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Singular vortices as building blocks of atmospheric low-frequency variability

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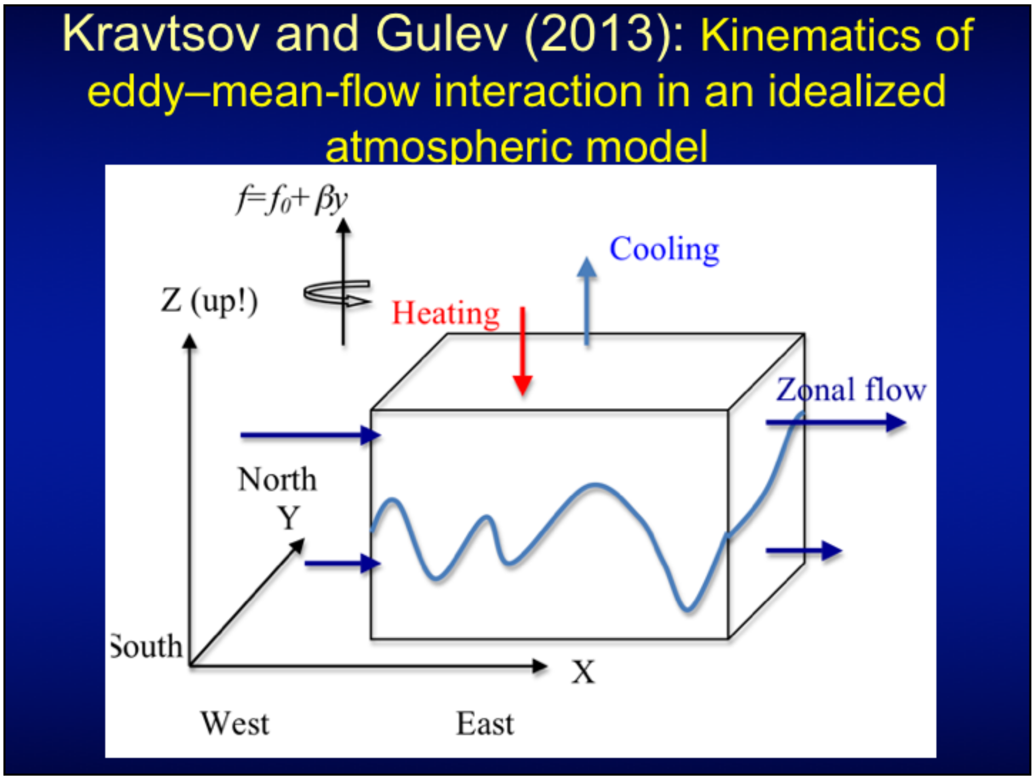


This work was primarily motivated by an earlier study (Kravtsov and Gulev 2013) of eddy–mean-flow interaction in an idealized atmospheric model. The two-layer quasi-geostrophic model used in that study is quite standard; it is arguably the minimal model to study interactions between eddies and zonal jets. The model is set up in a beta-plane channel, with periodic boundary condition in the zonal, east-to-west direction and no-slip conditions on the northern and southern boundaries of the channel. The bottom layer includes an idealized wavenumber-2 topography (not shown here). The model is thermally driven by the external forcing, which heats up the southern part of the channel and cools down the northern part, thus resulting in the south–north gradient of the interface eta between the model layers and, — due to thermal-wind relation, — in the large-scale zonal current with a vertical shear. Baroclinic instability of this current generates synoptic eddies, which interact with and further modify the original flow.

QG atmospheric model with topography (Kravtsov et al. 2005)

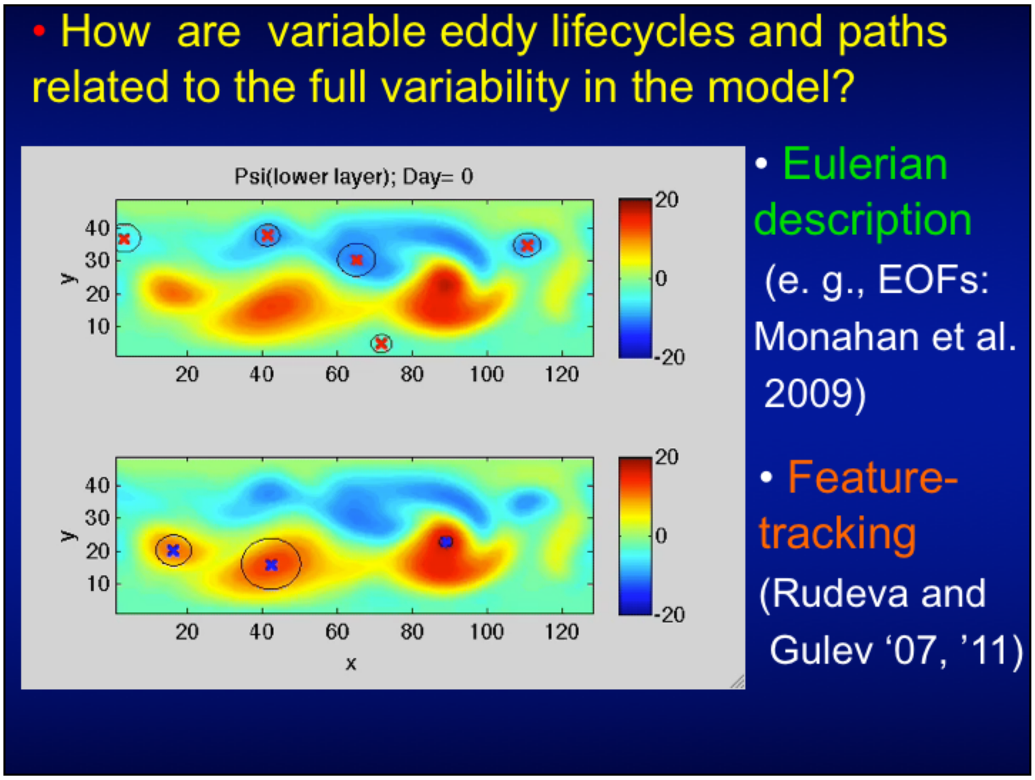
$$\frac{\partial \Pi_i}{\partial t} + J(\psi_i, \Pi_i) = Q_0 (-1)^i F_i(x, y, \eta) - \delta_{i,1} k \nabla^2 \psi_i$$

$$\Pi_i = \nabla^2 \psi_i + (-1)^i f_0 \frac{\eta - \delta_{i,1} h_B(x, y)}{h_i} + \beta y;$$

$$\eta = (\psi_1 - \psi_2)/g'; \quad i = 1, 2$$

- Zonally-symmetric thermal forcing acting on η
- A metaphor for zonal flows in the ocean/atmosph.
- Studied behavior in a range of Q_0, h_B and g'

The governing equations can be written in terms of layer streamfunctions and contain a few control parameters that we can change to study the sensitivity of the model's behavior. We perform long simulations of the model for each set of parameter and then study the output of these simulations.



There exist two complementary approaches to describing variability of the turbulent flows in the ocean and atmosphere. In one approach, which stems from classic Eulerian description of the fluids, one examines the evolution of the flow in a set of fixed spatial locations; for example, on a grid of a numerical model. Shown on this slide in color shading is a snapshot of the model's lower-layer streamfunction, with negative and positive values represented by the shades of blue and red, respectively. Note the existence of a region with strong gradient of the streamfunction, which separates the "red" and "blue" regions of the figure. This wavy jet is identifiable in the middle part of the basin and flows predominantly in the east-to-west (that is, left-to-right) direction along the axis of the channel.

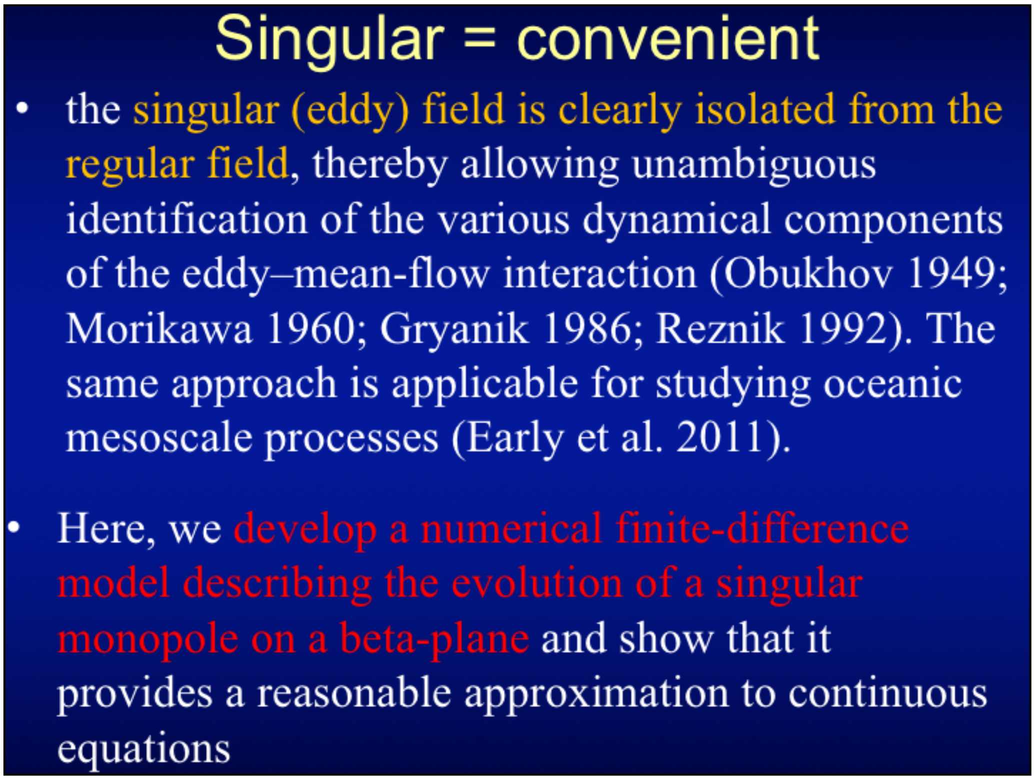
If we follow the evolution of this flow further, however (see the slide's animation), we will notice that at other times and/or other parts of the channel the existence of such a coherent predominantly zonal jet becomes far from obvious, and the flow is instead completely dominated by a smaller (zonal) scale eddy field. To recover the underlying large-scale structure of the flow in an Eulerian approach, one has to apply various filtering methods, such as low-pass filtering in time or zonal averaging in space, Empirical Orthogonal Function (EOF) analysis etc. Such analyses demonstrate that the dominant mode of flow variability in the model consists of persistent zonal-jet shifts to the north and south of its time-mean, climatological position.

An alternative, system-centered method of looking at the atmospheric general circulation is to recognize traveling synoptic eddies as natural building blocks of the atmospheric variability and perform their detection and tracking. An example of such tracking is visualized here by showing the paths and effective radii of cyclones (in the upper panel) and anticyclones (in the lower panel). The length of a typical eddy lifecycle, from birth to dissipation, is about 4 days, but **it turns out that lower-frequency climate variability can be diagnosed through analyzing the slow changes in the eddy lifecycles and trajectories. This is the main result of Kravtsov and Gulev (2013) study**, which establishes a detailed empirical connection between these two — Eulerian and system centered — approaches to describing climate variability.

The central result of Kravtsov and Gulev (2013) study is that the synthetic synoptic eddy field constructed by launching the composite-mean round cyclones and anticyclones along their actual simulated tracks in the model has the same time mean and very similar leading modes of variability compared to the full streamfunction field. Thus, the distribution of synoptic eddies carries a lot of information not only about the propagating high-frequency modes, but also about the LFV, which is dominated by that in the eddy paths.

Main idea (long-term goal):

- Since LFV seems to be unrelated to the detailed structure of individual synoptic eddies…
- … and LFV seems to be equivalent to the variability in synoptic eddy paths…
- … can we model LFV (=eddy paths) by randomly seeding the basin with interacting singular vortices, whose intensities and sizes vary according to self-similar composite life cycles?



In this study, we take the first step in the direction of modeling the midlatitude LFV in terms of interacting singular vortices and develop a numerical model which faithfully approximates the evolution of an isolated singular monopole on a beta-plane.

Governing equations (1.5-layer model)

$$\psi_s = -\frac{A}{2\pi} K_0(p|\boldsymbol{r} - \boldsymbol{r}_0(t)|) \qquad (1)$$

$$\partial_t \Omega + J(\psi + \psi_s, \Omega + \beta y)$$

$$+ (p^2 - a^2) J(\psi + \dot{x}_0 y - \dot{y}_0 x, \psi_s)$$

$$= -K \nabla^6 \psi \qquad (2)$$

$$\Omega = \psi - a^2 \psi;$$

$$\dot{x}_0 = -\partial_y \psi|_{\boldsymbol{r}=\boldsymbol{r}_0}; \quad \dot{y}_0 = \partial_x \psi|_{\boldsymbol{r}=\boldsymbol{r}_0}, \qquad (3)$$

$$\psi|_{t=0} = 0. \qquad (4)$$

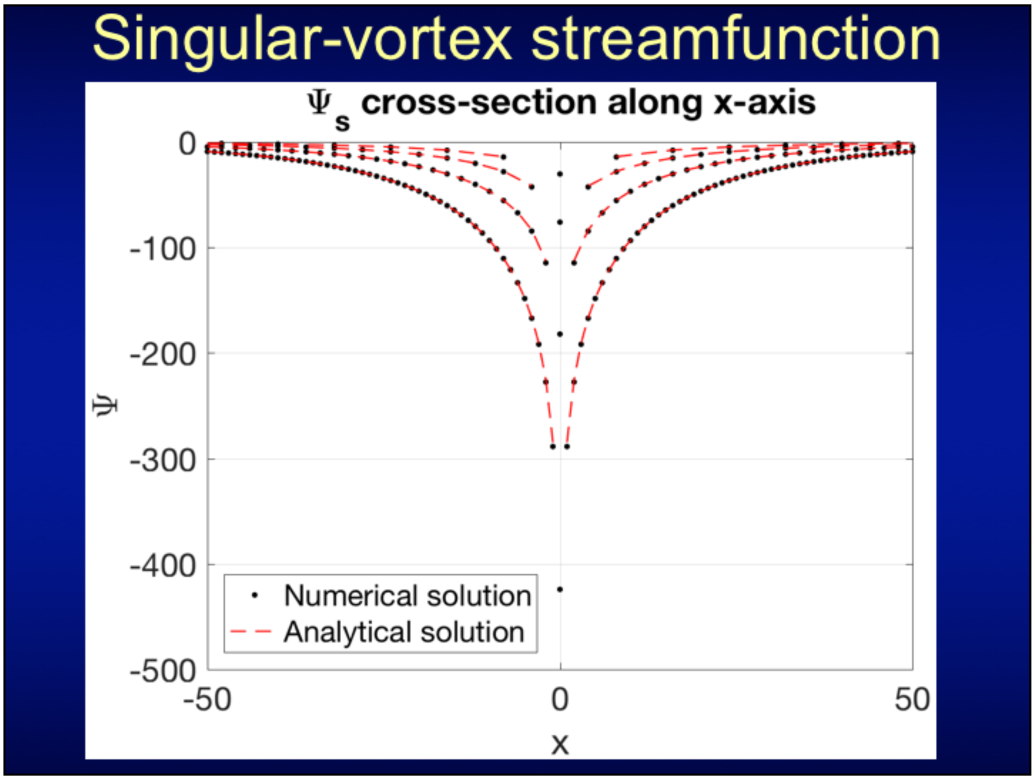
A is the singular vortex intensity, p is the SV inverse size, Psi is the regular flow streamfunction, a is the inverse Rossby radius. r0(t)=(x0(t),y0(t)) is the singular vortex path, J is the jacobian, the dot denotes time derivative. K is superviscosity acting on the regular-field only. On a beta-plane, even if the regular field is zero initially, it will be generated due to the eddy's emitting Rossby waves (see below). To solve (1-4) means to find the SV paths and the evolution of the regular streamfunction field.

Numerics

In numerical experiments, the Bessel vortex (1), which satisfies $\nabla^2\psi_s - p^2\psi_s = A\delta(x - x_0)\delta(y - y_0)$, is replaced by its non-singular counterpart $\psi_s{}^*$ consistent with the finite spatial resolution Δ of the model:

$$\left(\psi_{s\,i,j+1}^{*} + \psi_{s\,i,j-1}^{*} + \psi_{s\,i+1,j}^{*} + \psi_{s\,i-1,j}^{*} - (4 + p^2\Delta^2)\,\psi_{s\,i,j}^{*}\right)/\Delta^2 = A\,\delta_{i,i0}\delta_{j,j0}/\Delta^2. \quad (5)$$

Otherwise, things are standard: *x*-periodic wide channel geometry, second-order finite differences, leap-frog in time, cubic splines to interpolate SV fields on the model grid.



The numerical solution (5) is nearly identical to the analytical profile except at the center of the vortex (where the analytical singular vortex is unbounded), for every model resolution. Finer resolutions allow us to use lower superviscosity and better approximate available analytical solutions.

Model parameters-1

Parameter notation/value	Parameter description
$L_x = 51\ 200$ km	x-extent of the channel
$L_y = 30\ 000$ km	y-extent of the channel
$T = 80$ days	duration of each simulation
$\beta = 2 \times 10^{-11}$ m^{-1} s^{-1}	y-gradient of the Coriolis parameter
$R_d = 600$ km	Rossby radius of deformation
$A = 2\pi \times 5\beta R_d^3$	Amplitude of the vortex (intense vortex case; cf. Reznik 1990, 1992)
300 km	Vortex size: small-vortex case
$L_v =$ 600 km	Vortex size: point-vortex case
1200 km	Vortex size: large-vortex case
$a = R_d^{-1}$	Inverse Rossby radius
$p = L_v^{-1}$	Inverse singular-vortex size

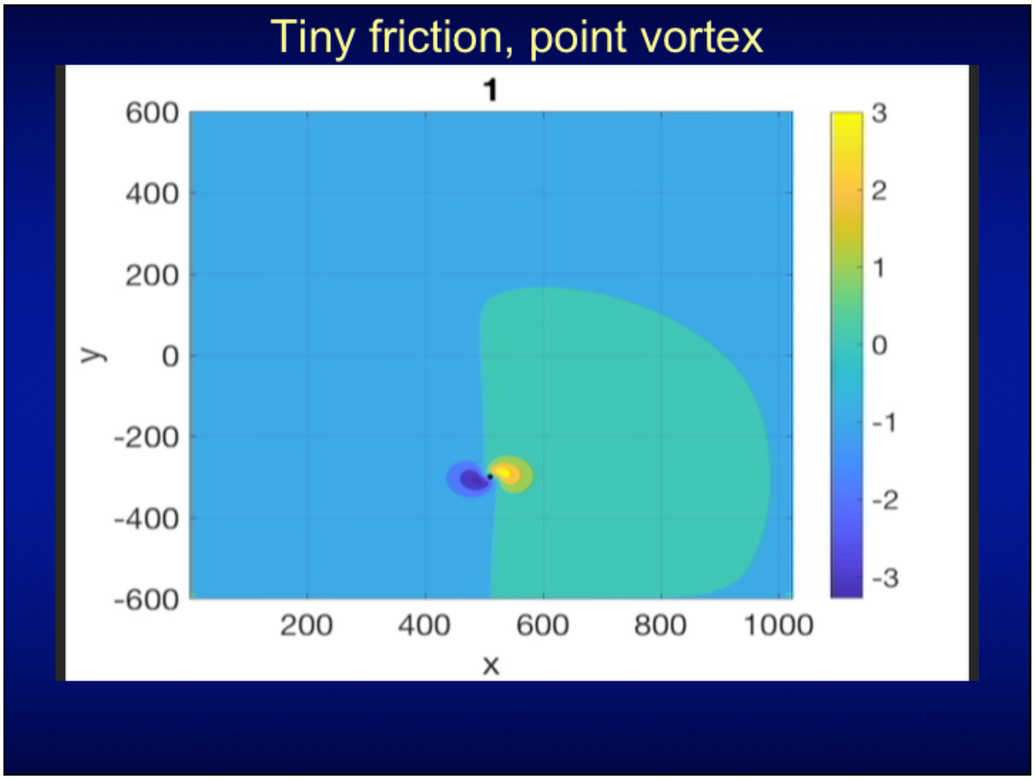
Resolution-independent parameters: atmospheric set up, small-vortex, point-vortex and large-vortex cases

Model parameters-2

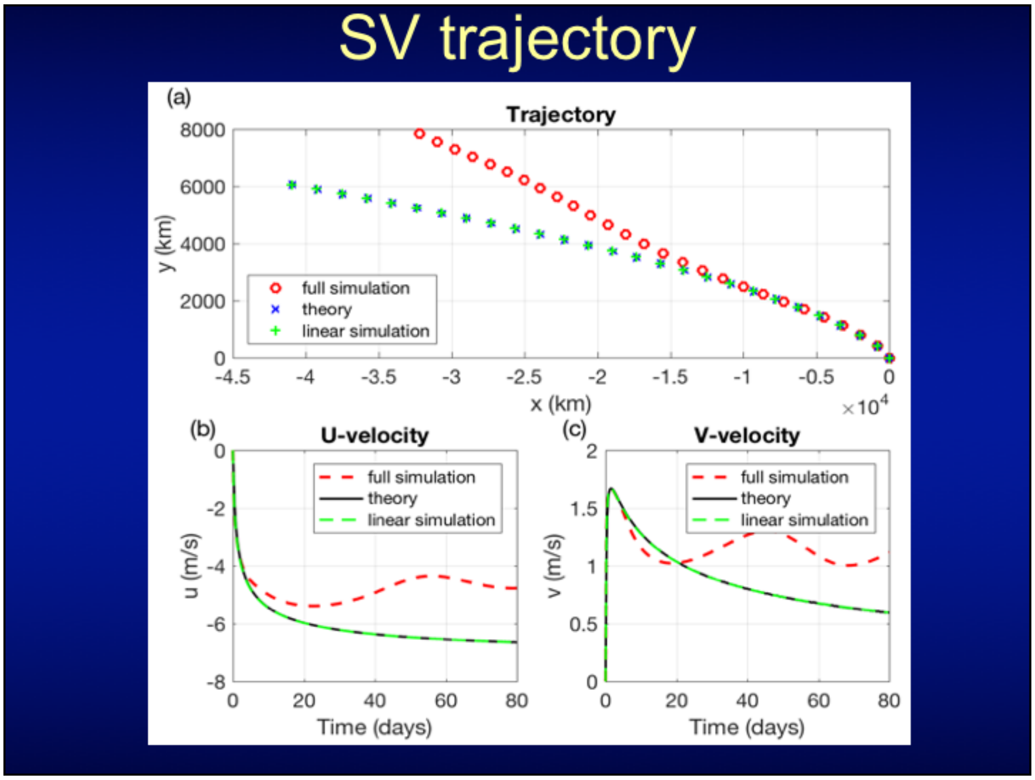
Table 2: Resolution-dependent model parameters.

Simulation name	**Model resolution** $\Delta x = \Delta y \equiv \Delta$ (km)	**Time step** Δt (s)	**Superviscosity** K ($m^4 s^{-1}$)
Large-friction case	200	300	2×10^{16}
Medium-friction case	100	150	2×10^{15}
Small-friction case	50	75	2×10^{14}
Tiny-friction case	25	30	2×10^{13}
Convergence check	50	30	2×10^{15}

We studied a wide range of superviscosities (three orders of magnitude difference between high-resolution and coarse-resolution cases).

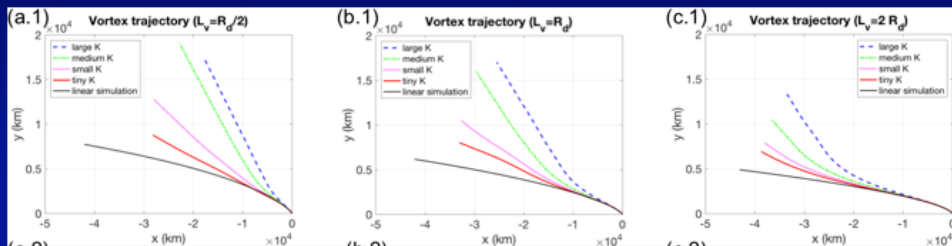


Initial stages of the SV/regular-flow system development are characterized by the generation of the so-called beta-gyres in the regular field, due to PV conservation and the ensuing Rossby-wave generation. This stage is linear and an approximate solution can be found analytically. Later on, during inertial stage, regular-flow self-advection becomes important and the dipolar beta-gyres regime gives way to an embedded-vortex regime, in which the SV is inside the regular monopole of the opposite sign. The northwestward propagation happens because the SV and the the center of the regular monopole are slightly offset. The change of the propagation mechanism during the inertial stage is one of the new results of this study.

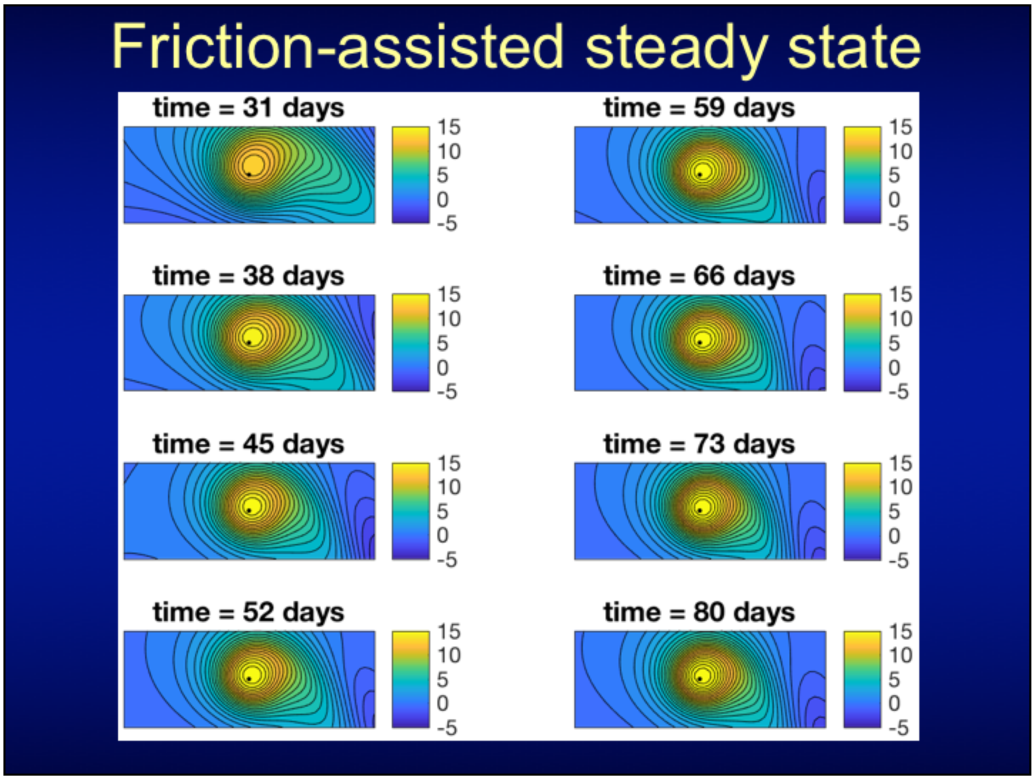


(1) Linear simulation matches the linear analytical solution; (2) inertial stage is characterized by the zonal slowdown and meridional speedup of the SV.

Sensitivity to SV size and friction



- More zonal trajectories for larger SV and smaller viscosities
- Existence of friction-assisted steady state for large viscosity



Large-friction point-vortex simulation: Regular streamfunction field in the reference frame associated with the vortex.

Summary

- We developed a numerical scheme for the "finite-difference" singular vortex evolution on a beta-plane
- The scheme approximates well known analytical solutions and conserves invariants of motion (not shown)
- Inertial stages of the SV development are characterized by the disintegration of beta-gyres and switch to "embedded vortex" self-propagation mechanism
- Existence of friction-assisted steady state

Discussion

These results lay a foundation for numerical consideration of systems of multiple singular vortices, which could provide further insights in our fundamental understanding of the processes underlying multi-scale atmospheric variability.

Acknowledgements

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I would like to conclude this presentation by putting up a few important references that are relevant to various parts of this talk and provide further information about the topics considered here.