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Synchronization and Causality across Time Scales in ENSO

Sergey Kravtsov

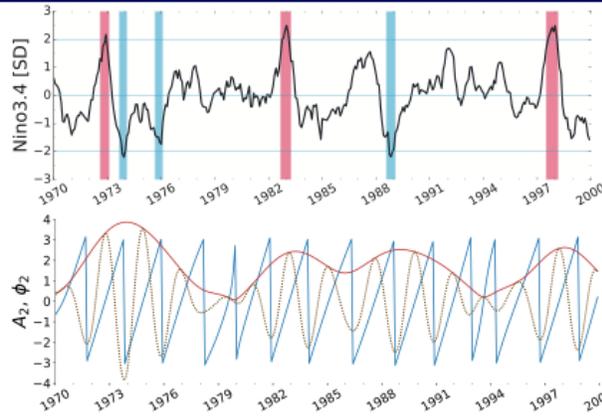
University of Wisconsin-Milwaukee, USA
P. P. Shirshov Institute of Oceanology, Russia

Collaborators: N. Jajcay, G. Sugihara, A. A. Tsonis and
M. Palus

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<https://people.uwm.edu/kravtsov/>

Study ENSO dynamics in Nino3.4 time series



Nino3.4
time series

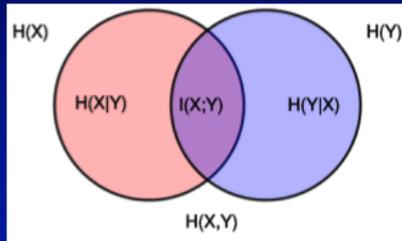
Reconstruction
of the biennial
mode (p=2 yr)
using CCWT

$$\psi_p(t) = s_p(t) + i\hat{s}_p(t) = A_p(t)e^{i\phi_p(t)}$$

- We use **Complex Continuous Wavelet Transform (CCWT)** to obtain **phase $\Phi(t)$** and **amplitude $A(t)$** time series associated with variability **at a given central period p** (Palus 2014a,b)
- We then examine causal relationships among pairs of the $\Phi(t)$ or $A(t)$ time series associated with different central periods

Minimal amount of input data to study a very complex phenomenon - just a single time series! It is amazing how much can be deduced via this methodology!

(Conditional) Mutual Information



$$I(X; Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log \left(\frac{p(x, y)}{p(x) p(y)} \right)$$

mutual information (MI)

$$I(X; Y|Z) = \mathbb{E}_Z(I(X; Y)|Z) = \sum_{z \in Z} \sum_{y \in Y} \sum_{x \in X} p_Z(z) p_{X,Y|Z}(x, y|z) \log \frac{p_{X,Y|Z}(x, y|z)}{p_{X|Z}(x|z) p_{Y|Z}(y|z)}$$

conditional MI

- For Nino3.4 analysis:

$$I(\phi_{p_1}(t); \phi_{p_2}(t))$$

— phase synchronization

$$I(\phi_{p_1}(t); \phi_{p_2}(t + \tau) - \phi_{p_2}(t) | \phi_{p_2}(t))$$

— phase–phase
causality

$$I(\phi_{p_1}(t); A_{p_2}(t + \tau) | A_{p_2}(t))$$

— phase–amplitude
causality

From Wikipedia. $I(X;Y)$ measures a degree of similarity between the joint distribution of two random variables and the product of their marginal distributions. If these two are similar, variables are independent, if not – the uncertainty of one variable is reduced given the knowledge of the other. For phases, in extreme case, we can deduce the phase of one variable knowing the phase of the other – phase synchronization. Synchronization can be intermittent and still lead to large values of mutual information on average. For bi-variate Gaussian distribution, MI has a one-to-one relationship with linear correlation; MI, however, more general, and comprises both linear and non-linear dependencies. Conditional mutual information involving future values of one of the variables measures causality (directed information, flow of information). Analogy to Granger causality, but, once again, CMI is a more general measure of causality. All of these are computed based on finite samples of data; need to bin and compute probabilities as frequencies of occurrence of events; advanced estimation methods have been developed.

Does it make sense? Conceptual example:

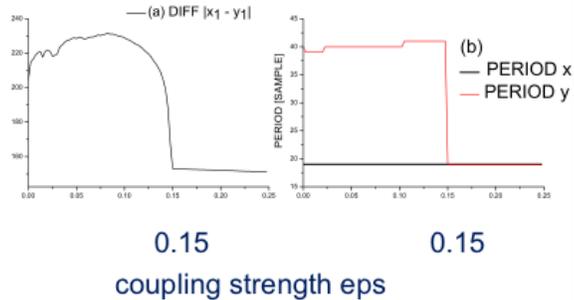
We illustrate the ability of our approach to detect causal relationships within complex systems using a toy example of unidirectionally coupled Rössler models (studied in detail by Paluš & Vejmelka⁷). Here, the driving, master system X is defined as

$$\begin{aligned}\dot{x}_1(t) &= -\omega_1 x_2(t) - x_3(t) \\ \dot{x}_2(t) &= \omega_1 x_1(t) + 0.15 x_2(t) \\ \dot{x}_3(t) &= 0.2 + x_3(t)(x_1(t) - 10)\end{aligned}$$

and the driven, slave, system Y as

$$\begin{aligned}\dot{y}_1(t) &= -\omega_2 y_2(t) - y_3(t) + \epsilon(x_1(t) - y_1(t)) \\ \dot{y}_2(t) &= \omega_2 y_1(t) + 0.15 y_2(t) \\ \dot{y}_3(t) &= 0.2 + y_3(t)(y_1(t) - 10)\end{aligned}$$

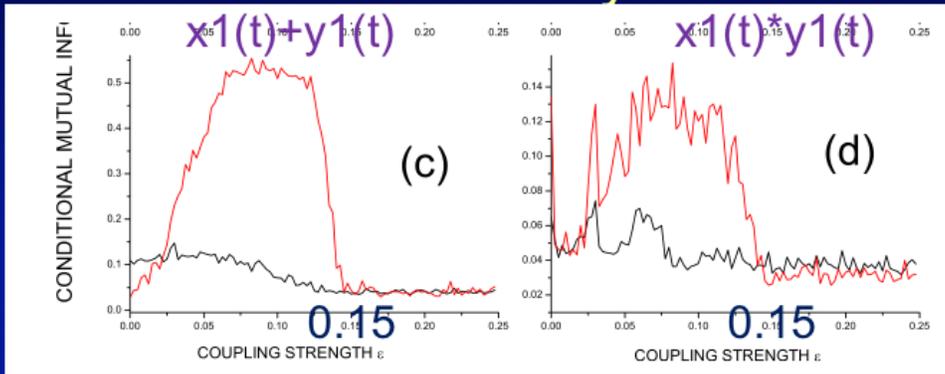
where $\omega_1 = 2.0$, $\omega_2 = 0.985$. The first component of Y contains the diffusive coupling term $\epsilon(x_1(t) - y_1(t))$, where ϵ is the coupling strength.



Question: Based on a **single observable**, e.g. linear $[x_1(t)+y_1(t)]$ or nonlinear $[x_1(t)*y_1(t)]$ combination of $x_1(t)$ and $y_1(t)$, **can we detect the direction of causality** between master and slave systems?

At large coupling strength the slaved system synchronizes with the master system and starts to oscillate with the period of the master system. Before this happens, can we deduce the causal direction by analyzing a single (linear or nonlinear) observable?

CCWT/CMI causality detection:



causal direction ($X \rightarrow Y$)

$$I(\phi_1(t); \phi_2(t+\tau) | \phi_2(t))$$

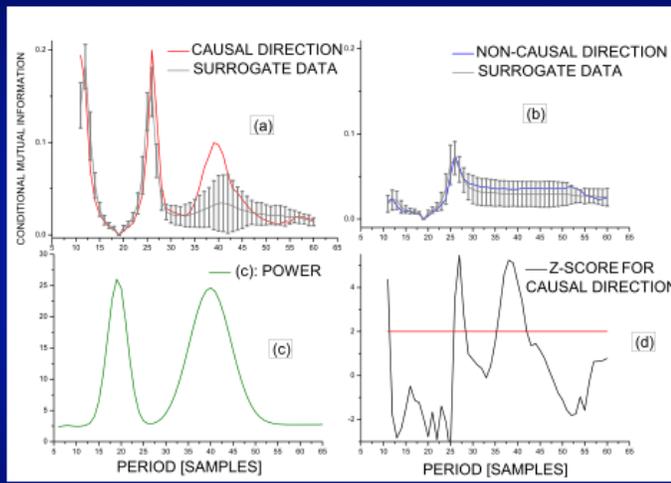
non-causal direction ($Y \rightarrow X$)

$$I(\phi_2(t); \phi_1(t+\tau) | \phi_1(t))$$

Note: For central periods p_1 and p_2 , we used true intrinsic periods of X and Y systems. What if p_1 and p_2 are unknown? Stat. significance?

Yes we can! Be mindful of spurious peaks though. Note that causality cannot be estimated in the synchronized regime (large coupling strength).

Statistical Significance of CMI estimates using Fourier Transform Surrogates

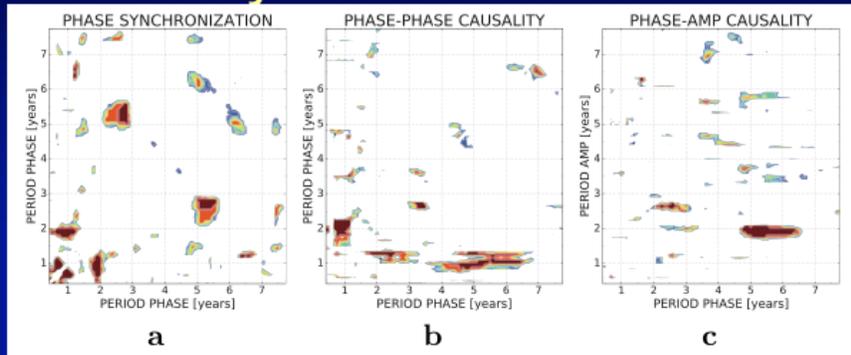


$x_1(t)+y_1(t)$,
 $\epsilon=0.065$,
 $p_1=20$ (true period of the driving system X),
 p_2 in the range from 10 to 60 (on the abscissa)

- Spurious CMI peaks are possible, but tend to be localized and sensitive to CCWT bandwidth (not shown)

In the following, we will only show the values of mutual information that are statistically significant at the 5% level relative to FT surrogates (Fourier transform time series, randomize phases of sines and cosines, transform back to the physical space). Small-scale “noise” in the parameter space of the plots is likely to be associated with spurious peaks in MI or CMI.

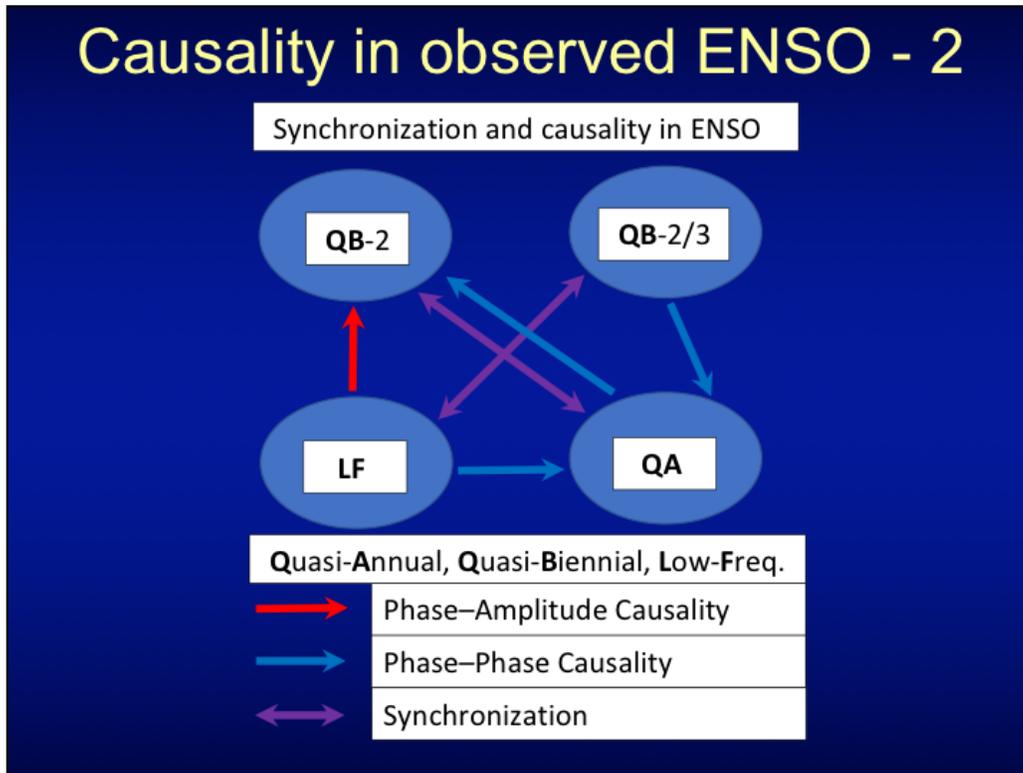
Causality in observed ENSO - 1



- Annual Cycle, Quasi-Biennial and Low-Frequency (periods of 4–6 yr) modes
- Combination Tones ($LF \pm AC = QA$, $LF \pm QB = QB2-3$)
- Synchronization: LF/QB2-3, QB/AC, QA
- Causality. Phase–phase: LF → QA, QA → QB, QB2-3 → QA; Phase–amplitude: LF → QB

The analysis identifies processes at three distinct time scales important in ENSO dynamics (consistent with previous knowledge): QA, QB, LF. AC → QB synchronization (ENSO's biennial events tend to peak in winter). Combination tones between LF and AC/QB modes lead to QB2-3 and QA variability. QB2-3 is synchronized with LF, QA with AC. No direct causal connection between LF phase and QB phase, but indirect connections via annual cycle. The only pronounced phase-amplitude causal connection is between LF and QB. Schematic on the next slide summarizes these points.

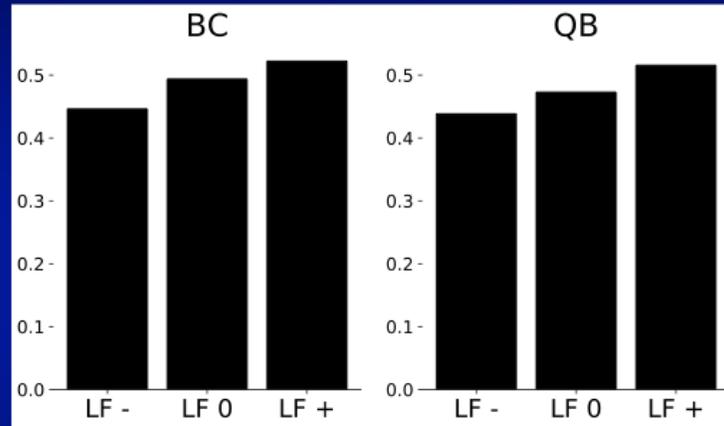
Causality in observed ENSO - 2



So, positive phase of LF, LF+ is associated with enhancement of biennial QB-2 variability and, through nonlinear interactions, enhancement of the biennial combination tones QB-2/3; the latter QB-2/3 variability is phase-synchronized with the LF variability, and both lead to changes in the shape of the annual cycle's main harmonic and combination tones (jointly QA variability), which fixes the phase of QB2 variability. Synchronization within a suite of different QB-2 modes leads to extreme ENSO events. This is the summary of causality in ENSO.

All of these points are illustrated, one by one, in the following three slides.

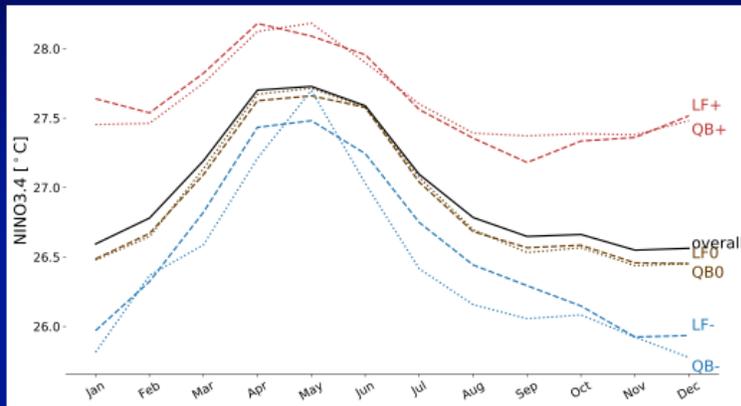
LF phase → QB2 amplitude



- Larger QB magnitudes for warm LF phase (contributing to stronger El Nino events)
- More pronounced LF/BC combination tones too (QB-2/3)!

Go back and forth between this slide and slide 8 (ENSO causality schematic).

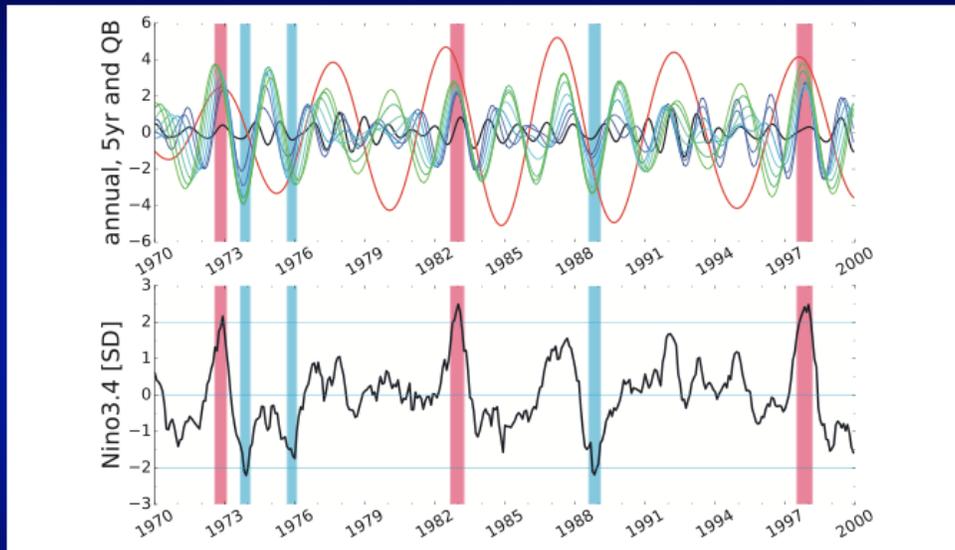
LF, QB2-3 → QA → QB2



- Seasonal cycle composites conditioned on the LF or QB-2/3 phases are very similar!
- Seasonal cycle in the +phases is more harmonic (and shorter-period) than that in -phases
- QA “fixes” the phase of QB2 (peaks in winter!)

Go back and forth between this slide and slide 8 (ENSO causality schematic). The shape of the seasonal cycle depends on the phase of the LF and QB2-3 variability (the latter two tend to be synchronized). The amplitude of the seasonal cycle is small though (compared to that of extreme ENSO events), in all cases.

Extreme ENSO events



- LF phase is immaterial
- QB events synchronize!
- Annual cycle is tiny
- El Niño vs. La Niña

In particular, during all of the strong El Niño events of years 72/73, 82/83 and 97/98, the QA, QB and LF modes were characterized by synchronous pronounced maxima. Note, however, that strictly speaking, the synchronization with LF mode is not really necessary for an extreme ENSO event to materialize, since the 'peak' of this mode spans a good part of the year (for example, the peak of LF mode did not really occur in winter 1982/83, but the LF wintertime 'background' during that time was still abnormally warm), while the amplitude of the QA mode is, in general, small, so that this mode does not contribute much directly to the magnitude of a given event. Instead, what appears to be essential for an extreme ENSO to occur is the synchronization of multiple QB modes with each other. We believe that this 'internal' QB synchronization is what has been picked up by our conditional mutual information analysis in the form of LF→QA→QB phase connections and also LF phase→QB amplitude connections (since synchronization of phases of different QB modes should automatically result in a large-amplitude event.) By contrast, during a moderate El Niño of 87/88, the LF, QB and QA modes exhibited phase shifts, with lower-frequency modes leading the higher-frequency modes (in particular a suite of QB modes) instead of being 'stacked' on top of one another, thus limiting the magnitude of this event. Notably, strong La Niña events do not seem to be associated with the minimum of the LF mode, but instead occur during near-neutral LF conditions when the minima of

the QA modes and the minima of the whole range of QB modes synchronize. Thus, in both El Niño and La Niña cases, the behaviour of the QB modes has a vital control on the magnitude of the ENSO events.

Summary and discussion

- Causality diagnosis of Nino3.4 time series points to a **dominant role of quasi-biennial variability in ENSO dynamics**.
- QB modes do not account for a large fraction of Nino3.4 variance on average, but **the spikes in QB variability can still lead to extreme ENSO events** when multiple QB modes synchronize.
- The **QB variability** during extreme ENSO events is clearly not due to alteration between weak and strong annual cycle (Meehl 1987), which is weak in both cases. **What is it then?**
- The observed causal connections in ENSO are absent from the simulations of state-of-the-art climate models (CMIP5), as well as from conceptual models of ENSO interactions with seasonal cycle (e.g., PRO model). **Hence, the existing explanations of multi-scale ENSO dynamics are inconsistent with observations.**
- **Empirical LIM SST models mimic some of these dynamics**

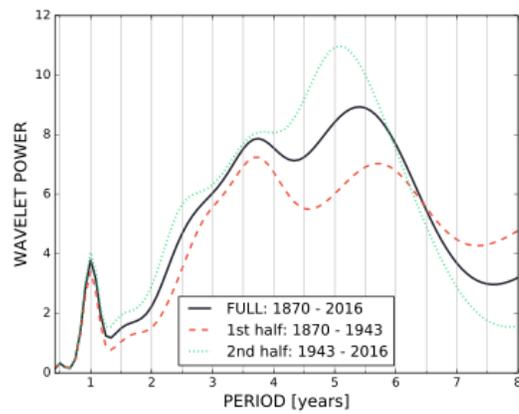
Back-up slides: Wavelet spectra of Nino3.4, Summary figure of comparison with CMIP5 models, LIM figures.

References

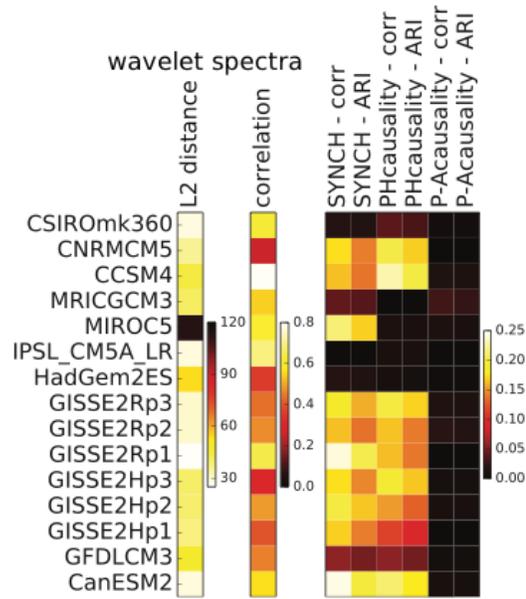
- Milan Paluřs, "Multiscale atmospheric dynamics: cross-frequency phase-amplitude coupling in the air temperature," *Phys. Rev. Lett.* 112, 078702 (2014).
- Milan Paluřs, "Cross-scale interactions and information transfer," *Entropy* 16, 5263–5289 (2014).
- Milan Paluřs and Martin Vejmelka, "Directionality of coupling from bivariate time series: How to avoid false causalities and missed connections," *Phys. Rev. E* 75, 056211 (2007).
- Gerald A Meehl, "The annual cycle and interannual variability in the tropical Pacific and Indian Ocean regions," *Mon. Weather Rev.* 115, 27–50 (1987).
- Wikipedia: Mutual Information (https://en.wikipedia.org/wiki/Mutual_information)

Back-up slides

Nino3.4 wavelet spectrum

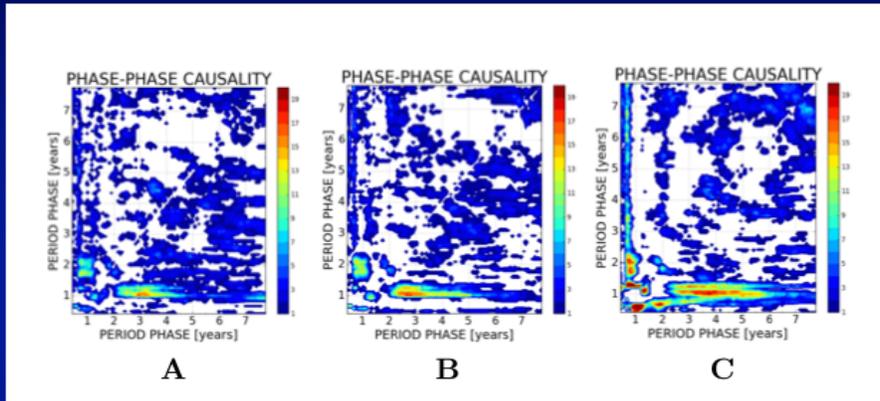


CMIP5 analyses



The synchronization and causality comparisons are based on thresholded binary maps.

Phase Causality in LIMs



- A – additive noise
- B – multiplicative noise
- C – multiplicative noise snippets

The Nino3.4 simulated in LIM models matches the observed causality much better than CMIP5 or conceptual models.