

Abner of Burgos: The Missing Link between Nasir al-Din al-Tusi and Nicolaus Copernicus?

Michael Nosonovsky

University of Wisconsin-Milwaukee

Abstract

The geometrical theorem known as the ‘Tusi couple’ was first discovered by Persian astronomer Nasir al-Din al-Tusi (1201–1274). The Tusi couple was believed to be discovered for Europeans by Nicolaus Copernicus (1473–1543) and it played an important role in the development of his planetary system. It has been suggested by Willy Hartner, that Copernicus borrowed it from al-Tusi, however, a particular way of transmission is not known. In this article I show that Spanish-Jewish author Abner of Burgos (1270–1340) was familiar with the Tusi couple and followed Tusi’s notation in his diagrams. This may provide a missing link in the transmission of the Muslim astronomical knowledge to Europe and advance our understanding of the European Renaissance as a multicultural phenomenon.

Keywords

astronomy – Hebrew medieval manuscript – Spain – Abner of Burgos

Many authors showed in recent decades that Persian astronomers of the 13th century from the Maragha school in southern Azerbaijan came very close to the discovery of the heliocentric Copernicus planetary system.¹ Although

1 E.S. Kennedy, ‘Late Medieval Planetary Theory,’ *Isis* 57 (1966) 365–378; S. Kren, ‘The Rolling Device of Nasir al-Din al-Tusi in the *De Sphaera* of Nicole Oresme?,’ *Isis* 62 (1971) 490–498; W. Hartner, ‘Copernicus, the Man, the Work, and Its History,’ *Proc. Amer. Phil. Soc.* 117 (1973) 413–422; N. Swerdlow, ‘Derivation of the First Draft of Copernicus’s Planetary Theory,’ *Proc. Amer. Phil. Soc.* 117 (1973) 423–512; N. Swerdlow and O. Neugebauer, *Mathematical Astronomy in Copernicus’s *De Revolutionibus** (New York 1984); R. Jamil, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on Astronomy* (New York 1993); G. Saliba, *A History of Arabic Astronomy: Planetary Theories*

these astronomers did not claim explicitly that the sun is located in the center of the planetary system, their mathematical formalism was identical with that of Nicolaus Copernicus and unlike the earlier formalism employed by the geocentric Ptolemaic system. Furthermore, there is indirect evidence that Copernicus (1473–1543) was familiar with the work of Nasir al-Din al-Tusi (1201–1274), the founder of the Maragha observatory, and that the discoveries of al-Tusi were the necessary key element to finalize the Copernicus system.² Willy Hartner showed that it is likely that Copernicus was familiar with al-Tusi's work, since there is a striking similarity between the drawings of Copernicus in his *De Revolutionibus orbitum celestium* (On revolutions of celestial spheres) and of al-Tusi in his *Tadhkira fi 'ulm al-ha'ya* (Memorandum on the science of astronomy).³ The Latin notation of Copernicus's diagrams followed the Arabic notation of al-Tusi.

The theorem or geometrical device coined by Kennedy the 'Tusi couple' is in the center of the discussion.⁴ The Tusi couple involves two spheres, one of which has twice the same diameter as the other. The small sphere is inside the big one and can roll upon it (see Fig. 1). Al-Tusi, who used this device for his lunar theory, showed that a point upon the small sphere would move back and forth along a straight line and thus the straight periodic motion can be presented as a superposition of two circular motions. Copernicus used the same mechanism for his (erroneous) theory of trepidation and in his theory of Mercury. Although it was believed that Copernicus re-invented the Tusi couple, Hartner showed conclusively that it was borrowed.⁵ However, direct evidence of the transition of this knowledge to Europe is very limited.

In search of possible pre-Copernicus references to the Tusi couple in Europe, Kren called attention to the manuscript *De Spera* by French scholar Nicole Oresme (c. 1323–1382), written before 1362.⁶ In this work, Oresme describes how to produce a back-and-forth rectilinear motion by combining three uniform circular motions and suggests the mechanism is very similar to the Tusi

During the Golden Age of Islam (New York 1994); M. di Bono, 'Copernicus, Amico, Fracastoro and Tusi's Device: Observations on the Use and Transmission of a Model,' *J. Hist. Astron.* 26 (1995) 133–154; G. Saliba, 'Revisiting the Astronomical Contacts between the World of Islam and Renaissance Europe: The Byzantine Connection,' in *The Occult Sciences in Byzantium*, ed. P. Magdalino (Geneva 2006), 361–374; E. Huff, *The Rise of Early Modern Science: Islam, China and the West* (Cambridge 1993); R. Morrison, 'A Scholarly Intermediary Between the Ottoman Empire and Renaissance Europe,' *Isis* 105 (2014) 32–57.

2 Hartner, 'Copernicus,' 421–422.

3 Hartner, 'Copernicus,' 422.

4 Kennedy, 'Planetary Theory,' 370.

5 Hartner, 'Copernicus,' 422.

6 Kren, 'Rolling Device,' 490–498.

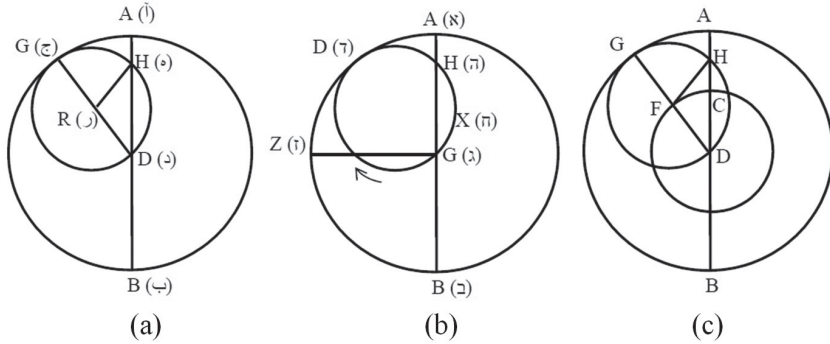


FIGURE 1 (a) The Tusi couple. As the small sphere (*DGH*) rotates upon the big sphere (*ADB*), the point *A* oscillates along the straight line *AB*. The original Arabic notation of al-Tusi is shown in parentheses according to Hartner 421. (b) Our reconstruction of the Statement 33 by Abner. (c) The schematics and original notation used by Copernicus (based on Hartner 422).

couple. The difference between the ‘Tusi device’ and the ‘Oresme device’ is that in the first, the small sphere touches the big one and rolls upon it, whereas in the second ‘the distance *B* descending in the motion of the deferent is the distance at which point *C* may ascend with the motion of the epicycle.’ However, kinematically this is the same motion. Oresme also does not state that the deferent has the same radius as the epicycle, which is required in order to have the same mechanism as that of al-Tusi. Kren cautiously concludes: ‘Confused and incomplete as it is, one cannot say with surety that it does represent al-Tusi’s mechanism. We must await the discovery of further material which may at least strengthen our conjectures.’⁷

There are many examples of translation of Hindu, Arabic, and Persian astronomical works into Latin, but particular ways of transmission of Eastern astronomical material to Europe are not known. Kennedy notes that the school in Maragha attracted students from as far as Muslim Spain as well as from the East.⁸ It was suggested also that Byzantium could have played the role of way station in the transmission of this material to Europe, because Hulagu, the grandson of Genghis Khan and the patron of al-Tusi and his observatory, was involved into diplomatic relations with the Byzantine Emperor Michael VIII Palaeologus.⁹

7 Kren, ‘Rolling Device,’ 498.

8 Kennedy, ‘Planetary Theory,’ 378.

9 Kren, ‘Rolling Device,’ 497.

Although the Byzantine channel of transmission of Muslim astronomical and mathematical concepts is quite plausible, I would like to investigate another typical way of transmission of the Arabic and Persian literature to Europe, that is, through Jewish translators.¹⁰ Some eighty years ago, Jacob Leveen and Solomon Luria called attention to the Hebrew mathematical treatise *Meyasher 'aqov* (Rectifying the curved) by an author named Alfonso, found in manuscript in the British Museum.¹¹ In 1960, Luria (1891–1964) obtained a copy of the manuscript and suggested that Gita Gluskina prepare a scientific edition and translation of the manuscript. Gluskina's book, which appeared in 1983 in Moscow after overcoming numerous difficulties, included a scientific edition of the text, translation into Russian, many explanatory notes, geometrical diagrams missing in the manuscript and a facsimile of the manuscript.¹² Some commentaries were prepared by the historian of mathematics Boris Rozenfeld.

The geometrical treatise is devoted to the quadrature of the circle, relation of the curved and the straight, and philosophical problems of the motion. The author, Alfonso, embraced Plato's viewpoint in the Aristotle-Plato controversy.¹³ He also believed that besides the potential (*koah*) and actual (*po'al*) causes, there is also the intermediate (*emtza'i*) cause. The intermediate cause corresponds to the motion, which relates a finite point to the actually infinite line.¹⁴

Gluskina undertook the difficult task of identifying the author, and came to the conclusion that Alfonso is Abner of Burgos (c. 1270–1340), a Spanish Jewish scholar, who converted into Christianity at the age of fifty and acquired the name Alfonso de Valladolid.¹⁵ Although this claim by Gluskina was at first skeptically accepted by some scholars, later Gad Freudental and Shlomo Pines

10 R. Singerman, *Jewish Translation History. Benjamins Translation Library* 44 (Amsterdam 2003).

11 J. Leveen, 'Note on Some Names in a MS in the British Museum,' *Jewish Quarterly Review* 13 (1922) 101; S. Luria, 'Die Infenitisimaltheorie der antiken Atomisten,' *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* 2 (1933) 106–185. The MS Add. 26984, vi, ff. 93b–128a, British Museum, London.

12 [G.M. Gluskina,] *Alfonso, Meyasher 'aqov* (Rectifying the Curved; in Russian). *Monuments of Literature of the Orient*, vol. 62, ed., trans., and comm. by G.M. Gluskina, S.Y. Luria, and B.A. Rozenfeld (Moscow 1983).

13 Gluskina, *Alfonso, Meyasher 'aqov*, 21.

14 Gluskina, *Alfonso, Meyasher 'aqov*, 23–24.

15 Gluskina, *Alfonso, Meyasher 'aqov*, 17–24. G.M. Gluskina, 'On the Authorship of the Mathematical Treatise *Meyasher 'aqov*' (in Russian), *Palestinskiy Sbornik* 25 (1974) 152–156.

found additional arguments in support of this identification and agreed that it is proven beyond any doubt that Abner of Burgos wrote *Meyasher 'aqov*.¹⁶

Abner of Burgos was a theologian, physician, and astrologist.¹⁷ In his philosophical concepts he was an opponent of Aristotle and supporter of Plato. He was familiar with astronomy; for instance, in 1334 he tried to convince the elders of Toledo that they had erred in fixing the date of Passover, which would require a significant astronomical knowledge.¹⁸ Though he wrote in Castilian too, most of his works were composed in Hebrew.¹⁹ It is questionable whether Abner knew Arabic; he read Greek and Arabic authors in Hebrew translations.²⁰ Unfortunately, the publication by Gluskina is not very well known outside of the former Soviet Union.

Statement 33 of the treatise²¹ was identified by Rozenfeld as Tusi's theorem.²² He states, however, that 'Alfonso's proof is incomplete (or not completely extent) and the correct solution of this problem was found by Copernicus in his *De Revolutionibus*.' Rozenfeld also identifies Ridwan mentioned in the Statement as Abu-l-Hasan Ali ibn Ridwan (998–1061), astronomer, philosopher, and physician from Cairo. He does not discuss, however, whether Abner could have read the Arabic original or must have used a translation of his work.²³

Apparently, the problem that concerned Abner was not whether the straightforward motion can be presented as a combination of circular motions (which is quite close to his main topic, 'rectifying the curved'), but whether the oscillating point stops at the moment when it changes the direction of its

16 G. Freudental, 'Two Notes on Sefer Meyasher 'aqov by Alfonso, Alias Abner of Burgos' (in Hebrew), *Qiryat Sefer* 63 (1990/91) 984–986, English trans. in G. Freudental, *Science in the Medieval Hebrew and Arabic Traditions* (Aldershot 2005) part ix, 1–4.

17 Z. Avnery, 'Abner of Burgos,' *Encyclopedia Judaica* (2nd ed., 2007) vol. 1, 97–98.

18 Avnery, 'Abner of Burgos,' 97.

19 Avnery, 'Abner of Burgos,' 98.

20 Gluskina, *Alfonso, Meyasher 'aqov*, 27; see also R. Glasner, 'Hebrew Translations in Medieval Christian Spain: Alfonso of Valladolid Translating Archimedes,' *Aleph* 13 (2013) 185–199 and S. Sadik, 'Abner de Burgos and the Transfer of Philosophical Knowledge between Judaism and Christianity,' *Medieval Encounters* 22 (2016) 95–112.

21 Gluskina, *Alfonso, Meyasher 'aqov*, 85–86, 196–197. Note that the letters *he*, *daleth*, and *heth* are similar to each other in the MS. Our schematic drawing (Fig. 1b) is slightly different from Gluskina. The notes in square brackets are by Gluskina. The points naming translation convention is: *A* for *aleph*, *B* for *beth*, *G* for *gimel*, *D* for *daleth*, *H* for *he*, *Z* for *zayin*, *X* for *heth*, *T* for *teth*, *N* for *nun*.

22 Gluskina, *Alfonso, Meyasher 'aqov*, 107. This topic was also discussed in Y. Langermann, 'Medieval Hebrew Texts on the Quadrature of the Lune,' *Historia Mathematica* 23 (1996) 31–53.

23 Gluskina, *Alfonso, Meyasher 'aqov*, 107–108.

motion. Note that this is very similar to the question raised by Nicole Oresme, whether 'at the point of return, the mobile cannot be said to be moving or to be at rest.' The time of completion of the works by Abner and by Oresme was very close. The treatise of Abner was written after he was baptized, most likely, in the 1330s, while the work by Nicole Oresme was completed before 1362.

An independent discovery of the Tusi couple by several scholars in the 14th century is unlikely. The form of the Tusi device considered by Abner that involves touching rolling circles is much closer to that of al-Tusi himself, rather than the one considered by Nicole Oresme, although there is no evidence that the work by al-Tusi was translated into Hebrew. Following Harter, who established that the Latin notation of Copernicus followed the Arabic notation by al-Tusi,²⁴ I compared the Hebrew notation by Abner with the two and found a similarity in notation. Al-Tusi used the first five letters of the 'abjad' system, *A, B, G, D, H* (which is actually the sequence of the letters of the Hebrew alphabet) and the letter *R*. Abner utilized the same first five letters plus the letters *Z, X, T, and N*. Like both al-Tusi and Copernicus, Abner used *A* and *B* for the large sphere and *G, D, H* for the small sphere, however, he only swapped the letters *G* and *D* at the small sphere. Oresme just used first letters of the Latin alphabet.

Transmission of Eastern astronomic knowledge via Spain seems likely, especially when keeping in mind that students from Muslim Spain studied in the Maragha school and that Arabic astronomic works (e.g. by ibn Ridwan) were well known to Jewish scholars in Christian Spain, such as Abner.

24 Harter, 'Copernicus,' 422.