Formation of buyer-seller trade networks in a quality-differentiated product market

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Abstract. We examine the formation of buyer-seller links when exchange can take place only if such a link exists. Sellers produce products of different qualities, and multiple sellers can form a sellers' association to pool their customers setting uniform prices. Buyers form trade links with individual sellers or sellers' associations. We show which buyer-seller links will form and find the set of links that are stable and show how these links influence prices. We also show that a trade network mismatch may occur where a high-quality good remains unsold even without an economy-wide excess supply of goods. JEL classification: D40, D51, D82

Formation de réseaux commerciaux acheteurs-vendeurs dans un marché de produits différenciés en terme de qualité. Les auteurs examinent le processus de formation de liens acheteurs-vendeurs quand l'échange ne peut s'effectuer que si de tels liens existent. Les vendeurs produisent des biens de différentes qualités et un nombre de vendeurs peuvent former une association pour mettre en commun leurs clients et imposer des prix uniformes. Les acheteurs forment des liens commerciaux avec des vendeurs individuels ou avec leurs associations. On montre quels liens acheteurs-vendeurs vont se former, quels liens vont s'avérer stables, et comment ces liens influencent le niveau des prix. On montre aussi qu'un réseau de commerce peut être mal adapté et un bien de haute qualité resté sans preneur même s'il n'y a pas offre excédentaire de biens dans toute l'économie.

1. Introduction

In the past decade, there has been a growing literature establishing theories about economic networks. Among various forms, trade networks are certainly

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important ones, as they are essential not only for exchange patterns but also for market organizations. While previous studies have created valuable insights towards understanding the emergence of trade networks, many interesting issues still remain unexplored, including, the formation of a coalition of sellers who can pool their customers and set prices cooperatively and the possibility of a trade-network mismatch where a good remains unsold even without an economy-wide excess supply of goods.

To address these issues, we develop a model of trade networks that specifies relationships or links between buyers and sellers producing goods/services of differentiated quality, where exchange can take place only if a link exists. Such a link may represent a formal relationship between a buyer and seller, or it may represent an informal tie where, for instance, buyers buy goods only from members of their family, ethnic, or social group.[1] We allow multiple sellers to form a coalition to pool their customers by setting uniform prices (called a 'sellers' association'), and such an association may consist of members of mixed quality. There are many examples of buyer-seller links where both sellers' quality differences and sellers' associations are important. For instance, a household (buyer) may form a tie (or link) with a cleaning service (sellers' association) to have a house cleaned at certain intervals, where the cleaning service may have multiple cleaners employed of different abilities or different work ethics. Alternatively, a trading company in a developing country is like a sellers' association in that it generally represents a group of domestic sellers. The trading company specializes in forming links with international buyers and can match such buyers with the sellers that it represents. These trading companies are common in the clothing industry, where domestic sellers have different abilities regarding how fast they can fill an order and the ability to fill an order quickly is highly valued by the buyer. Similarly, in the health care industry, groups of buyers may be represented by an HMO that forms contractual agreements (links) with hospitals and/or hospital chains of different qualities to provide health care service to the HMO's members.[2]

In this paper, we show which buyer-seller links will form and find the set of links (or networks) that are stable. A graph (or compilation of trade links) is called stable if no buyer or seller wants to exit the game (i.e., all payoffs must be non-negative) and no buyer would like to simultaneously sever her current link and form a new link. The set of prices that supports such a stable graph is called the set of active trading prices. We give conditions under which a sellers' association may form and conditions under which buyers may decide to link with

1 Examples include informal buyer-seller ties in the New York apparel industry (see Uzzi 1996) as well as formal contractual ties between HMOs (health care buyers) and hospitals (health care sellers).

2 See Hsing (1999), Dicken and Hassler (2000), and Cawthorne (1995) for discussion of trading companies in the clothing industry, and Town and Vistnes (2001) and Mathewson and Winter (1996) for discussion of HMO insurance. Many other examples abound, one of which is the International Trade Network established in 1996, which promotes sales of medical equipment and contains members of different specialties and qualities.

a sellers' association of mixed quality. We show how these buyer-seller networks influence the price paid for the good/service exchanged. In particular, we give conditions on the network for which the seller (or sellers' association) can set the price above his reservation price and conditions on the network for which the active trading prices are always below the buyer's reservation price.

There are a number of findings contrasting with conventional theories of trade between buyers and sellers under the Arrow-Debreu competitive framework. For brevity, we summarize but a few. First, a trade mismatch may occur in the absence of market or informational frictions, where a high quality good may remain unsold even without an economy-wide excess supply of goods. Second, given a set of active trading prices, there are generically multiple stable network configurations. Third, owing to price incentives, a sellers' association may naturally form in many circumstances and may consist of members of mixed quality even under complete information, where buyers choose to form links to this mixed-quality association. Fourth, prices are affected by whether a seller remains inactive, but not by the number of inactive sellers. The removal of inactive sellers affects both non-negligible net payoffs to sellers and buyers by raising the active trading prices. Fifth, the removal of inactive buyers lowers the minimum active trading price acceptable to sellers. To each seller's association, an increase in its customer base can result in a price compression in the sense that the minimum active trading price acceptable to sellers increases, whereas the maximum active trading price acceptable to buyers decreases. Even more surprisingly, the price determined by a symmetric Nash bargaining solution can turn out to be lower with a larger customer base. Finally, the sellers' (resp. buyers') net payoffs are influenced by the valuation of the high-quality good and the presence of an idle good (resp. the valuation of the low-quality good and the presence of an inactive seller), rather than the average quality, the quality differential or the relative market tightness (measured by the ratio of sellers to buyers).

1.1. The closely related literature

The most closely related work to the present paper is that of Kranton and Minehart (2000, 2001). In their pivotal studies, Kranton and Minehart (2001) focus on when the non-cooperative formation of buyer-seller networks leads to the formation of efficient graphs,[3] while Kranton and Minehart (2000) examine the competitive equilibrium prices in buyer-seller networks. Our paper adds to theirs in two significant ways. First, we examine the case of heterogeneous sellers and homogeneous buyers, while Kranton and Minehart (2000, 2001) consider

3 Jackson (2003) gives a further analysis of the Kranton and Minehart (2001) efficiency results for the case where the cost of a link is incurred by both the buyer and the seller. Under a framework similar to that in Kranton and Minehart (2001), Corominas-Bosch (2004) studies how buyers and sellers bargain in an exogenously given network and establishes useful conditions of networks under which the subgame perfect equilibrium of the bargaining game generates the short-side rule outcome in the Walrasian setting (i.e., the short side of the market extracts all the surplus).

the case of homogeneous sellers and heterogeneous buyers (specifically, buyers are heterogeneous in that they have different valuations in demand). Second, we introduce the possibility of links between sellers in the form of a sellers' association that contains a group of sellers, while Kranton and Minehart (2000, 2001) focus on links between individual buyers and sellers.

There is a trade network literature that concentrates on the value that trade networks create by decreasing search costs, reducing informational asymmetries, and facilitating cooperation.[4] In contrast, our paper focuses on the non-cooperative formation of trade links and the determination of price ranges that support active trade. Our work is also related to general theories of network formation and stability.[5] In particular, in our study of the endogenous formation of trade networks, players can form and sever links. However, our focus on the influences of sellers' quality differences and the patterns of sellers' associations is certainly very different from that in the existing literature. Finally, our analysis of sellers' associations is related to the coalition formation literature.[6] However, our focus on trade networks is different from that of the existing literature.

2. The environment

Consider a trading economy with indivisible goods of high and low quality where agents have full information. The central feature of the model is to determine endogenously both the structures of trade networks and the ranges of product prices for active trade.

2.1. Agents

Denote the set of buyers as B and the set of sellers as $S = H \cup L$, where H and L are the set of high- and low-quality sellers, respectively. Let $\mu(X)$ represent the cardinality of integer set X. We assume that there are at least two buyers and two

4 For example, Helper and Levine (1992) show how alternative sources of trade alter the terms of trade. While Rauch (1996) allows buyer-seller links to eliminate the search for a buyer, Riordan (1996) considers links that facilitate cooperation and hence create value, and Casella and Rauch (1997) allow family ties to reduce information asymmetries. For a recent survey of the trade network literature, see Rauch (2001).

5 See Aumann and Myerson (1988), Jackson and Wolinsky (1996), Dutta and Mutuswami (1997), Bala and Goyal (2000), Chwe (2000), Watts (2001), Furusawa and Konishi (2002), Jackson and Watts (2002a,b), Page, Wooders, and Kamat (2005), and Goyal and Joshi (2003). Aumann and Myerson (1988) were the first to take an explicit look at network formation in a strategic context where connections defined a communication structure that was applied to a cooperative game. Both Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997) concentrate on the tension between network stability and network efficiency. Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002a,b), Page, Wooders, and Kamat (2005), and Goyal and Joshi (2003) examine the endogenous formation of various types of social/economic networks when players can form and sever links. Chwe (2000) studies how network structure influences coordination games, whereas Furusawa and Konishi (2002) apply the network formation game to studying the establishment of international free trade agreements.

6 For example, see Kaneko and Wooders (1982), Greenberg and Weber (1993), and Demange (1994), to name but a few.

sellers, with at least one high-quality seller and one low-quality seller. Thus, we have: $\mu(B) = \beta \geq 2$, $\mu(S) = \sigma \geq 2$, $\mu(H) = \sigma^H \geq 1$ and $\mu(L) = \sigma^L \geq 1$, where $\sigma^H + \sigma^L = \sigma$. For illustration purposes, we will refer to a buyer as 'she' and a seller as 'he.'

2.2. Production

Assume that every seller of type $\tau \in \{H, L\}$ produces exactly one unit of an indivisible good per period, where the good produced has quality τ.[7] Goods are completely perishable and hence have no inventory value. Each buyer demands at most one unit of the good per period. Thus, if a buyer and seller are allowed to trade, they will trade at most one unit.

2.3. Trade network structure

In contrast with studies of bilateral trade, we allow buyers and sellers to form an *association*, which is denoted by A. An association is called, a *sellers' association* (denoted A^S) if $a \in S$ for all $a \in A^S$. Each seller may belong to at most one association. An association, A, is called *non-degenerate* if $\mu(A) > 1$. The cost of maintaining an association of size $m \geq 1$ equals $(m - 1)k$ for each association member, where $k \geq 0$. Thus, k is an individual association member's unit cost of maintaining a tie with every other member in the association.[8] Note that we focus our analysis on sellers' associations. Since buyers are homogeneous and sellers are not, the formation of buyers' associations is not as interesting as the formation of sellers' associations which may contain members of different types (see the companion working paper, Wang and Watts 2004, for a brief discussion of the formation of a buyers' association).

Trade can occur only if there is a trade *link* between a buyers' and a sellers' association A^S. The cost of maintaining such a trade link for each buyers' and sellers' association is c ≥ 0.[9] We assume that each association divides this cost

7 For simplicity we assume there are no costs of production. Alternatively, we could assume that there are, in fact, costs of production (and that the cost of producing low- and high-quality goods are different) but that the seller receives a benefit from consuming the good that is exactly equal to its production cost.

8 Alternatively, we could model association costs as a fixed cost or consider that association members pay a club fee, which would not qualitatively change the results. Here, the same set of graphs will be stable (as long as an updated version of assumption 1 is met, which replaces $2k$ with the fixed cost); the active trading prices which support these graphs will be slightly different, however, in that the fixed cost will replace $(m - 1)k$ in the active trading price equations. Similarly, any other change in the association cost (such as making costs non-linear in the size of the association) will change the active trading prices but will not change the set of stable graphs. We choose to allow association costs to be linearly increasing in the number of members as a simple way to capture the idea that larger associations are more costly to run.

9 We assume that the cost of a link is the same for both buyers and sellers' associations for simplicity. Alternatively, we could allow costs to be different or to be asymmetrically shared. This change would not qualitatively change the results, in that the same set of graphs would be stable, but the active trading prices supporting these graphs would be slightly different, since ow there would be two types of link costs in the active trading price equations instead of one.

evenly among its members. Note that if A^S is linked to buyer b and if $\mu(A^S) > 1$, then not all sellers in A^S are guaranteed a trade with b. In this case, we assume that an individual seller $s \in A^S$ sells a product to b with probability $1/\mu(A^S)$.

A *trade network* is a graph, G, consisting of trade links between buyers' and sellers' associations. If A^S is linked with b, we write $(A^S, b) \in G$. We assume in the basic framework that each buyer has at most one link, with a relaxation of this assumption provided in section 5.1. However, a sellers' association may have links to multiple buyers.

2.4. *Individual payoffs*

To each buyer, the consumption of one unit of the quality differentiated good yields a utility of q^i, for $i = H, L$ (and the consumption of any additional units generates no utility). We assume that

ASSUMPTION 1. $q^H > q^L > 2(c + k)$.

While the first inequality is trivial, the second inequality ensures that the value generated from trading a low-quality good is sufficient to cover the cost of maintaining a link between a buyer and a non-degenerate sellers' association of size 2.

Let the set of buyers be represented by $B = \{b_1, \ldots, b_m\}$ and the set of sellers' associations by $A^S = \{A_1^S, \ldots, A_n^S\}$, with G representing the trade network between the sets. Denote the price of a good produced by seller $s \in A_j^S$ as $p(A_j^S)$. We assume that all sellers in an association set a uniform price regardless of any differences in quality.[10]

Define the payoff for seller $s \in A_j^S$. Let B_j represent the set of buyers that A_j^S is linked with in graph G and let m_j be the number of individual buyers in the set B_j. Seller s's net payoff, $v(B_j, A_j^S)$, is then given by

$$v\left(B_j, A_j^S\right) = \min\left\{\frac{m_j}{\mu\left(A_j^S\right)}, 1\right\} p\left(A_j^S\right) - \frac{m_j}{\mu\left(A_j^S\right)} c - \left[\mu\left(A_j^S\right) - 1\right] k. \tag{1}$$

The first part of equation (1) represents the probability that seller s sells a product times the price he receives if his good is sold. The second part of (1) represents s's share of the trade link cost and s's share of the association cost, respectively. Let $p^L(A_j^S)$, or in short p_i^L, be the price that sets $v(B_j, A_j^S) = 0$, which is conveniently referred to as *sellers' reservation price* in the sense of active market participation.

10 This assumption is made to capture the motivating examples of a wholesaler or HMO that charges uniform prices for goods or services of different quality. If we assumed, instead, that the high-quality good charged a higher price, then the results would be similar to those we obtain for buyer-seller networks in which no association is formed.

The net payoff for a buyer is more complex, owing to different valuations of quality-differentiated goods. Assume that in graph G, buyer b maintains a single link with sellers' association A_j^S, and that A_j^S is linked with m_j buyers in G. Denote b's payoff as $u\,(b,\, A_j^S)$, whose value is given by

$$u\big(b,\, A_j^S\big) = \min\left\{\frac{\mu\big(A_j^S\big)}{m_j},\, 1\right\}\left[\frac{\mu\big(A_j^S \cap H\big)q^H + \mu\big(A_j^S \cap L\big)q^L}{\mu\big(A_j^S\big)} - p(A^S)\right] - c. \tag{2}$$

The first part of (2) represents expected quality of a good purchased from A_j^S minus the expected price (where if $m_j > \mu(A_j^S)$, buyer b is not guaranteed a purchase and thus only pays price p if she receives a good). The last part of (2) represents buyer b's trade link cost. Let $p^U(A_j^S)$, or in short p_i^U, be the price that sets $u(b,\, A_j^S) = 0$, referred to as *buyers' reservation price* in the sense of active market participation.

2.5. *The concept of equilibrium*

In equilibrium, prices must fall in the range in which both buyers and sellers are willing to participate. We normalize the value of outside alternatives for both buyers and sellers to zero.

Fix a set of buyers $\{b_1, \ldots, b_m\}$ and a set of sellers' associations $\{A_1^S, \ldots, A_n^S\}$ so that each seller is a member of at most one association. A graph G, which links each buyer to at most one sellers' association, is *stable* if for all $b_i \in B_k$ linked to A_k^S in G and for all b_u and A_u^S unlinked in G,

$$\begin{aligned} &\text{i) } u(b_i,\, A_k^S) \geq 0 \text{ and } u(b_i,\, A_k^S) > u(b_i,\, A_j^S) \text{ for all } A_j^S \in \{A_1^S, \ldots,\, A_n^S\};\\ &\text{ii) } u(b_u,\, A_j^S) < 0 \text{ for all } A_j^S \in \{A_1^S, \ldots,\, A_n^S\};\\ &\text{iii) } v(B_k,\, A_k^S) \geq 0 \text{ for all } A_k^S \in \{A_1^S, \ldots\, A_n^S\} \text{ and } v(\phi,\, A_u^S) \geq 0;\\ &\text{iv) } p(A_u^S) = c. \end{aligned} \tag{3}$$

Interpreting equation (3), G is stable[11] if

i) no buyer, who is linked to some sellers' association in G, wants to sever her current link (i.e., no buyer would prefer to sever her tie and receive a payoff of 0), or wants to simultaneously sever her link and form a new link;
ii) no buyer, who is unlinked in G, wants to form a link;

11 Note that in our definition of stability it is the buyer who initiates forming a link, not the seller. Since buyers are homogeneous and sellers are heterogeneous, it makes sense that buyers will care about which sellers' associations they are linked with but sellers will care only about the price, not about which particular buyers they are linked with. However, it is possible in our framework for sellers to have a more active role in forming links, as is discussed in section 5.

iii) no seller wants to exit the game (i.e., no seller would prefer to sever all ties to have a payoff of 0) and all unlinked sellers must receive a non-negative payoff.[12]
iv) any unlinked sellers set price equal to the cost of forming a link, c.

Importantly, by ruling out the case of simultaneously severing a link and forming a new link in part (i), our stability concept is stronger than that of pairwise stability, adopted in Jackson and Wolinsky (1996), Jackson and Watts (2002a) and many others. Further, we assume that if a buyer is indifferent between forming a link and having no links then she chooses to form a link. We use this in the proof of proposition 3 to rule out equilibria where there is at least one unlinked buyer and at least one linked buyer and where all buyers (linked and unlinked) receive a payoff of zero. The assumption that a buyer would always choose to sever a link when indifferent would also work. However, we assume a buyer will always form a link when indifferent for ease of exposition.

Our paper establishes a price range within which a particular stable network emerges.[13] We call the set of prices which supports a given stable graph the *set of active trading prices*. Specifically, this set defines a range of prices for which no linked buyer prefers to buy from a different sellers' association or prefers not to purchase the product at all (condition (i) of equation (3)), no unlinked buyer wishes to purchase from a sellers' association (condition (ii)), and no seller in an association that is linked prefers to not sell a product (condition (iii)).[14] Thus, the set of active trading prices are both necessary and sufficient for the corresponding network to be stable. For a particular sellers' association A_j^S, let us define the *effective minimum price*, $p_j^{\min}$, as the minimum possible price that supports a stable graph and the *effective maximum prices*, $p_j^{\max}$, as the corresponding maximum possible price. The set of active trading prices for A_j^S is thus given by $P_j = [p_j^{\min}, p_j^{\max}]$ and the set of active trading prices in the trading economy is denoted by $P = \{P_j\}$. These effective minimum and maximum prices ($p_j^{\min}$ and $p_j^{\max}$) need not be the same as the ex ante reservation prices (p_j^L and p_j^U) because they account for sellers' and buyers' endogenous outside options.

Since all the payoff functions are linear in price, one can easily pin down the *symmetric Nash price*, p_j^N. It is important to recognize that the outside alternatives to buyers and sellers depend on the initial network graph as well as those graphs associated with severing and forming a particular link. Since these outside alternatives are exactly those determining effective minimum and maximum prices, it

12 Thus, for $k > 0$ any unlinked sellers will be single sellers and there will be no unlinked sellers' associations containing multiple sellers.

13 The only other network paper that we are aware of that establishes a price range is Kranton and Minehart (2000).

14 Note that we are agnostic regarding how a sellers' association forms for two reasons. First, modelling the formation process would greatly complicate the analysis by adding further restrictions to the set of stable graphs (equation (3)). Second, even without these further restrictions we are able to obtain interesting results regarding the configuration of stable graphs and the prices that support these graphs.

is reasonable to assume that buyers and sellers simply bargain over prices in the set of active trading prices as long as they agree to be in a given graph when prices fall in this set. The symmetric Nash bargaining solution between a sellers' association A_j^S and its customers B_j is therefore $p_j^N = (p_j^{\min} + p_j^{\max})/2$.[15] Obviously, one may solve for any cooperative Nash prices under asymmetric Nash bargaining as a weighted average of $p_j^{\min}$ and $p_j^{\max}$ by attaching the weight associated with sellers to $p_j^{\max}$ and that with buyers to $p_j^{\min}$. Therefore, the set of active trading prices P contains the solutions of equilibrium prices under various mechanisms, including Nash bargaining and ascending-bid auctions.[16]

A sellers' association in a stable graph is called *inactive* if the sellers' association has no links and thus sells no goods. As stated in condition (iv) of (3), we assume that such an inactive sellers' association is willing to sell a good at its breakeven price, which is the cost of forming a link, c. Thus, if a stable graph contains an inactive sellers' association, no buyer wishes to purchase from them at price c.

A sellers' association in a stable graph contains an *idle good* if the association has a good that is unsold. Denote the set of idle goods as I. If a sellers' association is linked to fewer buyers than it has members, then some of the associations' goods will be unsold and are thus idle. So an association with an idle good may be linked and thus may be active. However, an inactive seller is by definition unlinked and thus always has an unsold or idle good.

We now define the concept of efficiency in our trading economy. Active trading is *efficient* if

$$\mu(I) \equiv \max\{\sigma - \beta, 0\}, \tag{4}$$

that is, if idle goods exist only when there is an excess supply of goods. However, since goods are of heterogeneous quality, it is less desirable to have a high-quality

15 We assume that B_j and A_j^S bargain over prices in the active trading range, and we take the threat point to be the utility vector generated when B_j faces the highest price $p_j^{\max}$ and A_j^S faces the lowest price $p_j^{\min}$. If we rescale utility so that the threat point is our origin, then B_j and A_j^S bargain over a utility vector (u, v) in $\{(u, v) \mid v \leq -u + p_j^{\max} - p_j^{\min}\}$. If we maximize $u \cdot v$ subject to this constraint, we get that $u^* = v^* = (p_j^{\max} - p_j^{\min})/2$, adding the respective threat point utilities to u^* and v^* yields that $p_j^N = (p_j^{\min} + p_j^{\max})/2$.

16 It is easily seen that in an ascending-bid auction with sellers' setting the auction reservation price at $p_j^{\min}$ and where only linked buyers are allowed to bid, the equilibrium auction price is equal to $p_j^{\min}$ whenever there is no excess demand for goods (or the number of buyers linked to the sellers' association is less than or equal to the number of sellers in the association) and is equal to $p_j^{\max}$ whenever there is excess demand for goods (or the number of buyers linked to the sellers' association is greater than the number of sellers in the association). Bidding will always stop at $p_j^{\max}$, since, if the price exceeds $p_j^{\max}$, the linked buyers will prefer to sever their current links (and thus drop out of the bidding with the current sellers' association) and link with one of the other sellers' associations (or drop out entirely if there is no other sellers' association). Note that, if we allow a sellers' association to choose its own auction reservation price, it will choose $p_j^{\max}$, since this is the maximum price for which they can sell the good. We chose to set the reservation price at $p_j^{\min}$ instead to illustrate the role that excess demand plays in setting ascending-bid auction prices.

good unsold. Accordingly, another useful efficiency criterion is that there cannot be a low-quality good sold when there is an idle high-quality good:

$$\mu(I \cap H) = \max\{\sigma^H - \beta, 0\}. \tag{5}$$

It is said that active trading is *weakly inefficient* if (4) fails to hold, but (5) is satisfied and *strongly inefficient* if both (4) and (5) are violated. We say that a given graph is weakly (respectively, strongly) inefficient if there is a positive probability that active trading in the graph is weakly (respectively, strongly) inefficient.

Like Kranton and Minehart (2001), we assume that only the buyer can initiate forming a link. Thus, under this definition of stable network, we have ruled out the possibility of active seller networking, because sellers are not allowed to discriminate against or seek out buyers and thus sellers are not allowed to form and/or sever individual links with buyers. This simplification is innocuous; moreover, one can easily see that even with relatively 'passive' sellers, a non-degenerate sellers' association may still emerge. This implies that the underlying mechanism must be direct price incentives rather than market power (which may influence prices indirectly, as in the case of cartels). A discussion of active seller networking is provided in section 5.2.

3. Specific examples

To gain more insights towards establishing general properties, we begin by illustrating our model with three buyers and three sellers (i.e., $\beta = \sigma = 3$, $B = \{b_1, b_2, b_3\}$ and $S = \{s_1, s_2, s_3\}$). In particular, we focus on two cases: (i) $\sigma^H = 2$ and $\sigma^L = 1$, with $s_1, s_2 \in$ H and $s_3 \in L$; (ii) $\sigma^H = 1$ and $\sigma^L = 2$, with $s_1 \in H$ and $s_2, s_3 \in L$. Again, we allow sellers, but not buyers, to form associations. There are four possible types of non-degenerate sellers' associations: the grand coalition $\{s_1, s_2, s_3\}$ and three different two-seller associations with either homogeneous quality types or heterogeneous quality types.

3.1. Formation of trade networks and sellers' association

In this economy we have at least eight stable trade networks, as depicted in figure 1. When there are more low-than high-quality sellers ($s_1 \in H$ and $s_2, s_3 \in L$), we have an additional stable graph with all three buyers linked to the high-quality seller and with both low-quality sellers inactive (call it case (ix); graph not shown). In the appendix, we provide the proof of an interesting case (iii) in which a non-degenerate sellers' association arises. When sellers' association $\{s_i, \ldots, s_j\}$ is formed, we represent the association's corresponding price as $p_{i\ldots j}$. One may quickly observe that in this trading economy, a non-degenerate sellers' association can easily emerge (see cases (iii), (iv), (v), (vi), (vii), and (viii)).

For each graph represented in figure 1 there are additional conditions required for stability, which are listed in the appendix. The graphs with less demanding

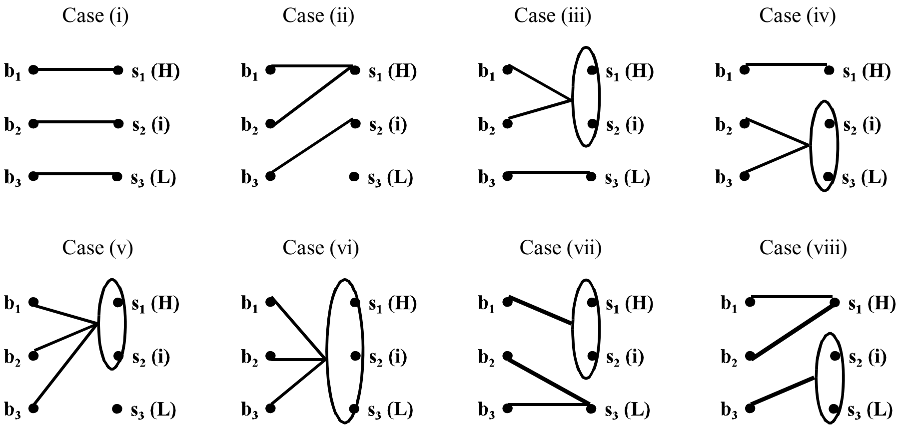

FIGURE 1 Patterns of trade networks with $\mu(B) = 3$, $\mu(S) = 3$, and $i = H$ (1A) or L (1B)

additional conditions may be more likely to emerge. To facilitate a comparison based on additional conditions we let the costs of maintaining a link within an association and that between the traders be the same ($c = k > 0$). When there are two active high-quality sellers and one inactive low-quality seller (cases (ii) and (v) of figure 1A), the formation of a high-quality sellers' association $\{s_1, s_2\}$ with which all buyers are linked (case (v)) is more likely to emerge than the graph without a non-degenerate sellers' association (case (ii)).[17] When there are two low-quality sellers and one high-quality seller (figure 1B), however, the formation of a mixed-quality sellers' association $\{s_1, s_2\}$ is less likely than that without a non-degenerate sellers' association.[18] While the sellers' association is more likely to emerge in the former case, owing to the reduction in competition, the sellers' association is less likely to arise in the latter case, owing to the reduction in sale price as a result of including a low-quality seller in the association. Furthermore, in the absence of a non-degenerate sellers' association, it is not surprising that the presence of two inactive sellers (case (ix) in figure 1B) is less likely than the presence of one inactive seller (cases (ii)).[19]

In graphs (i), (iii), (iv), and (vi) of figure 1 all goods are sold, while in each of the remaining graphs there is a mismatch in that not all goods are sold. The graphs in the first group satisfy the efficiency criterion. In the second group, graphs (vii) of figure 1 and (viii) of figure 1A are strongly inefficient as there exist idle goods of high-quality, whereas the remaining cases are weakly inefficient. While the additional stability conditions of the weakly inefficient stable networks

17 In addition to assumption 1, case (ii) in figure 1A requires a stronger condition, $q^H - 2q^L \geq 4c$, than that for case (v), $q^H - (3/2)q^L \geq 4c$.

18 In addition to assumption 1, case (ii) in figure 1B requires a weaker condition, $q^H - 2q^L \geq 0$, than that for case (v), $q^H - 2q^L \geq 4c$.

19 In addition to assumption 1, case (ii) in figure 1B requires a stronger condition, $q^H - 2q^L \geq 0$, than that for case (ix), $q^H - 3q^L \geq 0$.

require moderate quality differential, the strongly inefficient ones require that the value of the low-quality good be sufficiently higher than the link cost (see the appendix). Notice also that of these efficient stable graphs it is possible to have a one-to-one link pattern (case (i) of figures 1) and it is also possible for buyers to be linked to a sellers' association, which is either of high quality (case (iii) of figure 1A), low quality (case (iv) of figure 1B), or mixed quality (case (iii) of figure 1B, (iv) of 1A, and (vi) of 1A and 1B). We thus conclude that non-degenerate sellers' associations can naturally arise in stable networks to achieve first-best allocations, where the associations may be of various sizes containing members of uniformly high, uniformly low, or mixed quality.

3.2. Mismatch in stable trade networks

A 'mismatch' occurs whenever a graph with equal numbers of buyers and sellers has some goods that are unsold (see cases (ii), (v), (vii), (viii), and (ix) in figures 1A and 1B). The first type of mismatch occurs when a sellers' association has fewer buyers linked to it than it has members; this situation results in an excess supply of goods. For example, consider case (viii) of figures 1A and 1B. Here, the sellers' association has an average quality that is lower than that of the single seller, and the association has an excess supply of goods, while the single high-quality seller has an excess demand for his good. As shown in the appendix, in order for this type of trade network to be stable, one requires, in addition to assumption 1, that the value of the high-quality good be sufficiently larger than the cost of maintaining a trade link. An excessive number of buyers are thus willing to link with the single high-quality seller, since the expected payoff from maintaining such a link is large, causing the mismatch. Interestingly, an excess supply of goods can occur even if the sellers' association has a higher average quality than its competitors (see case (vii) of figures 1A and 1B). This may seem counter-intuitive at first glance, yet such a graph can be stable if the low-quality good is sufficiently valued and if the sellers' association maintains a high price.

The second type of mismatch features an inactive seller (cases (ii) and (v) in figures 1A and 1B) or even two inactive sellers (case (ix) in figure 1B). An immediate observation is that an inactive seller is always of low quality. A general analysis is delegated to section 4 (propositions 1 and 2). Such a graph with an inactive seller is stable only if the quality differential is large enough. To see this, recall that an inactive low-quality seller sets his price equal to c; thus, if a buyer forms a link with this seller, she will receive a payoff of $q^L - c$. This graph is stable if each buyer is currently receiving a payoff greater than or equal to $q^L - c$. So, for instance, in graph (ii) of figure 1, buyers 1 and 2 each receive a payoff of $(1/2)\,(q^H - p_1 - c)$. Stability requires that this payoff be greater than or equal to $q^L - c$ (to keep these buyers from severing their current ties and linking with the inactive seller), which is true if the quality differential is sufficiently large. Thus, graphs with inactive sellers require additional conditions to assumption 1.

3.3. Characterization of active trading prices

In table 1, we list the active trading price ranges that support every stable graph. From these ranges, we can determine the effective minimum prices and effective maximum prices, $p_i^{\min}$ and $p_i^{\max}$ (see the appendix). Next, we compare the effective minimum and maximum prices to the sellers' reservation price, p^L, and the buyer's reservation price, p^U. Focusing on the effective minimum prices, we find that the effective minimum price may exceed the corresponding sellers' reservation price. (We call such a minimum price an interior effective minimum price.) For example, consider case (ii) of figure 1A. Here, two buyers are linked with one high-quality seller, the third buyer is linked with another high-quality seller, while the low-quality seller remains inactive. The stability of this trade network requires that the second high-quality seller always sets a price higher than his reservation price. If he sets his price equal to his reservation price, then one of the two buyers linked with the first high-quality seller would prefer to link with seller 2 in order to receive a higher net payoff. This would not occur if the second seller were of low quality (case (ii) in figure 1B). Intuitively, with an inactive seller of identical (low) quality, an active seller cannot set the price higher than the reservation level, or its buyer would prefer to link with the inactive seller.

Another type of stable graph with interior minimum pricing occurs when a non-degenerate sellers' association has an idle good (cases (vii) and (viii) in figures 1). Here, the high minimum price keeps the graph stable by preventing buyers who are not linked to the association from severing their ties and forming ties with the association. With sufficiently large quality differential, this high price is necessary in case (vii) of figure 1, where the sellers' association provides goods of higher quality on average. In case (viii) of figure 1, the sellers' association provides goods of lower quality on average; thus, the association has an interior minimum price only if the quality differential is sufficiently small.

We next turn to effective maximum prices where an interior maximum price implies that the effective maximum price is below the buyer's reservation price. In contrast to the case of interior minimum prices, the case of interior maximum prices is much more straightforward. The only crucial factor now is whether or not there are inactive sellers. When there are inactive sellers (either one low-quality inactive seller as in cases (ii) and (v) in figures 1 or more than one inactive sellers as in case (ix) in figure 1B), an active seller cannot pin any of his customers down at their reservation utility level. This is because any of his customers would otherwise sever their link and form a link with the initially inactive seller to receive a higher net payoff.

3.4. Gains to sellers from forming a sellers' association

The final example shows that 'price incentives' are crucial for the formation of a non-degenerate sellers' association. Here, we contrast case (ii) with case (vi) in figure 1A. In case (ii), there are three buyers and three sellers, where the first two sellers are linked to buyers but the third seller is inactive. In case (vi), all

TABLE 1
Active trading prices for stable trade networks with three buyers and three sellers

	Case A: $H=\{s_1,s_2\}$, $L=\{s_3\}$	Case B: $H=\{s_1\}$, $L=\{s_2,s_3\}$
I	$c \le p_1 \le (1/2)\min\{q^H+p_2, 2q^H-q^L+p_3, 2q^H-2c\}$ $c \le p_2 \le (1/2)\min\{q^H+p_1, 2q^H-q^L+p_3, 2q^H-2c\}$ $c \le p_3 \le (1/2)\min\{2q^L-q^H+p_1, 2q^L-q^H+p_2, 2q^L-2c\}$	$c \le p_1 \le (1/2)\min\{2q^H-q^L+p_2, 2q^H-q^L+p_3, 2q^H-2c\}$ $c \le p_2 \le (1/2)\min\{2q^L-q^H+p_1, q^L+p_3, 2q^L-2c\}$ $c \le p_3 \le (1/2)\min\{2q^L-q^H+p_1, q^L+p_2, 2q^L-2c\}$
ii	$2c \le p_1 \le \min\{p_2, q^H-2q_L+2c\}$ $c \le p_2 \le \min\{(1/3)(2q^H+p_1), q^H-q^L+c\}$ $p_3 = c$	$2c \le p_1 \le q^H-2q^L+2c$ $p_2 = p_3 = c$
iii	$c+k \le p_{12} \le \min\{(1/2)(2q^H-q^L+p_3), q^H-c\}$ $c \le p_3 \le \min\{q^L-(2/3)(q^H-p_{12}), q^L-c\}$	$c+k \le p_{12} \le (1/2)\min\{q^H+p_3, q^H+q^L-2c\}$ $c \le p_3 \le \min\{(1/3)(2q^L-q^H+2p_{12}), q^L-c\}$
iv	$c \le p_1 \le \min\{(1/3)(2q^H-q^L+2p_{23}), q^H-c\}$ $c+k \le p_{23} \le (1/2)\min\{q^L+p_1, q^H+q^L-2c\}$	$c \le p_1 \le \min\{(1/3)(3q^H-2q^L+2p_{23}), q^H-c\}$ $c+k \le p_{23} \le \min\{(1/2)(2q^L-q^H+p_1), q^L-c\}$
v	$(3/2)c+k \le p_{12} \le q^H-(3/2)q^L+(3/2)c$ $p_3 = c$	$(3/2)c+k \le p_{12} \le 1/2(q^H-2q^L+3c)$ $p_3 = c$
vi	$c+2k \le p_{123} \le (1/3)(2q^H+q^L)-c$	$c+2k \le p_{123} \le (1/3)(q^H+q^L)-c$
vii	$c+2k \le p_{12} \le \min\{q^H-(1/3)q^L+(1/3)p_3, q^H-c\}$ $2c \le p_3 \le \min\{q^L-2(q^H-p_{12}), q^L-2c\}$	$c+2k \le p_{12} \le \min\{(1/2)(q^H+q^L)-c, (1/2)q^H+(1/6)q^{L+}(1/3)p_3\}$ $2c \le p_3 \le \min\{q^L-2c, 2p_{12}-q^H\}$
viii	$2c \le p_1 \le \min\{2p_{23}-q^L, q^H-2c\}$ $c+2k \le p_{23} \le \min\{(1/6)(q^H+3q^L+2p_1), (1/2)(q^H+q^L)-c\}$	$2c \le p_1 \le \min\{q^H-2q^L+2p_{23}, q^H-2c\}$ $c+2k \le p_{23} \le \min\{(1/3)(3q^L-q^H+p_1), q^L-c\}$
xi	n/a	$3c \le p_1 \le q^H-3q^L+3c$ $p_2 = p_3 = c$

sellers are part of an association that is linked to all buyers. By comparing the effective minimum and maximum prices (see the appendix), we find that if the cost of maintaining a link is not too large (specifically if $c \leq 2k$, then the effective minimum price for the two active sellers is larger with the association. And the sellers' association has a higher effective maximum price for all three goods as long as the quality differential is not too large (specifically, $4q^L \geq q^H + 6c$). This difference in the effective maximum price occurs because an inactive seller always sets his price as low as possible; so, if the active sellers want to keep their customers, they cannot set prices too high. Yet, if the sellers form an association that includes this inactive seller, then the sellers are able to eliminate the stiff competition of an inactive seller and thus can set prices higher. Therefore, if buyers and sellers have equal bargaining power (if price is split evenly between the effective minimum and maximum prices) or if sellers have greater bargaining power and if the cost of forming an association is not too big, then all sellers are strictly better off in the association.

3.5. Trade networks with unequal numbers of buyers and seller

Up to now, we have considered cases with equal numbers of buyers and sellers. Next, we consider what happens if we have more (or fewer) buyers than sellers? Basically, the aforementioned features concerning the stable trade network patterns and the active trading prices that support these network patterns remain qualitatively unchanged. Additionally, one may now compare the current active trading prices with those obtained previously. To begin, we compare the following stable graph under $\mu(B) = 3$ and $\mu(S) = 2$ (left panel of figure 2) with case (ix) of figure 2 (right panel). We find that the active trading prices that support the two graphs are identical, as are the symmetric Nash prices (see the appendix). The existence of just one inactive seller gives each buyer an alternative source from which to purchase the product; thus, active sellers must lower their prices in order to keep their existing customers. Since each buyer wishes to purchase only one item, the existence of an additional inactive seller does not influence a buyer's behaviour, and so prices do not fall further.

Next, what happens if an inactive seller is removed from the market? Here, we compare the graph where two buyers are linked to the same high-quality seller and a third buyer is linked to a low-quality seller (see the case depicted in the right panel in figure 3) to the identical graph where one additional idle low-quality seller is added (case (ii) in figure 1B or the left panel in figure 3). Comparing the effective minimum and maximum prices of the two graphs (see the appendix), we find that both the effective minimum and the effective maximum prices are higher for the case without the inactive seller. Thus, removing an inactive seller lessens competition and allows the remaining sellers to price higher. In this case, the symmetric Nash price is unambiguously higher.

On the contrary, what happens if we eliminate an unlinked buyer? Consider the case (left panel of figure 4), where the third buyer is idle with case (i) of

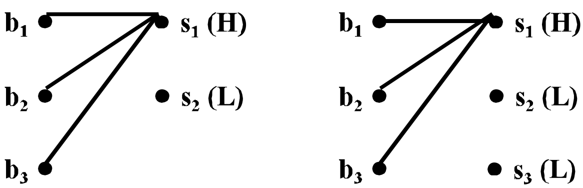

FIGURE 2 Trade network with increase in inactive sellers

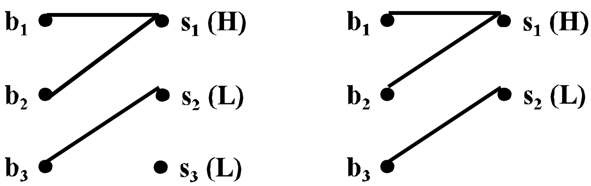

FIGURE 3 Trade network with inactive seller removed from market

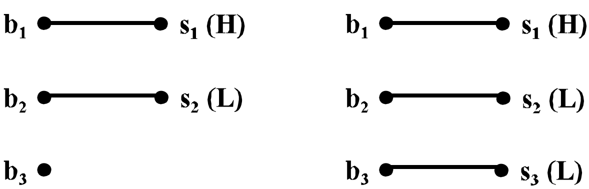

FIGURE 4 Trade network with unlinked buyer eliminated

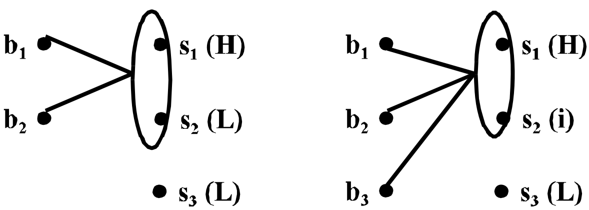

FIGURE 5 Seller's association with increase in number of buyers

figure 1B where each buyer is linked to one seller (right panel). Here, the effective maximum prices are the same for both graphs. However, the effective minimum price is lower for the case without an inactive buyer. Intuitively, to sustain the graph with the inactive buyer, the active sellers cannot set prices too low, or the inactive buyer will decide that it is better to compete with another buyer for the product rather than to remain idle. Thus, the symmetric Nash price decreases when an inactive buyer is removed from the market.

Finally, we compare the active trading prices supporting the following stable graph under $\mu(B) = 2$ and $\mu(S) = 3$, where two buyers are linked to a seller's association of size two and the third seller is idle (left panel of figure 5), with those supporting case (v) of figure 1B, where there is an additional buyer linked to the seller's association (right panel). This enables us to look at what happens if we increase the number of buyers purchasing from a sellers' association. We find that the effective minimum price of the goods sold by the seller's association is higher in the latter case, while the effective maximum price is lower. The effective minimum price is higher when there are more buyers linked to the seller's association, since maintaining such links is costly, and thus the seller's association must raise its minimum price to cover this increased cost. However, the maximum price is lower when there are more buyers, because more buyers leads to excess demand, and so the seller's association must lower its maximum price to prevent buyers from linking with the inactive seller. Thus, increasing the number of buyers purchasing from a sellers' association leads to a 'price compression' rather than a uniform rise in the effective minimum and maximum prices. Moreover, we can compute the symmetric Nash prices as $p_{12}^N = (1/4)(q^H - q^L) + (1/2)k + c$ (left panel of figure 5) and $p_{12}^N = (1/4)(q^H - q^L) + (1/2)k + c$ (right panel). Thus, a larger customer base of a sellers' association results in a lower symmetric Nash price due to the dominant effect of outside competition from the inactive seller.

4. General properties for the formation of trade networks

From the discussion in section 3 above, we can draw some general properties of trade networks and prove them formally for arbitrary numbers of buyers and sellers. As defined in section 2, an inactive seller (resp., inactive buyer) is a seller (buyer) who has no links to buyers (sellers) and an idle good is a good unsold. Notably, an idle good could either be a good produced by an inactive seller or be a good produced by a sellers' association where the number of buyers linked to the association is strictly less than the number of association members (thus the association is unable to sell all of its goods).

We begin by establishing the existence of a stable network and the existence of a non-degenerate sellers' association.

THEOREM 1. *Under assumption 1, a stable network always exists.*

Proof. It suffices to show that there always exists a one-for-one network that is stable. We consider three different cases: (i) $\mu(B) \le \mu(H)$, (ii) $\mu(H) < \mu(B) \le \mu(S)$, and (iii) $\mu(B) > \mu(S)$. First, we examine the case of $\mu(H) \ge \mu(B)$. Consider the graph where each buyer is linked to a different high-quality seller and all low-quality sellers and remaining high-quality sellers are inactive. If each seller sets price equal to c then no buyer has incentive to sever her current tie or to sever her current tie and link to someone else and no seller has incentive to sever his tie, thus the graph is stable.

Second, consider $\mu(H) < \mu(B) \le \mu(S)$. We show that the following one-for-one linked graph G is stable: $\mu(H)$ buyers are each linked to a different high-quality seller, the remaining buyers are each linked to a different low-quality seller, and all remaining low-quality sellers are inactive. Let each high-quality seller set price equal to p and each low-quality seller set price equal to c. The net payoffs of buyers linked with high-quality sellers are $u^H = q^H - p - c$, whereas those linked with low-quality sellers are $u^L = q^L - 2c$. The net payoffs of sellers of high and low-quality are $v^H = p - c$ and $v^L = 0$, respectively. In order for no buyer or seller to want to sever her/his link and exit the game, we must have $u^j \geq 0$ and $v^j \geq 0$ for $j = H, L$, which implies that $p \in [c, q^H - c]$ (recalling that $u^L = q^L - 2c \geq 0$ by assumption 1). Moreover, no buyer should want to sever her current link and link with someone else. In order for a buyer linked with a high-quality seller not to want to sever her link and form a link with an inactive low-quality seller, we must have: $p \leq q^H - q^L + c$ (note that for such a buyer to sever a link and form a link with an active low-quality seller is obviously not as good). Similarly, for a buyer linked with a low-quality seller not to want to sever her link and form a link with a high-quality seller, we must have $p \geq q^H - 2q^L + 2c$. Thus, under the range of active trading prices, $\max\{q^H - 2q^L + 2c, c\} \leq p \leq q^H - q^L + c$ (which exist as long as assumption 1 is met), G is stable.

Finally, consider $\mu(B) > \mu(S)$. In contrast to case (ii), a one-for-one linked graph G must have inactive buyers of size $\mu(B) - \mu(S)$, who receive zero payoffs, where no seller is inactive. Let each high-quality seller set price equal to $p(H)$ and each low-quality seller set price equal to $p(L)$. In order for no buyer or seller to want to sever her/his link and exit the game we must have $p(j) \in [c, q^j - c]$ for $j = H, L$. For an inactive buyer to have no incentive to form a link to a currently linked high- or low-quality seller, we must have: $p(j) \geq q^j - 2c$, which together with $p(j) \leq q^j - c$ implies $q^j - p(j) \in [c, 2c]$. For a linked buyer not to sever a link and to form a link with a seller of a different type, it is required that $p(H) \leq q^H - (1/2)(q^L - p(L))$ and $p(L) \leq q^L - (1/2)(q^H - p(H))$, which are not binding, given $p(j) \in [c, q^j - c]$ and $q^j - p(j) \in [c, 2c]$. Thus, G is stable, given $\max\{q^j - 2c, c\} \leq p(j) \leq q^j - c$, which exist under assumption 1. ■

THEOREM 2. *Suppose that assumption 1 holds true and that the costs of maintaining links (both between traders, c, and within the association, k) are low compared with the value of goods (q^H and q^L). Then, a stable network with a non-degenerate sellers' association of mixed quality exists.*

Proof. It suffices to show that a stable network may feature a sellers' association as the grand coalition of all sellers with which every buyer is linked. There are two cases, depending on whether the number of buyers β is greater than or smaller than the number of sellers σ. First, assume that $\beta \leq \sigma$. Denoting the price of the good as p, the net payoff of $b \in B$ is

$$u(\{b\}, \{S\}) = (\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L - p - c. \tag{6}$$

To ensure active participation by buyers $u(\{b\}, \{S\})$ must be non-negative. Since the number of buyers is less than the number of sellers, no seller is guaranteed to make a sale. To ensure active participation of sellers we must have $(\beta/\sigma)(p - c) - (\sigma - 1)\,k \geq 0$. Thus, active trading prices must satisfy

$$(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L - c \geq p \geq c + [\sigma(\sigma - 1)/\beta]k. \tag{7}$$

This constraint can be met only if

$$(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L \geq 2c + [\sigma(\sigma - 1)/\beta]k, \tag{8}$$

and thus holds true for q^H and q^L significantly larger than c and k.

Next, we must check the case where $\beta \geq \sigma$, and thus not all buyers are guaranteed of purchasing the product. Now, we have $u(\{b\},\{S\}) = (\sigma^H/\beta)q^H + (\sigma^L/\beta)q^L - (\sigma/\beta)p - c$, and active trading prices must satisfy $(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L - (\beta/\sigma)c \geq p \geq (\beta/\sigma)c + (\sigma - 1)k$. This requires

$$(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L \geq [(2\beta)/\sigma]c + (\sigma - 1)k, \tag{9}$$

which again holds true only if q^H and q^L are significantly larger than c and k. ■

Theorem 2 implies that the emergence of a stable trade network with a non-degenerate sellers' association containing producers of mixed quality requires fairly mild conditions. Under assumption 1, (8) holds true as long as k is sufficiently small, regardless of the magnitude of c. Yet (9) requires both k and c to be small. With excess demand, forming a large sellers' association of mixed quality thus requires low costs of maintaining links not only within the association but also between buyers and sellers.[20]

More notably, even with relatively passive sellers, a non-degenerate sellers' association may still emerge. Thus, the underlying mechanism for the formation of a sellers' association must be direct price incentives rather than market power. It is in this sense that our sellers' associations are in contrast with cartels in the conventional industrial organization literature.

The remainder of this section is devoted to studying the properties of stable buyer-seller networks. The three propositions in section 4.1 characterize the patterns of trade networks; in particular, propositions 1–3 establish conditions under which a stable network may feature inactive sellers or inactive buyers. In section 4.2, propositions 4–8 characterize the active trading prices supporting stable trade networks. We impose assumption 1 throughout without further references.

20 Theorem 2 gives conditions under which a graph where every buyer is linked to the grand sellers' association is stable. Since our definition of stability requires that no buyer wants to sever his tie and that no seller wants to exit the game, we know that every buyer and seller receives a non-negative payoff. and so this graph is not Pareto dominated by the empty network.

4.1. Stable network properties

PROPOSITION 1. *If the number of buyers is greater than or equal to the number of high-quality sellers (i.e., $\mu(B) \geq \mu(H)$), then there does not exist a stable network containing any inactive sellers of high quality.*

Proof. We prove this proposition by contradiction. Assume that G is a stable graph containing $s \in S \cap H$ who is inactive. If a buyer $b \in B$ severs her link with A^S to form a link with the inactive seller s, then the net gain equals $\Delta = q^H - 2c - u(b, A^S)$, where the inactive seller's price is fixed at the minimum, c. It remains to show that $\Delta > 0$ and thus that G is unstable. First, notice that if A^S is empty, then $\Delta = q^H - 2c > 0$; thus all buyers must be linked. Second, assume that the average quality of a good purchased from A^S is $q < q^H$. Then $u(b, A^S) \leq q - p(A^S) - c < q^H - 2c$ and thus $\Delta > 0$. (Note that $u(b, A^S) = q - p(A^S) - c$ if A^S is linked with one buyer and is less than or equal to $q - p(A^S) - c$ if A^S is linked with multiple buyers.) Therefore, if G is stable, then A^S must consist of only high-quality sellers and, similarly, all buyers must be linked to high-quality sellers' associations. By assumption $\mu(B) \geq \mu(H)$ and by assumption there exists at least one high-quality seller who is inactive; thus, there must exist an association, say A^S, that maintains links with m buyers such that $m > \mu(A^S)$. The stability of G requires that no seller in A^S wishes to sever a link; thus, we need $v(., A^S) \geq 0$ or, equivalently, $\min\{m/\mu(A^S), 1\}\ p(A^S) \geq mc/\mu(A^S) + [\mu(A^S) - 1]k$ or

$$p(A^S) \leq \max\{\mu(A^S)/m, 1\}\{mc/\mu(A^S) + [\mu(A^S) - 1]k\}. \tag{10}$$

This implies $u(\{b\}, A^S) \leq q^H - \max\{\mu(A^S)/m, 1\}\{mc/\mu(A^S) + [\mu(A^S) - 1]k\} - c$ and thus that

$$\begin{aligned} \Delta &\geq \max\{\mu(A^S)/m, 1\}\{mc/\mu(A^S) + [\mu(A^S) - 1]k\} - c \\ &\geq [\max\{\mu(A^S)/m, 1\}m/\mu(A^S) - 1]c > 0, \end{aligned} \tag{11}$$

which implies G is not stable. ■

PROPOSITION 2: *Under assumption* 1, *if* $\mu(H) \geq 1$ *and* $\mu(L) \geq 1$, *then a stable network may feature inactive low-quality sellers when the quality differential is sufficiently large.*

Proof. We consider two different cases, $\mu(H) > \mu(B)$ and $\mu(H) \geq \mu(B)$, and show that in both cases a graph with inactive low-quality sellers is stable when the quality differential is sufficiently large. First, we note that for the case of $\mu(H) > \mu(B)$, we consider the graph where each buyer is linked to a different high-quality seller and all low-quality sellers and remaining high-quality sellers are inactive. As shown in part (i) of the proof of the theorem, if each seller sets price equal to c, then the graph is stable. Second, consider $\mu(H) \geq \mu(B)$. We show that the following graph G is stable. Let all low-quality sellers be inactive. Let each high-quality seller be linked to either m or $m + 1$ buyers, where m and

$m+1$ are the integers closest to $\mu(B)/\mu(H)$. (If $\mu(B)/\mu(H)$ is an integer then let each high-quality seller be linked to $m+1=\mu(B)/\mu(H)$ buyers.) Let each seller linked to m(resp. $m+1$) buyers set the same price p_m (resp., p_{m+1}). Then, the net payoff of a buyer b is $(1/i)(q^H-p_i)-c$, for $i \in \{m, m+1\}$, depending on which type of high-quality seller she is linked to. In order to ensure active participation of all sellers, we must have $p_i \geq ic$, where $i \in \{m, m+1\}$. To ensure active participation of buyers we must have

$$(1/i)\big(q^H-p_i\big)-c \geq 0 \quad \text{for} \quad i \in \{m, m+1\}. \tag{12}$$

If the graph is stable, then no buyer wants to sever her current link and link with an inactive low-quality seller,

$$(1/i)\big(q^H-p_i\big)-c \geq q^L-2c \quad \text{for} \quad i \in \{m, m+1\}. \tag{13}$$

Moreover, no buyer can want to sever her current link and form a link with another high-quality seller; thus,

$$\begin{aligned}&(1/m)\big(q^H-p_m\big)-c \geq [1/(m+2)]\big(q^H-p_{m+1}\big)-c, \quad \text{and}\\&[1/(m+1)]\big(q^H-p_{m+1}\big)-c \geq [1/(m+1)]\big(q^H-p_m\big)-c.\end{aligned} \tag{14}$$

If we let $p_m = p_{m+1}+\varepsilon$, where $\varepsilon > 0$ and p_{m+1} satisfies

$$(m+1)c \leq p_{m+1} \leq \min\{q^H-(m+1)(q^L-c), q^H-(m+1)c\}, \tag{15}$$

then all conditions are met. So active trading prices exist as long as: $q^H \geq (m+1)q^L \geq \mu(B)q^L$ and $q^H \geq 2(m+1)\,c \geq 2\mu(B)c$. By assumption 1 we know that $q^L \geq 2c$; thus, active trading prices exist as long as the quality differential is sufficiently large (specifically, $q^H - q^L \geq mq^L$). ■

PROPOSITION 3. *Under assumption* 1,

- *i) if* $\mu(B) \leq \mu(S)$, *then there does not exist a stable network containing any inactive buyers*
- *ii) if* $\mu(B) > \mu(S)$, *then there exists a stable network containing at least one inactive buyer.*

Proof. We prove part (i) of the proposition by contradiction. Assume, to the contrary, that $\mu(B) \leq \mu(S)$ and that there exists a stable network, G, with at least one buyer, say, b_1, who is unlinked and thus does not purchase a product. Since $\mu(B) \leq \mu(S)$, there must also exist at least one seller, say, s_1, who does not sell his product. If s_1 is unlinked, then s_1 sets his price at c and, by assumption 1, b_1 would gain from linking to s_1; thus, G is not stable. If s_1 is linked but is not selling his product, then he must be part of a sellers' association. Since we assumed that G

is stable and that some buyers are already linked to this association, these buyers must expect to receive a payoff $u \geq 0$ (if $u < 0$, then the buyers would sever their ties to this association). Since s_1 is not selling his product, b_1 would also expect payoff $u \geq 0$ if he linked with this seller's association. Thus, b_1 will form the link and G is not stable.[21]

Next we prove part (ii) of the proposition. Assume $\beta = \mu(B) > \mu(S) = \sigma$. We show that graph $G = \{(b_1, s_1), (b_2, s_2), \ldots, (b_\sigma, s_\sigma), (b_{\sigma+1}), \ldots, (b_\beta)\}$ is stable. To show that G is stable, we need to find at least one active trading price vector that supports G. By assumption 1, we can select $0 < \varepsilon < c/2$ such that $q^L \geq 2(c + \varepsilon)$. Let $p(s_j) = q^j - c - \varepsilon$, where and $q^j = q^H$ if $s_j \in H$ and $q^j = q^L$ if $s_j \in L$. To prove that G is stable, we must show that no buyer wants to sever a link and/or form a new link and that no seller wants to sever a link. By construction,

$$v(b_j, s_j) = p(s_j) - c = q^j - 2c - \varepsilon > 0, \tag{16}$$

implying that no seller wants to sever a link. Similarly,

$$u(b_j, s_j) = q^j - p(s_j) - c = \varepsilon > 0 \quad \text{for } 0 \leq j \leq \sigma, \tag{17}$$

implying that no buyer wishes to sever a link. If a linked buyer j severs her current tie and simultaneously links with seller $k \neq j$ (who, by construction, is already linked to another buyer), then buyer j's payoff would equal $1/2q^k - 1/2p(s_k) - c = \varepsilon - 1/2c < 0$; thus, buyer j will stay linked to seller j. Lastly, we check that no initially unlinked buyer wishes to form a link with a seller, say seller j. Forming such a link would give the buyer a payoff of $1/2q^j - 1/2p(s_j) - c = \varepsilon - 1/2c < 0$; thus, the buyer will not form the link. ■

The results in propositions 1–3 suggest that high-quality sellers can never remain inactive in a stable trade network, whereas lower-quality sellers can be inactive if the quality differential is sufficiently large. A buyer can remain inactive only when there is global excess demand. When the number of buyers is greater than or equal to that of sellers, proposition 2 shows that it is always possible to have a trade-network mismatch with the coexistence of a buyer who chooses not to purchase and an unsold low-quality good.

4.2. Active trading price properties

In the next two propositions, we characterize the active trading prices supporting stable networks. In particular, we are interested in when an active price will be strictly above the sellers' reservation price p^L or strictly below the

21 Note that we assume here that if a buyer is indifferent between forming a link and having no links, then she forms a link. Alternatively, we could assume that if a buyer is indifferent, then she always severs the link. Using this assumption in the above proof, we would just need to change $u \geq 0$ to $u > 0$ and $u < 0$ to $u \leq 0$.

buyers' reservation price p^U. While the former implies a non-negligible positive surplus for sellers, the latter implies a non-negligible positive surplus for buyers.

PROPOSITION 4. *Let $c = k = 0$ and $\beta = \mu(B) \geq \mu(S) \geq 3$. Then, a high-quality seller's association A with at least one idle good always has an active trading price:*

$$p(A) \geq [1/(\beta - 1)]q^H > p^L(A). \tag{18}$$

Proof. Consider a stable graph G, which contains a high-quality sellers' association, say, A, which has at least one idle good. Since there exists an idle good and since $\beta \geq \sigma$, we know that there exists at least one buyer, say, b_1, who is not guaranteed a purchase in graph G. Thus, b_1 must be one of m' buyers who is linked to sellers' association A', where $m' > \mu(A')$. (Note that it is possible that A' is empty and thus that b_1 is linked to no one.) Let q' be the average quality of a good purchased from A'. Then, buyer b_1's expected payoff is

$$u(b_1, A') = [\mu(A')/(m')][q' - p(A')]. \tag{19}$$

Next, we compare $u(b_1, A')$ with the payoff b_1 would receive if she severed her tie to A' and linked to A. Let the number of buyers that A is already linked be $m > 0$ (as A is a high-quality association, we know from proposition 1 that A must be linked with at least one buyer). Since A has an idle good, we know that if b_1 links with A, b_1 purchases a high-quality good with certainty. So $u(b_1, A) = q^H - p(A)$. If G is stable, then we must have $u(b_1, A') \geq u(b_1, A)$, which implies

$$p(A) \geq q^H - [\mu(A')/(m')][q' - p(A')]. \tag{20}$$

By assumption, $m' \leq \beta - 1$ and $\mu(A')/(m') \leq (m' - 1)/m' \leq (\beta - 2)/(\beta - 1)$. Thus, $q^H - [\mu(A')/(m')][q' - p(A')] \geq q^H - [(\beta - 2)/(\beta - 1)]q^H = [1/(\beta - 1)]q^H$, and, hence, $p(A) \geq [1/(\beta - 1)]q^H > 0 = p^L(A)$. ■

PROPOSITION 5. *Let G be a stable graph with at least one inactive seller. Then, any active association, A, has an active trading price*

$$p(A) \leq p^U(A) - (q^L - 2c) < p^U(A). \tag{21}$$

Proof. Consider an active sellers' association A. Let b_1 be one of m buyers linked to A and q^A be the average quality of a good purchased from A. Then, buyer b_1's expected payoff becomes

$$u(b_1, A) = \min\{\mu(A)/m, 1\}[q^A - p(A)] - c. \tag{22}$$

Buyer b_1's reservation price sets $u(b_1, A) = 0$, thus implying

$$p^U(A) = q^A - \max\{m/\mu(A), 1\}c. \tag{23}$$

Let s_1 be an inactive seller in G of quality q^i, $i \in \{H, L\}$. (If $\mu(B) > \mu(H)$, then from proposition 1 we know that $i = L$.) If b_1 severs her link to A and links to s_1, her payoff would be $u(b_1, s_1) = q^i - 2c$ (since an inactive seller always sets price equal to c). By the stability of G, we must have $u(b_1, A) \geq u(b_1, s_1)$, thus implying $p(A) \leq q^A - \max\{m/\mu(A), 1\}(q^i - c)$ or, using (23),

$$p(A) \leq p^U(A) - \max\{m/\mu(A), 1\}(q^i - 2c) \leq p^U(A) - (q^L - 2c), \tag{24}$$

where the last inequality is a result of the fact that $q^i \geq q^L$ and that $\max\{m/\mu(A), 1\} \geq 1$. ■

PROPOSITION 6. *Let G be a stable graph with at least one inactive seller. Any new stable graphs where all inactive sellers exit the market are associated with higher effective maximum prices.*

Proof. Any new stable graphs that remove all inactive sellers from the game will no longer have to satisfy the first inequality of equation (24). Thus, the new graph is associated with weakly higher effective maximum prices. ■

PROPOSITION 7. *Let $c = k = 0$ and let $q = (\sigma^H q^H + \sigma^L q^L)/\sigma$. Let G be a stable graph featuring m (low-quality) inactive sellers and a sellers' association containing all remaining sellers with whom all buyers are linked. If $q < \max\{1+\beta/m, \sigma/m\}q^L$, then any new stable graphs that remove all inactive sellers from the game (by including them in the association) must be associated with a higher symmetric Nash price.*

Proof. Assume the initial stable graph features m inactive sellers (of low quality) and a sellers' association, A, of size $(\sigma - m)$, which is linked with all β buyers. We will compute the Nash price, p^N, for a good sold by association A. The utility buyer i has from linking to A is $\min\{(\sigma - m)/\beta, 1\}((\sigma q - mq^L)/(\sigma - m) - p)$, while the utility i would have if he linked instead to an inactive seller is q^L. Thus, $p^{\max} = (\sigma q - mq^L)/(\sigma - m) - \max\{\beta/(\sigma - m), 1\}q^L$. In order for no seller to want to exit the game, we need $p \geq 0$; thus, $p^{\min} = 0$. The Nash price $p^N = 1/2\ p^{\max} + 1/2\ p^{\min} = 1/2((\sigma q - mq^L)/(\sigma - m) - \max\{\beta/(\sigma - m), 1\}q^L)$.

Next, consider a stable graph where all σ sellers are members of association A' and where all buyers are linked to A'. We compute $p^{N'}$, the Nash price of a good sold by association, A'. Buyers i's utility from linking to A' is $\min\{(\sigma - m)/\beta, 1\}(q - p)$, while his utility if he severs this tie is 0; thus, $p^{\max} = q$. And, as above, $p^{\min} = 0$; thus, $p^{N'} = 1/2q$. Next we must show that $p^N \leq p^{N'}$ or that $1/2((\sigma q - mq^L)/(\sigma - m) - \max\{\beta/(\sigma - m), 1\}q^L) \leq 1/2q$. This inequality holds true if and only if $q \leq \max\{1 + \beta/m, \sigma/m\}q^L$. ■

PROPOSITION 8. *Let $\mu(B) > \mu(S)$ and let the initial stable network contain at least one inactive buyer but no idle goods. Then, any new stable graphs that allow all* *inactive buyers to exit the market are associated with lower effective minimum prices.*

Proof. We consider the case where the initial stable graph G features inactive buyers and features a sellers' association A of size $n \geq 1$, which is linked with $m \geq n$ buyers (no idle good) and supplies goods of average quality q at a price $p \in [p_{\min}, p_{\max}]$. Let A be the association achieving the highest value of $[n/(m+1)](q - p)$. The utility that an inactive buyer b would achieve by linking with A is $[n/(m+1)](q-p) - c$. Thus, in order for G to be stable, it must be that an inactive buyer b does not wish to form a link with A, implying that $[n/(m+1)](q-p) < c$ or $p > q - [(m+1)/n]c$. Next, we consider stable graphs where all inactive buyers have exited the game. Here, inequality $p > q - [(m+1)/n]\,c$ will no longer need to be satisfied, thereby yielding a weakly lower effective minimum price. ■

From proposition 4, we learn that a high-quality sellers' association with an idle good always receives a non-negligible payoff even if the cost of forming an association or a buyer-seller link is zero. This result implies that the sellers' non-negligible payoff is not the result of the sellers' trying to cover the cost of forming an association or a buyer-seller link. Instead, a high price is necessary to keep other buyers from linking with this association. Furthermore, the proposition suggests that such non-negligible payoffs to sellers increase with the valuation of the high-quality good (q^H) rather than the average quality or the quality differential; moreover, they decrease with the number of buyers (β) as opposed to the relative market tightness measured by the ratio of sellers to buyers. Finally, since the number of buyers is greater than or equal to the number of sellers, the existence of an idle good implies the existence of at least one buyer who does not purchase a good. Thus, this proposition gives the prices that will support a trade-network mismatch with the presence of both an unsold high-quality good and a buyer who does not purchase a good.

Proposition 5 suggests that every sellers' association in an economy with at least one inactive seller has active trading prices below its buyers' reservation price. So the existence of an inactive seller guarantees that linked buyers will receive a non-negligible positive payoff. Furthermore, the higher the valuation of the low-quality good over the buyer-seller link costs ($q^L - 2c$), the greater will the buyer's non-negligible positive payoff be. Since this payoff is mainly driven by the outside option facing each buyer in the presence of inactive low-quality sellers, it is not influenced by the average quality, the quality differential, or the relative market tightness.[22] Proposition 6 shows that if these inactive sellers exit

22 The industrial organization literature investigates the relationship between buyer-supplier contracts and outside supplier options, focusing on how sunk costs and supplier uncertainty may make an outside option more or less desirable to the buyer (see Helper and Levine 1992; Scheffman and Spiller 1992; Riordan 1996). Alternatively, proposition 6 shows how the outside supplier's network affects the buyer-supplier price.

the market, then the effective maximum price will increase and the buyers are no longer guaranteed a non-negligible positive payoff.

Proposition 7 shows that if a sellers' association allows low-quality inactive sellers to join the association, then the associations' symmetric Nash price will increase as long as the inactive sellers' quality is not too low. Thus, eliminating competition allows the association to raise its price, even though including the inactive sellers in the association decreases the association's average quality, as long as the quality gap between the original association and the inactive sellers is not too big. Finally, proposition 8 indicates that the removal of inactive buyers tends to lower effective minimum prices, as there is no need to set the price high to keep buyers inactive.

5. Extensions

This section extends the basic model by allowing buyers to form multiple links, discussing the implications of active seller networking, and presenting interesting avenues for future research.

5.1. Multiple links by buyers

For illustrative purposes, assume that each buyer is allowed to have at most two links. Consider the case where there are two buyers and three sellers and where the first two sellers are high quality and the third is of low quality. Rather than examining all possible graphs (as this becomes quite repetitive without adding much intuition), we consider the following interesting graph and show that it is stable: $G = \{(b_1, s_1), (b_1, s_2), (b_2, s_2), (b_2, s_3)\}$. This graph is chosen because each buyer maintains exactly two links, one with a seller who has no other links and one with a seller who has multiple links.

Recall that each buyer demands at most one unit of the good. Thus, when a buyer has multiple links she will buy from the seller who maximizes her net payoff. If one of these sellers has m links, then the buyer receives the product from this seller with probability $1/m$. If this buyer has a second link, she can tell the second seller she will purchase the product with probability $1 - (1/m)$. If graph G is stable, it must be that no buyer wants to sever one of her links, that no buyer wants to simultaneously sever one of her links and form a new link, and that no buyer or seller wants to simultaneously sever all of her/his links and become inactive. Thus, the more links a buyer has, the more stability conditions there are to check. We show in the appendix that graph G is stable if assumption 1 plus $q^L \geq 3c$ and $q^H \geq 4c$ hold true. Thus, it is possible to have a stable graph with multiple links by buyers if the cost of trade links is not too high.

5.2. Active seller networking

Throughout the paper, we assume, as do Kranton and Minehart (2001), that only the buyer can initiate forming a link. Sellers can close up shop and thus sell to no one, but sellers are not allowed to discriminate against or seek out buyers, and thus sellers are not allowed to form and/or sever individual links with buyers. What if the seller can sever individual links? The definition of a stable network must now be modified; in particular, condition (iii) in (3) becomes

$$\text{(iii)}' \quad v\left(B_k, A_k^S\right) > 0 \quad \text{and } v\left(B_k, A_k^S\right) > v\left(B_k \backslash \{b_i\}, A_k^S\right)$$
$$\text{for all} \quad b_i \in B_k \quad \text{and } A_k^S \in \left\{A_1^S, \ldots, A_n^S\right\},$$

saying that in a stable network the seller (or sellers' association) must not want to sever any of his links.

We claim that the set of stable networks found in section 3 will remain unchanged if the seller can pass on the buyer-seller link costs (in the form of a higher price) to the buyer. A simple way to model this is to set the seller's cost of maintaining a link equal to 0 and to set the buyer's link cost equal to $2c$ (thus, the buyer pays all link costs). It is easy to see that this assumption will quantitatively change the active trading price ranges of table 1 but will not qualitatively change the price ranges and will not change the set of stable networks.[23] That is, the qualitative results (of sections 3 and 4) concerning the configuration of stable networks and active trading prices remain valid. For example, consider graph $G = \{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$, which is stable under passive seller networking as long as $q^H \geq 4c$. The relevant question for stability under active seller networking would be whether s_1 would sever his link with b_2. If s_1 pays no link cost, then he is indifferent between severing the link or not and G is stable even with active seller networking.

An alternative question is whether a seller wants to leave an association and instead form a link with another buyer. In this case, we must again modify the definition of a stable network to permit such active seller networking. Call the original graph G and the alternative graph G'. For illustrative purposes, we do not check stability over the entire range of prices set in G but simply focus on examining whether a stable graph G under passive seller networking will remain stable under active seller networking when the symmetric Nash price set in G is maintained in the alternative graph G' (i.e., we assume myopic behaviour where individuals do not account for the effect of their networking and trading decision on equilibrium prices). To be more concrete, consider a graph $G = \{(b_1,\{s_1, s_2\}), (b_2,\{s_1, s_2\})\}$ where there is a third and inactive buyer b_3. We want to know whether s_1 or s_2 would leave the association to form a link with b_3 under myopic behaviour where an individual seller leaving the association $\{s_1, s_2\}$ continues to set the price at the previous symmetric Nash value given by $p_{12}^N = \frac{1}{2}(q^H + q^L) - 2.5c$. We assume that G is stable under passive seller networking, that is, conditions (i)–(iii) of (3) are met, which implies that $\frac{1}{2}(q^H + q^L) > 3.5c + k$. Further assume $k < 2.5c$. One can easily show that if s_2 (the low-quality seller) left the association to form a link with b_3 (thus creating graph

23 For instance, in table 1A case (i), the c term on the left-hand side of the inequality will now equal zero, while the c term on the right-hand side of the inequality will now equal $2c$.

$G' = \{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$) and set the price equal to p_{12}^N, then the resulting graph G' would be unacceptable to the sellers if $1/2(q^H + q^L) \in [3.5c + k, 4.5c + k)$ and would be unacceptable to the buyers if $(q^H - q^L) > 2.5c$. Similarly, one can show that if s_1 (the high-quality seller) left the association to form a link with b_3 (thus creating graph $G' = \{(b_3, s_1), (b_1, s_2), (b_2, s_2)\}$) and set the price equal to p_{12}^N, then the resulting graph would always be unacceptable to the buyers and would be unacceptable to the sellers if $1/2(q^H + q^L) \in [3.5c + k, 4.5c + k)$. Thus, G is stable,even with active seller networking, if q^H and q^L are in the appropriate ranges stated above.

Next, turning to the other side of the coin, we enquire what happens if a seller is allowed to join an association. For illustration purposes, consider case (v) of figure 1: $G = \{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, \{s_1, s_2\})\}$, where s_3 remains inactive. In particular, we are interested in learning whether it is always preferred to have s_3 join the association (the new graph is then captured by case (vi) of figure 1) if the new association $\{s_1, s_2, s_3\}$ sets the price at the previous symmetric Nash value given by $p_{12}^N = (1/2)\{[q^H - (3/2)q^L] + 3c + k\}$ (for figure 1A) and $p_{12}^N = (1/2)[(1/2)(q^H - 2q^L) + 3c + k]$ (for figure 1B). We suppose that G is stable under passive seller networking that is, assumption 1 and an additional condition are met ($q^H - (3/2)q^L \geq 3c + k$ for figure 1A and $q^H - 2q^L \geq 3c + k$ for figure 1B). Further assume $k > 2c$. We can easily show that forming $\{s_1, s_2, s_3\}$ by setting the price at p_{12}^N would be unacceptable to the sellers in this association if $q^H - (3/2)q^L \in [3c + k, 3k - c)$ (for figure 1A) and $q^H - 2q^L \in [3c + k, 2(3k - c))$ (for figure 1B). Thus, even if s_3 is allowed to join the association $\{s_1, s_2\}$, it would not occur in this case and G is stable even with active seller networking.[24] It is interesting to observe that G is (weakly) inefficient, so allowing for active networking need not alleviate such an inefficiency due to the presence of trade frictions.

In summary, with regard to a seller's decision to leave or join an association, our results remain qualitatively unchanged, even if we allow for coalitional deviations, as long as the assumption of myopic behaviour is maintained. Two remarks are now in order. First, we do not allow sellers to change associations and to take individual buyers with them when they change associations.[25] There are

24 Alternatively, one may re-examine this issue by considering case (ix) of figure 1B: $G = \{(b_1, s_1), (b_2, s_1), (b_3, s_1)\}$, where both low-quality sellers, s_2 and s_3, are inactive. Suppose that assumption 1 holds and $q^H > 3q^L$, so that G is stable under passive seller networking. If $q^H - 3q^L \in [0, 2k - 3c)$ for $k > (3/2)c$ or if $q^L \in [2(c + k), 3c)$ for $c > 2k$, then even though one of the inactive sellers (say, s_2) is allowed to join the association $\{s_1\}$, it would not occur in this case and G is stable even with active seller networking. In the first case, forming $\{s_1, s_2\}$ by setting the price at p_1^N would be unacceptable to the sellers; in the second, the buyers would rather sever the link with the association.

25 Since the focus of our paper is on the sellers' associations, it is natural not to consider coalitions of buyers. Should such a coalitional deviation be allowed, some inefficient stable graphs satisfying Conditions (i)–(iii) in (3) may become unstable. For example, consider case (ii) of figure 1, where both buyer b_1 and buyer b_2 are linked to seller s_1, b_3 is linked to s_2, and s_3 remains inactive. If b_1 and b_2 are allowed to collude, then b_1 can pay b_2 to sever her link with s_1 and instead form a link with s_3. With appropriate side payments, both b_1 and b_2 can be made better off, and so this type of buyer coalitional deviation grants case (ii) of figure 1 unstable.

many sellers' associations where sellers cannot take clients or buyers with them when they leave. For instance, consider the example mentioned in footnote 1 of a contractual tie between an HMO and a physicians' group. If a physician (seller) leaves the group, he will not take any patients (buyers) with him, since the patients have a contractual tie with the physicians' group, not that particular physician, and so the patients are unable to leave with the physician. The inability of sellers to exit with individual buyers is an interesting network trade friction to consider.

Second, we maintain the assumption that individuals are myopic, as do Jackson and Wolinsky (1996) and many others cited in footnote 5. Should individuals be farsighted, as in Page, Wooders, and Kamat (2005), the set of stable networks would be refined. However, an important point of the paper is that pairwise links between buyers' and sellers' associations can create trade frictions that result in inefficient outcomes. For example, a high-quality good may remain unsold even without an economy-wide excess supply of goods. These (weakly and strongly) inefficient outcomes not only are interesting in theory but also capture important network trade features in reality. To allow for a far-sighted network would be an interesting avenue for future research.

Another interesting avenue for future research would be to allow active seller networking while at the same time allowing both buyers and sellers to maintain multiple links. This might be particularly relevant if buyers were heterogeneous (as in Kranton and Minehart 2001). In such a case, in order to ensure a sale there would be an incentive for a seller of high quality to maintain links with buyers who value high quality, while additionally maintaining links to buyers who do not value quality.

Finally, one could also motivate active seller networking by allowing buyers and sellers to bargain over prices in the active trading range in a more general form than the simple symmetric Nash bargain specified in section 2 above (see, e.g., Laing, Palivos, and Wang 2002). For instance, graphs in which there is excess demand (resp., excess supply) for a good should lead to more bargaining power for the sellers (resp., buyers), and thus prices should end up in the upper end (resp., lower end) of the active trading range. Allowing for such bargaining would make it beneficial for sellers to maintain multiple links to buyers, since doing so would attract more customers. This is yet an additional agenda for further exploration.

Appendix

This appendix, organized by the respective sections, verifies the emergence of trade networks and derives active trading prices for the representative case (with the remaining data available upon request), summarizes additional assumptions and maximum/minimum pricing, and proves the general properties.

Proof of case (iii) with three buyers and one high-quality and two low-quality sellers

In case (iii) of figure 1B, $G = \{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, s_3)\}$ and the net payoffs of buyers are

$$u(\{b_i\}, \{s_1, s_2\}) = (1/2)(q^H + q^L) - p_{12} - c \quad \text{for}$$
$$i = 1, 2 \quad \text{and} \quad u(\{b_3\}, \{s_3\}) = q^L - p_3 - c,$$

where p_{12} denotes the price of a good produced by the sellers' associations $A^S = \{s_1, s_2\}$, which provides goods with mixed quality. The reservation value for s_1 and s_2 is $c + k$, so we must have $p_{12} \geq c + k$ for these sellers to be active. Similarly, $v(b_3, s_3) \geq 0$ implies $p_3 \geq c$. In order for b_1 or b_2 to be active, we must have $u(\{b_i\}, \{s_1, s_2\}) \geq 0$ $(i = 1, 2)$, or, $p_{12} \leq (1/2)(q^L + q^H - 2c)$. In order for b_3 to be active, we need $u(\{b_3\}, \{s_3\}) \geq 0$, or, $p_3 \leq q^L - c$. In order for b_1 or b_2 not to sever the link to form a link with the sellers' association, the following condition must be met: $p_{12} \leq (1/2)(q^H + q^L) - (1/2)(q^L - p_3) = (1/2)(q^H + p_3)$. In order for b_3 not to sever the link to form a link with the sellers' association, we need: $p_3 \leq q^L - (1/3)(q^H + q^L - 2p_{12}) = (1/3)(2q^L - q^H + 2p_{12})$. Thus, active trading under this network occurs under

$$c + k \leq p_{12} \leq (1/2) \min \{q^H + p_3, q^L + q^H - 2c\};$$
$$c \leq p_3 \leq \min \{(1/3)(2q^L - q^H + 2p_{12}), q^L - c\}$$

which require no additional conditions to assumption 1.

Summary of additional assumptions required for stable trade networks in section 3.1

Case	$s_1, s_2 \in H$ and $s_3 \in L$	$s_1 \in H$ and $s_2, s_3 \in L$
(i)	none	none
(ii)	$q^H - 2q^L \geq 4c$	$q^H \geq 2q^L$
(iii)	none	none
(iv)	none	none
(v)	$q^H - (3/2)q^L \geq 3c + k$	$q^H - 2q^L \geq 3c + k$
(vi)	none	none
(vii)	$q^L \geq 4c$	$q^L \geq 4c$
(viii)	$q^H \geq 4c$	$q^H \geq 4c$
(ix)	n/a	$q^H \geq 3q^L$

Summary of minimum and maximum pricing for stable trade networks in section 3.1

Solving the simultaneous equation system for each applicable lower and upper bounds for the active trading prices listed in tables 1A and 1B, we obtain the respective minimum pricing and maximum pricing for each stable trade network.

Case	$s_1, s_2 \in H$ and $s_3 \in L$ Minimum pricing	Maximum pricing
(i)	$p_1^{\min} = \max\{c, q^H - 2(q^L - c)\} \geq c = p^L$ $p_2^{\min} = \max\{c, q^H - 2(q^L - c)\} \geq c = p_2^L$ $p_3^{\min} = c = p_3^L$	$p_1^{\max} = q^H - c = p_1^U$ $p_2^{\max} = q^H - c = p_2^U$ $p_3^{\max} = q^L - c = p_3^U$
(ii)	$p_1^{\min} = 2c = p_1^L$ $p_2^{\min} = 2c > c = p_2^L$ $p_3^{\min} = c$(inactive)	$p_1^{\max} = q^H - 2(q^L - c) < q^H - 2c = p_1^U$ $p_2^{\max} = q^H - (q^L - c) < q^H - c = p_2^U$ $p_3^{\max} = c$(inactive)
(iii)	$p_{12}^{\min} = \max\{c + k, q^H - (3/2)(q^L - c)\}$ $\geq c + k = p_{12}^L$ $p_3^{\min} = c = p_3^L$	$p_{12}^{\max} = q^H - c = p_{12}^U$ $p_3^{\max} = q^L - c = p_3^U$
(iv)	$p_1^{\min} = c = p_1^L$ $p_{23}^{\min} = \max\{c + k, 3/2c - q^H + 1/2q^L\}$ $\geq p_{23}^L$	$p_1^{\max} = q^H - c = p_1^U$ $p_{23}^{\max} = (1/2)(q^H + q^L) - c = p_{23}^U$
(v)	$p_{12}^{\min} = (3/2)c + k = p_{12}^L$ $p_3^{\min} = c$(inactive)	$p_{12}^{\max} = q^H - (3/2)(q^L - c) < q^H - (3/2)c = p_{12}^U$ $p_3^{\max} = c$(inactive)
(vi)	$p_{123}^{\min} = c + 2k = p_{123}^L$	$p_{123}^{\max} = (1/3)(2q^H + q^L) - c = p_{123}^U$
(vii)	$p_{12}^{\min} = \max\{c + 2k, q^H - (1/2)(q^L - 2c)\}$ $\geq c + 2k = p_{12}^L$ $p_3^{\min} = \max\{2c, q^L - 3q^H + 3(c + 2k)\}$ $\geq 2c = p_3^L$	$p_{12}^{\max} = q^H - c = p_{12}^U$ $p_3^{\max} = q^L - 2c = p_3^U$
(viii)	$p_1^{\min} = \max\{2c, 3(c + 2k) - (1/2)(q^H + 3q^L)\}$ $\geq 2c = p_1^L$ $p_{23}^{\min} = \max\{c + 2k, (1/2)q^L + c\}$ $\geq c + 2k = p_{23}^L$	$p_1^{\max} = q^H - 2c = p_1^U$ $p_{23}^{\max} = (1/2)(q^H + q^L) - c = p_{23}^U$

Case	$s_1 \in H$ and $s_2, s_3 \in L$ Minimum pricing	Maximum pricing
(i)	$p_1^{\min} = \max\{c, q^H - 2(q^L - c)\} \geq c = p_1^L$ $p_2^{\min} = c = p_2^L$ $p_3^{\min} = c = p_3^L$	$p_1^{\max} = q^H - c = p_1^U$ $p_2^{\max} = q^L - c = p_2^U$ $p_3^{\max} = q^L - c = p_3^U$
(ii)	$p_1^{\min} = 2c = p_1^L$ $p_2^{\min} = c = p_2^L$ $p_3^{\min} = c$(inactive)	$p_1^{\max} = q^H - 2(q^L - c) < q^H - 2c = p_1^U$ $p_2^{\max} = c < q^L - 2c = p_2^U$ $p_3^{\max} = c$(inactive)
(iii)	$p_{12}^{\min} = \max\{c + k, (1/2)(3c + q^H - 2q^L)\}$ $\geq c + k = p_{12}^L$ $p_3^{\min} = c = p_3^L$	$p_{12}^{\max} = (1/2)(q^H + q^L) - c = p_{12}^U$ $p_3^{\max} = q^L - c = p_3^U$
(iv)	$p_1^{\min} = \max\{c, q^H + 2(c + k - q^L)\} \geq c = p_1^L$ $p_{23}^{\min} = \max\{c + k, (3/2)c - (3/2)q^H + q^L\}$ $\geq p_{23}^L$	$p_1^{\max} = q^H - c = p_1^U$ $p_{23}^{\max} = q^L - c = p_{23}^U$

(*Continued*)

	$s_1 \in H$ and $s_2, s_3 \in L$	
Case	Minimum pricing	Maximum pricing
(v)	$p_{12}^{\min} = (3/2)c + k = p_{12}^L$	$p_{12}^{\max} = (1/2)(q^H - 2q^L + 3c)$ $< (1/2)(q^H + q^L - 3c) = p_{12}^U$
	$p_3^{\min} = c$(inactive)	$p_3^{\max} = c$(inactive)
(vi)	$p_{123}^{\min} = c + 2k = p_{123}^L$	$p_{123}^{\max} = (1/3)(q^H + 2q^L) - c = p_{123}^U$
(vii)	$p_{12}^{\min} = \max\{(1/2)q^H + c, c + 2k\} \geq c + 2k = p_{12}^L$ $p_3^{\min} = \max\{3c + 6k - (3/2)q^H - (1/2)q^L, 2c\} \geq 2c = p_3^L$	$p_{12}^{\max} = (1/2)(q^H + q^L) - c = p_{12}^U$ $p_3^{\max} = q^L - 2c = p_3^U$
(viii)	$p_1^{\min} = \max\{2c, q^H + 3(c + 2k - q^L)\} \geq 2c = p_1^L$ $p_{23}^{\min} = \max\{c + 2k, q^L - (1/2)q^H + c\} \geq c/2 + k = p_{23}^L$	$p_1^{\max} = q^H - 2c = p_1^U$ $p_{23}^{\max} = q^L - c = p_{23}^U$
(ix)	$p_1^{\min} = 3c = p_1^L$ $p_2^{\min} = c$(inactive) $p_3^{\min} = c$(inactive)	$p_1^{\max} = q^H - 3(q^L - c) < q^H - 3c = p_1^U$ $p_2^{\max} = c$(inactive) $p_3^{\max} = c$(inactive)

Summary of minimum and maximum pricing for selected stable trade networks in section 3.5

	Three buyers and two sellers with $s_1 \in H$ and $s_2 \in L$	
Case	Minimum pricing	Maximum pricing
(i)	$p_1^{\min} = \max\{2c, q^H - 3(q^L - c)\} \geq 2c = p_1^L$ $p_2^{\min} = \max\{c, q^L - q^H + 2c\} \geq c = p_2^L$	$p_1^{\max} = q^H - 2c = p_1^U$ $p_2^{\max} = q^L - c = p_2^U$
(ii)	$p_1^{\min} = 3c = p_1^L$ $p_2^{\min} = c = p_2^L$	$p_1^{\max} = q^H - 3(q^L - c) < q^H - 3c = p_1^U$ $p_2^{\max} = c$(inactive)
(v)	$p_1^{\min} = \max\{c, q^H - 2c\} \geq c = p_1^L$ $p_2^{\min} = \max\{c, q^L - 2c\} \geq c = p_2^L$	$p_1^{\max} = q^H - c = p_1^U$ $p_2^{\max} = q^L - c = p_2^U$
(vi)	$p_{12}^{\min} = \max\{c + k, (1/2)(q^H + q^L - 3c)\} \geq c + k = p_{12}^L$	$p_{12}^{\max} = (1/2)(q^H + q^L) - c = p_{12}^U$

	Two buyers and three sellers with $s_1 \in H$ and $s_2, s_3 \in L$	
Case	Minimum pricing	Maximum pricing
(i)	$p_1^{\min} = c = p_1^L$ $p_2^{\min} = c = p_2^L$ $p_3^{\min} = c$(inactive)	$p_1^{\max} = q^H - q^L + c < q^H - c = p_1^U$ $p_2^{\max} = c = p_2^U$ $p_3^{\max} = c$(inactive)
(ii)	$p_{12}^{\min} = c + k = p_{12}^L$	$p_{12}^{\max} = (1/2)(q^H - q^L) + c$ $< (1/2)(q^H + q^L) + c = p_{12}^U$
	$p_3^{\min} = c$(inactive)	$p_3^{\max} = c$(inactive)

Proof of stable graphs in section 5.1 with multiple links by buyers

In the case where $G = \{(b_1, s_1), (b_1, s_2), (b_2, s_2), (b_2, s_3)\}$, the net payoffs of buyers are

$$u((\{b_1\}, \{s_1\}), (\{b_1\}, \{s_2\})) = q^H - \min\{p_1, (1/2)(p_1 + p_2)\} - 2c$$

$$u((\{b_2\}, \{s_2\}), (\{b_2\}, \{s_3\})) = \max\{q^L - p_3, (1/2)(q^H + q^L - p_2 - p_3)\} - 2c.$$

We must check that no buyer wants to sever a link and that no buyer would like to simultaneously sever a link and form a new link. First, note that in order for b_1 not to sever her link to s_2, we must have at least $\min\{p_1, (1/2)p_1 + (1/2)p_2\} = (1/2)(p_1 + p_2)$, and in order for b_2 not to sever her link to s_2 we must have at least $\max\{q^L - p_3, (1/2)(q^H + q^L - p_2 - p_3)\} = (1/2)(q^H + q^L - p_2 - p_3)$.

Next we systematically check all links. In order for b_1 not to sever her link to s_1 we must have: $p_1 \leq q^H - 2c$. In order for b_1 not to sever her link to s_1 and form a link with s_3 we must have: $p_1 \leq q^H - q^L + p_3$. In order for b_1 not to want to sever her link to s_2 or sever this link and form one with s_3 we must have $p_2 \leq \min\{p_1, q^H - q^L + p_3\}$. In order for b_2 not to want to sever her link to s_2 or sever this link and form one with s_1 we must have: $p_2 \leq q^H - q^L + p_3 - 2c$. In order for b_2 not to want to sever her link to s_3 or sever this link and form one with s_1 we must have: $p_3 \leq \min\{q^L - 2c, q^L - q^H + p_1\}$. Additionally, no buyer or seller wants to sever all of her/his links. Combining these results, we obtain the set of active trading prices:

$$c \leq p_1 \leq \min\{q^H - 2c, q^H - q^L + p_3\}$$

$$2c \leq p_2 \leq \min\{p_1, q^H - q^L + p_3 - 2c\}$$

$$c \leq p_3 \leq \min\{q^L - 2c, q^L - q^H + p_1\}$$

$$p_1 + p_2 \leq 2q^H - 4c$$

$$p_2 + p_3 \leq (1/2)q^H + q^L - 2c,$$

which exist if assumption 1 plus $q^L \geq 3c$ and $q^H \geq 4c$ hold true.

References

Aumann, R.J., and R.B. Myerson (1988) 'Endogenous formation of links between players and coalitions: an application of the Shapley value,' in *The Shapley Value*, ed. A. Roth (New York: Cambridge University Press)

Bala, V., and S. Goyal (2000) 'A noncooperative model of network formation,' *Econometrica* 68, 1181–229

Casella, A., and J. Rauch (1997) 'Anonymous market and group ties in international trade,' NBER Working Paper No. 6186

Cawthorne, P. (1995) 'Of networks and markets: the rise and rise of a south Indian town: the example of Tiruppur's cotton knitwear industry,' *World Development* 23, 43–56

Chwe, M. (2000) 'Communication and coordination in social networks,' *Review of Economic Studies* 67, 1–16

Corominas-Bosch, M. (2004) 'Bargaining in a network of buyers and sellers,' *Journal of Economic Theory* 115, 35–77
Demange, G. (1994) 'Intermediate preferences and stable coalition structures,' *Journal of Mathematical Economics* 23, 45–58
Dicken, P., and M. Hassler (2000) 'Organizing the Indonesian clothing industry in the global economy: the role of business networks,' *Environmental Planning A* 32, 263–80
Dutta, B., and S. Mutuswami (1997) 'Stable networks,' *Journal of Economic Theory* 76, 322–44
Furusawa, T., and H. Konishi (2002) 'Free trade networks,' mimeo, Boston College
Goyal, S., and S. Joshi (2003) 'Networks of collaboration in oligopoly,' *Games and Economic Behavior* 43, 57–85
Greenberg, J., and S. Weber (1993) 'Stable coalition structures with unidimensional set of alternatives,' *Journal of Economic Theory* 60, 62–82
Helper, S., and D. Levine (1992) 'Long-term supplier relations and product-market structure,' *Journal of Law, Economics, and Organization* 8, 561–81
Hsing, Y. (1999) 'Trading companies in Taiwan's fashion shoe networks,' *Journal of International Economics* 48, 101–20
Jackson, M.O. (2003) 'The stability and efficiency of economic and social networks,' in *Advances in Economic Design*, ed. M. Sertel and S. Koray (Heidelberg: Springer-Verlag)
Jackson, M.O., and A. Watts (2002a) 'The Evolution of social and economic networks,' *Journal of Economic Theory* 106, 265–95
— (2002b) 'On the formation of interaction networks in social coordination games,' *Games and Economic Behavior* 41, 265–91
Jackson, M.O., and A. Wolinsky (1996) 'A strategic model of social and economic networks,' *Journal of Economic Theory* 71, 44–74
Kaneko, M., and M. Wooders (1982) 'Cores of partitioning games,' *Mathematical Social Sciences* 3, 313–28
Kranton, R., and D. Minehart (2000) 'Competition for goods in buyer-seller networks,' *Review of Economic Design* 5, 301–31
— (2001) 'Theory of buyer-seller networks,' *American Economic Review* 91, 485–508
Laing, D., T. Palivos, and P. Wang (2002) 'Product diffusion and pricing with market frictions,' *Economic Theory* 19, 707–36
Mathewson, F., and R. Winter. (1996) 'Buyer groups,' *International Journal of Industrial Organization* 15, 137–64
Page, F., M. Wooders, and Z. Kamat (2005) 'Networks and farsighted stability,' *Journal of Economic Theory* 120, 257–69
Rauch, J. (1996) 'Trade and search: social capital, Sogo Shosha, and spillovers,' NBER Working Paper No. 5618
— (2001) 'Business and social networks in international trade,' *The Journal of Economic Literature* 39, 1177–203
Riordan, M. (1996) 'Contracting with qualified suppliers,' *International Economic Review* 37, 115–28
Scheffman, D., and P. Spiller (1992) 'Buyers' strategies, entry barriers, and competition,' *Economic Inquiry* 30, 418–36
Town, R., and G. Vistnes (2001) 'Hospital competition in HMO networks,' *Journal of Health Economics* 20, 733–53
Uzzi, B. (1996) 'The sources and consequences of embeddedness for the economic performance of organizations: the network effect,' *American Sociological Review* 61, 674–98
Wang, P., and A. Watts (2004) 'Formation of buyer-seller trade networks in a quality-differentiated product market,' working paper version, mimeo, SIU
Watts, A. (2001) 'A dynamic model of network formation,' *Games and Economic Behavior* 34, 331–41