IDENTIFICATION OF FORCED RESPONSE AND INTERNAL CLIMATE VARIABILITY USING ENSEMBLE LINEAR DYNAMICAL MODES

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Abstract

The problem of accurate detection of climate response to slow external forcing in 19-21 centuries is complicated by the presence of internal climate variability, which can also exhibit slow (decadal and multidecadal) large-scale dynamics, and also by the fact that there is only one observed climate realization available. At the same time, state-of-the-art Earth system models (ESMs) exhibit different spatiotemporal content on slow time scales, and their ability to estimate forced and internal climate variability needs further verification, especially given a relatively poor (short) observational statistics with respect to slow time scales. Here we present a method called ensemble linear dynamical mode (E-LDM) decomposition [1] which addresses the problem of forced signal and internal variability detection from small ensembles of ESM simulations. The method is based on the general assumption that the forced response is the same in all ensemble members and the internal variability is uncorrelated, while both of them can be essentially represented by a low-dimensional set of spatial patterns and corresponding forced and internal time series with certain time scales; the patterns, the time series and their time scales are optimized via the Bayesian framework. We compare the E-LDM method with other state-of-the-art methods of forced signal detection on synthetic and ESM-simulated data, and also discuss its applicability to the problem of intercomparison of ESMs and their verification with respect to real data. This research was supported by the state assignment of the Institute of Applied Physics of the Russian Academy of Sciences (Project No. FFUF-2022-0008).

1. The goals of this research

- Develop a method able to disentangle forced response and internal variability in ensembles of climate simulations (using previously developed linear dynamical mode (LDM) decomposition)
- Test the method performance with respect to other state-of-the-art methods for forced response estimation
- Address the problem of forced response estimation from single climate realization using ensembles of model simulations

2. E-LDM decomposition

Original LDM decomposition

- Data – single realization time series: $\mathbf{Y} = (\mathbf{y}_1, \dots, \mathbf{y}_N)$, $\quad \mathbf{y}_n \in \mathbb{R}^D$
- The LDM model to fit:

$$\mathbf{y}_n = \mathbf{A}\mathbf{p}_n + \mathbf{c} + \sigma\xi_n, \qquad \mathbf{p}_n \in \mathbb{R}^d, \xi_n \sim \mathcal{N}(0, \mathbf{I}) \tag{1}$$

- The parameters are: patterns $\mathbf{A} \in \mathbb{R}^{D\times d}$, time series $\mathbf{P} = (\mathbf{p}_1, \dots, \mathbf{p}_N)$, $\mathbf{c} \in \mathbb{R}^D$, $\sigma \in \mathbb{R}$.
- Bayesian methodology to train LDMs:
  1. Define parameters $\mu = (\mathbf{A}, \mathbf{c}, \mathbf{P}, \sigma)$
  2. Define prior probability density function PDF $P_{pr}(\mu) = P_{pr}(\mathbf{A}, \mathbf{c}, \sigma) \cdot P_{pr}(\mathbf{P})$
  3. The key point of LDM is the PDF for the time series $\mathbf{P}$:

$$\mathbf{p}_1 \sim \mathcal{N}(0, \mathbf{\Sigma}_p), \quad \mathbf{p}_{n+1} \sim \mathcal{N}(\hat{\alpha}\mathbf{p}_n, \mathbf{\Sigma}_p(\mathbf{I} - \hat{\alpha}^2)) \tag{2}$$

  with diagonal matrices $\hat{\alpha}$ and $\mathbf{\Sigma}_p$ defined by $\hat{\alpha}_{ii} = e^{-\frac{1}{(\tau_p)_i}}$, $(\mathbf{\Sigma}_p)_{ii} = (\sigma_p)_i^2$
  This PDF means that LDM components belong (by assumption) to the class of smooth linear stochastic processes with decay times $\tau_p \in \mathbb{R}^d$ and standard deviations $\sigma_p \in \mathbb{R}^d$: $p_{i,n+1} = \hat{\alpha}_{ii}p_{in} + \sqrt{(\sigma_p)_i^2(1-\hat{\alpha}_{ii}^2)}\eta_{in}$
  4. LDM parameters are found by maximization of the posterior PDF: $P(\mu|\mathbf{Y}) \propto P(\mathbf{Y}|\mu) \cdot P_{pr}(\mu)$
  5. The hyperparameters $d, \tau_p, \sigma_p, \dots$ are found by maximizing the Bayesian evidence: $P(\mathbf{Y}) = \int P(\mathbf{Y}|\mu) \cdot P(\mu)\, d\mu \underset{d,\tau_p,\sigma_p,\dots}{\longrightarrow} \max$
  6. See [2, 3] for further algorithm details

Ensemble LDM decomposition

- Data – ensemble of M realizations: $\mathbf{Y} = \left\{(\mathbf{y}_1^{(m)}, \dots, \mathbf{y}_N^{(m)})\right\}_{m=1}^{M}$, $\quad \mathbf{y}_n^{(m)} \in \mathbb{R}^D$
- The ensemble LDM model:

$$\mathbf{y}_n^{(m)} = \mathbf{A}\mathbf{p}_n^{(m)} + \mathbf{B}\mathbf{f}_n + \mathbf{c} + \sigma\xi_n^{(m)}, \qquad \mathbf{p}_n^{(m)} \in \mathbb{R}^{d_p},\ \mathbf{f}_n \in \mathbb{R}^{d_f}, \xi_n^{(m)} \sim \mathcal{N}(0, \mathbf{I}) \tag{3}$$

- We assume that ensemble members are generated by a climate model under the same forcing, so that:
  - The term $\mathbf{B}\mathbf{f}_n$ describes the forced response equal in all ensemble members (with spatial patterns $\mathbf{B}$)
  - The term $\mathbf{A}\mathbf{p}_n^{(m)}$ describes the modes of internal variability (with spatial patterns $\mathbf{A}$)
  - Forced component time series $\mathbf{F} = (\mathbf{f}_1, \dots, \mathbf{f}_N)$ are equal in all ensemble members
  - Internal variability time series $\mathbf{P} = \left\{(\mathbf{p}_1^{(m)}, \dots, \mathbf{p}_N^{(m)})\right\}_{m=1}^{M}$ are unique for each member
- Bayesian methodology to train E-LDMs remains the same. See [1] for further details.

References

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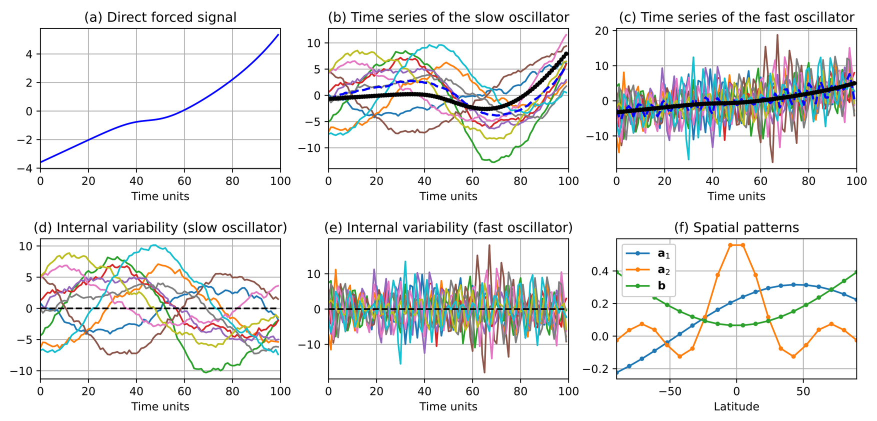
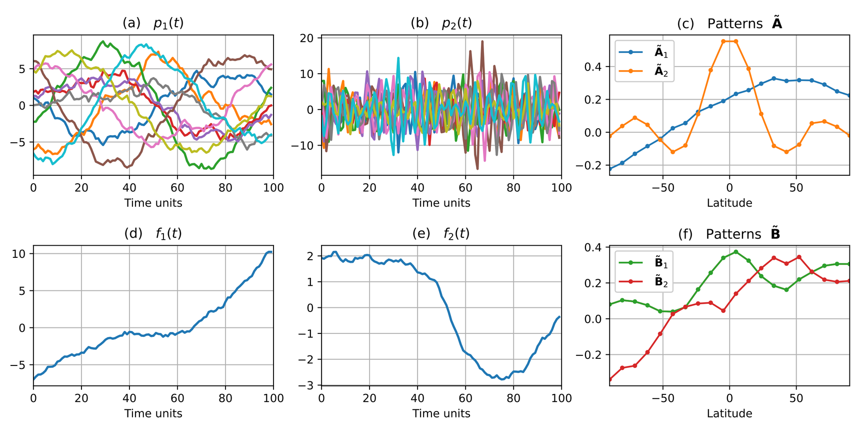
3. E-LDM: results

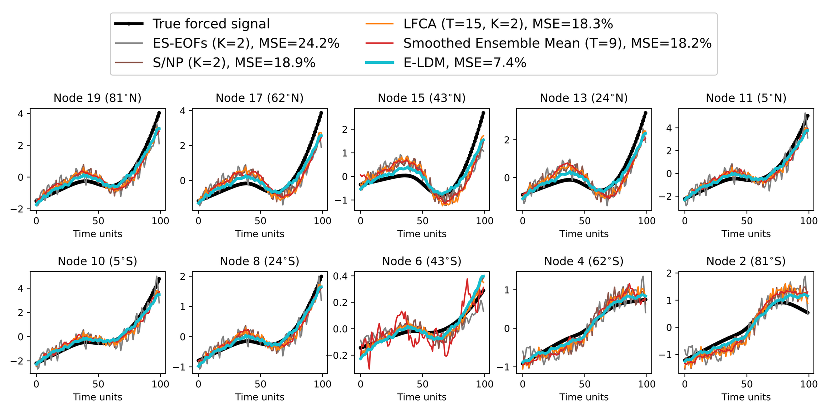
E-LDM: synthetic example



Data: 10 realizations of a prototype linear climate system with slow and fast modes damped by external forcing. The forced response is composed of the modes' responses and a direct response[1].



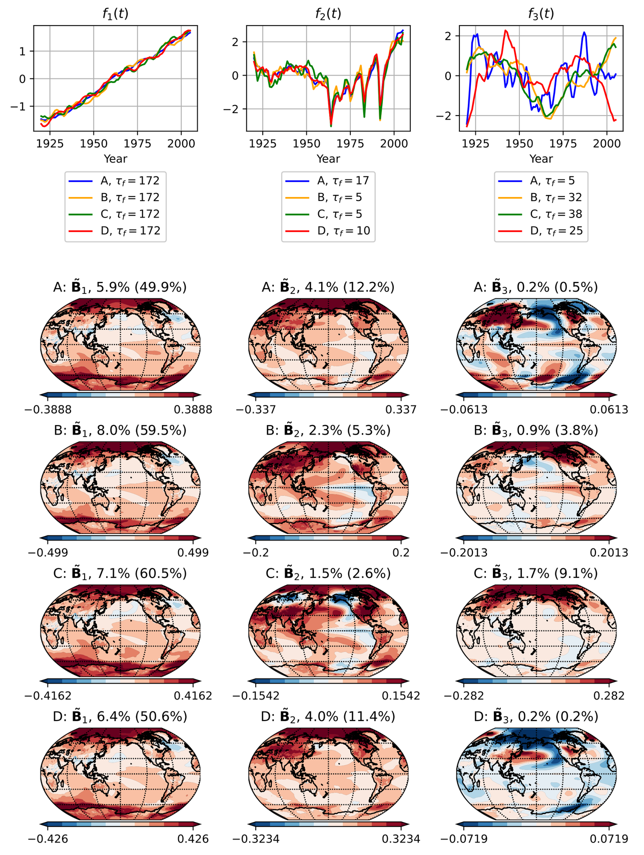
E-LDM decomposition: time series and patterns. The true forced response is two-dimensional



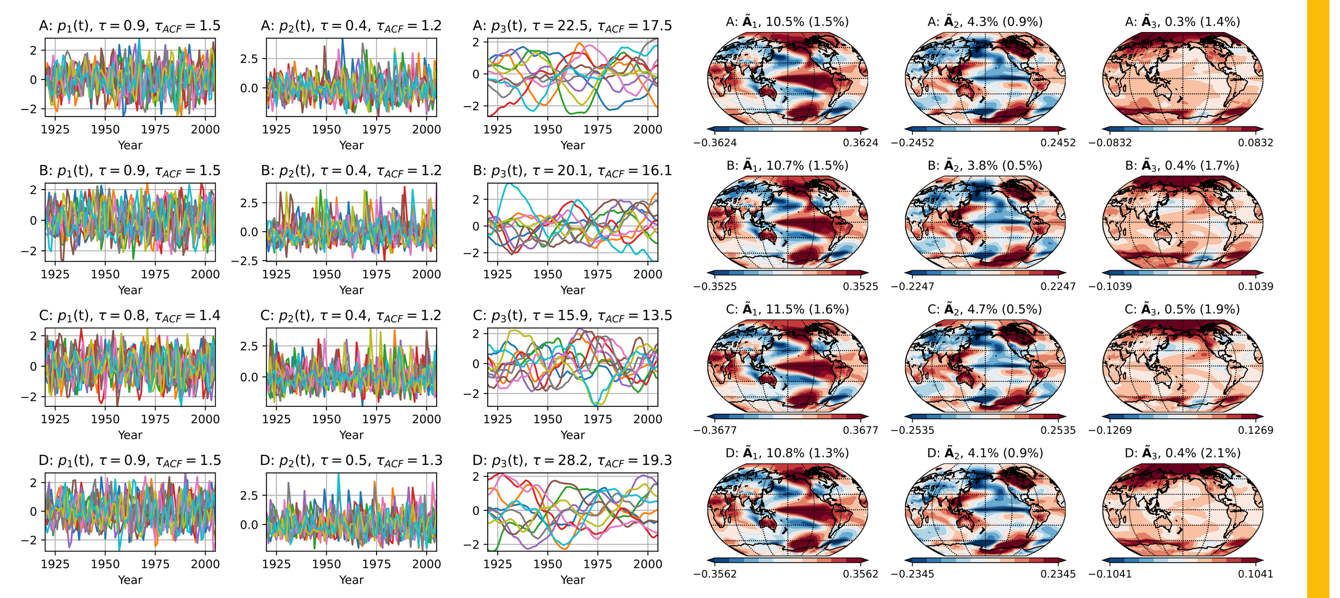
Forced response in different spatial nodes identified by different methods. The other methods are smoothed ensemble mean[1], extended-space EOFs[1], S/NP [4], LFCA [5]

E-LDM: CESM-LE example

Annual surface air temperature (SAT) data: 40 members of CESM-LE ensemble [6] were splitted into four sub-ensembles A, B, C, D, and E-LDMs were computed for each (after pre-normalization and ensemble EOF pre-compression).



E-LDM decomposition: forced response time series $f_1(t)$, $f_2(t)$, $f_3(t)$ and corresponding spatial patterns $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$ of the three ELDM forced modes identified in the sub-ensembles A–D of SAT simulations (see the legend, which also lists the optimal time scales τ_f of the modes). All patterns are in °C. The percentages in panel captions show the ratio of the corresponding component variance to the total SAT variance (in brackets – the same after applying 30-yr boxcar running mean smoother).



E-LDM decomposition: Time series $p_1(t)$, $p_2(t)$, $p_3(t)$ and patterns $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$ of three internal E-LDM modes, by columns. The rows, from top to bottom, show the results for sub-ensembles A, B, C, D. For the time series, the multiple curves within each panel show the results for 10 different sub-ensemble members. The optimal time scales τ_p of the modes are shown in the legend. Additionaly, the estimates of characteristic quarter-periods τ_{ACF} extracted from autocorrelation functions of each mode time series are shown in the legend. For the patterns, the notation is the same as for the E-LDM forced modes.

E-LDM: conclusions

- E-LDMs find both the forced response and the internal variability components
- E-LDMs provide optimal number of components
- E-LDMs perform at the level of the best methods in forced response estimation, while all their hyperparameters are determined by the Bayesian procedure
- In all CESM-LE sub-ensembles E-LDMs find 3 forced and 3 internal components
- Two fast internal components have ENSO-PDO-IPO-like structure. The slow component is weak and concentrated in high latitudes.
- Forced response is robust with respect to sub-ensembles (not shown)
- In both the synthetic example and the CESM-LE example the patterns of slow internal and forced modes overlap. It may be related with the interaction between forcing and multidecadal modes.

4. Forced response estimation from multi-model ensemble data

Multi-model ensemble LDM decomposition

- Data – multi-model ensemble (S models, with M realizations each):

$$\mathbf{Y} = \left\{\left\{(\mathbf{y}_1^{(m,s)}, \dots, \mathbf{y}_N^{(m,s)})\right\}_{m=1}^{M}\right\}_{s=1}^{S}, \qquad \mathbf{y}_n^{(m,s)} \in \mathbb{R}^D \tag{4}$$

- The multi-model ensemble LDM model:

$$\mathbf{y}_n^{(m,s)} = \mathbf{A}\mathbf{p}_n^{(m,s)} + \mathbf{B}\mathbf{f}_n^{(s)} + \mathbf{c} + \sigma\xi_n^{(m,s)}, \qquad \mathbf{p}_n^{(m,s)} \in \mathbb{R}^{d_p},\ \mathbf{f}_n^{(s)} \in \mathbb{R}^{d_f}, \xi_n^{(m,s)} \sim \mathcal{N}(0, \mathbf{I}) \tag{5}$$

- The forced component time series is made model-wise!
- The same Bayesian methodology to train multi-model ensemble LDMs

Forced response estimator from single realization

- Under assumption that the forced mode patterns are the same, but their time series are model-wise, let us train simple linear estimator of the forced components $\mathbf{f}_n^{(s)}$ from single realization data $\mathbf{y}_n^{(m,s)}$
- To avoid overfitting, project the data onto K ensemble EOFs:

$$\mathbf{x}_n^{(m,s)} = \mathbf{V}\mathbf{y}_n^{(m,s)}$$

- Since LDM modes are smooth, perform low-pass filtering with the time scale T:

$$\tilde{\mathbf{x}}_n^{(m,s)} = L_T(\mathbf{x}_n^{(m,s)})$$

- Train "smoothed linear regression" (SLR) to find fingerprinting patterns $\mathbf{U}$:

$$\mathbf{f}_n^{(s)} = \mathbf{U}\tilde{\mathbf{x}}_n^{(m,s)} + \epsilon_n^{(m,s)}$$

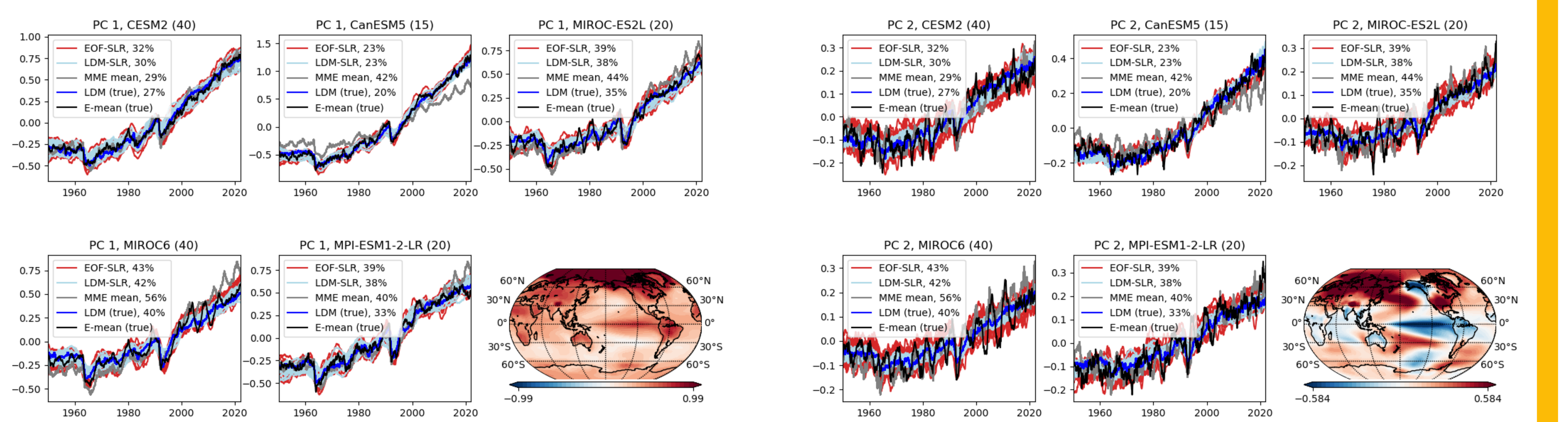
- Optimize hyperparameters K and T via cross-validation
- The forced response for the new realization $\mathbf{y}_n^{new}$ is estimated as follows:

$$\mathbf{y}_n^{forced} = \mathbf{B}\mathbf{U} \cdot L_T(\mathbf{V}\mathbf{y}_n^{new}) + \mathbf{c}$$

- The obtained estimator is robust with respect to internal variability and model differences because patterns $\mathbf{U}$ are trained to deal with it!

Example: ForceSMIP datasets

Monthly SAT data from the ForceSMIP training ensemble including 5 models. The first 10 realizations of each model were used to train LDM-SLR. The other 135 members were used for verification here. LDM preprocessing: seasonal cycle exclusion, normalization, EOF-compression.



The figures show the forced signal estimation by LDM-SLR, EOF-SLR and simple multi-model ensemble mean (MME mean) from testing realizations. Black: "true" forced signal estimation (from all members for each model). Blue: LDM forced signal estimation from 10 realizations of each model, which is then approximated by SLR mapping. The chosen indices are projections to the 1 and 2 EOF patterns (bottom right of each panel).

Multi-model analysis: conclusions

- Multi-model LDM algorithm is developed to detect forced response and internal variability in a multi-model ensemble.
- Smoothed linear regression is used to find a mapping from single realization to its forced response. It is robust with respect to internal variability and model uncertainty.
- Based on the preliminary simple tests with ForceSMIP SAT data, LDM-SLR estimator only slightly outperforms EOF-SLR estimator. It may reflect that the main uncertainty comes with the SLR part of the method.
- More evaluation is coming within the ForceSMIP project!