Synoptic Meteorology II: Isentropic Analysis

Readings: Chapter 3 of *Midlatitude Synoptic Meteorology*.

Introduction

Before we can appropriately introduce and describe the concept of isentropic potential vorticity, we must first introduce the principles of isentropic analysis.

What is meant by *isentropic*? Strictly, it refers to *constant entropy*, such that an isentropic surface is a surface of constant entropy. Entropy, however, is equivalent to the natural logarithm of potential temperature plus a scaling factor; in other words, entropy is directly proportional to potential temperature. As a result, the term isentropic refers to *constant potential temperature*, such that an isentropic surface is a surface of constant potential temperature.

Why do we care about isentropic analysis? The air streams associated with midlatitude, synoptic-scale weather systems are three-dimensional, and air parcels can move across isobaric surfaces. However, in the absence of diabatic processes, air parcels conserve potential temperature as they move. This implies that the flow along the air streams associated with midlatitude, synoptic-scale weather systems remains approximately confined to an isentropic surface. Thus, we care about isentropic analysis because it gives a more accurate depiction of the three-dimensional synoptic-scale motions through midlatitude weather systems than does isobaric analysis alone.

Vertical Structure of Potential Temperature

In the troposphere, potential temperature typically *increases* with increasing height (excepting the case of strong surface heating leading to a shallow near-surface layer where potential temperature decreases with increasing height). Through Poisson's equation, colder temperatures on an isobaric surface correspond to lower potential temperature and warmer temperatures on an isobaric surface correspond to higher potential temperature. Thus, *isentropic surfaces slope **upward** over colder air*.

On the planetary scale, colder air is typically found toward the poles and warmer air is typically found toward the Equator. Thus, to first order, isentropic flow from the Equator toward the pole will ***ascend*** while isentropic flow from the poles toward the Equator will ***descend***. This is depicted in Fig. 1. On the synoptic scale, the same vertical cross-section in Fig. 1 can apply to both warm and cold fronts (as in Fig. 2), albeit with much tighter packing and steeper sloping of the isentropes: poleward flow ahead of a cyclone ascends over a warm front and equatorward flow in its wake descends as it approaches a cold front. The former is sometimes called *isentropic upglide*, whereas the latter is sometimes called *isentropic downglide*. This pattern of vertical motion matches our conceptual model of midlatitude cyclone structure, with extensive cloudiness poleward and minimal cloud cover rearward of the surface low.

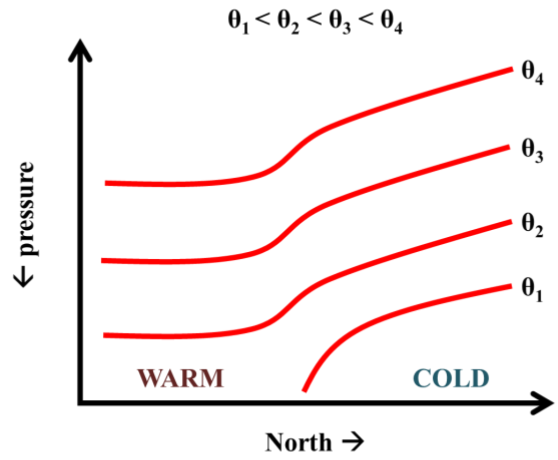

Figure 1. Idealized vertical cross-section (in the meridional direction) of potential temperature in the Northern Hemisphere. See text for details.

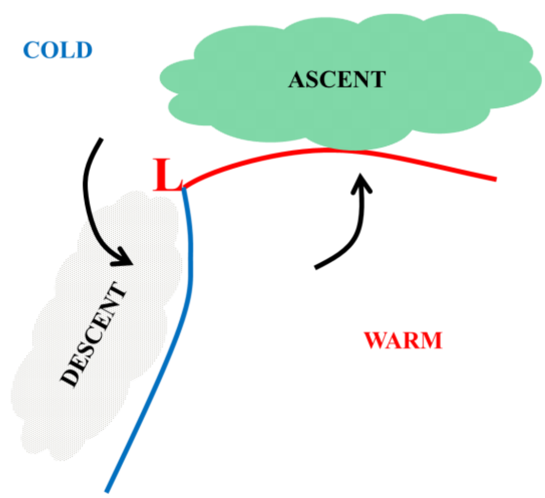

Figure 2. Idealized schematic of the horizontal flow and regions of ascent and descent found in conjunction with a mid-latitude, synoptic-scale surface cyclone/front system in the Northern Hemisphere. The warm (cold) front is denoted by the red (blue) line.

Advantages and Disadvantages of Isentropic Analysis

With the above in mind, what are the key advantages and disadvantages of isentropic analysis?

Advantages

- The synoptic-scale flow (including the flow through midlatitude weather systems) is approximately isentropic. The degree of accuracy of this approximation is about the same as that the geostrophic approximation.

- Isentropic surfaces better depict three-dimensional flow than do isobaric surfaces, yet they also depict the quasi-horizontal nature to the tropospheric flow as well as isobaric surfaces. [Recall: synoptic-scale scaling states that vertical motions are small but non-negligible compared to horizontal motions.]

- As we will demonstrate shortly, vertical motion can be inferred directly from analyses of pressure and wind on isentropic surfaces.

- Transforming the governing equations from isobaric into isentropic coordinates simplifies the vorticity equation into a form given by *isentropic potential vorticity*, the analysis and interpretation of which we will cover in the coming weeks.

Disadvantages

- Synoptic-scale flow is not entirely isentropic. This is especially true near the surface, where vertical mixing, sensible heating, and radiation are important, and in regions of latent heat release at any altitude.

- When potential temperature decreases with height – most common at and near the surface due to strong sensible heating – a single potential temperature value may correspond to two different vertical levels. This complicates isentropic analyses; which altitude, or some average thereof, should be represented by such a potential temperature?

- When potential temperature is nearly constant with height – most common near the surface due to vertical mixing – it may be difficult to determine the precise height of that isentropic surface. Fortunately, vertical mixing makes most other quantities (wind, mixing ratio, etc.) fairly uniform in the vertical, so determining their precise values on that isentropic surface is not as difficult as for the case where potential temperature decreases with height.

- Unlike constant height surfaces, isentropic surfaces may intersect the ground (as depicted in Fig. 1 above). Because they exhibit more vertical tilt than isobaric surfaces, they intersect the ground more frequently than do isobaric surfaces.

- Atmospheric data require a coordinate transformation from the constant height or isobaric coordinate on which they are obtained or modeled to create an isentropic analysis. This is often done by computer algorithms, but the interpolation used by these algorithms results in the data not representing true observations.

- The range of isentropic surfaces to look at varies by season and thus from event to event. Unlike with isobaric charts, where we typically examine a small set of isobaric surfaces (850 hPa, 500 hPa, etc.), there is no one isentropic chart to consider for all events. Typically, warmer isentropic surfaces (300+ K) are considered during the warm seasons while colder isentropic surfaces (290-295 K) are considered during the cool seasons.

Geostrophic Flow and Isentropic Analysis

On an isobaric surface, the geostrophic wind blows parallel to the isohypses. What is the analogous relationship on isentropic surfaces, however?

We start by introducing the concept of dry static energy (*s*). Dry static energy can be expressed as:

$$s = \Phi + c_p T \tag{1}$$

The dry static energy on an isentropic surface is equivalently known as the *Montgomery streamfunction*, or *M*.

Dry static energy is conserved for dry-adiabatic motions, whether horizontal or vertical in nature. Thus, on an isentropic surface, the wind should blow parallel to contours of constant dry static energy (such that dry static energy does not change along the wind) so long as the flow remains dry adiabatic. As we stated earlier that the dry-adiabatic assumption has similar accuracy to the geostrophic approximation, we will refer to the flow parallel to contours of constant dry static energy as the geostrophic flow on an isentropic surface.

With this in mind, we state the geostrophic wind on isentropic surfaces (without derivation) as:

$$v_{g\theta} = \frac{1}{f_0}\frac{\partial M}{\partial x} \tag{2a}$$

$$u_{g\theta} = -\frac{1}{f_0}\frac{\partial M}{\partial y} \tag{2b}$$

Note that this is directly analogous to the geostrophic wind on isobaric surfaces, except with *M* replacing Φ and with the calculation being carried out on an isentropic surface.

Fundamentals of Isentropic Analysis (Neglecting Diabatic Processes)

Conceptual Development

On an isobaric surface, the thermodynamic equation can be written as:

$$\frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + \vec{\mathbf{v}} \cdot \nabla_p \theta + \omega \frac{\partial \theta}{\partial p} \propto Q \tag{3}$$

In (3), the subscript of *p* on the horizontal potential advection term denotes that it, like the rest of the terms of (3), is computed on an isobaric surface. This equation denotes that changes in potential temperature following the motion are exclusively tied to diabatic heating (the rate of which is given by *Q*).

There are four primary causes of diabatic heating in the troposphere:

- Latent heating due to phase changes of water (e.g., condensation and evaporation).
- Sensible heating at the surface.
- Absorption and emission of radiative energy (nominally, radiative processes).
- Turbulent vertical mixing, particularly in the boundary layer.

Let us presume that latent heating is absent; i.e., no precipitation, cloud cover, or deep, moist convection in the synoptic-scale environment. Sensible heating and turbulent mixing may be large near the surface but are typically negligible in the mid- to upper troposphere. If we either constrain ourselves to looking above the boundary layer, or otherwise neglect sensible heating and turbulent mixing, the only remaining diabatic forcing is radiative cooling.

Long-term satellite-based observations suggest that radiative cooling contributes to 1-2 K cooling per day throughout the troposphere in the midlatitudes. The precise magnitude of this cooling varies with the seasons; it is greatest in winter and lowest in summer. How does this compare to the adiabatic forcing terms in (3), however? A typical synoptic-scale value for the horizontal gradient of potential temperature in the midlatitudes is 1 K over 100 km, such that a wind speed of 10 m s^{-1} blowing along this gradient from cold to warm air will result in a local cooling (due to horizontal advection) of 8.64 K per day. Since this cooling is several times larger than that typically observed with radiative processes in the midlatitudes, we can neglect diabatic cooling.

With this, (3) becomes:

$$\frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + \vec{\mathbf{v}} \cdot \nabla_p \theta + \omega \frac{\partial \theta}{\partial p} = 0 \tag{4}$$

Equation (4) states that potential temperature is *conserved* following the three-dimensional motion. Because we are dealing with purely isentropic, dry-adiabatic flow, we also know that mixing ratio is conserved following the motion (e.g., no gain or loss of water vapor along the flow). To confirm this, think about ascent and descent in the context of a skew T, ln p diagram: dry-adiabatic ascent and descent follow dry adiabats and lines of constant mixing ratio for temperature and moisture, respectively. Mixing ratio being conserved for isentropic flow will prove useful when we look at moisture transport on isentropic surfaces.

If we solve (4) for ω, we obtain:

$$\omega = \frac{\frac{\partial \theta}{\partial t} + \vec{\mathbf{v}} \cdot \nabla_p \theta}{-\frac{\partial \theta}{\partial p}} \qquad (5)$$

In (5), the denominator is a measure of static stability, or how tightly packed isentropes are in the vertical. The more tightly packed that isentropes are in the vertical, such as across a frontal zone or the tropopause, the greater the static stability.

(5) states that ω is a function of the sum of potential temperature advection on an isobaric surface and the local change of potential temperature, all divided by the static stability. This form of ω is known as the *isentropic vertical motion* or *adiabatic omega*. Departures in vertical motion from the isentropic vertical motion occur when the flow is non-isentropic (when diabatic processes cannot be neglected).

We typically do *not* use (5) to obtain an estimate for ω on an isobaric surface. Instead, as we demonstrate in the next section, we transform the vertical coordinate of (5) from pressure p to potential temperature θ to obtain an equation for ω along an isentropic surface.

Vertical Motion on an Isentropic Surface

We can transform (5) so that it is applicable on isentropic rather than isobaric surfaces. To do so, we need to transform the partial derivatives in (5) – those with respect to t, x, and y – that appear in the numerator of (5). We can do so by making use of the following relationships:

$$\left(\frac{\partial \theta}{\partial t}\right)_p = -\frac{\partial \theta}{\partial p}\left(\frac{\partial p}{\partial t}\right)_\theta \qquad \left(\frac{\partial \theta}{\partial x}\right)_p = -\frac{\partial \theta}{\partial p}\left(\frac{\partial p}{\partial x}\right)_\theta \qquad \left(\frac{\partial \theta}{\partial y}\right)_p = -\frac{\partial \theta}{\partial p}\left(\frac{\partial p}{\partial y}\right)_\theta \qquad (6)$$

Subscripts of p reflect evaluation on isobaric surfaces whereas subscripts of θ reflect evaluation on isentropic surfaces. In (6), static stability acts as the transform operator between isobaric (p) and isentropic (θ) coordinates.

Notably, (6) demonstrates that *horizontal gradients of potential temperature on isobaric surfaces are proportional to horizontal gradients of pressure on isentropic surfaces*! As applied to synoptic-scale analysis, what does this mean? Let us first consider a cold front as an illustrative example. A cold front separates relatively cool air behind it from relatively warm air ahead of it, with the front itself located along the leading edge of the relatively cool air. This implies the presence of a horizontal potential-temperature gradient along and behind the cold front on an isobaric surface.

Since a horizontal gradient of potential temperature on an isobaric surface is proportional to a horizontal gradient of pressure on an isentropic surface, from the above, there must be a horizontal pressure gradient along and to the rear of the cold front on an isentropic surface. Thus, *when analyzed on an isentropic surface,* a cold front separates relatively low pressure behind it from relatively high pressure ahead of it, with the front itself located along the leading edge of relatively low pressure. Similarly, a warm front separates relatively low pressure ahead of it from relatively high pressure behind it, with the front itself located along the trailing edge of retreating lower pressure. This can be visualized through Fig. 1 by identifying values of p along isentropic surfaces: lower in the cold air behind the front, higher in the warm air ahead of the front.

(6) also demonstrates that an isobar is equivalent to an isotherm on an isobaric surface. This can be demonstrated using Poisson's equation; if both p and θ are constant, so is T. Because of this, isentropic vertical motion can be viewed as akin to a thermal forcing term. Where have we seen such a forcing term before? In the quasi-geostrophic omega equation, under the assumption that the synoptic-scale motion on isobaric surfaces is largely geostrophic!

If we substitute (6) into (5) and simplify, we obtain:

$$\omega = \left(\frac{\partial p}{\partial t}\right)_{\theta} + \vec{\mathbf{v}}_{\theta} \cdot \nabla_{\theta} p \qquad (7)$$

As before, subscripts of θ reflect evaluation on isentropic surfaces. (7) shows that isentropic vertical motion can be determined from the distribution of pressure on an isentropic surface. There are two forcing terms: the local rate of change of pressure on the isentropic surface and the advection of pressure by the horizontal wind on the isentropic surface.

(7) can be simplified by using the "frozen wave" approximation. This approximation states that the local rate of change of pressure is due entirely to a synoptic-scale weather system's motion (such that changes in the system's intensity, or other processes such as the diurnal cycle that can cause pressure to change, are neglected). Thus, in a reference frame moving with the synoptic-scale weather ***system*** (not flow) the first term on the right side of (7) is zero.

In this reference frame, it is also necessary to replace the advective velocity $\vec{\mathbf{v}}_\theta$ in (7) with the *system-relative* advective velocity, $\vec{\mathbf{v}}_\theta - \vec{\mathbf{c}}$. Here, $\vec{\mathbf{c}}$ represents the motion of the synoptic-scale weather system, which is typically estimated from a time series of meteorological charts.

With all of this in mind, (7) becomes:

$$\omega = \left(\vec{\mathbf{v}}_\theta - \vec{\mathbf{c}}\right) \cdot \nabla_\theta p \tag{8}$$

(8) states that, at a fixed location, the isentropic vertical motion is a function of the advection of pressure by the *system-relative horizontal wind* on the isentropic surface.

Based on the definition of the advection operator, higher pressure advection is defined where $-\left(\vec{\mathbf{v}}_\theta - \vec{\mathbf{c}}\right) \cdot \nabla_\theta p > 0$. Because (8) lacks the leading negative sign, higher pressure advection results in $\omega < 0$, signifying ascent. Higher pressure advection refers to the situation where the wind is blowing from higher toward lower pressures. For an air parcel moving with this wind, it starts at a higher pressure and ends at a lower pressure, thus ascending as it moves.

Conversely, lower pressure advection is defined where $-\left(\vec{\mathbf{v}}_\theta - \vec{\mathbf{c}}\right) \cdot \nabla_\theta p < 0$. Again, because (8) lacks the leading negative sign, lower pressure advection leads to $\omega > 0$, signifying descent. Lower pressure advection refers to the situation where the wind is blowing from lower toward higher pressures. For an air parcel moving with this wind, it starts at a lower pressure and ends at a higher pressure, thus descending as it moves.

From (8), the magnitude of the vertical motion is directly proportional to the magnitude of the pressure advection. Thus, stronger vertical motions result from larger magnitudes of horizontal pressure advection. This is accomplished by having a large horizontal change in pressure over a short distance (large ∇p) and/or a strong system-relative wind (large $\vec{\mathbf{v}}_\theta - \vec{\mathbf{c}}$) blowing across the isobars. Thus, air parcels that change pressure rapidly over a short distance on an isentropic surface (i.e., as seen with a steeply sloped isentropic surface) are associated with stronger vertical motions.

For synoptic-scale weather systems that move much slower than the synoptic-scale flow on an isentropic surface (e.g., those that are quasi-stationary), (8) can be approximated by:

$$\omega \approx \vec{\mathbf{v}}_\theta \cdot \nabla_\theta p \tag{9}$$

However, for faster-moving weather systems, making use of this approximation can substantially change the magnitude and/or sign of the vertical motion from that obtained via (8).

Impact of Diabatic Processes

If diabatic heating is included as a forcing term in deriving (8), we obtain:

$$\omega = \left(\vec{\mathbf{v}}_\theta - \vec{\mathbf{c}}\right) \cdot \nabla_\theta p + \frac{\partial p}{\partial \theta}\frac{d\theta}{dt} \tag{10}$$

In isolation, the impact of diabatic heating on vertical motion in the isentropic framework is given by:

$$\omega = \frac{\partial p}{\partial \theta}\frac{d\theta}{dt} \tag{11}$$

Since potential temperature typically increases upward, the $\partial p/\partial \theta$ term is typically negative ($\partial p < 0$, $\partial \theta > 0$). Thus, diabatic warming ($d\theta/dt > 0$) results in $\omega < 0$, or ascent, whereas diabatic cooling ($d\theta/dt < 0$) results in $\omega > 0$, or descent. This is unsurprising, as this is the same conclusion that we arrived at in the context of the omega equation.

However, the physical meaning is slightly different, owing largely to (11) being cast on isentropic rather than isobaric surfaces. Diabatic warming results in the increase of an air parcel's potential temperature. Because potential temperature typically increases upward away from the surface, this causes the air parcel to rise as it moves from one isentropic surface to another. Conversely, diabatic cooling results in the decrease of an air parcel's potential temperature. This causes the air parcel to descend as it moves from one isentropic surface to another.

Dry-adiabatic (or isentropic) motion is not strictly valid in regions of clouds and precipitation, although the vertical distribution of moisture is such that the dry-adiabatic approximation is not significantly erroneous in the mid- to upper troposphere in the midlatitudes. Nevertheless, most synoptic-scale weather systems are associated with condensation and thus latent heat release and the non-conservation of potential temperature. Therefore, the principles of isentropic analysis may be fundamentally invalid with the very features that we are most interested in. While this is *quantitatively* true, it is not a significant *qualitative* hindrance, as demonstrated in Section 3.3 of the Lackmann text. Let us demonstrate this concept utilizing a figure, Fig. 3.12 from the Lackmann text (reproduced below as Fig. 3).

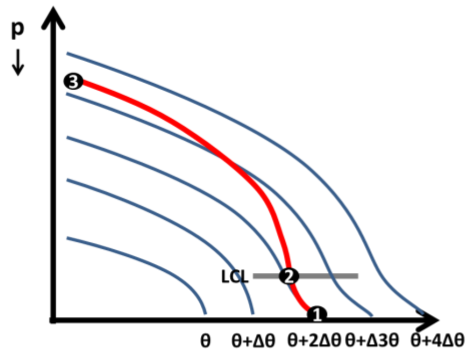

Figure 3. Idealized cross-section showing isentropes, in solid black contours, and three positions of a hypothetical air parcel, labeled from 1 to 3 in temporal order. The lifting condensation level, or LCL, is indicated by a thin horizontal line that corresponds to position 2 of the air parcel. Reproduced from *Midlatitude Synoptic Meteorology* by G. Lackmann, their Fig. 3.12.

Consider an air parcel near the ground with an initial potential temperature of $\theta+2\Delta\theta$. Presuming that the wind on this isentropic surface is from right to left (e.g., east to west), this parcel ascends along this isentropic surface. This ascent is along a dry adiabat, conserving potential temperature until it reaches its lifting condensation level (LCL). Thus far, if we neglect friction and turbulent vertical mixing, the isentropic analysis principles expressed above are entirely valid.

Upon reaching its LCL, the air parcel may continue to rise (if it is forced to do so, or if it has also reached its LFC). However, now that it has reached its condensation level, the air parcel ascends along a moist adiabat. The latent heat release associated with condensation causes the potential temperature of the air parcel to increase as it ascends. While the dry adiabatic assumption expressed earlier is no longer precisely valid, we see that the motion is merely more strongly upward than one would infer based on isentropic ascent alone. Therefore, violation of the dry-adiabatic assumption results in an *underestimation* of the isentropic ascent and does not change the qualitative aspects of the analysis.

An Example

With all of this in mind, let us examine an example isentropic analysis for insight, as depicted in Fig. 4. In so doing, consider a few noteworthy remarks related to isentropic analyses in general:

- An isentropic analysis will typically contain analyses of mixing ratio (conserved following the motion on an isentropic surface), wind, and pressure. Less commonly, it will include contours of constant Montgomery streamfunction and, by extension, the geostrophic wind applicable on isentropic surfaces.

- The sign of the isentropic vertical motion can be visually approximated by evaluating how the wind is oriented with respect to the horizontal pressure gradient, as noted above.
- Frontal boundaries are located along horizontal pressure gradients on isentropic surfaces, with cold fronts found on the leading edge of advancing lower pressure and warm fronts found on back edge of retreating lower pressure.
- For greatest accuracy, the wind on an isentropic surface should be that the *system-relative* wind rather than the full wind. However, it can be difficult to reliably estimate this wind, as synoptic-scale weather systems often change their speed or direction of motion, and their redevelopment can often be confused for motion. Likewise, it is entirely possible that two weather systems on a given chart may move in different directions or at different velocities, making a single system motion estimate unreliable at best. Therefore, most isentropic analyses are constructed using the full wind.
- Isentropic charts may have large areas of missing data where the isentropic surface is below ground or corresponds to two different pressure levels. Data in such areas, if plotted, should be discarded.
- The actual construction of an isentropic analysis is typically handled by computer algorithms. It is time-consuming to for a human to manually interpolate data from an isobaric surface to an isentropic surface and subsequently analyze the data.
- Isentropic surfaces closer to the ground are on constant surfaces of relatively low potential temperature. Conversely, isentropic surfaces higher above the ground are on constant surfaces of relatively high potential temperature.

Fig. 4 below contains two panels, both depicting pressure (contours) and wind (barbs; half: 5 kt, full: 10 kt; pennant: 50 kt) on the 296 K isentropic surface at 0000 UTC 3 January 2002. The wind in panel (a) is the full wind, whereas that in panel (b) is the system-relative wind. In both analyses, the wind is blowing from higher pressure to lower pressure along the Gulf Stream from Florida toward North and South Carolina, signifying ascent along the 296 K isentropic surface. Likewise, in both analyses, the wind is blowing from lower to higher pressure across the northwestern Gulf of Mexico, signifying descent along the 296 K isentropic surface.

However, subtle differences appear between the two analyses across Alabama and Mississippi. The full wind is approximately parallel to the isobars on the 296 K isentropic surface across Alabama and Mississippi, signifying no vertical motion. However, the system-relative wind blows from higher to lower pressure across Alabama and Mississippi, signifying ascent on the

296 K isentropic surface. The importance of this difference is apparent upon a comparison to Fig. 5.

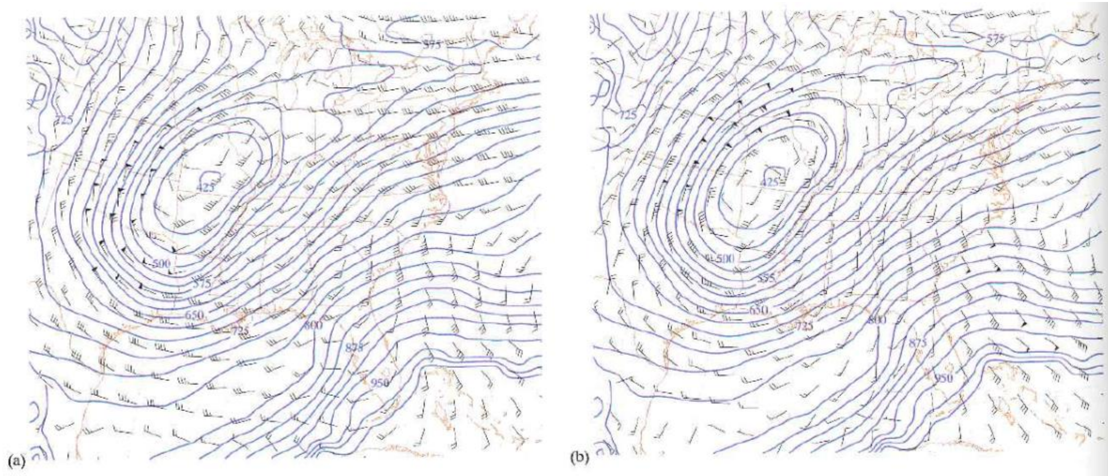

Figure 4. Pressure (hPa, contours) and wind (kt, barbs; full wind in panel (a), system-relative wind in panel (b)) on the 296 K isentropic surface, valid at 0000 UTC 3 January 2002. Reproduced from *Midlatitude Synoptic Meteorology* by G. Lackmann, their Fig. 3.9.

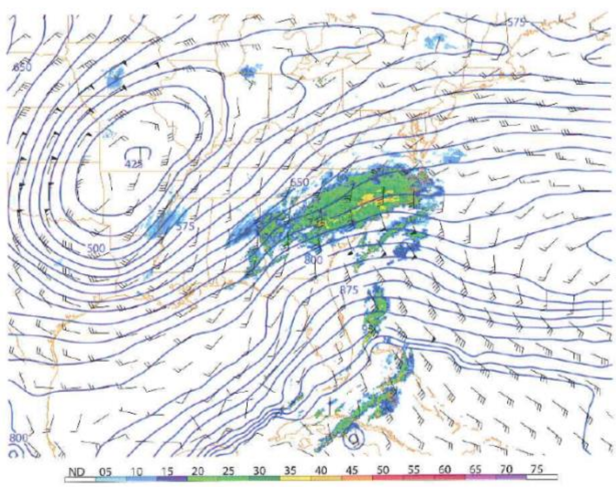

Figure 5. As in Fig. 4b, except with composite radar reflectivity (shaded; dBz) superimposed.

Areas of condensation and precipitation align very closely to the regions of ascent inferred from dry adiabatic motion along the 296 K isentropic surface(s). Non-zero composite reflectivity returns extend into Alabama and, to lesser extent, the lower Mississippi River valley. As noted above, these are areas of weak ascent implied by the system-relative isentropic analysis, helping to highlight shortcomings associated with the use of (9) instead of (8). Regardless, the analyses in Fig. 4 demonstrate the value of isentropic analysis, even under conditions in which the underlying dry-adiabatic assumption is violated.

Real-time isentropic analysis charts may be found for selected isentropic surfaces on the College of DuPage website linked from the course webpage.