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Product diffusion and pricing with market frictions*

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Summary. We study pricing and product diffusion in a dynamic general equilibrium framework with product market frictions. Ongoing R&D activity leads, with an endogenously determined probability, to continual improvements in product quality. We characterize the steady-state equilibrium with endogenous product diffusion in which a number of different goods co-exist on the quality ladder. We show that the severity of the economy's market frictions is a crucial determinant of the pricing structure, the product diffusion pattern, the level of R&D investment, the rate of endogenous growth, the length of Schumpeterian product cycles and the possibility of multiple growth paths. Eliminating market frictions leads to a degenerate product ladder of precisely one step, containing only the most recent product, as in the monopolistic competition literature.

Keywords and Phrases: Market frictions, Product diffusion, Vertical innovations, Schumpeterian growth.

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1 Introduction

With the advent of the industrial revolution, the previous two centuries have witnessed striking advances in not only the quality of goods available to consumers but also in the intermediate inputs used to produce them.[1] Despite the indubitable importance of this process for economic growth and welfare, it is only recently, with the development of the neo-Schumpeterian growth literature, that economists have begun to formally model this phenomenon.[2] Building upon the Schumpeterian notion of "creative destruction," these models conceive of a world in which on-going R&D, driven by the attempts of innovators to capture the monopoly rents that accrue from their discoveries, leads to an inexorable vertical step-by-step advance of each good along the rungs of a product ladder. Producers of today's cutting edge products displace ("destroy") older vintages, while simultaneously recognizing their own impending demise in the wake of subsequent technological advances.

While many goods are evidently characterized by this type of on-going process of creative destruction, it is equally clear that many of them display non-trivial product *diffusion* as well. Thus, at any given point in time, both cutting edge products and lower-quality vintages of goods are often simultaneously available to consumers and are being purchased by them. Furthermore, this process leads to the emergence of interesting *pricing* relationships among the goods that occupy the various rungs of a given product ladder. The most elementary of these is the simple observation that "state-of-the-art" products generally command higher prices than other, lower-quality versions, which are located further down the ladder.

Although the Schumpeterian framework offers an excellent forum for understanding the level of R&D activity, it is arguably less suited for analyzing product diffusion. The reason is that, in this class of models, diffusion often takes a rather extreme form in which the introduction of the most recent state-of-the-art product immediately renders all other vintages obsolete. This feature stems from the monopolistically-competitive foundation of the canonical model, wherein the leading firm *limit prices* so as to just exclude lower-quality producers from the market. Yet, this behavior is clearly at odds with what we see in practice. Thus, to give one example, in today's PC market it is currently possible for consumers to purchase computers ranging anywhere from a 486 machine to one embodying PIII architecture with a 1000 MHZ processor. One obvious candidate explanation for this behavior is to appeal to some form of heterogeneity among

[1] In this paper, we focus on vertical innovations that increase product quality. Indeed, Scherer (1980) notes that firms devote the lion's share (approximately 59%) of their research outlays to product improvement. Of course, horizontal innovations, leading to greater product *variety*, have also been critical (see, for example, Stokey, 1995). Grossman and Helpman (1991), demonstrate a formal equivalence between the (positive) results of models with quality improvements along given product ladders and horizontal innovation models, in which the number of product varieties increases through time.

[2] See, for example, the seminal papers of Romer (1990), Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991), Segerstrom (1991), Aghion and Howitt (1992), and the further developments of the model reported in Cheng and Dinopoulos (1992) and Romer (1994).

buyers, such as differences in their tastes or in their income levels. While there is obviously some validity to this, it is surely not the whole story. The reason is that with enough heterogeneity it becomes easy to explain diffusion, but difficult to account for obsolescence in any satisfactory manner.

In this paper, we construct a dynamic-general equilibrium Schumpeterian growth model that exhibits non-trivial product diffusion *and* creative destruction, despite the assumption of homogeneous buyers. Our primary goal is to understand how R&D activity affects (and is affected) by both pricing patterns as well as the range of quality-differentiated products offered for sale at each point in time. The key point of departure of this paper, relative to earlier work, is that we assume that trade (search) frictions are an integral part of the trading environment. Thus, instead of simply positing monopolistically-competitive markets (in which trade is coordinated by the Walrasian auctioneer), we assume that exchange is difficult and time consuming.[3] We show that in this setting, product diffusion and obsolescence arise in a rather natural way. The intuition leading to these results is simple. In the presence of costly search, lower-quality versions of the product *may* remain viable, since the producers of these goods agree to terms of trade that ensure further *buyer* search is sub-optimal. As an immediate corollary of this, lower-quality versions of the good, located towards the bottom of the product ladder, become obsolete for the simple reason that there is no price that can accomplish this goal, while simultaneously yielding their producer non-negative profits. We show that the price of each good depends upon both its intrinsic quality as well as the ease with which buyers and sellers can locate alternative trading partners. In turn, these latter search costs are endogenized and shown to depend upon the severity of the underlying trade frictions in the market.

Our framework also offers a rather transparent illustration of the intertemporal effects of on-going R&D activity on both pricing and diffusion patterns. The reason is that the logic described above, concerning the determinants of the terms of trade, carries over to *as-yet-undeveloped* products. More specifically, in order to sell any output, producers must set their prices so that each buyer elects to purchase their product, rather than simply waiting for higher-quality goods that will be assuredly available in the future.[4] It follows that for sufficiently rapid rates

[3] One interpretation of these frictions is that they reflect the time agents spend acquiring information about goods prior to purchasing them. Another interpretation is that once a consumer decides to purchase a good, availability and hence delivery times are random. We consider both of these mechanisms in this paper. Obviously, technological changes in both communications and computational power as well as increased "market making" activity have, arguably, dramatically reduced these transactions costs over the last fifty years or so. For instance, over this period the employment share of the retail sector (relative to private sector employment) grew annually at a rate of 0.28% (t-value 22.4) and search related consumption expenditures (on, amongst other things, private transportation, telephone services, telegraphs and postage) relative to other non-durable purchases grew at 0.18% (t-value 3.9). Moreover, the relative price of retail trade services has declined annually at a rate of 4.5% (t-value 30), which (partially) reflects the effect of reduced transactions costs.

[4] As Rosenberg (1976) has emphasized, in the context of technological improvements, on-going quality- improvements form a double edged sword. In particular, buyers must balance the incentives to acquire current "state-of-the-art" products with those of waiting for even better, as yet undeveloped, ones in the future.

of R&D, all but the most advanced products are driven from the market. The reason is that, under these circumstances, buyers either wait for more advanced products to appear or else search only for state-of-the-art goods.

We believe that our search-equilibrium approach also contributes to the current "market structure" debate in the growth literature [see, in particular, Aghion and Howitt (1998, chapter 7) and the references cited therein]. A basic prediction of the Schumpeterian approach is that greater competition among firms lowers profits and blunts the incentives for firms to conduct R&D. However, several recent empirical studies have cast the validity of this core premise into doubt, by uncovering what appears to be a *positive* correlation between product market competition and R&D activity [see, for example, Nickell (1996) and Blundell et al. (1999), in which competition is found to induce inventive activity and to spur growth]. *Inter alia* this has led to the emergence of an evolutionary hypothesis to explain this result, whereby greater competition forces firms to innovate in order to survive. In contrast, the model advanced here is distinctly Schumpeterian in nature (in particular, innovative activity is spurred on by the desire of agents to earn the monopoly rents that accrue to the holders of intellectual property rights). Yet, from the search perspective, the measures of increased competition (i.e., number of innovation-good producers) used in the empirical literature may simply reflect increases in *market thickness*, which *raise* profits and encourage R&D activity. In this light, it is interesting to note that Babbage (1832), Schumpeter (1942), Schmookler (1966), and Rosenberg (1994) have all forcefully articulated that conditions in the product market are crucial determinants of inventive activity and economic growth.[5]

In addition to the neo-Schumpeterian papers mentioned earlier, our model is also related to the literature on the diffusion of knowledge and technology (for example, Stokey, 1988; Jovanovic and Rob, 1989; Chari and Hopenhayn, 1991; Jones and Newman, 1995). In Stokey, knowledge accumulation through economy-wide learning-by-doing is the driving force behind economic growth. Knowledge growth leads to the introduction of newer goods and the obsolescence of older ones, by reducing the marginal cost of production of the former. Jovanovic and Rob examine knowledge diffusion in a random-matching framework, where agents "meet" and share information. In contrast, our model attempts to capture "informational barriers," in the product market, where consumers acquire information through search. Importantly, and in contrast to their work, since *prices* are endogenous so are the incentives to gather information through search. Chari and Hopenhayn construct a model of technological diffusion, in which technology diffuses slowly because of complementarities between current investments and existing vintages of human capital. In contrast in our paper, there are no such complementarities and diffusion arises as a result of trade frictions. Finally, Jones and Newman develop a search and matching model of technological diffusion. Here, new technologies slowly enter the market as agents

[5] Laing, Palivos, and Wang (1995a) study the link between labor market frictions and output growth, in an environment with endogenous human capital accumulation. For a survey of the literature on growth and market frictions, see Laing, Palivos, and Wang (1995b).

gradually learn about them through search and experimentation. In contrast, in our framework agents know precisely what the cutting edge is. However, search frictions can allow several products to be produced and traded simultaneously.

Section 2 describes the theoretical model. We consider the production and sale of a single quality-differentiated product. This market admits two alternative interpretations. According to one view, the good is a quality-differentiated intermediate input manufactured by upstream producers and used by downstream firms. In another interpretation, the market is simply a final goods' market populated by consumers ("buyers") and producers ("sellers"). Although these two interpretations lead to formally identical descriptions of the tastes and technologies of the agents in the model, they suggest two quite distinct approaches to modelling search frictions, which we refer to as the "random matching" and "directed search" variants of the model.[6] The key difference between them is that in the former case buyers are uninformed *ex ante* about either the quality or prices at given sellers, while with directed search this information is available to them. With random matching, buyers are conceived of, heuristically, as gradually acquiring information on product quality and the terms of trade are agreed *ex post* after search has taken place. In contrast, with directed search sellers set prices strategically to attract customers. However, buyers' search decisions are uncoordinated, so their demands may be frustrated if too many of them visit a given seller during the period.[7] Section 3 examines the partial-equilibrium properties of the model, under both search schemes, assuming: (*i*) only two varieties of the innovation good are traded at any given point in time (*ii*) the number of producers is given and (*iii*) the level of R&D expenditures is pre-determined. Section 4 derives, through a process akin to "backwards induction," the general-equilibrium properties of the model. More precisely, this Section endogenizes the entry decisions of consumers as well as the R&D activities of firms. Furthermore, we circumscribe conditions under which two and only two products are traded at a time. Section 5 discusses the welfare properties of the model, while Section 6 offers some conclusions.

2 The model

Time is continuous and is indexed by $t \in \mathbb{R}_+$. All agents are risk neutral and discount the future at the common rate, $r > 0$. There are two theaters of eco-

[6] The random-matching approach follows the spirit of Diamond (1982a, b), Mortensen (1982), and Pissarides (1985), whereas the directed-search approach relies on the competitive search framework developed by Peters (1991), Moen (1997), Burdett, Shi, and Wright (2000), and Shi (2000).

[7] For concreteness, we interpret the random-matching environment as best suited to the intermediate goods' market view of the model. The reason is that, in practice, final producers are often forced to search among potential upstream suppliers for a firm that is both available and capable of producing, for example, a highly specialized intermediate input. The specialized - possibly one off - nature of such transactions obviously preclude effective price positing, implying that the terms of trade must be agreed ex post. In contrast, the directed search approach is arguably best suited for modelling search frictions in final consumer goods' markets, as consumers may know the prices, location, and quality of each seller *ex ante.*

nomic activity: (*i*) a goods' market and (*ii*) an R&D sector. In the goods' market a quality-differentiated indivisible product is produced and exchanged, while innovations in the R&D sector lead to the drafting of "blue-prints" that improve product quality. At each point in time, t, the state of the economy is described by $n \in \mathbb{N}$, denoting the number of existing blue-prints. The quality of the j^{th} innovation is,

$$\tilde{q}(j) = q_0\gamma^j, \tag{1}$$

where $j = 1, 2, ..., n$, and $\gamma > 1$ is a (constant) relative increment in product quality. In what follows, it is convenient to index the time interval commencing with innovation n and terminating with innovation $n + 1$ by "n." The basic structure of the model is depicted in Figure 1.

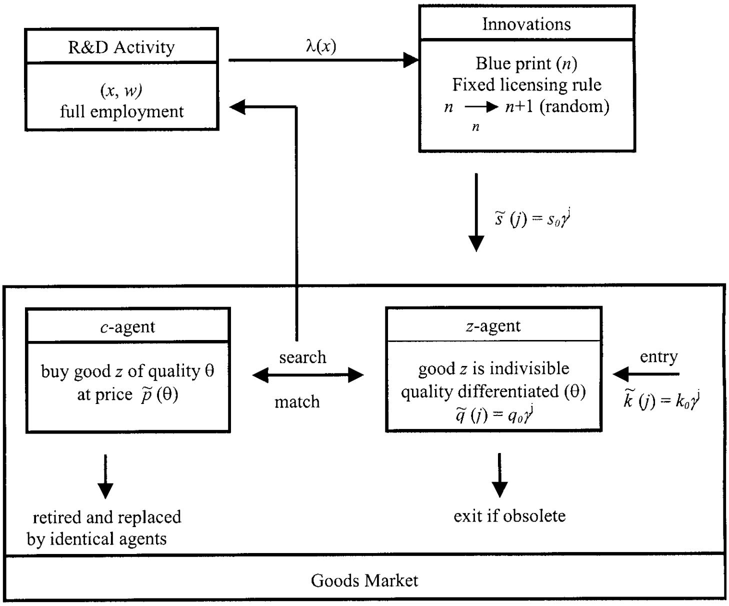

Figure 1. The structure of the economy

2.1 The environment

The goods' market is populated by consumers ("buyers"), indexed c, and producers ("sellers"), indexed z, with respective population masses C and Z. Gains from trade arise because of differences in the valuations these agents place upon consuming the good. More specifically, the consumption of a good with quality $\tilde{q}$ provides zero benefit to producers and, through a suitable choice of units, utility $\tilde{q}$ to consumers. As in Diamond (1982b), this formulation is intended to capture the benefits of specialization and trade over self-sufficiency.

Immediately after consumption, c-agents are assumed to retire from the market and are replaced by identical agents. The former of these features is the simplest way of modelling the presence of life-time wealth constraints: the decision to purchase and consume a good in the present limits future consumption possibilities, while the latter ensures constant populations in steady-state.[8]

The production technology is simple. In order to produce a given product quality, $j \in \{1, 2, ..., n\}$ a z-producer must: (*i*) own a unit of capital specific to that product, and (*ii*) reach agreement with the product's patent holder (see below). Thus, our model is one of vintage capital, in which z-producers become "locked-in" (through their choice of technology) to manufacture a specific product. Marginal production costs are normalized to zero. Furthermore, each producer is capable of producing at most one unit of output at a time. We assume both the free entry and exit of producers. Consequently, producers can instantly enter the product market and produce any good $j \in \{1, 2, ..., n\}$ once they have procured a unit of capital and reached agreement with the patent holder. The costs of the unit of capital necessary to produce good z evolve according to: $\widetilde{k}(j) = k_0\gamma^j$. The justification for the increase in $\tilde{k}$ with j is that more advanced products require correspondingly more sophisticated technologies for their manufacture, while the particular form ensures the existence of a balanced growth equilibrium or, equivalently, the existence of a stationary equilibrium in the transformed model analyzed below. There is free disposal of capital, so that producers can instantly dispose of their capital and exit the market if it should prove optimal to do so.

At each point in time producers (may) occupy different rungs of the quality ladder. In what follows let $N \in \mathbb{N}$ denote the length of the product ladder (i.e., the number of different z-goods *actively* offered for sale) and let $\theta = 1, 2, \ldots, N$ denote a particular rung of the ladder.[9] We adopt the convention that $\theta = 1$ (the top of the ladder) corresponds to the most recent innovation. The total mass of producers *active* in the market is then, $Z = \sum_\theta Z(\theta)$, where $Z(\theta)$ is the mass of producers at rung θ. In what follows we denote the fraction of type θ producers in the market by $\pi(\theta) \equiv Z(\theta)/Z$.

2.2 Search and matching

Although up to this point we have, for concreteness, interpreted the goods' market as a product market populated by consumers and firms, an equally valid interpretation is that of an intermediate goods' market. In this context z-agents are

[8] This formulation implies that the *birth rate* of c-producers endogenously adjusts to ensure a constant mass of traders C. This assumption simplifies the analysis somewhat. We could equally well treat the birth rate as *exogenous* and let the population C adjust in steady-state. In this case, the flow matches of c-producers would equal their birth rate and C would be inversely related to the endogenous contact rate.

[9] The personal computer market provides a helpful illustrative example of such a ladder. The number of innovations n' is exceedingly large (ranging, at a certain point of time, from the original IBM machine to ones currently utilizing a Pentium *PIII* processor). Yet, as a practical matter, in the mid 90's only three classes of product were traded ($N = 3$): the 386 ($\theta = 3$), 486 ($\theta = 2$), and $P5$ ($\theta = 1$). The subsequent introduction of the *PII* processor led to the obsolescence of the 386. This raised n from n' to $n' + 1$. The number of traded goods, N, remained 3. The *PII* processor then corresponds to $\theta = 1$; the original $P5$ to $\theta = 2$ etc.

upstream producers of a quality-differentiated intermediate input and c-agents are downstream producers, who accrue incremental revenues $\tilde{q}$ from using an input of quality $\tilde{q}$. Although, from a formal point of view, these two interpretations are equivalent from the perspective of describing the tastes and (production) technologies of the agents in our model, they suggest two quite distinct ways of modelling the processes underlying trade, which we refer to as *random matching* and *directed search*.[10] Before formally describing each of these in turn, we first present notation common to both. Let $\mu(\theta)$ denote the flow probability with which a c-agent locates a good at position θ on the product ladder and let $\eta(\theta)$ denote the corresponding flow probability with which a z-agent finds a customer. Finally, $m \equiv C/Z$ is used to parameterize the "tightness" of the product market from the perspective of z-producers. A "tight" market is consequently one in which m takes on a low value.

2.2.1 Random matching

In the prototypical random matching framework of Diamond (1982a, b), Mortensen (1982), Pissarides (1985): (*i*) the flow contact rate between traders is determined according to an exogenously specified matching technology $m_0 M(C, Z)$, which depends upon the mass of each type of trader in the market and (*ii*) the terms of trade are determined *after* search and matching takes place. In this paper we assume that, bargaining is instantaneous and that it is conducted according to a symmetric Nash bargaining rule. Furthermore, the matching technology is assumed to possess the following properties.

Assumption 1. *(Random matching technology). The matching technology, M : $\mathbb{R}^2_+ \to \mathbb{R}_+$, (i) is strictly increasing and strictly concave (ii) is homogeneous of degree one and (iii) satisfies the Inada conditions* $\lim_{Z\to 0} M_Z(C, Z) = \infty$, $\lim_{Z\to\infty} M_Z(C, Z) = 0$ *and the boundary condition* $M(C, 0) = 0$.

An increase in the number of participants on either side of the market raises the instantaneous matching rate (but at a diminishing rate). The constant-returns-to-scale assumption is made for convenience. The boundary condition on M ensures an interior steady-state solution. Finally, the severity of market frictions is parameterized by m_0; a higher value of m_0 raises the instantaneous matching rate for any given population levels C and Z, and so corresponds to a lowering of search frictions.

[10] Canonical examples of the random matching and bargaining framework are due to, Diamond (1982a, b), Mortensen (1982), and Pissarides (1985). At present, there is no fixed nomenclature for what we have termed directed search. Butters (1977) describes the environment as one of "search and advertising," while Moen (1997) names it "competitive search;" Peters (1991) views it as "ex ante price offers," while Shi (2000) terms it "price posting." However, the key seems to be, not so much that prices are posted (on this see Coles and Eckhout, 1999), but rather that buyers know *something* of the whereabouts of sellers and that search decisions are uncoordinated.

2.2.2 Directed search

Following the seminal work of Butters (1977), we conceive of *directed search* as a trading environment in which *ex ante* (prior to search): (i) sellers (z*-agents*) announce their price, $\widetilde{p}(\theta)$ *and* the probability of attracting a given customer, $\eta(\theta)$, (ii) buyers (c-agents) know both the locations and the type of product sold by each z-producer (iii) capacity limitations imply sellers may be unable to service demand (iv) buyers' search decisions are uncoordinated, with each independently chooses the probability, $\phi(\theta)$, of visiting producers at a given point on the product ladder, $\theta = 1, \ldots, N$. Two features of this approach are particularly noteworthy. The first concerns the ability of c-agents to direct their search towards finding specific qualities' through their choice of $\phi(\theta)$. The second is the *ex ante* strategic use of prices by sellers (z-producers) to influence the number of buyer visits during a given period.

2.2.3 Two interpretations: Intermediate and final goods' markets

In practice, in intermediate goods' markets, downstream producers must often gather information from several potential upstream suppliers in order to find one that is capable of producing, for example, a highly specialized input that meets certain precise specifications. In this case, the search process is perhaps best conceived of as one of random matching, in the sense that the final producer is uninformed *ex ante* about which, if any, upstream firm is available and capable of producing a satisfactory input. Furthermore, the specialized - possibly one off - nature of the transaction, often precludes effective price posting, forcing the transacting parties to agree to the terms of trade *ex post* (i.e., after search has taken place). In contrast for many final goods, matching is through directed search, since consumers may know both the location of sellers as well as the prices offered by them *ex ante*. In this case, the search friction is the time taken to find a seller currently holding a given product in stock. In this setting sellers set prices strategically, recognizing that the price they post may influence the number of consumers that visit them during a given period. Furthermore, consumers are aware that a firm offering a relatively "low" price, may also offer a correspondingly low probability of service.[11]

2.3 Research and development

We assume as in, for example, Aghion and Howitt (1992) that research and development takes place within a large number of specialist firms. The probability that a firm successfully innovates in a small epoch of time, $h > 0$, is $\lambda(x, R)h$ where R denotes a fixed level of research capital, which is hereafter normalized

[11] The classification offered here is intended only to motivate our approach. Clearly, there are intermediate goods whose purchase is perfunctory and final consumer goods which are procured only after considerable search.

to 1, and x is the level of R&D expenditure. We assume that the function λ is homogenous of degree one, strictly increasing and strictly concave in x and that it satisfies the Inada conditions, as well as the boundary condition $\lambda(0, 1) = 0$. With this formulation greater R&D expenditure, x, increases the likelihood of a discovery.

The R&D sector is characterized by free entry. However, once a discovery is made the innovator obtains exclusive rights over the idea (patent laws are assumed to be well defined and to be perfectly enforced). An innovator with the rights to a blue-print n licenses it to new entrants into the product market. The license entitles the holder of the patent (i.e., the research firm) to an exogenous royalty payment, on each unit of output sold, of: $\widetilde{s}(j) = s_0\gamma^j$ for each $j = 1, ..., n$.[12] Observe that, even though s_0 is exogenous the returns to R&D activity are*endogenous* and depend upon the volume of trade in the product market. Throughout, we impose the condition: $q_0 > s_0$, ensuring positive gains from production and trade.

2.4 Asset values

We are now ready to define the asset values of the participants in the good's market. At any point in time, c- and z-producers are either matched (M) and have a production/consumption opportunity or are unmatched (U) and searching for one. Each z-producer is located on some rung $\theta \in \{1, 2, \ldots, N\}$ of the quality ladder. We now proceed to define the "asset values" associated with occupying these states. Let $\widetilde{J}(n)_U$ denote the value to a producer of good c in the "unmatched state" and let $\widetilde{J}(\theta, n)_M$ denote the asset value of matching with a producer of good z who is on rung θ of the quality ladder. Finally let, $\widetilde{K}(\theta, n)_U$ and $\widetilde{K}(\theta, n)_M$ denote the corresponding asset values of type z producers (on rung θ of the ladder) in the matched and unmatched states respectively. Define $\rho \equiv r - \lambda(\gamma - 1)$ as the effective discount rate. In order to ensure that individual optimization problems are bounded and that there is non-degenerate growth we impose:

Assumption 2. *(Positive growth and bounded optimization)*

$$\lambda(2 - \gamma) > \rho > 0.$$

The first part of the inequality guarantees that the arrival of innovations is frequent enough for R&D to be undertaken; the second part ensures that the costs of delaying current consumption r exceed the expected net benefits of waiting for

[12] In Laing, Palivos, and Wang (2000) we endogenize the royalty payment by allowing z-producers and the patent holder to bargain over the payment before an agreement is reached. As shown, all of our main findings remain qualitatively unchanged. An alternative method would be to have the patent holder choose the size of z-producers (franchisees) and the royalty payment, by maximizing his/her expected royalty income subject to z-producers' reservation values. This approach, however, departs substantively from the current set-up and is therefore beyond the scope of the present paper.

the next innovation $\lambda(\gamma - 1)$. Assumption 2, then, requires $\gamma \in (1, 2)$. Dynamic programming arguments can be used to show that

$$r\widetilde{J}(n)_U = \mu(\theta)\left[\sum_{\theta=1}^{N} \pi(\theta)\widetilde{J}(\theta,n)_M - \widetilde{J}(n)_U\right] + \lambda\left[\widetilde{J}(n+1)_U - \widetilde{J}(n)_U\right], \quad (2)$$

$$\widetilde{J}(\theta,n)_M = q_0\gamma^{n+1-\theta} - \widetilde{p}(\theta,n), \quad (3)$$

$$\widetilde{K}(\theta,n)_M = \widetilde{p}(\theta,n) + \widetilde{K}(\theta,n)_U - s_0\gamma^{n+1-\theta}, \quad (4)$$

$$r\widetilde{K}(\theta,n)_U = \eta(\theta)\left[\widetilde{p}(\theta,n) - s_0\gamma^{n+1-\theta}\right] + \lambda\left[\widetilde{K}(\theta+1,n+1)_U - \widetilde{K}(\theta,n)_U\right]. \quad (5)$$

If $\theta = N$, then (by our free disposal assumption) $\widetilde{K}(N+1, n+1)_U = 0$. Expressions such as these have, by now, a familiar interpretation. For instance, consider $\widetilde{J}(n)_U$. The term $r\widetilde{J}(n)_U$ is the flow value to an unmatched producer of good c (given a total of n extant innovations): it equals the flow probability of becoming matched with a z-producer (μ) multiplied by the expected capital gain from this event [the first term on the right-hand-side of (2)] plus, in the event the consumer does not match, the flow probability of a new innovation (λ) multiplied by the capital gain from this change of state [the second term on the right-hand-side of (2)]. The other asset value equations (3)-(5) possess similar routine interpretations.

In what follows, when characterizing the properties of the model, it is convenient to express everything in terms of "effective units" by dividing by the common growth factor γ^n. Therefore, let $J_U \equiv \widetilde{J}(\theta)_U/\gamma^n$, $J(\theta)_M \equiv \widetilde{J}(\theta,n)_M/\gamma^n$, $p(\theta) = \widetilde{p}(\theta,n)/\gamma^n$, etc. This transforms the system into a stationary one and is valid due to the assumptions that payoffs are linear and that both the licensing, $\tilde{s}$, and capital, $\tilde{k}$, costs grow with n.

3 Decentralized trade

In this Section, we study pricing and product diffusion, in both the random matching and directed search environments described above. Throughout the Section, we assume that: (i) the rate of $R\&D$, λ, is given (ii) there are only two (adjacent) goods on the product ladder, $N = 2$ and (iii) at each point in time there are given and equal populations of z-producers on each rung of the product ladder (Together (ii) and (iii) obviously imply that, $\pi(1) = \pi(2) = \pi$). In Section 4, we analyze the general-equilibrium properties of the model, at which point both R&D activity, x, and z-producers entry decisions are endogenized. Section 3.1 considers random matching, while Section 3.2 examines directed search. Comparisons between the results of the two models are made at the end of this Section.

3.1 Random matching

We now determine the trading price $p(\theta, n)$, which is the payment – in effective units γ^n – for a unit of the indivisible good z of quality $q(n)$. A match between a c- and a z-agent may lead to a positive surplus, which can be divided between the two parties. We assume that negotiations are instantaneous and that the surplus is divided in accordance with a symmetric Nash bargaining rule. Thus, the surplus accruing to trade is equally divided between the bargaining units. (Of course, even under an equal division rule, the returns to trade are endogenous and depend upon both the matching parameters as well as the rate of R&D.) This gives:

$$J(\theta, n)_M - J(n)_U = K(\theta, n)_M - K(\theta, n)_U \geq 0. \tag{6}$$

Proposition 1a summarizes the properties of the price agreed in the Nash bargain.

Proposition 1*a*. *(The pricing schedule $p(\theta)$ and its properties).*
If Assumption 2 holds and $N = 2$, then, for arbitrary values of $(\mu, \eta, \pi, \lambda)$,
(i) the price of good z of quality $\theta = 1, 2$ is

$$p(\theta) = (2\gamma)^{-1} \left[q_0(\gamma^{2-\theta} - P_0) + s_0(\gamma^{2-\theta} + P_0)\right], \quad P_0 \equiv \frac{\mu[1 + (\gamma - 1)\pi(1)]}{\mu + 2\rho}. \tag{7}$$

(ii) The price differential between the two goods, Δp, is

$$\Delta p = p(1) - p(2) = [s_0 + q_0](\gamma - 1)/2\gamma > 0, \tag{8}$$

and satisfies: $\partial(\Delta p)/\partial\gamma > 0$.
(iii) The steady-state pricing function possesses the following properties:

$$\partial p(1)/\partial\lambda = \partial p(2)/\partial\lambda < 0, \quad \partial p(1)/\partial\mu = \partial p(2)/\partial\mu < 0,$$
$$\partial p(1)/\partial\gamma > 0 > \partial p(2)/\partial\gamma,$$

where $\partial p(1)/\partial\gamma > 0$ iff $2/\lambda > 4[(\pi(1)/\pi(2))\gamma^2 + 2\gamma - 1]/(\mu + 2r)$.

Proof. All proofs are presented in Laing, Palivos, and Wang (2000).

Parts (i) and (ii) of the Proposition indicate that the price of the good at the "top" of the quality ladder exceeds the price of goods further down the ladder (it is easy to show that this holds for arbitrary $N \geq 2$). In essence, the further a good is up the quality ladder the greater the size of the "cake" to be divided between the two parties and the trading price rises in recognition of this fact. An increase in γ raises the relative value of the good at the top of the ladder, which magnifies the price differential. An increase in the innovation rate λ weakens the bargaining position of all z-producers. This is because higher values of λ increase the flow probability of a new innovation, which will either make the product obsolete or else "bump" it a rung down the product ladder. Either of these possibilities is sufficient to lower the relative price of good z. An increase in μ, the contact rate of c with z, enhances c-agents' bargaining position by making alternative/better options more accessible, which lowers the price of all goods on the ladder. Finally, an increase in γ has two distinct effects: (*i*) it stiffens intertemporal competition (c-agents have an incentive to wait for the next innovation) and (*ii*) it makes good 1 relatively more attractive, by increasing the distance between the rungs on the product ladder. Both effects serve to lower the price $p(2)$. There is a potential ambiguity concerning the effect on the price of good 1, since, although effect (*i*) tends to lower $p(1)$, the increased distance between the rungs of the ladder tends to raise it. However, if the length of the product cycle (which with two goods equals, $l \equiv 2/\lambda$) is long enough the value of waiting is small, so that effect (*ii*) dominates and $p(1)$ increases with γ.

In Section 4 below, we characterize the general equilibrium properties of the model. We show that search frictions may allow less than state-of-the-art z-producers to remain in the market. Thus, at least for certain parameters, the equilibrium is characterized by *product diffusion* in which several varieties of the quality-differentiated product are actively at a given point. Rather than focusing simply on the shape of this diffusion curve, our paper is unique in that it characterizes the *pricing* structure of goods at various rungs on the quality ladder as well.

3.2 Directed search

One unsatisfactory feature of the random matching environment considered above is that prices can play no allocative role, since they are determined *ex post* after search is undertaken. Accordingly, in this Section, we examine directed search. This not only broadens the scope of the applicability of our approach, but also provides a check of its robustness.[13] The basic theme of this approach is that: (*i*) buyers know the locations of alternative sellers (*ii*) search among buyers is uncoordinated (*iii*) sellers post prices at the beginning of each period and (*iv*) (some) sellers are unable to satisfy the demands of an arbitrary number of consumers. The approach is intuitive and appealing, as sellers may use prices to influence the number of customers visiting them and buyers are aware that their demands may be frustrated for the period, if too many buyers arrive (relative to the firm's capacity). The model of competitive search we develop is closest in spirit to that of Burdett, Shi, and Wright (2000) and Shi (2000), since each involves heterogeneity on (at least) one side of the market.[14] However, an important distinction is that our framework is explicitly dynamic, involving on-going R&D, product innovation, and diffusion.

We first discretize time in intervals of length $h > 0$. During each of these periods, each c-agent can visit at most one z-producer and each z-producer

[13] Several recent papers, building upon Butter's (1977) seminal contribution, construct and explore the properties of such models (Peters, 1991; Moen, 1997; Burdett, Shi, and Wright, 2000; Cao and Shi, 2000; Lagos, 2000; Shi, 2000).

[14] In Burdett et al. firms differ in their production capacities, whereas in Shi firms use heterogenous technologies and workers differ in their skills.

can produce at most one unit of output. The discount rate is: $(1 - rh)$ and the populations of c- and z-type agents are: $C \in \mathbb{N}$ and $Z(\theta) \in \mathbb{N}$ respectively, where $Z = Z(1) + Z(2)$. We assume from the outset that C and Z are large enough that each agent ignores any aggregate effects of its actions. (Indeed, we consider the limits $C \to \infty$ and $Z \to \infty$, where each agent has precisely zero effect in the aggregate). During the course of a given period, h, the probability of an innovation is λh. As before, the latest innovation increases maximum product quality from $q_0\gamma^n$ to $q_0\gamma^{n+1}$ at the beginning of the next period.

Under directed search, c-agents know the locations and quality offered by each z-producer. At the beginning of each period z-producers set $(p(\theta), \eta(\theta))$, $\theta = 1, 2$, where $p(\theta)$ is the product price and $\eta(\theta)$ is the probability of attracting a given customer (i.e., a given c-agent).[15] It follows that the probability of *failing* to attract one of the c-customers is $(1 - \eta(\theta))^C$. From this, the probability of attracting at least one customer is

$$H(\eta(\theta)) = 1 - [1 - \eta(\theta)]^C. \tag{9}$$

In the event that more than one customer arrives at a z-firm, each faces an equal probability of service. Thus, from a customer's perspective the probability of service is

$$\mu(\eta(\theta)) = H(\eta(\theta))/[\eta(\theta)C]. \tag{10}$$

where $\eta(\theta)C$ is the expected queue length. We consider a *competitive* search environment, in which the market provides a reservation life-time utility U_0 to each c-agent. The value of U_0 is determined in equilibrium, but treated as parametric by each decision maker. Each period, z-producers' expected discounted profits are given by $\Pi(\theta) = H(\eta(\theta))[p(\theta) - s(\theta)]$, where $p(\theta) - s(\theta)$ equals the net returns from the sale (i.e., the price minus royalty payment in effective units) and $H(.)$ is the probability the firm attracts at least one customer. To facilitate comparability with the previous sub-Section, it is convenient to deal with the continuous-time Bellman equations, which we define as the limit when the interval between successive time periods approaches zero,[16]

$$K(1)_U = \frac{\Pi(1)}{r + \lambda} + \frac{\lambda\gamma\Pi(2)}{(r + \lambda)^2}, \tag{11}$$

$$K(2)_U = \frac{\Pi(2)}{r + \lambda}. \tag{12}$$

For customers the value of visiting a firm that sets $\eta(\theta)$ and posts the price, $p(\theta)$, is

$$U(\theta) = \frac{\mu(\eta(\theta))}{\rho + \mu(\eta(\theta))}[q(\theta) - p(\theta)], \tag{13}$$

[15] More naturally, firms pick the probability of *serving* a given customer. As we shall see below, this uniquely determines the probability of attracting given customers, so that choosing the latter amounts to picking the former.

[16] For the purposes a compact presentation, we write down directly the asset values in effective units (i.e., divided by γ^n), rather than going through their derivation as was done previously. Consequently, in effective units, $s(1) = \gamma s_0$, $s(2) = s_0$, $q(1) = \gamma q_0$, and $q(2) = q_0$.

where, from Assumption 2, $\rho > 0$. During each period, z-producers choose the pair $(p(\theta), \eta(\theta))$ to maximize their expected discounted profits, $\Pi(\theta)$, subject to offering consumers at least their reservation utility U_0.[17] Thus, firms solve

$$\max_{(p(\theta),\ \eta(\theta))} \quad \Pi(\theta) = H(\eta(\theta))[p(\theta) - s(\theta)], \tag{14}$$

subject to

$$U(\theta) = \frac{\mu(\eta(\theta))}{\rho + \mu(\eta(\theta))}[q(\theta) - p(\theta)] \geq U_0. \tag{15}$$

With (15) holding as equality, we can utilize (10) to obtain

$$p(\theta) - s(\theta) = (q(\theta) - s(\theta)) - U_0 \left(1 + \frac{\rho C \eta(\theta)}{H(\eta(\theta))}\right). \tag{16}$$

From (9) and (16), we have $\Pi(\theta) = [q(\theta) - s(\theta) - U_0]H(\eta(\theta)) - (\rho U_0 C)\eta(\theta)$. The first-order condition from maximizing $\Pi(\theta)$ with respect to $\eta(\theta)$ therefore gives $[q(\theta) - s(\theta) - U_0][1 - \eta(\theta)]^{C-1} - \rho U_0 = 0$, which yields,

$$U_0 = \frac{q(\theta) - s(\theta)}{1 + \rho[1 - \eta(\theta)]^{1-C}}, \qquad \text{for } \theta = 1, 2. \tag{17}$$

Since all z-producers in position θ on the product ladder are symmetric, they pick the same value of $\eta(\theta)$. Recall that $\phi(\theta)$ is the probability with which a given c-agent chooses to shop for a product at position θ on the product ladder. It follows that the probability with which a given c-agent picks a given z-producer is simply, $\phi(\theta)/[\pi(\theta)Z]$. Yet, from our earlier definition this *is* precisely, $\eta(\theta)$, so that

$$\eta(\theta) = \phi(\theta)/[\pi(\theta)Z]. \tag{18}$$

Note that, since $\eta(\theta)$ may differ across goods on the product ladder, our model of quality differentiated products captures the (potentially complex) relationships between prices, quality, and availability.

We are not interested in the properties of this finite economy *per se*. Rather, it is our objective to study a large economy, wherein agents (legitimately) treat U_0 as exogenous. To this end, recall that $C = mZ$. If m is treated as a positive constant, we can substitute (18) into (17) and take the limit $Z \to \infty$ to derive

$$U_0 = \frac{q(\theta) - s(\theta)}{1 + \rho \exp[\Phi(\theta)]}, \quad \text{for } \theta = 1, 2, \tag{19}$$

where $\Phi(\theta) \equiv \phi(\theta)m/\pi(\theta)$ and "low" values of $\Phi(.)$ indicate a "tight" market, from the perspective of $Z(\theta)$ producers. To see that this is so, rewrite $\Phi(\theta) = \frac{\phi(\theta)C/Z}{Z(\theta)/Z}$, in which case the numerator is the relative (to Z) numbers of customers visiting intermediate-good market θ and the denominator is the relative (to Z) number of θ intermediate-good producers.

[17] Note that maximizing Π_2 automatically implies the maximization of K_2. Then, given the maximized value of Π_2 in K_1, the producer maximizes K_1 by maximizing Π_1 (which is the dynamically consistent solution).

In general equilibrium, the ratio m of buyers to sellers adjusts to ensure that the *ex ante* zero profit condition $K(1)_U = k_0$ is satisfied. Yet, many of the most interesting features of the directed search variant of our model can be most simply understood by considering its properties for fixed m. Accordingly, we analyze the properties of prices, $p(\theta)$, and c-agents' search strategies $\phi(\theta)$, treating m as exogenous.[18]

Since c-agents know the locations and type of each z-producers, they adjust $\phi(\theta)$ according to the relative prices of the two goods, their relative qualities, as well as the probability of service in each market. Applying (19) for both $\theta = 1$ and $\theta = 2$ to eliminate U_0 yields

$$\rho\{\exp[\Phi(1)] - \gamma \exp[\Phi(2)]\} = \gamma - 1 > 0, \tag{20}$$

where $\Phi(1) = m\phi(1)/\pi(1)$ and $\Phi(2) = m\phi(2)/\pi(2)$. Since there are only two goods on the quality ladder it is convenient to define $\phi \equiv \phi(1)$ and $1 - \phi \equiv \phi(2)$. It is straightforward from (20) to verify that:

Lemma 1. *(Behavior of c-agents).* Given, $\pi(\theta) = \pi$, each c-agent's choice of ϕ satisfies:

(i) $\phi > \pi > (1 - \phi)$*; (ii)* $\partial\phi/\partial\lambda > 0$, $\partial\phi/\partial m < 0$, $\partial(\phi m)/\partial m > 0$*, and* $\partial[(1 - \phi)m]/\partial m < 0$.

Thus, given equal populations of each $Z(\theta)$ producer type (i.e., $\pi(\theta) = \pi$), consumers direct their search toward the goods at the top of the product ladder (i.e., $\phi > \pi$). For *given m*, a higher innovation rate, λ, encourages consumers to search for the newest products. The reason for this is that a greater value of λ not only makes goods further down the ladder (relatively) less attractive, but also forces the producers of the newest goods to offer more attractive terms of trade as they, in essence, face stiffer *intertemporal competition* from (as yet) undeveloped higher quality products. An increase in m raises, for given ϕ, the demand at each $Z(\theta)$-producer (the "scale" effect). However, with consumers *already* concentrating their search efforts at the highest quality producers, the increase in m - via a congestion externality - reduces the efficacy of search for these products. At the margin, households adjust by substituting toward good $\theta = 2$, so that $\phi(2) = 1 - \phi$ rises. Yet, as indicated by Lemma 1, the scale effect dominates so that $\partial(\phi m)/\partial m > 0$.

From equations (16) and (19), we have, as $Z \to \infty$,

$$p(\theta) = q(\theta) - [q(\theta) - s(\theta)]\Gamma(\Phi(\theta)), \tag{21}$$

where $\Gamma(\Phi(\theta)) \equiv \left[\frac{1-\exp[-\Phi(\theta)]+\rho\Phi(\theta)}{\{1-\exp[-\Phi(\theta)]\}\{1+\rho\exp[\Phi(\theta)]\}}\right] \in [0, 1]$. A simple application of L'Hospital's rule shows that $\lim_{\Phi(\theta)\to 0} \Gamma(\Phi(\theta)) = 1$ and that $\lim_{\Phi(\theta)\to\infty} \Gamma(\Phi(\theta)) = 0$. This, in conjunction with Lemma 1, may be used to prove:

[18] This approach enables us to determine $K(\theta)_U$ as a function of m. The general-equilibrium analysis endogenizes m, which is determined by the free-entry condition, $K(1)_U = k_0$. Once m is determined, we evaluate the set of parameters for which only two products, $N = 2$, appear on the product ladder.

Proposition 1*b*. (*Pricing schedule* $p(\theta)$ *and its properties).*

(i) (*Limiting behavior*) $\lim_{\Phi(\theta)\to 0} p(\theta) = s(\theta)$ *and* $\lim_{\Phi(\theta)\to\infty} p(\theta) = q(\theta)$*;*
(ii) (*Characterization of prices*) $\partial p(\theta)/\partial\Phi(\theta) > 0$, $\partial p(\theta)/\partial q(\theta) > 0$, *and* $\partial p(\theta)/\partial s(\theta) > 0$*;*
(iii) (*Top of the product ladder,* $p(1)$) $\partial p(1)/\partial\phi > 0$, $\partial p(1)/\partial m > 0$, *and* $\partial p(1)/\partial\lambda > 0$*;*
(iv) (*Bottom of the product ladder,* $p(2)$) $\partial p(2)/\partial\phi < 0$, $\partial p(2)/\partial m > 0$, *and* $\partial p(2)/\partial\lambda < 0$*;*
(v) (*Price differential*) $q_0(\gamma - 1) > \Delta p > 0$ *and* $\partial(\Delta p)/\partial\lambda > 0$.

It is helpful to write, (21) as:

$$q(\theta) - p(\theta) = [q(\theta) - s(\theta)]\Gamma(\Phi(\theta)), \tag{22}$$

The left-hand-side of (22) is the payoff accruing to c-agents from trade and consumption, while the term, $q(\theta) - s(\theta)$ represents the total available (static) gains from trade. In this light, $\Gamma(.)$ is simply the fraction of the surplus accruing to c-agents. As $\Phi(\theta) \to \infty$, the market becomes very "tight" from the perspective of c-agents, implying $p(\theta) \to q(\theta)$. Consequently, under conditions of such extreme competition, c-agents' (consumers') surplus is driven to zero. Similar, but opposite, considerations apply as $\Phi(\theta) \to 0$. The derivatives in (*ii*) possess routine interpretations. In (*iii*) note that an increase in $\phi = \phi(\theta)$ automatically raises $\Phi(\theta)$ and lowers $\Phi(\theta)$ (the first result in (*iii*) and (*iv*) then follows from part (*ii*) of the Lemma). An increase in m raises the demand for each type of product, so that each price rises. Finally, an increase in the innovation rate λ raises $p(1)$ and lowers $p(2)$. The reason is that as λ rises, the effective discount rate ρ falls, since waiting offers the chance of consuming better products in the future. This affects goods at the top and bottom of the quality ladder asymmetrically. Heuristically, if consumers must 'wait' (i.e., must search) they might as well do so while looking for the highest-quality products. The resultant re-allocation of c-agents' search efforts towards the top of the quality ladder, $\theta = 1$, raises $p(1)$ and lowers $p(2)$. Finally, part (*v*) of the Proposition examines the determinants of the *difference* between the prices of goods at the top and the bottom of the ladder. Naturally, the most recent products command the highest price. The price differential is also bounded above by $q_0(\gamma - 1)$ and depends positively on the innovation rate λ. The former of these results says that producers at the top of the product ladder fail to capture *all* of the incremental consumer surplus, $q_0(\gamma - 1)$, provided by their product. This stems directly from the existence of search frictions and competition from lower-quality producers. The latter result indicates that there is a direct link between the pace of R&D and the price differential. This feature is absent from the random matching variant of the model considered earlier. It arises here, because c-agents are informed of the locations of the newest products and can re-direct their search efforts, by adjusting ϕ, in light of any changes in the pace of R&D.

Given m, the expected profits from potential matches between type z-producers and c-agents are $\Pi(\theta) = [p(\theta) - s(\theta)]\{1 - \exp[-\Phi(\theta)]\}$, where

$1 - \exp[-\Phi(\theta)] = \lim_{Z\to\infty} H(\eta(\theta))$ is the probability that at least one customer arrives during the course of the period. Using equation (21) to substitute for $p(\theta)$ yields,

$$\Pi(\theta) = (q(\theta) - s(\theta))\Upsilon(\Phi(\theta)), \quad (23)$$

where $\Upsilon(\Phi(\theta)) \equiv [1 - \Gamma(\Phi(\theta))]\{1 - \exp[-\Phi(\theta)]\} \in [0,1]$ and $\Upsilon(0) = 0$, $\lim_{\Phi\to\infty}\Upsilon = 1$, $\partial\Upsilon/\partial\Phi(\theta) > 0$ and $\partial\Upsilon/\partial\rho > 0$.[19] The function $\Upsilon(\Phi(\theta))$ captures the fraction of the gains from trade, $q(\theta) - s(\theta)$, accruing to z-producers as a function of the measure of market tightness, $\Phi(\theta)$, and other exogenous parameters of the model. Under previous assumptions, $q_0 - s_0 > 0$, from which $\Pi(\theta) \geq 0$. Consider,

Corollary. *(Instantaneous profits). The instantaneous profits, $\Pi(\theta)$, accruing to producer θ satisfy:*

(i) *(Limiting behavior)* $\lim_{m\to 0}\Pi(\Phi(\theta)) = 0$ *and* $\lim_{m\to\infty}\Pi(\Phi(\theta)) = q(\theta) - s(\theta)$;

(ii) *(effect of R&D)* $\frac{d\Pi(\theta)}{d\lambda} = \left(\frac{\partial\Upsilon(\theta)}{\partial\rho}\right)\left(\frac{\partial\rho}{\partial\lambda}\right) + \left(\frac{\partial\Upsilon(\theta)}{\partial\Phi(\theta)}\right)\left(\frac{\partial\Phi(\theta)}{\partial\lambda}\right) \gtreqless 0$ for $\theta = 1$ and < 0 for $\theta = 2$.

These results are intuitive: as $m \to 0$, competition among z-producers for limited customers forces $p(\theta) \to s_0$, at which point profits approach zero. Similar, but opposite, considerations apply as $m \to \infty$. Consider next part (*ii*), which looks at the effects of R&D on profits. By definition, $\partial\rho/\partial\lambda = -(\gamma - 1) < 0$. In view of this, the first term in the expression captures the effects of *intertemporal* competition on profits. As λ increases, consumers become more "patient" and capture a greater fraction of the gains from trade, so that *ceteris paribus* $\Pi(\theta)$ falls. Yet, the increase in λ also induces a shift in the way consumers direct their search toward products at the top and the bottom of the ladder. Specifically, ϕ rises so c-consumers sample more often from goods at the top of the product ladder. The effect of this for good $\theta = 2$ is unambiguous: both the increased intertemporal competition and the *intratemporal* reduction in demand lower $\Pi(2)$. The picture is different for good $\theta = 1$ at the top of the ladder. Although producers face stiffer intertemporal competition, the demand for their product also rises. This makes it difficult to be precise about the effects of an increase in λ on $\Pi(1)$.

It is not difficult to see that the qualitative features of the comparative-static results and the general-equilibrium properties of the random matching and directed search variants of the model are very similar. Consequently, to avoid excessive duplication, we report only results pertaining to the former of these trading processes [the general-equilibrium analysis of the latter can be found in Laing, Palivos, and Wang (2000)].

4 Steady-state equilibrium

In this Section we establish the conditions for the existence of a steady-state equilibrium with: (*i*) free entry of z-producers (*ii*) endogenous R&D activity and

[19] After manipulation, $\Upsilon(\Phi(\theta)) = \frac{\rho\{\exp[\Phi(\theta)] - (1+\Phi(\theta))\}}{1+\rho\exp[\Phi(\theta)]}$.

(*iii*) two goods diffusing at a given point in time (i.e., $N = 2$). We demonstrate that the main findings of the partial-equilibrium analysis, considered previously, carry through in this more general setting. Our discussion proceeds according to a process akin to "backwards induction." Specifically, in Section 4.1 we treat the level of R&D expenditure, x, as exogenous (which pre-determines the arrival rate of innovations, $\lambda = \lambda(x)$) and examine the conditions under which there is a free-entry equilibrium with two and only two goods diffusing at a given point. Section 4.2 then characterizes optimal innovative activity, and Section 4.3 derives the steady-state equilibrium with endogenous R&D and free entry.

4.1 Steady-state equilibrium with exogenous R&D

Using equation (5), the asset values of unmatched producers of $Z(1)$ and $Z(2)$ (in effective units) are

$$K(2)_U = (\eta/(\lambda + r))(p(2) - s_0/\gamma), \tag{24}$$

$$K(1)_U = (\eta/(\lambda + r))(p(1) - s_0) + (\lambda\eta/(\lambda + r)^2)[\gamma p(2) - s_0]. \tag{25}$$

Using these we can define a steady-state equilibrium:

Definition 1. (*Steady-state random-matching equilibrium with exogenous* R&D). *A steady-state random-matching equilibrium with exogenous* R&D *expenditure and* $N = 2$*, is a tuple:*
$(Z^*, \mu^*, \eta^*, \pi^*(1), p^*(\theta))$ *such that:*

(i) *(Symmetric Nash bargain)* $J^*(\theta)_M - J^*_U = K^*(\theta)_M - K^*(\theta)_U \geq 0$*;*
(ii) *(Unrestricted entry)* $K^*(1)_U - k_0 = 0$*;*
(iii) *(Two goods on the quality ladder) (a)* $p^*(2) - s_0/\gamma > 0$ *and (b)* $J^*_U > q_0/\gamma^2$*;*
(iv) *(Steady state)* $\eta^* Z^* = \mu^* C = m_0 M(C, Z^*)$ *and* $\pi^*(1) = \pi^*(2) = 1/2$.

Part (*i*) of the definition ensures that the steady-state pricing schedule is consistent with the equal division rule. Part (*ii*) is the free-entry condition for z-producers: upon purchasing a unit of capital for k_0, any number of firms can (once they have agreed a license) enter the product market. If the expected value from entering the product market exceeds the costs from doing, $K(1)_U > k_0$, there is a large influx of new entrants which lowers $K(1)_U$, by making it more difficult and time consuming for them to find trading partners. Conversely, $K(1)_U < k_0$ is impossible (in steady-state) since entrants expect to make a loss. One can easily show that $K(1)_U > K(2)_U$, which implies that it is optimal for new entrants to acquire the technology needed to manufacture the good at the top of the ladder. Condition (*iii*) guarantees that only two goods diffuse at any one time. In particular, part (*iiia*) asserts that it is individually rational for the producers of the lowest quality good ($\theta = 2$) to remain there and to continue searching for trading partners; part (*iiib*) ensures that a third good does not remain on the ladder. Finally, (*iv*) is necessary for constant and equal populations of z-searchers. The first equation, $\eta^* Z^* = \mu^* C$, is an identity which simply says that the two sides of the market contact each other at the same flow rate. The second equation says that this flow rate is governed by the matching technology described in Assumption 2.

The model possesses a convenient recursive structure which can be used to prove the existence of an equilibrium and to characterize its properties. First, the equilibrium values of the matching rates μ^* and η^* can be determined from the entry condition (*ii*) and the steady-state conditions (*iv*). Once these have been determined the pricing schedule, $p^*(\theta)$, is determined from Proposition 1a and the population Z^* is derived from the steady-state conditions (part (*iv*) of Definition 1). It is then a simple matter to check whether or not the proposed pair (μ^*, η^*) satisfies restrictions (*iii*), given in Definition 1, ensuring $N = 2$.

The condition $K(1)_U = k_0$ [noting (7) and (25)] implicitly defines an equilibrium-entry ('*EE*') locus $\eta = \eta^{EE}(\mu; .)$. This determines, for each μ, the value of η at which firms are just indifferent between entering and not entering the product market.

Lemma 2. *(The EE locus). The function* $\eta = \eta^{EE}(\mu; .)$ *possesses the following properties:*

(i) $\lim_{\mu \to 0} \eta^{EE} > 0$; (*ii*) $\partial \eta^{EE}/\partial \mu > 0$, $\partial \eta^{EE}/\partial \lambda > 0$, and $\partial \eta^{EE}/\partial k_0 > 0$.

These results may be understood as follows. Consider, for example, an increase in μ (the matching rate of c- with z-agents). This lowers both $p(2)$ and $p(1)$ (Proposition 1*a*), which, in turn, reduces (expected) profits. To ensure that z-producers are once more just indifferent between entering and not entering the market, η must rise, which reduces the waiting time between trades. Thus, the *EE* locus slopes upwards. The other comparative-static results reported in Lemma 2 have analogous interpretations.

The constant-returns-to-scale property of the matching function and the steady-state condition $\mu C = \eta Z$, yields $\eta = m_0 M(\eta/\mu, 1)$. This implicitly defines a relationship (referred to as the *SS* locus) $\eta = \eta^{SS}(\mu, m_0)$, along which the population of z-producers, Z, is constant. Lemma 3 summarizes the properties of this locus.

Lemma 3. *(The SS locus). The function* $\eta^{SS}(\mu; m_0)$ *possesses the following properties:*

(i) $\partial \eta^{SS}/\partial \mu < 0$, $\partial \eta^{SS}/\partial m_0 > 0$; (*ii*) $\lim_{\mu \to 0} \partial \eta^{SS}/\partial \mu = -\infty$ *and* $\lim_{\mu \to \infty} \partial \eta^{SS}/\partial \mu = 0$.

Starting at any point on the *SS* locus, where $\mu C = \eta Z = m_0 M(C, Z)$, an increase in μ to μ' raises the instantaneous contact rate between c- and z-agents: $\mu' C > m_0 M(C, Z)$. In order to restore the steady state, Z must rise and η must decline (since M is concave), which explains why the *SS* locus is downward sloping. The limiting properties of the *SS* locus follow directly from the Inada conditions.

By exploiting the properties of the *SS* and *EE* loci, we can prove the existence of a steady-state equilibrium and characterize its properties.

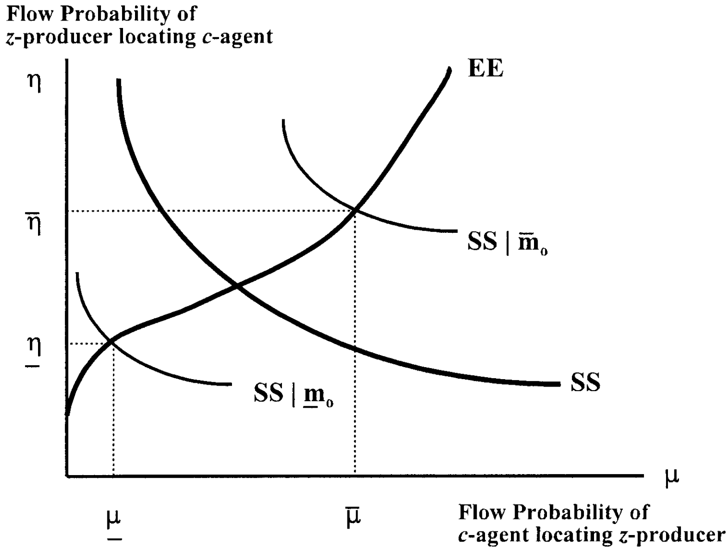

Figure 2. Steady-state equilibrium

Proposition 2. *(Steady-state equilibrium with exogenous* R&D *expenditure). Given* $m_0 \in \left(\underline{m}_0, \overline{m}_0\right)$*, a steady-state equilibrium exists with* $N = 2$.

Figure 2 illustrates the equilibrium ($\overline{\mu} \equiv 4\rho/(\gamma - 1)$ and $\underline{\mu} \equiv \overline{\mu}/(\gamma + 2)$). The restriction on matching efficacy m_0 simply ensures that, as part of the equilibrium, only two goods are traded at a given point. This property is of particular interest, since it implies the search equilibrium is characterized by non-trivial *product diffusion*. This result depends crucially on the presence of market frictions (see Section 4.4) and holds despite the fact that all consumers value the good identically. In particular, it is the presence of "informational-barriers" (search costs) that enables producers of less than "state-of-the-art" products to remain viable and to continue trading. Once μ^* and η^* have been determined, the equilibrium price schedule is obtained from equation (7) in Proposition $1a$.

With the level of R&D exogenously given, the arrival rate of new innovations, λ, is predetermined. Nevertheless, before we turn into the determinants of investment in R&D and subsequently the steady-state equilibrium with endogenous R&D activity, it is useful to consider first Lemma 4.

Lemma 4. *(Effects of the innovation hazard rate).*

$$d\mu^*/d\lambda < 0,\ d\eta^*/d\lambda > 0,\ dp^*(\theta)/d\lambda \gtreqless 0, \qquad \theta = 1, 2.$$

An increase in λ, by lowering instantaneous profits, shifts the *EE* locus up (Lemma 2). The resulting decline in the population of z-producers makes it more difficult for c-agents to locate trading partners, so that μ^* falls, η^* rises and the intermediate-good market is less tight for z-producers. An increase in λ has, on the other hand, two opposing effects on the relative prices of good z. First, it directly enhances type c-agents' bargaining strength which lowers the price (Proposition 1a). However, the induced price reduction discourages entry of z-producers (thus lowering μ^*) which tends to undermine c-agents' bargaining position somewhat and tends to increase the negotiated price. These considerations make it difficult to be precise about the effect of a change in λ on $p(\theta)$. However, if the indirect effect via the decline in μ^* is not too strong (as, for example, in the case in which the *SS* locus is nearly vertical) the partial-equilibrium results reported in Proposition 1a remain valid.

4.2 Determinants of innovative activity

We next examine the determinants of the optimal level of R&D expenditure which enables us to characterize the endogenous rate of economic growth and the length of the product cycle based on the primitives of the model. Given the price w of research capital (assumed exogenous throughout the analysis), innovators choose their R&D expenditure, x, to maximize the expected flow returns (recall that the amount of research capital has been normalized to one):[20]

$$\max_x \lambda(x)\gamma V(\theta, .) - x - w, \tag{26}$$

where $V(\theta; .)$ is the asset value (in effective units) of holding the rights to a patent which is in position θ of the quality ladder (newly developed products optimally enter at the top of the ladder: $\theta = 1$). Simple arguments show that (at an interior maximum):

$$\Lambda(x) \equiv \frac{\partial\lambda(x^*)}{\partial x} V(1; .) = 1/\gamma. \tag{27}$$

The asset value (in effective units) from possessing the rights to the patent for a good positioned at $\theta \in \{1, 2, \ldots, N\}$ on the quality ladder can be calculated using simple dynamic programming arguments:

$$rV(\theta; .) = s_0\gamma^{1-\theta}(Z^*/N)\eta^* - \lambda(x)\gamma[V(\theta; .) - V(\theta + 1; .)]. \tag{28}$$

Equation (28) says that the flow value of possessing the patent rights to good θ equals the flow return from royalties (Z^*/N is the product's market share) *minus* the expected capital loss from being pushed a rung down the ladder in the event of a new innovation. Given $N = 2$, it is possible, with the aid of (28) and the steady-state matching condition $\eta Z = \mu C$, to show that:

$$V(1; x) = \frac{(\lambda + r + \lambda\gamma)[s_0\mu^* C/2]}{(\lambda\gamma + r)^2}. \tag{29}$$

[20] As in Aghion and Howitt (1992), the assumption that λ is homogeneous of degree one simplifies matters, since it allows us to equate the solution of individual research firm's optimization problem with one in which a representative (price-taking) firm employs all research capital and finances all R&D expenditures.

Using (27) and (29), we can derive the R&D schedule, $x^*(\mu, C)$, which determines the optimal level of R&D expenditure undertaken by firms given μ and the exogenous variable C.

Lemma 5. *(Innovative activity). The R&D schedule $x^*(\mu, C)$ possesses the following properties:*

$$\partial x^*/\partial \mu > 0 \quad \textit{and} \quad \partial x^*/\partial C > 0.$$

An increase in either μ or C increases the number of matches and hence the total flow of royalties obtained from winning the patent race. In turn, this enhances the incentives firms have to engage in R&D activity.

4.3 Steady-state equilibrium with endogenous R&D

We are now prepared to study the properties of the steady-state equilibrium with endogenous R&D expenditure (in which $N = 2$).

Definition 2. *(Steady-state random-matching equilibrium).*
A steady-state random-matching equilibrium with $N = 2$ is a tuple $(Z^*, \mu^*, \eta^*, \pi^*(2), p^*(.), x^*)$, satisfying the conditions set out in Definition 1 in conjunction with the R&D *schedule, $x^* = x^*(.)$, described by Lemma 5.*

Definition 2 emends Definition 1 to allow for optimal (steady-state) R&D expenditure. Although the *SS* locus is unaffected by endogenous R&D activity, the *EE* locus must be modified to take into account the endogenous response of x^* (and hence λ) to changes in μ. Nevertheless, it can be shown that $\partial \eta^{EE}/\partial \mu > 0$ and hence Figure 2 remains a qualitatively valid representation of the equilibrium. Note that the *SS* locus is strictly decreasing and the *EE* locus is strictly increasing implies that a steady-state equilibrium with $N = 2$ and $\pi(1) = 1/2$ is unique.

Proposition 3 describes the effects of changes in the parameters of the model on the rate of growth and the length of the product cycle.

Proposition 3. (*Characterization of equilibrium* R&D*, endogenous rate of growth, and the length of the product cycle). In a steady-state equilibrium with endogenous R&D expenditure, a reduction in entry or search frictions (lower k_0 or higher m_0) or an autonomous increase in the tightness of intermediate-good market to the suppliers (higher m or C) encourages inventive activities $[x^*(\mu^*, C, k_0)]$, promotes economic growth $[g^*(\mu^*, C, k_0) \equiv \lambda(x^*)\log(\gamma)]$, and shortens the length of the product cycle $[l^* \equiv 2/\lambda(x^*)]$.*

A reduction in k_0 or an increase in m_0 encourages the entry of z-producers. This raises the flow rate of contacts in the product market and raises the size of the prize to the winner of the patent race. This spurs on R&D activity, which increases the rate of economic growth. An increase in m_0, by lowering market frictions, raises μ^*. This increases the contact rate between c- and z-agents which increases the returns to innovative activity. The effect of an increase in m_0 is therefore to increase x^* and the rate of economic growth g^*. This result suggests that short-run market frictions are indeed an integral part of the long run growth process. Any change that increases the level of R&D activity, by raising the arrival rate of new products, reduces the length of the product cycle (which is measured by $l^* \equiv 2/\lambda$). This feature of the model captures the idea of Schumpeterian creative destruction, in which the introduction of new, more advanced, products leads to the obsolescence of existing ones. Nevertheless, trade frictions imply non-trivial product diffusion, with *less than* cutting edge products coexisting with those currently on the frontier.

4.4 Multiple steady-state growth paths

Up to this point, the main propositions of the paper are derived for a steady-state equilibrium in which exactly two goods appear on the product ladder and there are equal numbers of each type of trader [i.e., $\pi(1) = \pi(2)$]. Such an equilibrium has been shown to exist for $m_0 \in (\underline{m}_0, \overline{m}_0)$, corresponding to a proper range of the matching rate $\mu \in (\underline{\mu}, \overline{\mu})$. However, other steady-state equilibria are admissible which correspond to: (*i*) a "mixed" case, in which $N = 2$ goods appear on the product ladder, but in different proportions [i.e., $\pi(1) \neq 1/2$] and (*ii*) product ladders with $N \neq 2$ rungs. Figure 3 plots an *EE* locus which encompasses the two-rung, mixed, and one-rung cases in an apparently zig-zag fashion. The lower portion (*EF*) of the *EE* locus corresponds to the previously discussed case in which two goods appear on the product ladder in equal proportions (i.e., $\pi(1) = 1/2$); the upper portion (segment *GH*) corresponds to one rung; and the middle segment (*FG*) pertains to the mixed case in which there are two rungs on the product ladder, but in different proportions: $\pi(1) \in (1/2, 1)$.

The single-rung portion *GH* is derived by substituting $\pi(1) = 1$ in the $p(1)$-function defined in Proposition 1*a*. This yields the product price, $p(1)|_{N=1}$ (to simplify the notation we do *not* write $p(1)|_{N=2}$ etc.). This is then substituted in (24) to yield the free-entry condition $K(1)^*_U\ |_{N=1} = k_0$ which, in conjunction with the R&D schedule $x^*(.)$, determines the *EE* locus $\eta = \eta^{EE}(\mu; .)\ |_{N=1}$. The resultant locus *GH* starts at $\overline{\mu}/2$ [at which point profits are exactly zero: $p(1)\ |_{N=1} = s_0$], possesses qualitatively similar features to those reported in Lemma 2, and lies everywhere *above* the two-rung equilibrium entry condition (*EF*). This latter result reflects the fact that *ceteris paribus* the length of the product cycle - and hence expected profits - *increases* with the number of rungs on the quality ladder. Thus, in order to satisfy the free-entry condition the matching rate η must, for given μ, be greatest in the one-rung case to raise the number of successful transactions over the product's (shorter) life-time.

Consider next the middle portion *FG* of the *EE* locus. In the event that, $p(2) - s_0/\gamma = 0$, producers of the low-quality product earn exactly zero profits, which gives rise to the possibility that some proportion may exit the market in equilibrium. The above zero-profit condition holds if and only if $\pi(1)\mu = 2\rho/(\gamma - 1)$ (i.e., $P_0 = 1$ - see Proposition 1*a*), which defines an inverse relationship

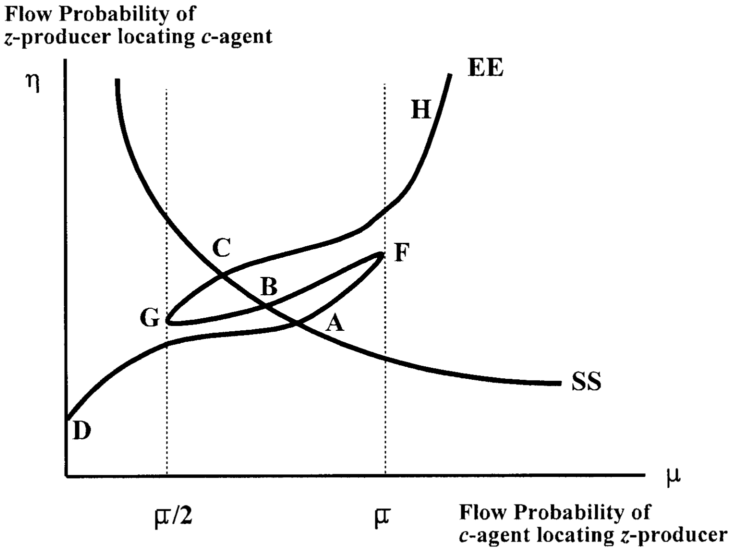

Figure 3. Multiple steady-state growth paths

between μ and $\pi(1)$. The shape of the portion *FG* of the *EE* locus is explained as follows. Starting at *G* in Figure 3, at which point $\pi(1) = 1$, an increase in μ (holding η fixed): (*i*) lowers $\pi(1)$ and (*ii*) raises the level of R&D activity $x^*(.)$ (see Lemma 5). The resultant increase in the arrival rate of new innovations, λ, reduces the length of the product cycle and ceteris paribus lowers z-producers' expected profits below k_0. The free-entry condition is restored at a higher value of η, which raises the volume of sales over the product's (shorter) life. This gives rise to the upward sloping portion *FG*.

Figure 3 illustrates multiple steady-state equilibria, with points *A*, *B*, and *C* corresponding to the two-rung, mixed, and one-rung cases, respectively. In contrast to other papers in the search literature (see, for example, Diamond, 1982b), multiple equilibria emerge without imposing an assumption of increasing-returns-to-scale in the matching technology. The existence of discrete jumps in the length of the product cycle (corresponding to differing number of goods on the product ladder) and search frictions are crucial for the existence of multiplicity. If search frictions disappear (m_0 becomes very large) there is a unique equilibrium with only the most recent innovation on the ladder. Thus, product diffusion takes place only if trade frictions are severe enough, since it is only under these conditions that less than cutting edge products remain viable.

Moreover, unlike previous work on multiple equilibria which is typically conducted within a stationary economic environment, each outcome depicted in Figure 3 is associated with a different balanced-growth path. The rate of growth at point *A* exceeds that at *C*, since not only is the volume of sales greatest at *A* (μ is large) but also two goods appear on the product ladder (implying a longer product cycle).

5 Consumer welfare and policy implications

The comparative-static results presented in Proposition 3 can be used to study issues pertaining to consumer welfare and policy. With free-entry, the *net* value of entry into either the R&D or z-sector is exactly zero in steady state. It follows that, provided we confine our attention to *comparative steady states*, we can identify per-capita economic welfare in effective units, Ω, with the (expected) "market life-time" utility of a searching c-agent. Consider some given point in time t' with n' extant innovations. Consumer welfare in *effective* units is: $\Omega \equiv E_n\{J_U \gamma^{n+n'}\}/\gamma^{n'} = J_U E_n\{\gamma^n\}$ (by construction, J_U is independent of $n \in \mathbb{N}$). Exploiting well-known properties of the Poisson distribution yields,

$$\begin{aligned}\Omega(\lambda;\mu;.) &\equiv J_U \sum_{n=0}^{\infty}\{\lambda^n \exp[-\lambda]/n!\}\gamma^n = J_U \exp[\lambda(\gamma-1)] \qquad (30)\\ &= [\mu/(\mu+\rho)]H(\lambda,\mu)\exp[\lambda(\gamma-1)],\end{aligned}$$

where $H(.) = (1/2)\{q_0(1+\gamma^{-1}) - [p(1)+p(2)]\}$, is the expected value of utility conditional upon a successful match: it equals the expected utility from consumption, $(1/2)q_0[1+\gamma^{-1}]$, minus its expected cost, $(1/2)[p(1)+p(2)]$ (the probability that a c-agent encounters either producer is $\pi = 1/2$). The term in braces is the effective discount rate. An increase in either the flow matching probability (μ) or the innovation rate (λ) lowers the discount rate: the former by reducing the waiting time until trade is consummated and the latter by increasing the probability of a new innovation (with attendant benefit), while the consumer remains unmatched. Finally, $\exp[\lambda(\gamma-1)]$ is the expected benefit of additional innovations during the consumer's market life-time.

It is straight-forward to show that Ω is increasing in its arguments λ and μ. Define the level of consumer welfare in steady-state equilibrium (see Definition 2) by $\Omega^* = \Omega(\lambda^*,\mu^*)$. Totally differentiating Ω^* with respect to $\zeta \in \{m_0, k_0\}$ yields:

$$\partial\Omega^*/\partial\zeta = \lambda_x^*(\partial\Omega^*/\partial\lambda)(\partial x^*/\partial\zeta)+[\partial\Omega^*/\partial\mu^*+\lambda_x^*(\partial\Omega^*/\partial\lambda)(\partial x^*/\partial\mu)](\partial\mu^*/\partial\zeta). \qquad (31)$$

The first term on the right-hand-side of this expression is the *direct* effect on welfare of changes in the level of R&D activity induced by the parametric change. The second term, captures the *general-equilibrium* effects that operate through the matching rate μ^*. The steady-state contact rate μ^* affects welfare directly (by influencing prices and determining the expected waiting time until a successful trade) and has an indirect impact on welfare through its effect on the level of R&D activity (captured by the first and second terms in square brackets respectively). From Propositions 1*a* and 3 and equation (31), it is immediate that raising m_0 or decreasing k_0 promotes growth and welfare.

Equation (30) can be used to determine the social efficiency of private R&D activity. A social planner might be envisioned as choosing a level of R&D expenditure (financed by a lump sum tax on c-agents) to maximize $\Omega(.)$, taking into account endogenous changes in μ^*.[21] The term $\lambda(\gamma - 1)$ is the net *social* benefit of an innovation: i.e., the value of the new good (γ) *minus* the lost value of the good rendered obsolete. As in Aghion and Howitt (1992), R&D firms fail to account for their destructive impact on currently traded products (the "business stealing effect") as well as their beneficial effect upon *future* innovators: the former of these engenders a tendency toward over-investment, while the latter toward under-investment. In addition, unique to the search equilibrium approach adopted here, there is a *market thickness* externality that is an additional source of *over-investment* in R&D. It stems from the failure by R&D firms to internalize the costs of their research activities on the contact rate μ^*. This link arises because increases in x (and hence λ) accelerate the rate of creative destruction, which deters entry by z-producers. In turn, this leads to a *thinner market* which lowers μ^* and consequently Ω^*.

As in Diamond (1982b), we can use our model to study the stabilizing potential of aggregate demand management policies. In Diamond's analysis such policies affected the *level* of economic activity; it is natural, in the context of our framework, to assess whether such policies can influence the *rate* of economic growth as well. In order to address this possibility we assume, somewhat starkly, that: (*i*) the government enters the product market (in mass G) and purchases the z-product (*ii*) government expenditure is financed by a lump-sum tax on c-agents (*iii*) any output procured by the government is "destroyed" (our arguments hold *a fortiori* if the government re-distributes goods to c-agents) and finally (*iv*) c-agents and the government reach the same bargaining agreement and have the same flow matching rate, μ, with z-producers. The total number of consumers of good z is now, $C + G$, which gives a flow rate of government expenditure: $(1/2)\mu G[p(1) + p(2)]$. With a balanced budget, this also equals the flow rate of (lump-sum) taxation. A c-agent's discounted expected life-time tax burden (in effective units) is therefore,

$$(1/2)[\mu/(\mu + \rho)](G/C)[p(1) + p(2)] \exp[\lambda(\gamma - 1)].$$

This formulation takes into account, with the inclusion of the term $exp[\lambda(\gamma - 1)]$, *growth* in the tax burden resulting from expenditure on new innovations. The expression for the tax burden in conjunction with (30), yields the following dynamic general-equilibrium aggregate-demand management problem:

$$\max_{G\geq 0} \Omega(G) = [\mu/(\mu+\rho)]\{H(\lambda, \mu) - (G/C)[p(1)+p(2)]/2\} \exp[\lambda(\gamma - 1)], \quad (32)$$

where the endogenous variables μ, η, x, and λ are evaluated at their equilibrium values with $N = 2$. Consider,

[21] Stokey (1995) compares, for a wide class of consumer preferences, the aggregate level of R&D activity in a competitive economy with the optimal rate. An interesting exercise - but one outside the scope of the present study - would be to compare the optimal rate with the level undertaken in a search-equilibrium environment, since our results indicate that R&D can impact on consumer welfare by affecting matching rates.

Proposition 4. *(Aggregate demand management).*

(i) A balanced budget increase in G raises the growth rate g^*, lowers the contact rate μ^* and, over some interval, $[0, \overline{G}]$ raises the steady-state tax burden.
(ii) A solution to the government's objective, (32), may entail activist policy: $G^* \equiv \arg\max \ \Omega(G) > 0$.

Part (i) of the Proposition indicates that an increase in government activity has three separate welfare effects: first, it promotes economic growth by making the product market thicker and hence spurring on R&D activity; second, it lowers the contact rate μ by enhancing the rate of creative destruction; and, third, over an interval $[0, \overline{G}]$, it raises the tax burden necessary for financing government expenditure.[22] The first of these effects raises consumer welfare, $\Omega(G)$, while the latter two lower it. Part (ii) of the Proposition says that there are circumstances under which the first (growth enhancing) effect dominates and that as a result activist fiscal policy is potentially warranted. Thus, our model illustrates that aggregate demand management policies may (beneficially) affect not only the *level* of economic activity but also the *rate* of economic growth. We refer to this phenomenon as the "Tang" effect, in reference to the government sponsored development of the famous 1960's citrus drink designed for the NASA space program.[23]

It is of interest to note that, as one can show with the aid of Proposition 3, an increase in government activity also reduces the length of the product cycle and thus accelerates the process of creative destruction, $\partial l^*/\partial G < 0$.

6 Concluding remarks

This paper develops a neo-Schumpeterian growth model in which short-run search frictions are an integral part of the trading environment. Higher levels of R&D investment increase the probability of a "break-through" that improves the quality of a differentiated final good and hence raises the rate of economic growth. Our results point to a close connection between the extent of market development (measured by the severity of search frictions) and the rate of economic growth. In particular, we show that a reduction in trade frictions spurs on R&D activity, and consequently growth, by enhancing market thickness. In this sense such frictions are central to the length of product cycles as well as to the rate of economic growth.

[22] In general, it is possible that tax revenues may decline as G is increased, since the government's tax revenue schedule may include decreasing segments. This, however cannot occur over the range of $[0, \overline{G}]$.

[23] In late 1960's, NASA announced a desire to purchase a juice drink that retains a high proportion of vitamins even under high heat. In response to this "if you produce, we will buy" policy, a tart drink, Tang, was designed and manufactured. Later, the popularity of the space program with children induced commercialization of Tang. Similar aggregate demand management has occurred elsewhere; an example is the development of Kowloon in Hong Kong as a business center during the era of Governor Milton.

In Laing, Palivos, and Wang (2000) we endogenize the royalty payment scheme, the innovative increment, γ, and allow R&D firms to account for their own impact on the asset value of holding the rights to a patent, $V(\theta;.)$. As we show there, our main findings remain robust to these extensions.

Our analysis may be further extended in a number of ways and for brevity we describe only two of them. First, it would be of interest to use our exchange structure and address issues pertaining to international trade and economic integration. This can be done by regarding c-agents (z-producers) as residing in the South (North). In this case, economic integration tends to reduce trading frictions. Second, it is possible to incorporate "knowledge spillovers" in our framework and allow for imitation, upon the expiration of patent protection, at a low cost (relative to the innovation). Since patent laws encourage innovative activity but discourage broader (and cheaper) imitation, such a tension can serve to determine the optimal length of patent protection.[24] Moreover, this generalization would lead to a positive feedback between market thickness and the profitability of entry by imitators, which could potentially be another source of multiple steady-state growth paths.

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[24] We would like to thank Ken Judd for suggesting this extension.

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