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# FACTOR ACCUMULATION AND TRADE: DYNAMIC COMPARATIVE ADVANTAGE WITH ENDOGENOUS PHYSICAL AND HUMAN CAPITAL*

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This article develops a two-country endogenous growth model with accumulation of both physical and human capital. We establish the existence of two-country balanced growth equilibria with physical and human capital in which a static and dynamic version of the Heckscher–Ohlin (HO) hypothesis hold true. We also show the existence of unbalanced growth equilibria in which the static and dynamic HO hypotheses can be violated. The multiplicity of paths with international trade emerge as a result of the intertemporal no-arbitrage condition when factor prices are equalized across countries.

## 1. INTRODUCTION

This article analyzes the short- and long-run determinants of the pattern of trade in a model of endogenous growth with physical and human capital accumulation. Although there has been an extensive literature on the determinants of comparative advantage and long-run trade patterns in dynamic settings (as surveyed in Smith, 1984), this literature has focused almost exclusively on models in which physical capital is the only accumulating factor. However, recent work in growth theory emphasizes that human capital plays an important and distinct role in explaining cross-country differences in income growth.[2] In the light of the central role of human capital as an "engine of growth," we feel that it is important

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[2] Lucas (1988) spurs interest in this area by arguing for the role of human capital accumulation as the "engine of growth." Romer (1990) and Barro (1991) provide empirical support for the importance of human capital to economic growth in cross-sectional studies. Evidence in support of the role of human capital for the case of Taiwan is found by Tallman and Wang (1994) who calculate that growth in the stock of human capital accounted for fully 45% of output growth over the period of their study. Young (1995) argues that nearly all of output growth in the East Asian economies can be accounted for by traditional measures of growth in inputs when those inputs include human capital.

to analyze long-run patterns of trade in models which allow for the interactions between physical and human capital accumulation as part of the growth process. Thus, one objective of our work is to fill in a gap in the international trade literature by analyzing the determinants of comparative advantage in models in which both physical and human capital are accumulating.

A second objective of our work is to fill a gap in the macroeconomics literature, which has used two-sector, closed-economy models of endogenous growth to analyze the growth process and the transitional dynamics of physical and human capital. The omission of international trade makes it difficult to study the consequences of the globalization of the world economy or the causes of the Asian miracles in which a high degree of trade dependency has generally been observed.[3] Furthermore, the existence of international trade can have a substantial impact on the dynamics of capital accumulation. In closed-economy endogenous growth models, the convergence of relative factor supplies results from the fact that a country with a relative scarcity of human capital will have a relatively higher return on human capital and a higher rate of human capital accumulation. In contrast, an open economy can make up for the relative scarcity of a particular factor by importing goods that use the scarce factor relatively intensively. Indeed, if the conditions for factor price equalization are met, local factor rewards will be independent of local factor supplies and hence will not serve as a force toward convergence in relative factor supplies. Thus, adding to the conventional channels of technological spillovers and reverse engineering through which globalization influences open economies, our article can elaborate on the role of international trade via its effects on factor accumulation and sectoral reallocation.[4]

We consider a three-sector endogenous growth model with two traded goods (the consumption and investment good) and a nontraded good (the education good). This model is an extension of the widely used closed-economy endogenous growth model (e.g., Rebelo, 1991; Caballé and Santos, 1993; and Bond et al., 1996). In order to focus on the role of factor supplies in the dynamic process, we assume that all countries have identical tastes and technologies. These assumptions are in the spirit of the static Heckscher–Ohlin (HO) model and ensure that countries have a common autarkic balanced growth path. Our analysis focuses on how factor abundance affects the pattern of trade when initial endowments of the countries are not on the autarkic balanced growth path.[5]

[3] Two important exceptions are Grossman and Helpman (1991) and Ventura (1997). While Grossman and Helpman emphasize technological innovations under monopolistic competition with physical capital as the lone reproducible factor, the contrasting role of human capital with physical capital is largely ignored. Ventura (1997) has used the factor price equalization property in a model of growth with accumulation of physical capital to argue that East Asian countries could experience high rates of physical capital accumulation without experiencing diminishing returns because they could change the composition of their production toward more capital intensive goods.

[4] Rivera-Batiz and Romer (1991) illustrate how international technology spillovers may advance developing countries, whereas Lucas (1993) suggests that learning-by-doing is crucial for Asian development miracles.

[5] A second type of model assumes that there are differences across countries in parameters such as savings rates (Smith, 1984), tax policies (Baxter, 1992), or time preferences (Stiglitz, 1970) that lead

We will examine two types of results relating factor abundance to the pattern of trade. The first is the familiar *static Heckscher–Ohlin hypothesis*, which relates factor supplies to the pattern of comparative advantage. Letting $k^i(t) \equiv K^i(t)/H^i(t)$ denote the aggregate physical/human capital ratio in country $i$ at time $t$, this hypothesis can be expressed as:

*Static H–O: If $k^A(t) > k^B(t)$ then*

(a) *under free trade, country A exports the physical capital intensive traded good at time t;*
(b) *under autarky, country A has a lower price of the physical capital intensive good and lower return on physical capital.*

These results hold in static models with two traded goods under our assumptions on tastes and technologies, and (a) has played a particularly important role because it is amenable to empirical testing. We would also like to know whether initial factor endowments can be used to predict future patterns of trade, which suggests the following definition of a *dynamic Heckscher–Ohlin hypothesis*:

*Dynamic H–O: If $k^A(t) > k^B(t)$, then $k^A(t') \geq k^B(t')$ for $t' > t$ and the physical capital abundant country will not export the human capital intensive good for $t' > t$.*

The statement of this hypothesis allows for the possibility that all countries converge to the same factor endowment ratio, so that trade disappears asymptotically, although this is not necessarily a long-run property of dynamic models with identical preferences and technologies.[6] Notably, the Ventura (1997) model satisfies both the static and dynamic H–O hypotheses regardless of whether relative factor endowments are convergent or nonconvergent.

countries to have different autarkic steady states. In these models, the emphasis is on how differences in these parameters influence steady-state values of factor prices and trade patterns. For example, Smith (1984) shows that in models of capital accumulation with fixed savings rates, the high savings rate country will export the capital intensive good in the long run. Stiglitz (1970) analyzes the case in which patience is reflected in different rates of time preference with infinitely lived consumers and shows that the patient country will export the capital intensive good in the steady state. At least one country will specialize in production in the steady state in the Stiglitz model, and international factor prices will not equalize. In contrast, Galor and Lin (1997) show that in an overlapping generations model, differences in rates of time preference may be consistent with factor price equalization in the long-run equilibrium even though the patient country still exports the capital intensive good. Bond and Trask (1997) examine how technological differences affect the long-run trade pattern in a variant of the model in this article.

[6] Two possibilities arise regarding the steady state. One is that the country's capital supplies converge to the same capital/labor ratio, so that factor endowments based trade is eliminated as countries approach the steady state (e.g., Findlay, 1970, 1995, Chapter 2; Deardorff and Hansen, 1978). Another possibility is that the country's factor supplies converge to different ratios in steady state, allowing international trade (and differences in per capita income) to persist in the steady state. (e.g., Ventura, 1997). Whether or not the economies converge to the same steady state under trade, there can be factor proportions based trade along the transition path to the steady state.

We will illustrate that our framework can generate violations of both the static and dynamic H–O hypotheses. Violations of part (a) of the static proposition can arise if the growth rate of human capital differs across countries. As the education good is nontraded, alterations in the growth rate of human capital require changes in the resources devoted to the production of traded goods and hence to the relative production of traded goods. A violation of part (b) of the static H–O hypothesis can also arise in the dynamic model because of intertemporal linkages. The potential failure of the dynamic version of the H–O hypothesis arises in cases where international trade results in the equalization of factor prices across countries. The intertemporal no-arbitrage condition for factor accumulation makes investors indifferent between accumulation of physical and human capital, and international trade with factor price equalization allows a continuum of (utility equivalent) unbalanced growth paths. This multiplicity of unbalanced growth paths for capital accumulation is consistent with a variety of different growth experiences across countries and a lack of convergence in relative factor supplies. The constraint on possible growth experiences is provided by the requirement that the economy remain incompletely specialized.

## 2. THE INTEGRATED WORLD EQUILIBRIUM

We begin by studying the closed-economy version of a three-sector model of endogenous growth in which there are two reproducible factors of production, physical capital ($K$) and human capital ($H$). We establish conditions under which there will exist a unique, stable equilibrium characterized by a *balanced growth path* (BGP) in which consumption and the stocks of physical and human capital all grow at a common rate. The closed economy framework can be thought of as representing the equilibrium for the world economy if factors of production were freely mobile across countries possessing identical preferences and technologies. Following Dixit and Norman (1980), we will refer to this as the solution of the *integrated world equilibrium* (IWE).

2.1. *The Model.* The three sectors will be referred to as the investment goods sector, $X$, the education sector, $Y$, and the consumption goods sector, $Z$.[7] All sectors are assumed to have production functions exhibiting constant returns to scale, with perfect competition in goods and factor markets. The production functions are specified as: $X = F(s_X K, u_X H) = u_X Hf(k_X)$, $Y = G(s_Y K, u_Y H) = u_Y Hg(k_Y)$, and $Z = J(s_Z K, u_Z H) = u_Z Hj(k_Z)$, where $s_i$ and $u_i$ are the shares of physical and human capital, respectively, allocated to production in sector $i$ and $k_i \equiv (s_i K)/(u_i H)$ is the physical to human capital ratio in sector $i$ for $i = X,Y,Z$. The output per unit human capital functions $f$, $g$, and $j$ are assumed to be strictly increasing and strictly concave.

[7] The model thus generalizes the two-sector endogenous growth models studied by Caballé and Santos (1993) and Bond et al. (1996) in which consumption and investment goods are treated as perfect substitutes.

The investment (education) good will provide additions to the stock of physical (human) capital and the evolution of the stocks will then be given by

$$\dot{K} = u_x H f(k_x) - \delta K \tag{1a}$$

$$\dot{H} = u_Y H g(k_Y) - \eta H \tag{1b}$$

where $\delta$ ($\eta$) is the rate of depreciation of physical (human) capital.[8] For the IWE, consumption ($C$) must be no greater than the output of the consumption good sector ($Z$)

$$C \leq u_Z H j(k_Z) \tag{1c}$$

With full mobility of factors across sectors, the resource constraints require that

$$\sum_{i=X,Y,Z} u_i \leq 1 \qquad \sum_{i=X,Y,Z} u_i\, k_i \leq k \quad \text{with} \quad u_i \in [0,1] \tag{2}$$

where $k \equiv K/H$ is the aggregate physical/human capital ratio. Denoting $\sigma \in [0,\infty)$ as the inverse of the constant intertemporal elasticity of substitution, the problem of the representative agent is

$$\max_{C,s_i,u_i} V(C) = \int_0^\infty \frac{C(t)^{1-\sigma}}{1-\sigma} \mathrm{e}^{-\rho t}\, dt \tag{3}$$

subject to (1), (2), $H(0) = H_0 > 0$ and $K(0) = K_0 > 0$

Since our primary concern is to characterize the BGP of the IWE in which all sectors are in operation, we concentrate on interior solutions to (3) for which $s_i$, $u_i \in (0,1)$.[9] Applying standard optimal control techniques, we can manipulate the necessary conditions to yield three fundamental relationships. First, letting $\varphi_i$ denote the unit cost function for sector $i$, perfect competition in goods and factor

[8] We treat education as a private good, whose returns are fully internalized by the owner. Grossman and Helpman (1991, Chapter 5) consider the case in which there are external effects from the development of new products and examine accumulation of two "factors": technology and a general capital. Their model differs from ours in the sense that knowledge is the engine of growth due to the presence of scale economies. Also, we assume that all workers experience the same increase in effective units of labor. In contrast, Stokey (1996) considers a model in which labor is heterogenous and classified as skilled or unskilled. The economy accumulates human capital or skilled labor through the transformation of unskilled labor.

[9] The transversality conditions are: $\lim_{t\to\infty} e^{-\rho t}\mu(t)K(t) = 0$ and $\lim_{t\to\infty} e^{-\rho t}\, \lambda(t)H(t) = 0$, with $\mu$ and $\lambda$ denoting the costate variables. In this section, we establish the existence of a unique BGP in which the control variables are in the interior of the feasible set. It is possible, however, that for some initial values of $K$ and $H$ there will be regions of the path for which $u_X = s_X = 0$ or $u_Y = s_Y = 0$. These cases can readily be handled by incorporating the constraints on control variables into $P(5)$ and making the appropriate modifications in the Hamiltonian. For a discussion of the validity and sufficiency of the transversality conditions, see Bond et al. (1996).

markets requires that price equal unit cost when a sector is in operation. This yields the *competitive profit relationships*

$$\varphi_X(w,r) = p_X \quad \varphi_Y(w,r) = p_Y \quad \varphi_Z(w,r) = 1 \tag{4a}$$

where $p_X, p_Y, r$, and $w$ are, respectively, the relative prices of the investment and education goods and the return to a unit of physical and human capital in terms of the numeráire consumption good. Second, letting $v_i$ denote the growth rate of variable $i$ and using the definition of relative prices, an *intertemporal no-arbitrage condition* can be derived as

$$\pi \equiv \frac{r}{p_X} + v_{p_X} - \delta = \frac{w}{p_Y} + v_{p_Y} - \eta \tag{4b}$$

which requires equalization of the net rates of return to investment (rental rate plus rate of capital gain less depreciation) in physical and human capital. Finally, the evolution of consumption along the optimal path is governed by a *modified Keynes–Ramsey equation*:

$$v_C = (1/\sigma)(\pi - \rho) \tag{4c}$$

Defining $\Omega(t) = p_X(t)K(t) + p_Y(t)H(t)$ to be wealth at time $t$, the preferences imply a consumption function for households with a marginal propensity to consume out of wealth at each time $t$, given by[10]

$$C(t) = \psi(t)\Omega(t) \tag{5}$$

where $\psi(t)^{-1} \equiv \int_t^\infty q(\tau)^{\frac{\sigma-1}{\sigma}} e^{\frac{-\rho\tau}{\sigma}}\, d\tau$ and $q(\tau) = \exp[-\int_t^\tau \pi(s)\, ds]$ is the price of consumption at time $\tau$ (relative to time $t$). Along the optimal path, the growth rate of wealth along the optimal path will then satisfy

$$v_\Omega = \pi - \psi \tag{6}$$

Note that if the economy is made up of $n$ households with initial endowments $\{K_0^i, H_0^i\}$, the evolution of household consumption and wealth will also be governed by (5) and (6). Since household consumption is determined by the level of wealth, and not its composition, households will be indifferent between all pairs $\{v_K^i(t), v_H^i(t)\}$ that yield the same the optimal growth rate of wealth because the intertemporal no-arbitrage condition equalizes returns across assets. Thus, the solution to (3) will determine the aggregate accumulation of factors, $\dot{K}(t)$ and $\dot{H}(t)$, but not the values for individual households. This contrasts with the case where the stock of labor is exogenously given (or grows at an exogenously given rate). With exogenous growth of $H$, (6) will uniquely determine $v_K^i$

[10] Barro and Sala-i-Martin (1995, pp. 66–67) provide a derivation of the consumption function in a simple Ramsey model.

as a function of household endowments and the exogenously given value of $v_H$. The indifference over the composition of wealth accumulation where both factors are accumulated will play a significant role in the open economy case considered below.

2.2. *Existence and Uniqueness of Balanced Growth.* We now establish the conditions under which there will exist a unique balanced growth path for the IWE satisfying (1)–(6) in which $C$, $H$, and $K$ grow at a common rate. First note that if a BGP exists, the relative prices of goods and factors must be constant and (4b) must be satisfied with $v_{p_X} = v_{p_Y} = 0$. With constant relative prices, (4a) and (4b) then represent a system of four equations that can be solved for the price vector $P = (w, r, p_X, p_Y)$ consistent with balanced growth for the IWE. Defining $\Gamma \equiv \{P \mid \varphi_i(w, r) = p_i, \text{ for } i = X,Y,Z\}$ to be the set of price vectors satisfying the competitive profit conditions and $v_{\max}$ to be the maximal attainable rate of consumption growth, we establish three sufficient conditions for the existence of a unique, nondegenerate balanced growth equilibrium:

CONDITION E:

(a) $\sup_{P\in\Gamma} \frac{r}{p_X} - \frac{w}{p_Y} > \delta - \eta > \inf_{P\in\Gamma} \frac{r}{p_X} - \frac{w}{p_Y}$
(b) $\rho > (1-\sigma)\, v_{\max}$
(c) $r^*/p^*_X - \delta > \rho$

where the superscript $*$ denotes the BGP value of the given variable. Part (a) of Condition E ensures the existence of a set of prices at which investment in both types of capital is profitable, part (b) guarantees bounded lifetime utility, and part (c) is needed for the balanced growth equilibrium to be nondegenerate. Provided that Condition E is satisfied, we can establish the following result.

PROPOSITION 1. *Under Condition E, there exists a unique balanced growth equilibrium for the integrated world economy in which:*

(a) $v_r = v_{p_X} = v_{p_Y} = 0$,
(b) *$H$, $K$, $C$, and $\Omega$ grow at the common rate,*

$$v^* = (1/\sigma)[\pi^* - \rho] > 0 \tag{7}$$

(c) *the aggregate physical to human capital ratio, $k^*$, and the consumption/wealth ratio on the BGP are given by*

$$k^* = \frac{f^*}{g^*}\left[\frac{k^*_Z(g^* - v^* - \eta) + k^*_Y(v^* + \eta)}{f^* - (v^* + \delta)\,k^*_X + k^*_Z(v^* + \delta)}\right]; \quad \frac{c^*}{p^*_Y + p^*_X k^*} = \pi^* - v^* \tag{8}$$

PROOF. See the Appendix.

In particular, the proof of Proposition 1 establishes that the uniqueness of the prices $\{w^*, r^*, p_X^*, p_Y^*\}$ satisfying $v_r = v_{p_X} = v_{p_Y} = 0$ consistent with balanced growth holds even in the presence of factor intensity reversals. In order for a BGP to exist at these prices, we must show that there exist sectoral allocations $u_i \in (0,1)$ for $i = X, Y, Z$ and $(c, k) > 0$ that satisfy equations (1) and (2) with equality when evaluated using $v_H = v_K = v^*$ and $k_i = k_i^*$ where $c = C/H$. This yields a system of five linear equations to be solved for the balanced growth values $(c^*, k^*, u_X^*, u_Y^*, u_Z^*)$, which generates the results in (8).

## 3. EQUILIBRIUM IN THE WORLD ECONOMY WITH INTERNATIONAL TRADE

We now consider the equilibrium in a world economy of $n$ countries with the initial endowment of factor supplies of country $i$ denoted by $\{K_0^i, H_0^i\}$, where factors are immobile between countries. We assume that the consumption and investment good are traded internationally whereas the education good is a nontraded service. We restrict our attention to the case where countries are incompletely specialized. Our objective in this section is to examine the conditions under which the equilibrium of the world economy with free trade in consumption and investment goods will be identical to the corresponding IWE as derived in Proposition 1. One way to simplify the analysis is to assume that the initial world factor supplies satisfy $K_0/H_0 = k^*$, so that the world is on the BGP characterized in Proposition 1 and the world relative price of investment goods at each $t$ is equal to $p_X^*$. We establish a set of factor endowments $R^*$ such that if $k^i \in R^*$ for all $i$, then there will exist a balanced growth path in which $v_H^i(t) = v_K^i(t) = v^*$ for all $i$. We also characterize the set of initial endowments for which there will exist unbalanced growth paths with $v_H^i(t) \neq v_K^i(t) \neq v^*$. We then examine how growth in this setting can affect trade patterns by establishing the conditions under which the static and dynamic H–O hypotheses will hold.

3.1. *Balanced and Unbalanced Growth Equilibria under Trade.* We begin by examining the optimization problem for the representative agent in country $i$, given free trade at world prices $p_X^*$. Since the education good is nontraded, the accumulation of human capital in country $i$ continues to be determined by the output of sector $Y$ as described in (1b). With free trade in the output of the $X$ and $Z$ sectors, consumption and the accumulation of physical capital are now governed by the trade balance condition

$$c^i + p_X^*(v_K^i + \delta)k^i = u_Z^i j(k_Z^i) + p_X^* u_X^i f(k_X^i) \tag{9}$$

which assumes no international lending or borrowing.[11] The optimization problem for country $i$ is to maximize the objective function in (3), subject to (1b), (2), (9) and initial factor supplies $\{K_0^i, H_0^i\}$. The necessary conditions for this problem are derived by applying the techniques used in Section 2.

[11] Since we will be focusing on distributions of factor supplies across countries such that factor prices are equalized by trade, the assumption that trade is balanced will not be restrictive.

If the equilibrium with international trade is to duplicate that of the BGP, then the vector of domestic prices in country $i$ at time $t$, $P^i(t) = \{w^i(t), r^i(t), p^*_X, p^i_Y(t)\}$, must equal the values on the BGP, $P^* \equiv \{w^*, r^*, p^*_X, p^*_Y\}$. In addition, consumption in country $i$ must equal that in the IWE. It follows from (4a) and the derivation of the IWE that the competitive profits conditions can be satisfied at time $t$ with $P^i(t) = P^*$. In order for these prices to be an equilibrium, it must also be the case that the factor markets and nontraded goods markets clear at these prices. Full employment requires the existence of values $u_i \in [0,1]$ such that $\sum_{j=X,Y,Z} u_j k^*_j = k^i$, where $k^*_j$ is the cost-minimizing physical/human capital ratio in sector $j$ at $w^*/r^*$. Since output of the education sector is nontraded, the share of human capital devoted to the $Y$ sector is determined from (1b) to be $u^i_Y = (v^i_H + \eta)/g(k^*_Y)$. Substituting this result into the full employment conditions yields the range of $k^i$ consistent with full employment for a given $v^i_H(t)$ at BGP prices:

$$(10)\quad k^i \in R \equiv \left[ u^i_Y k^*_Y + \left(1 - u^i_Y\right) \min\left(k^*_X, k^*_Z\right), u^i_Y k^*_Y + \left(1 - u^i_Y\right) \max\left(k^*_X, k^*_Z\right)\right]$$

Equation (10) requires that the relative physical capital endowment of country $i$ be no greater (less) than that required when country $i$ produces only the most (least) physical capital intensive traded good and the nontraded good in the proportions required to achieve $v^i_H(t)$. In this region, international trade results in the equalization of factor prices for efficiency units of labor.[12]

Our second requirement is that consumption in the free trade equilibrium equal that on the BGP, which from Proposition 1 requires that $C^i(t) = \pi^*(p^*_X K^i_0 + p^*_Y H^i_0)\, e^{v^* t}$. It follows from (5) and (6) that this consumption path can be attained if the wealth of country $i$ grows at rate $v^*$. To satisfy the wealth accumulation condition, the $v^i_H$ *and* $v^i_K$ must be such that

$$(11)\qquad p^*_Y v^i_H + p^*_X k^i\, v^i_K = \left(p^*_Y + p^*_X k^i\right) v^* \qquad \text{with} \quad v^i_H \geq -\eta \quad v^i_K \geq -\delta$$

Country $i$'s consumption path is equivalent to that on the BGP if there exists a path $\{v^i_H(t), v^i_K(t), \mathrm{k}^i(t)\}$ such that (10) and (11) hold for all $t$. If this condition holds for all countries, then the equilibrium with trade will duplicate the IWE.

We will examine two types of growth paths for country $i$: (1) *balanced growth paths* for country $i$ in which $v^i_H(t) = v^i_K(t) = v^*$ and $k^i(t) = k^i$ for all $t$, and (2) *unbalanced growth paths* in which $v^i_H(t) \neq v^i_K(t) \neq v^*$ and $k^i(t) \neq k^i$ for some $t$. The following result identifies the set of endowments that are consistent with balanced growth for country $i$ and establishes the possibility of unbalanced growth equilibria.

[12] Interpreting human capital as efficiency units of labor, this result is consistent with differences in wages per worker across countries that reflect differences in efficiency units per worker. Trefler (1993) argues that a simple modification of the traditional H–O model may fit the data well once factor endowments are adjusted to reflect international productivity differences. Our version of factor price equalization is consistent with this empirical finding.

PROPOSITION 2.

(a) *If the world price of the investment good is constant at the value* $p_X^*$ *obtained from the BGP of the IWE and country i has an endowment*

$$k^i \in R^* \equiv \left[u_Y^* k_Y^* + \left(1 - u_Y^*\right)\min\left(k_X^*, k_Z^*\right), u_Y^* k_Y^* + \left(1 - u_Y^*\right)\max\left(k_X^*, k_Z^*\right)\right] \quad (12)$$

*where* $u_Y^* \equiv (v^* + \eta)/g^*$, *then there exists a BGP for country i along which its consumption and stocks of physical and human capital all grow at the rate* $v^*$.

(b) *Choose any* $k_1, k_2 \in int\ R^*$. *If* $k^i(t_1) = k_1$, *then there exists a* $t_2 > t_1$ *and a feasible path* $v_{\mathrm{H}}^i(t)$ *such that* $k^i(t_2) = k_2$.

Proposition 2(a) specifies the range of endowments that are consistent with a balanced growth equilibrium for a small open economy, where (12) follows immediately from evaluation of $k^i \in R^i$ at $v_H^i = v^*$. Proposition 2(b) shows that any physical/human capital ratio in the interior of the region consistent with balanced growth can be reached from any other point in the region in finite time through an appropriate path of human capital investment. Since outputs of all three goods are strictly positive on the interior of the range defined in (12), it is possible to raise or lower $v_H(t)$ in the neighborhood of $v^*$ while maintaining full employment and factor price equalization. This creates a continuum of (utility equivalent) unbalanced growth equilibria in which $k^i$ varies over time.

In addition to the conditions described by Proposition 2(b), unbalanced growth may be consistent with the BGP utility level when $k^i(t) \notin R^*$. For example, suppose that $k^i$ is less than the lower bound of $R^*$. If there exists a time path $\{v_H^i(t), v_K^i(t)\}$ which satisfies (10) and (11) with $\dot{k}^i(t) > 0$, then $k^i$ will eventually enter $R^*$. Since $\dot{k}^i(t) > 0$ requires $u_Y^i \in [0, u_Y^*)$, it can be seen by comparison of (10) and (12) that the lower bound of $R^i$ will be less than the lower bound of $R^*$ if $\min(k_X^*, k_Z^*) < k_Y^*$. If this condition is satisfied, then unbalanced growth with $\dot{k}^i(t) > 0$ is consistent with full employment for $k^i \in [\min(k_X^*, k_Z^*), u_Y^* k_Y^* + (1 - u_Y^*)\min(k_X^*, k_Z^*)]$. Intuitively, the initial position is one in which country *i* possesses too little physical capital relative to human capital to support a balanced growth equilibrium. When the nontraded good uses physical capital relatively intensively compared to $\min(k_X^*, k_Z^*)$, decreases in the production of the nontraded good will increase the relative amount of physical capital available to produce traded goods by "relaxing" the full employment conditions.

A similar argument can be used to show that if $k_Y^* > \max(k_X^*, k_Z^*)$, full employment can be attained for a range of values of $k^i$ exceeding $u_Y^* k_Y^* + (1 - u_Y^*)\max(k_X^*, k_Z^*)$ with $\dot{k}^i(t) < 0$. Let $v_H^{\max} \equiv v^* + p_X^* k(\delta + v^*)/p_Y^*$ be the maximum growth rate for human capital consistent with (11), which is attained when $v_K = -\delta$. Defining $u_Y^{\max} \equiv (v_H^{\max} + \eta)/g^*$, the upper bound on this region occurs where the growth rate of human capital is at the maximum valued defined in (11), which yields the following result:

PROPOSITION 3. *If* $k_Y^* > \min(k_X^*, k_Z^*)$, *the possibility of unbalanced growth expands the range of endowments for which the BGP utility level can be obtained in the following cases.*

(a) *If* $k^i \in [\min(k_X^*, k_Z^*), u_Y{}^* k_Y^* + (1 - u_Y{}^*) \min(k_X^*, k_Z^*))$, *then there exists a continuum of paths* $\{v_H^i(t), v_K^i(t)\}$ *satisfying* $k^i \in R^i$ *with* $c^i(t) = c^*$ *for all t. Along any of these paths,* $\dot{k}^i(t) > 0$ *for all t for which* $k^i(t)$ *is in this interval.*

(b) *If* $k_Y^* > \max(k_X^*, k_Z^*)$ *and* $k^i \in (u_Y{}^* k_Y^* + (1 - u_Y{}^*) \max(k_X^*, k_Z^*), u_Y{}^{\max} k_Y^* + (1 - u_Y{}^{\max}) \max(k_X^*, k_Z^*)]$, *then there exists a continuum of paths* $\{v_H^i(t), v_K^i(t)\}$ *satisfying* $k^i \in R^i$ *with* $c^i(t) = c^*$ *for all t. Along any of these paths,* $\dot{k}^i(t) < 0$ *for all t for which* $k^i(t)$ *is in this interval.*

For initial endowments outside the union of the intervals described in Propositions 2 and 3, the BGP utility level will not be attainable.[13]

The IWE can be duplicated by free trade in the consumption and investment goods if the initial endowments $k_0^i$ lie in the range identified in Propositions 2 and 3 for all countries. To illustrate these results, Figure 1 depicts a production box diagram, which is frequently used to illustrate the region of endowments consistent with factor price equalization in the static H–O model. The dimensions of the box are the supplies of physical and human capital in the world economy at time $t$, normalized by dividing by $e^{v^* t}$, which will be constant in the IWE. The diagonal of the box will have a slope of $k^*$, the relative factor endowment of the world economy on the BGP. Denoting the countries as $A$ and $B$ respectively, any point in the box denotes an initial distribution of world endowments between the two countries, with country $A$'s ($B$'s) endowment measured relative to the origin $O_A$ ($O_B$). The line segments $O_A C$ and $O_A D$ are drawn with slopes equal to the upper and lower bounds in (12), so that any initial endowment lying within the cone defined by these line segments is consistent with a balanced growth equilibrium for country $A$ as defined by Proposition 2(a). Similarly, endowments contained within the cone defined by the segments $O_B D$ (parallel to $O_A C$) and $O_B C$ (parallel to $O_A D$) are consistent with balanced growth for country $B$. Thus, if the initial endowments are contained in the parallelogram $O_A C O_B D$, the equilibrium with international trade will yield the same utility level for each country as could be achieved on the BGP of the IWE.

Suppose that country $A$ has an initial endowment at point $E$, with $\Omega_E$ denoting the value of wealth at point $E$ evaluated at BGP prices. On the BGP, (11) requires that factors accumulate such that normalized wealth is constant, $\Omega_E = [p_X^* K(t) + p_Y^* H(t)]\, e^{-v^* t}$ for all $t$. The $\Omega_E$ line in Figure 1 illustrates the locus of values of factor supplies for which normalized wealth is constant. In a

[13] If endowments lie outside this range and there are no factor intensity reversals, then factor prices would not equalize and the country would produce only one of the traded goods (and possibly the nontraded good). The BGP utility level would not be attained in this case. If there are factor intensity reversals, it is possible for a country with endowments outside these intervals to be to be incompletely specialized. However, factor prices would not equal ($w^*$, $r^*$) and the BGP utility level would not be attainable.

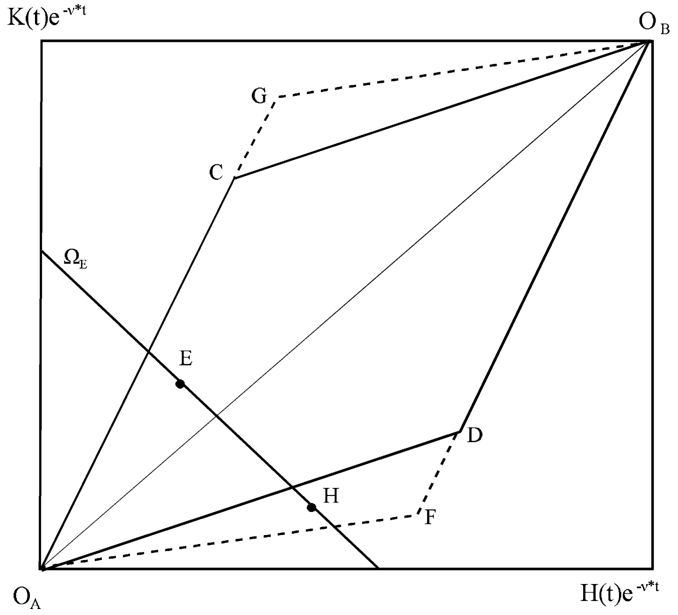


FIGURE 1

REGION OF FACTOR PRICE EQUALIZATION AT BGP PRICES WITH $k_X^* > k_Y^* > k_Z^*$: BALANCED GROWTH ($O_A D O_B C$) AND UNBALANCED GROWTH ($O_A F O_B G$)

balanced growth equilibrium for country $A$, physical and human capital will grow at rate $v^*$ and factor supplies in $A$ will remain at point $E$ over time. In the unbalanced growth equilibria for country $A$ described in Proposition 2(b), factor supplies will grow at differential rates and the factor supplies in $A$ will move along the segment of $\Omega_E$ contained in $O_A C O_B D$. In order to maintain the BGP for the world as a whole, unbalanced growth in country $A$ must be matched by unbalanced growth in Country $B$.

Proposition 3 establishes that there are two possible cases in which an economy whose initial endowment is outside the parallelogram $O_A C O_B D$ can attain the BGP utility level. If $k_Y^* > \min(k_X^*, k_Z^*)$, then the BGP utility level can be maintained for physical/human capital ratios as low as $\min(k_X^*, k_Z^*)$ by Proposition 3(a). This ratio is represented by the lines with slope $O_A F$ and $O_B G$. In this case, the region of initial endowments consistent with factor price equalization for the world economy is given by the parallelogram $O_A G O_B F$. If the initial endowment point is at $H$, the growth rate of human capital in country $A$ must be less than $v^*$ along any equilibrium path and $k^A(t)$ must be rising along the $\Omega_E$ line until it enters $O_A C O_B D$. Finally, if the education sector is the most physical capital intensive sector, $k_Y^* > \max(k_X^*, k_Z^*)$, the region consistent with the BGP of the IWE through trade will be expanded to include a region above $O_A G$ and a region below $O_B F$ as indicated by Proposition 3(b).

The analysis above established the indeterminacy of the pattern of factor accumulation when the world is on the BGP. It is important to note that our assumption in this section that the world is on the BGP is not critical to that result. We show in Bond et al. (2000) that in cases where factor prices in the IWE are changing over time, it is possible to define a set $R(t)$ such that factor price equalization will hold at time $t$ for $k^i \in R(t)$. The IWE can be duplicated with free trade for country $i$ if there exists a path $\{v^i_H(t), v^i_H(t), k^i(t)\}$ such that $k^i \in \mathrm{R}(t)$ for all $t$ and wealth grows at the same rate as in the IWE. Furthermore, if this initial path is in the interior of the set of feasible paths, then it will be possible to construct a continuum of paths that duplicate the IWE.

3.2. *Factor Supplies and Trade Patterns.* Having characterized the factor accumulation paths consistent with open economies attaining the BGP utility level, we now study the pattern of trade along such paths and examine the validity of the static and dynamic versions of the H–O hypothesis in this model. Assuming that free trade equilibrium duplicates the IWE, imports of the consumption good per unit of human capital in country $i$ are given by $m^i_Z = \psi(p^*_X k^i + p^*_Y) - u^i_Z j^*$. The market clearing condition for the world economy requires that world consumption equal world output of the consumption good, which from (1c) and (5) yields $\psi^* = u^*_Z j^*/(p^*_X k^* + p^*_Y)$, where $u^*_Z$ is the share of world human capital allocated to production of the consumption good. Substituting this into $m^i_Z$ and using the full employment condition, we obtain

$$m^i_Z = j^*\left\{\left[\frac{p^*_X k^*_X + p^*_Y - u^*_Y p^*_X(k^*_X - k^*_Y)}{p^*_X k^* + p^*_Y}\right]\left(\frac{k^* - k^i}{k^*_Z - k^*_X}\right) + \left[\frac{v^* - v^i_H}{g^*}\right]\left(\frac{k^*_X - k^*_Y}{k^*_Z - k^*_X}\right)\right\} \tag{13}$$

If human capital is growing at the same rate in country $i$ as in the world economy, then the pattern of trade will be determined completely by the first term in (13). The term in brackets must be positive since $u^*_Y < 1$. If the consumption good is physical (human) capital intensive relative to the investment good, then the first term will be positive if country $i$ is human (physical) capital abundant relative to the rest of the world. A sufficient condition for part (a) of the static H–O hypothesis to hold is that the growth rate of human capital in country $i$ equal that of the world economy, as would occur if country $i$ were experiencing balanced growth.

The second term in (13) captures the effect of variations in the growth rate of human capital on the pattern of trade when $k^*_X \neq k^*_Y$. Suppose that the consumption good is the physical capital intensive traded good and country $i$ is physical capital abundant relative to the world economy, so that the first term in (13) is negative. Part (a) of the static H–O hypothesis could be violated if the second term is positive and sufficiently large. The second term will be positive if $v^i_H(t) > v^*$ and $k^*_Y > k^*_X$ or if $v^i_H(t) < v^*$ and $k^*_Y < k^*_X$. In the case where the world is on the BGP, it is shown in the Appendix that for endowments sufficiently close to $k^*$, it is always possible to find feasible values of $v_H$ such that the pattern of trade violates that predicted on the basis of factor abundance. The failure of the H–O hypothesis occurs because variations in the rate of human capital growth alter the relative supply of physical capital available to produce traded goods, as the education good must be produced locally. For example, in the former case where $v_H^i(t) > v^*$ and $k_Y^* > k_X^*$, a relatively large amount of physical capital is being withdrawn from the traded goods sector to produce the physical capital-intensive nontraded good and meet the high growth rate human capital. The source of the failure of the static H–O hypothesis is that although country $i$ is physical capital abundant relative to the world as a whole in terms of its gross factor endowment, the net endowment available for the production of traded goods is human capital abundant relative to the world as a whole. Thus, with unbalanced growth, the difference in rates of human capital accumulation across countries may cause the ranking of countries on the basis of "net of human capital accumulation" factor abundance to differ from the ranking on the basis of "gross" factor abundance. It is the former abundance that determines the pattern of trade. Summarizing,

PROPOSITION 4.

(a) *If $v_H(t) = v^*$, then part (a) of the static H–O hypothesis will hold at time t.*
(b) *Suppose the world is on the BGP of the IWE. If $k^i(t)$ is contained in the range consistent with balanced growth in* (10), *then part (a) of the static H–O hypothesis will hold if $v_{\rm H}(t) = v^*$. However, if $k_X^* \neq k_Z^* \neq k_Y^*$, it is possible to find values of $k^i(t)$ in the neighborhood of $k^*$ for which there exist feasible values of $v_{\rm H}(t) \neq v^*$ such that part (a) of the static H–O hypothesis is violated at time t.*

Proposition 4 can also be combined with the earlier results on feasible paths for the open economy to examine the relationship between a country's initial endowments and its long-run trade pattern. If an economy experiences balanced growth for all $t$, then a country's relative factor abundance will remain constant and its trade pattern will reflect the static H–O hypothesis for all $t$. This case is consistent with the dynamic H–O hypothesis as stated in the introduction. Note however that there are unbalanced growth paths defined in Proposition 2(b) along which a country can switch from being physical capital abundant to being human capital abundant, so there can be no long-run relationship between trade patterns and initial factor endowments.

PROPOSITION 5. *For any endowment ratio satisfying (12), it is possible to find an unbalanced growth path such that the dynamic H–O hypothesis is violated.*

The failure of the dynamic H–O hypothesis results from the combination of the intertemporal no-arbitrage condition and the factor price equalization result that arises when open economies are incompletely specialized in production. With imcomplete specialization, the returns to factors in the open economy will be determined on world markets independently of domestic factor supplies. Open economies can alter their production patterns and raise the rate of physical relative to human capital accumulation, thereby altering their relative factor endowment without changing the relative return to human capital (which contrasts with closed economy models). This results from the fact that the open economy can absorb the increase in supply of physical capital by increasing its exports of goods that use physical capital more intensively. Moreover, the open economy is indifferent among these various relative factor endowments as factor prices are such that the intertemporal no-arbitrage, is satisfied and the returns to the two factors are equalized. When there is unbalanced growth, violations of the static H–O hypothesis arise as there exist paths for which a physical capital abundant country at time $t$ may import the physical capital intensive good at time $t$ for some factor proportions. The dynamic H–O hypothesis is violated as there may exist an unbalanced growth path along which an initially physical capital abundant country can become human capital abundant.

## 4. GAINS FROM TRADE

In this section, we examine the relationship between factor supplies and factor rewards in an autarkic economy. The path of prices and factor supplies in an autarkic economy will be given by the closed economy equilibrium analyzed in Section 2. By linearizing this system in the neighborhood of the BGP, we can derive the relationship between factor endowments and domestic prices during the transition to the BGP. We use these results to obtain the relationship between factor prices and goods prices for two countries that are at different points on the transitional path.

The following result characterizes the transitional dynamics of the closed economy model analyzed in Section 2.

PROPOSITION 6. *The BGP will exhibit saddle path stability for all factor intensity rankings.*

(a) *If $k_X > k_Y, p_X/p_Y$ will be a nonincreasing function of k along the transition path.*
(b) *If $k_X < k_Y$, then the relative price of physical to human capital, $p_X/p_Y$, will be constant along the transition path.*

This stability result extends that established in Bond et al. (1996) in a two-sector endogenous growth model in which there is a unified consumption/investment good sector and an education sector (a formal proof is provided in Bond et al., 2000). When $k_x > k_y$, an increase in the supply of human capital will expand the relative output of human capital. This makes the quantity adjustment process unstable at fixed prices, so the relative prices must be adjusting along the transition path. In contrast, when $k_x < k_y$, the quantity adjustment process is stable but the relative prices must jump immediately to the BGP value during the transition process.

Proposition 6 can be used to compare the autarkic factor prices and goods prices in two countries, $A$ and $B$, with $k^A > k^B$. First consider the case with $k_X > k_Y$. Assuming that the endowments of the countries are such that both countries are incompletely specialized in autarky, we must have $(p_X/p_Y)^A < (p_X/p_Y)^B$ from part (a) of the result. This confirms the intuition that the relative value of a unit of physical capital must be lower in the physical capital abundant country in autarky. In addition, the Stolper–Samuelson theorem of international trade theory will apply. Thus, $(p_X/p_Y)^A < (p_X/p_Y)^B$ implies $r^A < r^B$ when $k_X > k_Y$ (see the Appendix for proof). Finally, examination of the competitive profit conditions demonstrates that if both of the traded goods are being produced in each country, $r^A < r^B$ implies that $p_X^A < p_X^B$ $(p_X^A > p_X^B)$ if $k_X > k_Z$ $(k_X > k_Z)$ on $[r^A, r^B]$. This establishes that the price of the physical capital intensive traded good will be higher in autarky in the physical capital abundant country, confirming part (b) of the static H–O hypothesis.[14]

Next consider the case with $k_X < k_Y$. Proposition 6(b) establishes that for all endowments such that open economies are diversified in production at BGP prices, the autarky price will jump immediately to the BGP value. This implies that if $k^A$ and $k^B$ lie within the region consistent with incomplete specialization in autarky, there will be no differences in autarky prices across countries and there will be no gains from trade because national consumption levels are at their BGP values in autarky. The range of initial factor supplies consistent with incomplete specialization is obtained by utilizing the full employment conditions, (2), the trade balance condition, (9), subject to the requirement that output of the consumption good sector per unit of human capital be consistent with attainment of the BGP utility level which requires $u_Z^i(v^*, k_i) = c^*/j^*$. This yields a feasible range initial relative factor supplies of

$$k^i \in \left[u_Z^i(k^i, v^*)\, k_Z^* + \left(1 - u_Z^i(k^i, v^*)\right) k_X^*,\, u_Z^i(k^i, v^*)\, k_Z^* + \left(1 - u_Z^i(k^i, v^*)\right) k_Y^*\right] \tag{14}$$

Part (b) of the static H–O hypothesis will not hold in this region, as local factor endowments have no impact on prices and there will be no gains from trade.

It should be noted that the range of factor endowments in (14) is a subset of the range that is consistent with attaining the BGP utility level as derived in Propositions 2 and 3. The lower bound in (12) will be strictly greater than the lower bound consistent with the BGP utility level in Proposition 3(a) for the case $k_Y^* > k_X^*$. The upper bound in (14) will equal (exceed) the upper bound in Proposition 3(b) when $k_Z^* > (<)\ k_X^*$. This means that the opening of trade will expand the range of initial endowments for which the BGP utility level is attainable in the case.

PROPOSITION 7.

(a) *If $k_X > k_Y$, part (b) of the static H–O hypothesis holds. Gains from trade will exist for all $k^i \neq k^*$ in this case.*

[14] Note that the validity of part (b) of the static H–O imposes some restrictions on the technology because it requires that there be no factor intensity reversals on the region being considered.

(b) *If $k_X < k_Y$, factor prices are unaffected by local factor supplies and there are no gains from trade for countries whose endowments lie in the range given by (14). Trade will expand the range of endowments consistent with the BGP utility level.*

## 5. CONCLUDING REMARKS

In a dynamic H–O model where both human and physical capital endogenously accumulate, we have shown that when the conditions for factor price equalization are met, the composition of factor accumulation will be indeterminate. This indeterminacy results from the fact that if an open economy were to switch its investment toward human capital, it could adjust its output to produce more human capital intensive goods and increase its net exports of human capital intensive goods. Factor price equalization ensures that this change in composition of output does not lead to diminishing returns to human capital accumulation, and the intertemporal no-arbitrage condition ensures that the two paths are utility equivalent. While this indeterminacy results in the potential failure of the dynamic H–O hypothesis, the possibility of unbalanced growth leads to the violation of the static H–O hypothesis.

Several extensions are suggested by our results. In light of the general indeterminacy in long- and short-run trading patterns in this framework, it is of great interest to ask what additional features are required in this model in order to generate determinate long-run comparative advantage. In related work, Trask (1998) addresses this issue and illustrates that in a model that adds a third, fixed factor of production, although there remains no tendency for relative factor endowments to converge across countries, determinate long-run comparative advantage based on initial endowments will emerge. A second extension is to generalize household decision over human and physical capital, which may raise the possibility that households have preferences over the composition of their wealth and could possibly lead to convergence in relative factor endowments in a two-country model. One way to capture this is to follow Findlay and Kierzkowski (1983) and Stokey (1996), who differentiate between skilled and unskilled labor and endogenize the choice of whether to become skilled or to remain unskilled.[15] Finally, we could reexamine these issues for the case with positive human capital spillovers in production of traded goods. The presence of positive externalities may further differentiate roles of human from physical capital to break utility equivalence in the decentralized equilibrium and establish different dynamic properties to alter the gains from trade propositions.

[15] Findlay and Kierzkowski (1983) differentiate the accumulation of human capital from physical capital by considering an environment in which education requires time impacting the decisions of finitely lived agents. Stokey (1996), by contrast, incorporates time required for education by specifying a costly adjustment process for the accumulation of skilled labor. Her model is characterized by only one traded good and the international mobility of factors. Thus, all countries will converge to identical relative factor endowments as factors rather than goods are mobile.

## APPENDIX

PROOF OF PROPOSITION 1. From the necessary condition for (3), the wage can be expressed as the marginal product of human capital in the numeráire sector $Z$:

$$w = j(k_Z) - k_Z j'(k_Z) \tag{A.1a}$$

It is convenient to express the competitive profit conditions expressed in (4a) in the following form:

$$p_X f = w + rk_X \qquad p_Y g = w + rk_Y \qquad j = w + rk_Z \tag{A.1b}$$

The competitive profit condition for the $Z$ sector can be inverted to yield $w = \omega(r)$. With the interval $A \in \mathbb{R}^1$ denoting the domain of $\omega$, $\omega$ is continuous with

$$\omega'(r) = -k_Z < 0 \tag{A.2}$$

and can be used to solve for the prices consistent with operation of all sectors as a function of the rental rate, $p_i(r) = \varphi_i(\omega(r), r)$ for $i = X, Y$. This function $p_i(r)$ will be continuous on $A$, where differentiation of (A.1b) together with (A.2) gives

$$dp_X/dr = (k_X - k_Z)/f \qquad dp_Y/dr = (k_Y - k_Z)/g \tag{A.3}$$

Define the function

$$\xi(r) = \frac{r}{p_X(r)} - \frac{\omega(r)}{p_Y(r)} + \eta - \delta \tag{A.4}$$

which is continuous on $A$ by construction. Combining (A.2) and (A.3) yields

$$d(r/p_X(r))/dr = j/\left(p_X^2 f\right) > 0 \quad d(\omega(r)/p_Y(r))/dr = -jk_Y/\left(p_Y^2 g\right) < 0 \tag{A.5}$$

which implies $\xi'(r) > 0$. Part (a) of Condition E ensures that there exist $r$ sufficiently small that $\xi(r) < 0$ and $r$ sufficiently large that $\xi(r) > 0$, so there will be a unique value $r^*$ such that $\xi(r^*) = 0$. Since $\xi'(r) > 0$ holds regardless of the factor intensity rankings of the goods, this uniqueness result holds in the presence of factor intensity reversals. The growth rate of consumption on the BGP is obtained from (4) with $v_{p_X} = 0$.

The above analysis and part (c) of condition E ensures the existence of an $r^*$ consistent with $v^* > 0$. From (1a) and (1b), we have

$$u_X f^* = (v^* + \delta)k \qquad u_Y g^* = v^* + \eta \tag{A.6}$$

In order to have a feasible solution for the BGP, (2) must have a solution with $0 < u_i < 1$ for $i = X, Y, Z$. $u_Y > 0$ is ensured by (1b) with $v^* > 0$. $u_X > 0$ follows from (1a) if (2) has a solution with $k > 0$. Substituting (A.6) and (1c) with equality into (2) (with equalities), we can obtain the first equation of (8). A sufficient condition for $k^* > 0$ will be $g^* - v^* - \eta > 0$ and $f^* - (v^* + \delta)\, k_X^* > 0$. Since from (A.1) $f^* > (r^* k_x^*)/ p_x^*$, the latter condition will be satisfied if $(r^*/p_X^*) > v^* + \delta$. Utilizing (7), this can be rewritten as $(r^*/p_X^*) - \delta = \rho + \sigma v^* > v^*$. This inequality is obviously satisfied for $\sigma > 1$. For $\sigma < 1$, the assumed upper bound on the consumption growth rate can be used to rewrite the left-hand side of the inequality $(r^*/p_X^*) - \delta > (1 - \sigma)\, v_{\max} + \sigma v^*$ which must exceed $v^*$. A similar argument can be used to show that $g^* - v^* - \eta > 0$, using $g^* > w^*/p_Y^*$ and (7). Therefore, there will exist a unique $k^* > 0$ consistent with balanced growth, which ensures a unique $u_X > 0$.

It remains to be shown that $u_Z > 0$, which is equivalent to $c^* > 0$. Multiplying each of competitive profit conditions by the respective $u_i$ $(i = X, Y, Z)$ and summing up with the use of (2) in equalities, we get the national income identity, $p_X\, u_X f + p_Y u_Y\, g + u_Z j = w + rk$. Substituting into this equation (1c) with equality, (A.4) (recall that $w = \omega(r)$) and (A.6) give the solution for $c^*$. Since the right-hand side of this expression was shown to be positive above, we have $c^* > 0$. ■

PROOF OF PROPOSITION 4. From (13), consider a case in which $k^i = k^*$ and thus there does not exist a factor proportions motivation for trade. A violation of the static H–O theorem only requires that there exist values of $v_H^i$ such that $m_Z^i \neq 0$. Such values will exist under the conditions stated in the proposition. The condition $k_X^* \neq k_Z^*$ is necessary for one of the traded goods to be physical capital intensive. The knife edge case with $k_X^* = k_Y^*$ is one in which variations in the growth rate of human capital relative to physical capital have no effect on the physical/human capital ratio available for producing traded goods. ■

PROOF OF PROPOSITION 7(a). For the case of $k_X > k_Y$, the proposition requires that when all goods are produced, (i) $(p_X/p_Y)^A < (p_X/p_Y)^B$ implies $r^A < r^B$ and (ii) $r^A < r^B$ implies that $p_X^A < p_X^B$ $(p_X^A > p_X^B)$ if $k_X > k_Z$ $(k_X < k_Z)$ on $[r^A, r^B]$. Utilizing (A.1) and (A.3), we can derive

$$\text{(A.7)} \qquad \frac{d p_X /\, p_Y}{dr} = \frac{p_X}{p_Y} \frac{j(k_X - k_Y)}{p_X p_Y f g}$$

which implies (i). Then (ii) follows immediately from the differentiation results in (A.3). ■

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