in this sense, dot density gives an estimate of the advection magnitude (larger where more densely packed, smaller where less so).