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Heterogeneous borrowers, liquidity, and the search for credit

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Abstract

We develop a general equilibrium model of credit formation where borrowers and lenders must search for matches and where the composition of borrowers adjusts to satisfy equilibrium entry conditions. When market liquidity dries up as a result of fundamental shocks to the system, fewer borrowers will participate in the credit market with low-quality borrowers suffering disproportionately because of a flight to quality. However, less liquid credit markets need not be associated with lower social output, because the effect of higher average quality may outweigh that of reduced market participation, depending crucially on the source of the liquidity shock.

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1. Introduction

There is little doubt for economists these days that credit matters. Following Goldsmith (1969) and later King and Levine (1993), a number of empirical studies

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have found that credit market activity is positively related with the overall state of the economy.¹ The findings are broadly consistent with Walrasian models where financial intermediaries promote real activity by identifying high-return investments (Greenwood and Jovanovic, 1990) and through liquidity management (Bencivenga and Smith, 1991). However, recent empirical work (De Gregorio and Guidotti, 1995; Demetriades and Hussein, 1996; Luintel and Khan, 1999; Cunningham 1999; Rajan and Zingales, 2003) suggests that current theories have difficulties explaining reversals in financial development across time and across countries, reflected by situations where financial development does not move in conformity with real activity. Thus, a closer look at the processes and institutions that frame real and credit market activities is called for.

In this paper, we examine the nexus between credit market activity and real activity when credit markets are non-Walrasian. Our view builds on the pivotal work of Diamond (1990) who emphasizes that the ‘organization of the availability of credit is a critical determinant of the extent of liquidity.’ Specifically, *market liquidity*, or the ease by which funds can be raised in the aggregate, depends to some extent on the process of matching borrowers and lenders. This process is not instantaneous or costless because credit market participants have imperfect information regarding their economic opportunities. Thus, we observe entrepreneurs, especially new entrants and small business owners, who are constantly on the lookout for funds to finance their activities.² We also observe that lenders, particularly individual investors, loan brokers, as well as smaller and less established financial institutions, devote significant resources to identifying viable borrowers. Because agents must search for matching opportunities, market liquidity or the flow of new credit arrangements is endogenous.³ This non-Walrasian view of liquidity forces attention on the role of structural changes for the comovement of liquidity and real activity.

We develop a general equilibrium model of credit activity with pairwise meeting between borrowers and lenders. For illustrative purposes, we only consider investment loans that transform lenders’ endowments into productive projects by borrowing firms. Search frictions prevent instantaneous trading so that not all market participants are matched at a given point in time. Entry costs restrict unprofitable firms from participating in credit markets and prevent unmatched firms from searching too long. Once matched, borrowers and lenders enter into a

¹For a more comprehensive overview of the development of the literature, the reader is referred to recent survey articles by Becsi and Wang (1997) and Levine (2000).

²Access to financing as a barrier to entry for small firms is also stressed by Gertler and Gilchrist (1994). Evans and Jovanovic (1989) find that many entrepreneurs in the U.S. have been denied a loan but continued searching for funding. Blanchflower and Oswald (1998) report that interview surveys indicate raising capital is the primary problem of potential entrepreneurs.

³Goldsmith (1969) uses the assets of financial intermediaries (relative to GNP) to measure financial depth and liquidity. This empirical measure is closer in spirit to our idea of liquidity than the measures used later by McKinnon (1973) based on the liabilities of financial intermediaries and monetary aggregates. Recently, Dell’Ariccia and Garibaldi (2002) operationalize Goldsmith’s notion of liquidity and refer to it as aggregate credit.

partnership for the duration of a relationship, where Nash bargaining determines loan contract terms as a division of the surplus accrued between matched partners. Because the relative number of lenders and borrowers determines the probabilities of a successful match, the bargaining positions of market participants depend crucially on the relative *thickness of the credit market* (measured by the average funds available per lender).⁴ Since individuals focus on the effect of their actions on the surplus accrued and the threat points (or market values) but ignore the implications of their actions for the contact rates of other unmatched agents, a matching externality arises. Thus, the stripped-down version of our search model of credit markets has many features in common with Diamond's (1982) search model of goods markets and Pissarides' (1990) model of labor markets. Applying the search-theoretic approach to credit markets is useful as it emphasizes that one cost of allocating funds to their most productive use is idle projects and capital unemployment.⁵

Our paper extends the basic Diamond framework in three significant ways. First, we allow both market participation and matching probabilities to be determined endogenously, following the learning and labor matching framework developed by Laing et al. (1995). While market participation determines the size of the credit applicant pool, matching probabilities determine the capital unemployment rate. The importance of firm entry and exit for reallocating inputs has been stressed by Caballero and Hammour (1994). Here, entry helps endogenize the 'tightness' of the credit market, where tightness is not only an indicator of excess demand in the non-Walrasian setting but also determines the bargaining power of agents. Second, similar to Laing et al. (2002) for goods markets and Acemoglu (2001) for labor markets, we consider heterogeneous borrowers facing different risk, productivity and cost profiles, which are all common knowledge. This enables us to examine the interrelationship between liquidity and the 'composition' of borrowers when entry and exit of market participants are determined in equilibrium. That the composition of loans varies with economic activity has been documented by Bernanke et al. (1996) and Lang and Nakamura (1995) who find a flight-to-quality in credit markets when real activity weakens. Third, we compare the benchmark random matching technology (as in Diamond, 1990, and Rubinstein and Wolinsky, 1987) to an alternative framework with 'assortative' matching (see Becker et al., 1977) where lenders give priority to matches with high quality borrowers. This allows us to compare the proportion of productive and unproductive firms when markets select through competitive entry and exit

⁴Nash bargaining can be interpreted as the bilateral contractual arrangement used in Townsend (1978), though in his framework, the matching process by which trading agents are paired is not explicitly modeled. This is also consistent with the view (as expressed by Jaffee and Stiglitz, 1990) of the credit market as a customer market where contractual relationships between borrowers and lenders are established.

⁵See Ramey and Shapiro (2001) for a complementary explanation for idle or displaced capital that emphasizes thin resale markets for physical capital and costly search. Eisfeldt and Rampini (2003) also study capital reallocation but emphasize agency costs rather than search frictions.

with the proportion that arises when individual lenders are allowed to select (as in Greenwood and Jovanovic, 1990).⁶

We prove the existence of a unique nondegenerate steady-state search equilibrium with endogenous entry and heterogeneous agents. Then we analyze the effects of various fundamental shocks to the economy and show that the comparative static results of the model depend crucially on the response of endogenous matching rates and differential entry. Generally, we find that any shock that enhances matching rates causes aggregate liquidity to rise. While an increase in liquidity increases market participation by all firms, we show that low quality firms benefit disproportionately and the average quality of firms falls. Thus, fundamental shocks to the economy affect aggregate output by influencing not only the volume of trades (*market participation effect*) but also their average quality (*composition effect*). Accordingly, liquidity is procyclical for a given country and positively related to economic development across countries as long as composition effects are sufficiently small. Reversals, on the other hand, are a result of strong composition effects either in time series or cross sections. In other words, reversals reflect strong ‘flight to quality’ during a downturn or ‘flight from quality’ during an upturn.⁷

Our analysis also indicates that the balance of market participation and composition effects depends on the *source* of increased market liquidity. Fundamental shocks to firm profitability that enhance aggregate liquidity usually have strong market participation effects. Fundamental shocks to the organization of markets, however, may have strong composition effects. For instance, enhanced matching efficacy increases market liquidity and participation, but output and welfare may still fall when the composition effect is sufficiently strong. When the market shock is due to lower contract quit rates, however, the outcome is enhanced liquidity and strong market participation effects.

Apart from the aforementioned work by Diamond (1990), a few recent studies use a search framework to analyze various aspects of credit markets. Den Haan et al., (2003) consider a model of matching between borrowers and lenders that is also built on the framework of Pissarides. In their paper, borrowers have heterogeneous outcomes ex post as a result of an exogenous random funds allocation mechanism; in our paper, borrowers are intrinsically heterogeneous and face competitive equilibrium entry. Thus, some key findings are different. For example, a negative shock to market liquidity in their paper reduces market participation and output because it induces additional breakups of borrowing-lending relationships. In our paper, by contrast, fundamental shocks that reduce market liquidity also reduce

⁶One might also want to compare results with the middleman literature, which focuses on the emergence and equilibrium pattern of intermediated trade. Rubinstein and Wolinsky (1987) consider random matching under complete information, where middlemen emerge and capture a share of the matching surplus. Biglaiser (1993) constructs a bargaining model with asymmetric information where middlemen overcome informational inefficiencies. For Yavas (1994), middlemen become active when random matching is too ineffective.

⁷Since welfare is tied to output, welfare may fall when the composition effect is large, so that it is possible to have outcomes in a complete information framework that resemble results found in models with asymmetric information (cf. Biglaiser and Friedman, 1999).

market participation, though the blow to social output is offset because average quality also increases. Dell’Ariccia and Garibaldi (2002) compile data on gross credit flows in the U.S. that indicate dynamic patterns consistent with patterns predicted by search models in which the interaction between aggregate and idiosyncratic shocks generates simultaneous occurrence of credit expansion and contraction. Finally, Wasmer and Weil (2002) emphasize the interaction between loan and labor markets when both markets are distinguished by conventional search frictions. They emphasize that shocks to credit markets are transmitted to labor markets through changes in the creation of new firms and find that credit market frictions widen equilibrium unemployment.⁸

2. The basic model

To focus on the important aggregate variables, we first develop a benchmark framework without agent heterogeneity. Time is continuous and the world is populated with a continuum of identical lenders of unit mass and a continuum of identical borrowers (or firms) of mass I . The benchmark framework is similar to that of Diamond (1990) and features lenders and borrowers who are brought together by an anonymous random matching technology. Upon a successful match, a relationship is created and bilateral credit arrangements are negotiated. Specifically, symmetric Nash bargaining between lenders and borrowers determines the terms of the credit contract. In contrast to Diamond, the entry of firms and the rates at which borrowers contact lenders and lenders contact borrowers are determined endogenously.

Our lenders and the structure of the credit market are deliberately kept simple in order to later accommodate borrower heterogeneity. Lenders may be thought of as a rudimentary financial intermediary with a simplified liability side of the balance sheet. One may also think of them as a fusion of household and financial intermediary where the flow of funds between household and intermediary is certain and uninterrupted.⁹ A literal interpretation is that our lenders resemble the moneylenders found in informal rural credit markets in developing countries where capital markets are largely absent or in rudimentary form (Chandavarkar, 1987) or the merchant moneylenders that were prevalent in medieval Europe (Kohn 1999a). Merchant moneylenders prior to 1600 initially invested their own surplus funds by choosing between the relatively safe investment in their own business or the riskier investment for outside working capital. Over time merchant moneylenders expanded

⁸Caballero and Hammour (1999) emphasize some of the themes of the search-based credit market literature. Similar to Wasmer and Weil (2002), Caballero and Hammour stress the importance of financial markets for unemployment via the exit and entry of firms. They argue that two mechanisms drive factor reallocation. Though the details are different, their financial mechanism resembles our market participation effect and their selection mechanism resembles our composition effect.

⁹For a different type of liquidity allocation rule see Den Haan et al. (2003).

their scope and gradually developed into modern financial institutions; however, we limit ourselves to their original functions.¹⁰

We imagine that lenders are each endowed with a Lucas apple tree that generates flow income normalized to one unit. What lenders do with their income depends on the environment. Because there are many borrowers and lenders, the probability of rematches in a random environment is zero. Thus, IOUs from borrowers are not possible because they have no value for lenders. This captures the notion that firms can only obtain funds when they are in a relationship because only then does the lender have the required amount of control on the borrower's actions.¹¹ Autarky is the natural outcome under these circumstances and each lender is limited to consuming their flow value of one.

More interesting outcomes are possible, if lenders can make credit arrangements with firms that have access to a production technology. In this case, the lenders' unit flow income may earn positive returns. This is possible provided that lenders' saving can be converted into productive uses that yield a gross rate of return of R and provided that R exceeds the unit endowment flow.

Borrowing firms are rudimentary as well. They pop into existence by entering the loanable funds market. To enter, they must first pay a one-time fixed cost, v_0 , after which they then search for working capital to run their investment project.¹² This entry cost is a sunk cost and captures the setup costs associated with starting a project or business such as equipment and structural capital. The assumption of a fixed entry cost is not only realistic, but simplifies the analysis greatly.¹³ When matched with a lender, a firm has access to working capital, which allows it to produce flow output A with probability p and flow output zero with probability $(1 - p)$.

Our loanable funds market also features spatially separated borrowers and lenders. Therefore, pairwise meetings are not instantaneous and we assume a Poisson arrival process for contacts between lenders and borrowers. Specifically, we designate μ as the lender's contact rate (or flow probability of meeting a firm) and η as the firm's contact rate (or flow probability of meeting a lender). For finite contact rates, individual lenders or firms may either be matched or unmatched. We denote H as the mass of searching and unmatched lenders and F as the mass of searching firms. In addition, the mass of matched lenders is denoted by S , which by

¹⁰Kohn (1999a) provides a historical overview of the role of moneylenders who intermediated loans for investment in outside working capital as well as for dowries (such as Shylock in Shakespeare's *Merchant of Venice*). In our paper, we focus primarily on production loans.

¹¹According to Kohn (2000), lending in the Middle Ages was limited by the difficulty of finding people and by high trading costs, which arose because of slow communications, high costs of carriage, and a general vulnerability to predation.

¹²In this model, we do not allow lenders to have an entry choice by fixing their mass. This is because the consideration of the endogenous entry of homogeneous lenders would not add any additional insight towards understanding the working of the credit market.

¹³Thus, our fixed entry costs differ from the flow entry costs considered in Pissarides (1990), but they resemble the costs used in Laing et al. (1995).

construction equals the mass of matched firms. Thus, if we set the mass of lenders to one, we have $S = 1 - H$.

In the event that lending is unprofitable, or $R \leq 1$, unmatched lenders will choose to consume their flow endowment. But if lending is profitable or $R > 1$, lenders choose to search for a match in the loanable funds market. Once matched with a firm, they lend their unit endowment and consume the returns of their savings, R . For simplicity, the length of the lending contract is fixed at $1/\delta$. Thus, the flow probability of separation, or the contract quit rate, of a matched lender–borrower pair is δ and the expected total repayment to the lender over the life of the contract can be computed as R/δ . Upon separation, lenders and firms go back to face an anonymous matching process in a credit market where enduring relationships are not guaranteed. In other words, matched lenders provide firms with relationship-specific capital in a partnership of fixed duration.¹⁴

We now formalize the flow value associated with searching/unmatched and matched lenders. Denote J_u as the value associated with an unmatched lender and J_m as the value associated with a lender matched with a firm. We then have:

$$rJ_u = 1 + \mu(J_m - J_u), \quad (1a)$$

$$rJ_m = pR + \delta(J_u - J_m). \quad (1b)$$

Eq. (1a) says that the flow value associated with an unmatched lender is the sum of the flow rate of consumption of the endowment good and the net values gained from being matched with a firm ($J_m - J_u$) which arrives at rate μ . Eq. (1b) says that the flow value of a lender matched with a firm is the sum of the expected returns to the match generated from the loan contract R and the net value of terminating the lending contract and re-entering the unmatched state.¹⁵

Similarly for firms, let Π_u and Π_m denote, respectively, the unmatched and matched value associated with a firm. These asset values can be specified as:

$$r\Pi_u = \eta(\Pi_m - \Pi_u), \quad (2a)$$

$$r\Pi_m = p(A - R) + \delta(\Pi_u - \Pi_m). \quad (2b)$$

Eq. (2a) gives the flow value of an unmatched firm as the product of the rate by which firms contact searching lenders, η , and the net value of becoming matched ($\Pi_m - \Pi_u$). Eq. (2b) specifies the flow value of a matched firm as the sum of the net expected productivity of the investment project made possible by the loan contract, less the interest costs, and the net value of terminating the lending contract.

¹⁴As Kohn (1999b) indicates, the financing form favored by commercial enterprises prior to 1600 was the partnership, which featured periodic liquidation to distribute returns, non-transferable shares, and personal oversight to protect investor interest.

¹⁵If repayment is delayed until the time of contract termination, (1b) changes to $rJ_m = p\delta R + \delta(J_u - J_m)$ and δR becomes the appropriate measure of the gross interest rate. This entire analysis would go through with any substantive changes.

Using (1b) and (2b), the potential (ex ante) gains that accrue from a successful match become:

$$J_m - J_u = \frac{pR - rJ_u}{r + \delta}, \quad (3a)$$

$$\Pi_m - \Pi_u = \frac{p(A - R) - r\Pi_u}{r + \delta}. \quad (3b)$$

Since atomistic borrowers face unrestricted entry, they will take their threat points, the unmatched value Π_u , as parametric in the process of bargaining. The threat point of the borrower can be thought of as the firm's reservation value and will be determined as part of equilibrium through competitive entry of firms.¹⁶

Consider a cooperative Nash bargain which gives a share β of the matched surplus to lenders and $1 - \beta$ to firms. The bargaining weight β parameterizes the relative bargaining power of the lenders. Bargaining entails solving $\max_R (J_m - J_u)^\beta (\Pi_m - \Pi_u)^{1-\beta}$ subject to (3a) and (3b), taking both J_u and Π_u as given. Thus, the bargaining outcome must satisfy the following first-order condition:

$$\frac{\Pi_m - \Pi_u}{1 - \beta} = \frac{J_m - J_u}{\beta}. \quad (4)$$

Because firms are atomistic and competitive with their entry decisions based on the expected bargaining outcome, the market value of Π_u must be treated parametrically when solving for R . However, we can substitute (1a) into (3a) to eliminate J_u . Thus, the interest offer function, or the loan rate schedule agreed upon by a matched borrower and lender via Nash bargaining, is contingent on the state of the economy

$$pR - 1 = \frac{\beta(r + \delta + \mu)(pA - 1) - r\Pi_u}{r + \delta + \beta\mu}. \quad (5)$$

Specifically, the interest offer function increases with the lender contact rate, μ , and with productivity, A , but it decreases with the unmatched value of firms, Π_u , and the contract quit rate, δ . Intuitively, more contact possibilities for lenders increases their bargaining position relative to borrowers and leads them to charge higher loan rates. Increased productivity by firms raises the joint surplus and allows lenders to charge more. By contrast, a higher reservation value of firms enhances their bargaining position and reduces the loan rate that firms are willing to pay. Finally, greater contractual fragility and shorter relationships lower lenders' bargaining position vis-a-vis borrowers.

Steady-state matching in the loanable funds market requires that the flow of firms seeking loanable funds ηF equals the flow of lenders providing loanable funds μH . Either of these measures new credit arrangements or the flow of funds from lenders

¹⁶For a detailed description of the atomistic bargaining process with competitive or unrestricted entry, see Pissarides (1984) and related work cited in Laing et al. (1995). By contrast, the search-and-bargaining literature typically assumes an exogenous reservation value of zero. This rules out fixed entry costs for firms and hence makes it necessary to assume flow entry costs in order to tie down the number of firms in equilibrium.

to (new) borrowers and we use them as our flow measure of market liquidity, ℓ . The flow liquidity measure must also by construction equal the flow of matches according to a random matching technology. These arguments imply:

$$\mu H = \eta F \equiv \ell, \tag{6a}$$

$$\ell = m_0 \tilde{M}(H, F), \tag{6b}$$

where $\tilde{M}(\cdot, \cdot)$ is a random funds-matching function that satisfies the following properties: strictly increasing and strictly concave in each argument, homogeneity of degree one, standard Inada conditions and boundary conditions [i.e., $\tilde{M}(0, \cdot) = \tilde{M}(\cdot, 0) = 0$]. The scaling parameter m_0 measures the efficacy of the credit market, which can be thought of as an indicator of structural or technological reforms in the credit market. For example, an increase in m_0 may indicate an improvement in the efficiency of financial intermediation arising from being able to identify easier the sources of loanable funds and lending opportunities.

We create a population balance condition that is consistent with the matching technology by combining the last two equations. Specifically, if we divide through by the second argument in the matching function and substitute for H/F , we have a ‘Beveridge curve’ for the loanable funds market

$$\eta = m_0 M\left(\frac{\eta}{\mu}\right). \tag{7}$$

This relationship characterizes the firm contact rate η as a negative function of the lender contact rate μ . For convenience later, we express our Beveridge curve using matching probabilities (as in Laing et al., 1995) rather than populations (as in Pissarides, 1984).

With these specifications, we can characterize the Beveridge curve with

Lemma 1. (Beveridge curve). *The Beveridge curve is downward-sloping in (μ, η) -space and convex, asymptotes to both axes, and shifts away from the origin as the matching parameter, m_0 , increases.*

Another steady state requirement is that the flows into the loanable funds market must equal flows out of the market. If we combine this population balance condition with a stock measure of market liquidity, it is possible to characterize liquidity quite simply by the number of agents with loans. Specifically, recall that for a given contract quit rate δ , the duration of the contract is measured by $1/\delta$. Our flow measure of liquidity is therefore translated into a stock equivalent by $L = (1/\delta)\ell$. Because the lender population in steady state is fixed at unity, the inflow of lenders who enter to search for projects (after having been separated from other projects) must equal the outflow from the market. We thus obtain:

$$\mu H = \delta S, \tag{8a}$$

$$L = \frac{\ell}{\delta} = \frac{\mu H}{\delta} = S, \tag{8b}$$

where (8b) follows by applying the steady-state conditions (6a) and (8a). We conclude that the stock measure of market liquidity in our framework is simply the population of matched market participants. Because it measures the stock of lenders' assets, our stock measure of liquidity is consistent with the conventional empirical measure of liquidity originated in Goldsmith (1969).

To close the model, we must consider the endogenous entry of firms. Firms will enter the credit market as long as the unmatched value of participating in the credit market exceeds their initial cost of entry. Because competitive entry by firms causes the unmatched values of firms to be driven down to their entry cost, we have¹⁷

$$\Pi_u = v_0 \quad (9)$$

Competitive entry by firms also affects their profitability. Substituting (3b) into (2a) and combining the result with (9) yields the zero-profit or ZP condition

$$\eta^{ZP} = \frac{rv_0(r + \delta)}{p(A - R) - rv_0}. \quad (10)$$

Straightforward differentiation of the ZP condition implies:

Lemma 2. (Unrestricted entry). *The firm contact rate that satisfies the zero profit condition rises with the entry cost, v_0 , the interest offer, R , and the quit rate, δ ; it falls when the expected productivity, pA , rises.*

The underlying intuition is clear-cut once we keep in mind that zero profit requires a negative relationship between the net gains of firms accrued from a successful match and the firm contact rates. As net gains rise, more firms tend to participate in the credit market (to restore zero profit). However, having more firms lowers the probability that an individual firm will locate a lender.

We now put forth a formal definition of equilibrium that is congruent with our model:

Definition. A steady-state equilibrium is a tuple $R, \mu, \eta, H, S, F, \Pi_u$ that satisfies: (i) Nash bargaining, (5); (ii) steady-state matching and separation, (6), (7) and (8); (iii) free entry and zero profit, (9) and (10); and, (iv) population identity, $S + H = 1$.

Note that the free entry conditions immediately pin down Π_u at v_0 , whereas (5) gives R as an increasing function of μ . The latter relationship can be substituted into (10) to yield an upward-sloping ZP locus in (μ, η) space. We graph this ZP locus in Fig. 1 together with a downward-sloping Beveridge curve from (7). The intersection of the ZP locus and the Beveridge curve determines the equilibrium contact rates in steady state, μ^*, η^* . Straightforward comparative-static analysis using Fig. 1 shows that the equilibrium lender contact rates depend positively on matching efficacy (m_0) and productivity (A), whereas the effects of the entry cost (v_0) and the quit rate (δ) on μ^* are ambiguous.

¹⁷This, of course, requires that the ex-ante population of firms I is sufficiently large.

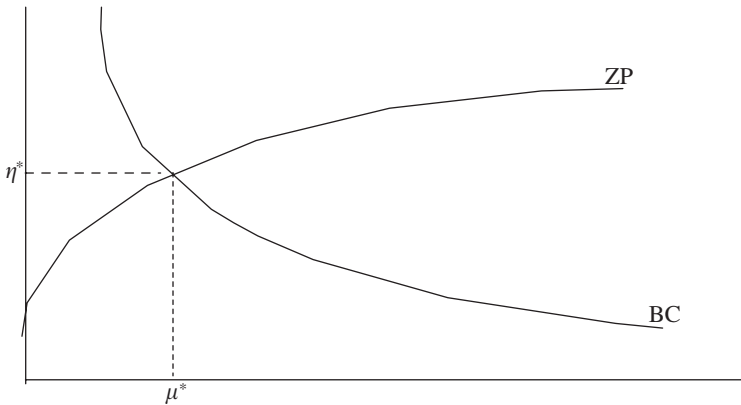


Fig. 1. Steady-state search equilibrium with homogeneous borrowers.

One can then use these equilibrium contact rates with (6), (8) and (9) to solve for the equilibrium masses of firms and lenders:

$$H^* = \frac{\delta}{\delta + \mu^*}, \tag{11a}$$

$$S^* = 1 - H^* = \frac{\mu^*}{\delta + \mu^*}, \tag{11b}$$

$$F^* = \frac{\delta \mu^*}{(\delta + \mu^*) \eta^*}. \tag{11c}$$

These three equations imply that the mass of searching lenders H is negatively related to the lender contact rate. Also, the mass of matched lender-firm pairs S and hence (the stock measure of) market liquidity depends positively on the lender contact rate. Moreover, the mass of searching firms F is increasing in the lender contact rate but decreasing in the firm contact rate. However, higher quit rates appear to have conflicting direct effects, because they increase flow liquidity (or the outflow δS) and also reduce stock liquidity. The explanation is simply that as the outflow increases, the inflow μH must also rise through an increase in H . Thus, heightened contractual fragility leads to more churning and search behavior by destroying existing relationships but also increases the funds available for loans. Finally, it is useful to point out that Eq. (11b) pins down the equilibrium value of social output, $Y^* = p S^* A$. Social output is increasing in the productivity of firms and the lender contact rate but decreasing in the quit rate. In summary, we have:

Proposition 1. (Equilibrium with homogeneous borrowers). *Provided that $(pA - 1) - rv_0 > 0$, a unique nondegenerate steady-state equilibrium with homogeneous borrowers exists, where the interest rate, the lender contact rate and social output are increasing in matching efficacy and firm productivity.*

3. From homogeneous to heterogenous borrowers

In this section we extend our model and allow heterogeneous firms to search for loanable funds. We are mainly concerned with developing the complications and some of the intuition introduced by ex ante heterogeneity and proving existence and uniqueness of the heterogeneous agent equilibrium. Then, in the next section, we show how the equilibrium responds to various exogenous shocks to the system.

We limit ourselves to two types of firms indexed by i with mass I^i . Firms have different riskiness, productivity, and set-up costs. While a firm's type is known to all, the number of firms of each type is determined by unrestricted entry with differential costs. The type 1 firm has access to a low-risk, low-return investment project, whereas the type 2 firm has access to a high-risk, high-return project. Membership in the population set I^i is determined by a random lottery. One must therefore identify firm types by adding superscripts i to the notation defined previously. Let N^i ($N^1 + N^2 = 1$) denote the (endogenous) fraction of type i firms entering the loanable funds market, which need not be the same as the ex ante population share I^i . Thus, $N^i F$ represents the population of type i firms searching for funds.

More specifically, we assume throughout the paper that the type 2 firms are more productive both in absolute terms and on average, but face a lower success rate. In other words, we assume more productive firms are also riskier. Moreover, the type 2 firms pay a higher entry fee, yet their production gains after subtracting entry costs still remain above that of the type 1 firms. This captures the idea that more productive technologies and projects often require greater set-up costs and face greater riskiness of failure. As an aid to memory, we will sometimes refer to the type 1 firm as the low quality firm and the type 2 firm as the high quality firm. Furthermore, to ensure that firms actively produce we assume that the net production gain (after subtracting entry costs) of all types exceeds the return of being idle. This means that we ignore the generic degenerate equilibrium under which some firms remain inactive. Summarizing, our basic assumptions are:

- (A1) (productivity) $A^2 > A^1$,
- (A2) (success rate) $p^2 > p^1$,
- (A3) (entry cost) $v_0^2 > v_0^1$,
- (A4) (net production gain) $p^2 A^2 - r v_0^2 > p^1 A^1 - r v_0^1$,
- (A5) (active operation) $p^1 A^1 - 1 > r v_0^1$.

Clearly, Assumptions (A4) and (A5) together are stronger than simply having expected output ordered as $p^2 A^2 > p^1 A^1 > 1$. As to be discussed later, this stronger set of assumptions is sufficient to rule out the possibility of multiple equilibria.

With heterogenous borrowers, the value functions remain the same with superscript i added to all relevant variables. The only exception is the value function for unmatched lenders must be amended to allow for the possibility of contacting one or the other type of firm

$$rJ_u = 1 + \mu^1(J_m^1 - J_u) + \mu^2(J_m^2 - J_u). \quad (12)$$

With different types of firms, the lender contact rates must be proportional to the relative masses of the firms, or $\mu^i = \mu N^i$ for $i = 1, 2$ whereby $N^1 + N^2 = 1$. In other words, firms do not possess differential abilities to search out lenders and the same goes for lenders, hence lenders have a common contact rate.

Under these circumstances, Nash bargaining implies an interest offer function for a borrower of type i when borrowers are heterogeneous ex ante

$$p^i[A^i - R^i] - r\Pi_u^i = \frac{1 - \beta}{\beta} \frac{r + \delta}{r + \delta + \mu} \left\{ (p^i R^i - 1) + \frac{\mu N^j}{r + \delta} [(p^i R^i - 1) - (p^j R^j - 1)] \right\}. \tag{13}$$

It can be verified that this equation reduces to (5) when borrowers are homogeneous. The most interesting aspect of this expression, is that it clearly illustrates that the outcomes of both types of firms are interdependent. The reason for this interdependence is simply that the threat point of lenders depends on the expected returns of both types.

Before conducting an equilibrium analysis, we first totally differentiate (13) to characterize the ‘interest offer function,’ conditional on the interest offer for borrowers of another type. This step is informative mainly because it allows us to understand the direct effects in the Nash bargain in isolation from the interactive effects. Thus, we find:

Proposition 2. (Interest offer). *The interest offer function $R^i(\mu, N^i; A^i, \Pi_u^i, \delta)$ is increasing in the lender contact rate, μ , and the own-type productivity, A^i , but decreasing in the fraction of low-type firms, N^1 , the own-type unmatched value of firms, Π_u^i , the other-type interest offer, R^j , and the quit rate, δ .*

While the results are straightforward, the response of the interest offer to the (endogenous) composition of firms deserves further comment. If more type-1 or low quality firms enter the loanable funds market, the share of low quality firms rises. According to (12), a rise in N^1 lowers the unmatched value of lenders (J_u), which is their bargaining threat point. Because lenders’ bargaining power falls, the interest offer falls.

Next, we note that all the steady-state conditions also remain valid. However, the zero-profit conditions for type- i firms ($i = 1, 2$) can be written as:

$$\eta^{ZPi} = \frac{rv_0^i(r + \delta)}{p^i[A^i - R^i] - rv_0^i}. \tag{14}$$

This equation is the same as (10) except for the subscripts that distinguish different firms. Hence, the zero-profit matching rate η^{ZPi} has identical properties to those described in Lemma 2.

We have already defined the most important features of steady-state equilibrium in the homogeneous case. With heterogeneous borrowers, the definition needs to be modified slightly. In particular, the composition of types is summarized by N^i and is determined by $\eta^{ZPi} = \eta$ and $N^1 + N^2 = 1$. The condition on η states that all firms

face the same contact rate in the anonymous random matching environment since they do not possess differential search abilities. Although the population masses can still be solved recursively using (11a)–(11c), the remaining equations that define an equilibrium all involve the composition variable, N^1 . The remainder of this section will use two bargaining equations, two zero profit conditions (with identical η) and the Beveridge curve to jointly solve for two interest rates, two contact rates, and N^1 .

Eq. (13) describes a system of equations that we would like to solve. In order to proceed, we define for notational convenience, let $B \equiv [(1 - \beta)/\beta](r + \delta)/(r + \delta + \mu)$, $\bar{a}^i \equiv p^i A^i - rv_0^i$ and $a^i \equiv \bar{a}^i + B$, where we note that \bar{a}^i defines the expected net output of firm i and $dB/d\mu < 0$. Also, we let $u^i \equiv N^i u$, and $u \equiv \mu/(r + \delta) = u^1 + u^2$ where $du^1/dN^1 > 0$, $du^2/dN^1 < 0$ and $du/d\mu > 0$. Then we can rewrite the Nash bargaining conditions (13) as

$$a^i = (1 + B + Bu^i)p^i R^i - Bu^i p^j R^j, \quad i = 1, 2, \quad i \neq j.$$

This system yields the solution

$$p^i R^i = \frac{\bar{a}^i + B}{1 + B} + \frac{(1 - \beta)\beta\mu}{r + \delta + \beta\mu} (\bar{a}^j - \bar{a}^i) N^j. \tag{15}$$

It is clear that Assumption (A5) allows low type firms to operate actively, i.e., $p^1 R^1 > 1$, which is given by $\bar{a}^1 > 1$.¹⁸ Assumptions (A4) and (A5) imply the following ordering of expected net outputs $\bar{a}^2 > \bar{a}^1 > 1$. This ordering is sufficient to guarantee active operation for both high and low quality firms, or $J_m^i > J_u$. Thus, we rule out the possibility that a particular type of firm is shut out of the market.

In order to work out the full comparative statics of the model, it is useful to document some intermediate results:

Lemma 3. (Expected interest rates). *Under Assumptions (A1)–(A5), we have the following:* (i) $\partial p^1 R^1 / \partial N^1 = \partial p^2 R^2 / \partial N^1 < 0 < \partial p^1 R^1 / \partial N^2 = \partial p^2 R^2 / \partial N^2$; (ii) $\partial p^1 R^1 / \partial \mu = \partial p^2 R^2 / \partial \mu > 0$; (iii) $\partial p^1 R^1 / \partial \delta = \partial p^2 R^2 / \partial \delta < 0$; and, (iv) $\partial p^i R^i / \partial v_0^i < 0$ for $i = 1, 2$.

Proof. See Appendix. \square

Intuitively, a change in the proportion of types of firms in the market, N^i , impacts the threat points of lenders and firms. An increase in N^1 lowers the threat point of a matched low quality firm and tends to increase the expected returns to the lender implied by the bargaining solution. However, it also lowers the threat point of lenders matched to low quality firms by lowering their unmatched value J_u . Because this latter effect dominates, $\partial p^1 R^1 / \partial N^1 < 0$. An increase in N^1 also lowers J_m but increases the threat point of high quality firms, both effects leading to $\partial p^2 R^2 / \partial N^1 < 0$. Similarly, an increase in the fraction of high quality firms, N^2 ,

¹⁸This is clear by rewriting (15) as:

$$p^1 R^1 - 1 = \frac{\bar{a}^1 - 1}{1 + B} + \frac{(1 - \beta)\beta\mu}{r + \delta + \beta\mu} (1 - N^1)(\bar{a}^2 - \bar{a}^1).$$

strengthens the relative bargaining position of lenders matched with both high and low quality firms so that $\partial p^1 R^1 / \partial N^2 > 0$ and $\partial p^2 R^2 / \partial N^2 > 0$. By contrast, an increase in v_0^1 lowers the net gains to a match for both the lender and firm. However, firms lose disproportionately more and hence a reduction in the expected returns to the lender is required to satisfy the bargaining rule. The effects of entry costs are entirely symmetric and opposite to the effects of changes in productivity.

Using (15), we can compute the expected interest rate spread between high and low quality firms ($p^2 R^2 - p^1 R^1$) and the actual interest rate spread ($R^2 - R^1$). It is important for internal consistency to verify that these spreads are nonnegative. Thus,

$$p^2 R^2 - p^1 R^1 = \beta(\bar{a}^2 - \bar{a}^1), \tag{16}$$

$$R^2 - R^1 = \beta \left(\frac{\bar{a}^2}{p^2} - \frac{\bar{a}^1}{p^1} \right) + \frac{1 - \beta}{r + \delta + \beta\mu} [\beta\mu(N^1 \bar{a}^1 + N^2 \bar{a}^2) + r + \delta] \left(\frac{1}{p^2} - \frac{1}{p^1} \right). \tag{17}$$

From the way that we have written them, it is clear that the spreads are nonnegative. This is true even in a frictionless economy where search and entry frictions disappear, because as we can see $\lim_{m_0 \rightarrow \infty, v_0^i \rightarrow 0} R^2 - R^1 = \beta(A^2 - A^1) + (1 - \beta)(1/p^2 - 1/p^1)(N^1 p^1 A^1 + N^2 p^2 A^2 - 1) > 0$, which depends on the productivity as well as the risk differential. Also, it is evident that the difference between the potentially unobservable ex ante credit spread and the observed ex post credit spread is that the latter varies systematically with aggregate economic conditions. This agrees with recent work by Collin-DuFresne et al. (2001) that credit spreads widen in times of economic uncertainty or recessions. Specifically, we can show:

Proposition 3. (Interest rate spreads). *Under Assumptions (A1)–(A5), both the expected and the actual interest rate spreads are positive. While the expected interest rate spread is only determined by the expected profitability differential, the actual interest rate spread also depends negatively on the share of low quality firms and positively on the lender contact rate.*

Proof. See Appendix. □

Under Assumption (A4), the productivity differential between high and low quality borrowers is sufficiently large relative to the entry cost differential so that $\bar{a}^2 > \bar{a}^1$. Hence, the interest paid by type 2 always exceeds the interest paid by type 1 in both expectations and realization. The reason why the actual interest rate spread depends on the share of low quality firms (N^1) and the lender contact rate (μ), in addition to the expected net productivity differential, is because participation and matching externalities are regarded as parametrically given by individual players.¹⁹

¹⁹In a frictionless economy where matching is instantaneous (as in Yavas, 1994) and firm entry is costless (i.e., $m_0 \rightarrow \infty$ and $v_0^i \rightarrow 0$), productivity and risk differentials alone pin down the actual rate spread. When matching is not instantaneous (as in Rubinstein and Wolinsky, 1987) and when there are entry frictions, the actual rate spread becomes smaller because of the composition effect. Also, it is interesting to note that both the composition effect and contact-rate effect on the actual rate spread diminish as firms' bargaining power $(1 - \beta)$ decreases.

We are now ready to consider the determination of steady-state equilibrium using the properties of the interest rate function considered above. From Proposition 2, we can write $R^i = R^i(\mu, \eta, N^1)$ where $R^i_\mu > 0, R^i_\delta < 0, R^i_{N^1} < 0$. Inserting the interest functions R^i into (7) and (14), we show that the steady-state tuple $\{\mu^*, \eta^*, N^{1*}\}$ thus satisfies the Beveridge curve and the individual ZP conditions. The proof is similar to the homogeneous borrower case, except that now two ZP conditions must be met simultaneously in order to determine the equilibrium share N^{1*} . With this, we offer the following result:

Lemma 4. (Equilibrium zero-profit trace). *Both the ZP¹ and ZP² loci from (14) are downward sloping in (N^1, η) -space with $|d\eta^*/dN^{1*}|_{ZP^1} > |d\eta^*/dN^{1*}|_{ZP^2}$. Furthermore, there exists a unique and upward sloping equilibrium zero-profit trace EZ in (N^1, η) -space, $\eta = \eta^Z(N^1)$ that satisfies (14) for a given μ such that there is a $N^{1*}_{min} > 0$ yielding $\eta^{Z(N^{1*}_{min})} = 0$ and $\infty > \eta^{max} \equiv \eta^Z(1) > 0$.*

Proof. See Appendix. □

To better understand Lemma 4, we graph the ZP¹ and ZP² loci and the EZ trace in (N^1, η) -space in Fig. 2. How these curves relate to the Beveridge curve (denoted BC) in (μ, η) -space is also shown in Fig. 2. Together these relationships pin down the steady-state equilibrium $\{\mu^*, \eta^*, N^{1*}\}$. However, it must be noted that Assumption (A4) is critical for Lemma 4. Without this assumption, multiple equilibria may emerge because we cannot guarantee that the two ZP loci only intersect once. Once multiple crossings of the ZP loci are allowed, uniqueness of the zero-profit trace is not warranted.

It is instructive to understand intuitively why the economy operates on the EZ trace where the ZP loci intersect. Define the expected net surplus differential between high and low type firms, or, in short, the firm net surplus differential as:

$$D \equiv \frac{p^2[A^2 - R^2]}{rv_0^2} - \frac{p^1[A^1 - R^1]}{rv_0^1}.$$

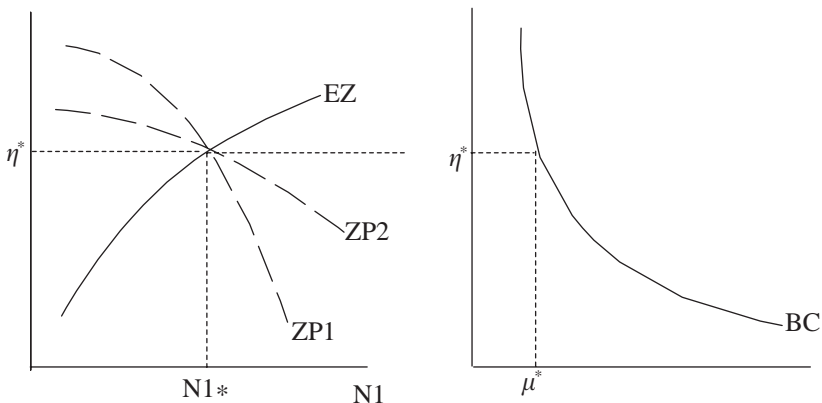


Fig. 2. Steady-state search equilibrium with heterogeneous borrowers.

Then we have:

Lemma 5. (Expected net surplus differential). *The net surplus differential between high and low type firms, D , possesses the following properties:*

- (i) $D = 0$ when both ZP conditions in (14) hold true, $\dot{N}^1 > 0$ and $\partial D / \partial N^1 < 0$ for any given $D > 0$.

Proof. Part (i) follows directly from Eq. (14). To prove part (ii), note that if we rewrite D using (16), D can be shown to only depend on the expected return of the low-quality firm,

$$D = \{(1 - \beta)(\bar{a}^2 - \bar{a}^1) - [p^1(A^1 - R^1) - rv_0^1](v_0^2 - v_0^1)/v_0^1\} / (rv_0^2).$$

The share of low-quality firms N^1 affects D through R^1 since relative entry costs differ by Assumption (A3). Thus, when N^1 rises, D falls because, as Lemma 3 demonstrates, having relatively more low-quality firms will drive down their expected return $p^1 R^1$. When D is positive, N^1 must rise to drive the firm surplus differential back down to equilibrium. To see this, notice that when N^1 rises, lenders are more likely to contact low type firms. This weakens the ability of lenders to extract a surplus from low-quality firms. The reduced bargaining position of lenders vis-a-vis low quality firms means that the expected return that can be extracted from their production will fall so that D falls accordingly and zero profit is restored. \square

Concerning part (ii) of Lemma 5, we note that N^1 is endogenous, so it will respond to other forces. For example, an increase in μ^* or a decrease in δ will ultimately increase N^1 .²⁰ These arguments are useful for understanding the comparative statics derived in Section 5. We are now prepared to establish:

Theorem. (Existence and uniqueness). *Under Assumptions (A1)–(A5), there exists a unique, non-degenerate steady-state equilibrium with full information if the expected production gains are sufficiently high such that $\eta^* \in (0, \eta^{\max})$.*

Proof. Existence and uniqueness will be proved in two steps. First, we claim that the BC and EZ loci uniquely determine steady-state $\{\mu^*, \eta^*, N^{1*}\}$. It is clear from the proof of Lemma 3 and expression (14) that as long as the expected production gains are sufficiently high such that $\eta^* \in (0, \eta^{\max})$, N^{1*} is bounded in the interval $(0, 1]$. Then, because the determinant of the pre-multiplied matrix of systems (7) and (14) is strictly positive, the implicit function theorem implies a unique steady-state $\{\mu^*, \eta^*, N^{1*}\}$. Thus, for a given pair $\{\mu, \eta\}$ satisfying (BC), there exists a unique pair $\{\eta, N^1\}$ that satisfies (EZ). Once we obtain the equilibrium matching rates μ^* and η^* and the fraction of low quality firms N^{1*} , we can use (11a)–(11c) to solve for the equilibrium masses $\{H^*, S^*, F^*\}$, and market liquidity L^* as well as $F^{1*} = N^{1*} F^*$ and

²⁰We note that

$$\frac{dD}{d\delta} = \frac{v_0^2 - v_0^1}{rv_0^1 v_0^2} \frac{dp^j R^j}{d\delta} < 0 < \frac{dD}{d\mu} = \frac{v_0^2 - v_0^1}{rv_0^1 v_0^2} \frac{dp^j R^j}{d\mu}.$$

$F^{2*} = (1 - N^{1*})F^*$. Because Eqs. (11a)–(11c) are all well-defined monotone functions, the determination of these masses is also unique. \square

4. Comparative statics under differential entry of heterogeneous borrowers

We are now prepared to characterize how the steady-state equilibrium with heterogeneous borrowers responds to various fundamental shocks. We consider fundamental shocks to structure of the credit market (such as changes in the efficacy of search and contractual fragility) and fundamental shocks to the firms' profitability from changes in productivity and entry costs. We are particularly interested in the general equilibrium response of the matching rates, the composition and the mass of the matched firms, and gross interest rates. We are also interested in the fraction of unmatched projects, the aggregate output produced by matched firms, and welfare.

First, we discuss various measures of the depth and breadth of the credit market. From (11b) we note that the equilibrium number of matches S^* is positively related to μ^*/δ . As discussed above, this term reflects market liquidity L^* because it is also equal to the aggregate share of lender funds that is channeled to firms and production. The size of the credit market is measured by $S^* + F^*$. This sum adds market participants that are matched to those that are unmatched and still searching. Also, we define $U^* = F^*/(F^* + S^*)$, which is the share of unmatched projects in the credit applicant pool. Because U^* measures the tightness of the credit market much like the unemployment rate in the labor market, we will call it the 'capital-unemployment rate.' Because $F^* = S^*(\delta/\eta^*)$, we find that $U^* = 1/(1 + (\eta^*/\delta))$. Thus, our measure of capital-unemployment depends on search and entry frictions solely through the factor δ/η^* .

Next, we compute social output, based on the steady-state masses of matched firms, S^*N^i ($i = 1, 2$):

$$Y^* = S^*[N^{1*}p^1A^1 + (1 - N^{1*})p^2A^2]. \quad (18)$$

This aggregate output measure can be decomposed into two components. First, S^* reflects aggregate matches and enhanced market liquidity (and enhanced market participation). Second, the square bracket term reflects the composition of output and can be interpreted as the average output over all matched firms. Because the two components need not always move in the same direction, the comparative statics with respect to the responses of interest rates and social output may at times be ambiguous.

Now we are ready to present our first set of comparative statics results.

Proposition 4. (Credit market shocks). *Under the circumstances described in the theorem, the effects of matching efficiency (m_0) and the contract quit rate (δ) on steady-state $\{h^*, m^*, N^{1*}, R^*, S^*, U^*, L^*, Y^*, J_u^*\}$ are given by:*

- (i) *An improvement in matching efficiency generates more matches and leads to a greater fraction of low-quality firms.*

Table 1
Summary of Comparative Statics

Effect of	Credit market shocks		Firm profitability shocks			
	Credit matching efficacy	Contract-quit rate	Type 1 productivity	Type 2 productivity	Type 1 entry costs ^a	Type 2 entry costs ^a
Effect on	m_0	δ	A^1	A^2	v_0^1	v_0^2
1. Contact rates and population masses						
η^*	0	+	+	–	–	+
U^*	0	–	–	+	+	–
μ^*	+	–	–	+	+	–
S^*	+	–	–	+	+	–
2. Market liquidity						
L^*	+	–	–	+	+	–
3. Composition of low-type borrowing firms						
N^{1*}	+	–	–	+	+ ^a	–
4. Interest rates and differential						
R^{i*}	0	– ^b	+	+ ^b	–	– ^b
$R^2 - R^1$	0	– ^b	?	+ ^b	?	– ^b
5. Social output						
Y^*	?	– ^b	+ ^b	+ ^b	?	?

^aTo sign the effect of entry cost of the low-quality firm v_0^1 on the composition of borrowers N^{1*} we require a regularity condition that the positive direct effect dominates the negative indirect effect via negotiated interest rates.

^bAssume production normality (market participation effect dominates the composition effect).

(ii) *An increase in the contract quit rate will raise the firm contact rate and reduce the capital unemployment rate but lower lender contact rates and market liquidity. Also, the share of low quality firms will fall. When the market participation effect dominates the composition effect, interest rates and output fall; otherwise, the effect is uncertain.*

Proof. See Appendix. □

Table 1 summarizes the comparative statics results.²¹ First, we discuss what happens when matching efficacy increases as a result of structural or technological improvements in the credit market. Intuitively, an increase in matching efficiency increases the contact rate for lenders μ^* and encourages the entry of firms. The number of matches increases (as captured by a rise in S^*) because of higher lender contact rates. From Proposition 3 we know that a rise in the lender contact rate raises the interest offer to each firm by an equal amount. This causes the firm surplus differential (D) to widen, and therefore low quality firms enter disproportionately

²¹While Fig. 2 is useful in illustrating the uniqueness of steady state equilibrium, the comparative statics cannot be easily captured graphically. This is because there are significant feedback effects between the matching rates and the fraction of firm types in the market.

and N^{1*} rises. The composition effect, which is summarized by the increase in N^{1*} , puts downward pressure on loan rates and is sufficiently strong that rates return to where they originally were. Thus, there is no net effect on loan rates and on the firm matching rate η^* . Because η^* is unchanged, the capital unemployment rate is unaffected. Finally, recall from our discussion of Proposition 1 that N^{1*} and μ^* have opposing effects on the unmatched value of lenders. Because of the presence of the composition effect, an increase in matching efficiency creates two offsetting forces on output. More matching means higher output because of greater market participation, but this effect on output is offset by the fact that there are relatively more low quality firms in the economy so that the average quality of firms falls. Thus, the positive effects of improved matching efficacy are dampened by composition effects. Our results also lend theoretical support to the empirical finding documented by Bernanke et al. (1996) and Lang and Nakamura (1995) that the composition of loans varies with economic activity and that a flight-to-quality in credit markets occurs when real activity weakens.

Next we ask, what happens following an increase in the contract quit rate δ ? An increase in δ reduces the unmatched value of all firms relative to their entry cost, lowers their threat points, and induces firms to exit. Thus, firm contact rates of surviving firms rise by the zero-profit condition, thereby reinforcing the negative direct effect of the contract quit rate on capital unemployment. From the Beveridge curve relationship, lender contact rates fall which causes a reduction in market liquidity and a fall in the overall level of matching S^* . Because there are relatively fewer productive firms, output falls; however, output could rise if the average productivity of the remaining firms rises. Average productivity is determined by the firm composition effect. As explained previously, low quality firms enter relative to high quality firms when the firm surplus differential is positive. Contract quits have a negative direct effect on loan rates and a negative indirect effect on loan rates because lender contact rates are reduced. Because lower loan rates reduce the surplus differential, N^1 falls. The net effect on output and welfare balances a negative effect from reduced market liquidity with a positive composition effect from an increase in the average productivity of remaining firms. Under the assumption of ‘production normality’ or when the market participation effect dominates the composition effect, loan rates, liquidity, and output all fall with a rise in contract quits. Interestingly, our results suggest that long-term relationships (i.e., lower contract quit rates) increase aggregate output and raise loan rates to all firms (although disproportionately more for high quality firms). However, when production normality is not imposed, it is possible that output rises with a rise in contract quits, though liquidity still falls. That is, reversals are possible in the absence of production normality. This scenario then provides plausible theoretical explanation for the empirical finding documented by De Gregorio and Guidotti (1995), Demetriades and Hussein (1996), Luintel and Khan (1999), and Rajan and Zingales (2003).

We now consider the effects of fundamental shocks to firm profitability from changes to their productivity or entry costs. Concerning results for entry cost shocks, we assume that the impact effect of market participation on the composition of firms

dominates secondary effects via (expected) interest payments.²² Under these circumstances, we can show:

Proposition 5. (Firm profitability shocks). *Under the circumstances described in the theorem, the effects of productivity (A^i) and entry costs (v_0^i) on steady-state $\{h^*, m^*, N^{1*}, R^*, S^*, U^*, L^*, Y^*, J_u^*\}$ are given by:*

- (i) *Productivity and entry cost shocks that raise the profitability of high quality firms increase lender contact rates, market liquidity, and the share of low quality firms, but lower firm contact rates which raises the capital unemployment rate. Productivity and entry cost shocks that raise the profitability of low quality firms will have the opposite effect on these variables.*
- (ii) *When the market participation effect dominates the composition effect, productivity shocks raise loan rates and output. Cost shocks tend to have the opposite effect (except for output where the outcome is open), if the market participation effect dominates the composition effect.*

Proof. See Appendix. \square

To understand the effects of shocks that increase firm profitability \bar{a}^i (due to either an increase in A^i or a reduction in v_0^i), recall that there are two mechanisms at work. First, when \bar{a}^1 (or \bar{a}^2) rises, $\bar{a}^1 - p^1 R^1$ (or $\bar{a}^2 - p^2 R^2$) rises less (or more) than proportionately because of differences in net expected productivity.²³ Zero-profit thus requires η to rise (or fall) which causes the capital unemployment rate to fall (rise). The Beveridge Curve translates the change of η into a fall (or rise) of μ so that both market liquidity and market participation fall (rise). Following the discussion of Proposition 1, the direct effect of an increase in \bar{a}^1 (or \bar{a}^2) raises loan rates, but the indirect effect of lower (or higher) μ causes them to fall (or rise). Loan rates rise when \bar{a}^1 or \bar{a}^2 rises, where production normality guarantees that indirect effects are not too large when the profitability shock benefits high quality firms. Second, N^1 rises whenever shocks induce a positive surplus differential between high and low type firms. From Propositions 1 and 3, an increase in \bar{a}^1 leads to higher profitability for low quality firms ($A^1 - R^1$) and higher loan rate for high quality firms (R^2) – the latter results in lower profitability for high quality firms. As a consequence, the firm surplus differential decreases, implying a fall in N^1 so as to restore zero profit. By similar arguments, an increase in \bar{a}^2 gives rise to a higher N^1 . As before, the effect of profitability shocks on output balances market participation effects and average productivity effects, whereby the latter is a sum of individual productivity effects and the composition effect. Productivity shocks tend to enhance average productivity

²²As shown in the Appendix, this assumption is needed only when analyzing shocks to the entry cost of the low quality firm.

²³A critical relationship for understanding how the zero profit conditions respond to shocks is given by

$$\bar{a}^i - p^i R^i = \frac{B}{1+B} (\bar{a}^i - 1) - \frac{(1-\beta)\beta\mu}{r+\delta+\beta\mu} N^j (\bar{a}^j - \bar{a}^i),$$

where the second term of the right-hand side is negative (or positive) for $i = 1$ (or 2).

directly, while their indirect effects tend to be offsetting. Similar results apply to changes in entry costs.

Overall, any shock that enhances credit matching causes aggregate liquidity to rise. While an increase in liquidity increases market participation by all firms, low quality firms enter disproportionately and the average quality of firms falls (unless the shock raises the profitability of low quality firms). Thus, liquid credit markets may or may not be associated with high output and welfare, depending on whether the composition effect on average quality outweighs the effect on market participation. Shocks to profitability from changes in productivity or entry costs that benefit high quality firms will enhance aggregate liquidity but create a negative composition effect. By assuming production normality, positive productivity shocks are generally associated with higher output and welfare. By contrast, positive credit market shocks will increase market liquidity and market participation, but because of a strong composition effect social output may rise or fall. Our findings contrast sharply with those in Den Haan et al. (2003), where borrowers have different outcomes ex post as a result of an exogenous random funds allocation mechanism rather than intrinsic heterogeneity. In their model, a positive shock to market liquidity encourages market participation and raises output because it lowers the rate of breakups of borrowing–lending relationships. In our paper, fundamental shocks that enhance market liquidity also induce market participation, but may reduce social output as a consequence of flight-from-quality.

Finally, we observe that because of the presence of composition effects, the world with heterogeneous borrowers differs greatly from the simpler world with homogeneous borrowers. Composition effects give rise to results in an environment without informational asymmetry that are similar to those found in economies with adverse selection (cf. Biglaiser and Friedman, 1999). Moreover, because of the presence of differential entry, an improvement in credit-market matching efficacy no longer generates an unambiguously positive effect on social output. The distinctive effects of changes in composition versus changes in market participation provide a fertile ground for future applied work in the area of credit markets.

5. Assortative financial matchmaking

Until now we have assumed that matching is random even though there is full knowledge about firm types. Suppose this knowledge is used to improve the performance of the loanable funds market. Specifically, we allow assortative matching in the credit market with priority given to loans to high quality firms. To justify assortative matching, it is necessary to assume that there is excess demand for funds (i.e., $H < F$). Moreover, we assume that the supply of funds exceeds the demand for funds by high quality firms. Otherwise, assortative matching would yield a corner solution with only high quality firms receiving loans. This latter restriction requires $N^2F < H$, which together with $H < F$ implies $N^1F > (H - N^2F)$.

Under these regularity conditions, all high-quality firms receive loans. The mass of low quality firms who receive loans is equal to the residual $H - N^2F$. Therefore,

assortative matching can be thought of as matching in two segmented markets, yet lenders do not undertake directed searches *ex ante*.

For notational convenience, we define the tightness of the loanable funds market as $\tau = F/H > 1$. We also reinterpret the contact rates as the arrival rates of the opportunities for funds. We thus have the following relationships: $\mu^1 = \mu(H - N^2F)/H = [1 - \tau(1 - N^1)]$, $\mu^2 = \mu(N^2F)/H = \tau(1 - N^1)$, and $\eta^1 = \eta^2 = \eta$. The last expression indicates that arrivals of opportunities are non-discriminating even though matches are assortative. With the modifications to the lender contact rates, we find that the analysis of Sections 3 and 4 remains valid. Rather than going through the entire analysis again, it is therefore sufficient to simply focus on the main difference between assortative and random matching.

To facilitate comparison of assortative and random matching, notice that $\mu^1 = [1 - \tau(1 - N^1)] < N^1$ and $\mu^2 = \tau N^2 > N^2$. That is, lenders are more likely to meet with high-quality borrowers under assortative matching than under random matching. This has two immediate consequences. First, as a result of the greater rate of contact with high-quality firms, the loan rate of high quality firms increases, which leads to a widening of the interest rate spread between the high- and low-quality firms. Second, due to assortative matching, more high-quality firms are granted loans and hence social output increases unambiguously. This highlights the funds-allocation role of credit markets when modeled as a non-Walrasian search market.

6. Summary and extensions

This paper has studied a dynamic general equilibrium search model of credit markets with heterogeneous borrowers and endogenous rates of entry and contact. The analysis identifies channels through which decentralized and assortative matching affects the size and quality of credit flows. Our results suggest that shocks that increase credit market liquidity also lead to increased market participation by firms and a composition effect whereby the participation of low-quality firms rises disproportionately. However, more liquid markets only increase output and welfare when the market participation effect dominates the composition effect. This generally is the case when shocks enhance the profitability of firms or when they make contracts less fragile (and financial relationships longer lasting). By contrast, structural shocks that make matching more efficient may have large composition effects. Moreover, both market participation and composition channels are crucial for influencing the *ex post* actual interest rate spread, but not the *ex ante* expected interest rate spread, yielding empirically testable implications.

Two natural extensions come to mind. First, one could allow credit markets to have a more active role than just sorting borrowers by quality. In addition, lenders could choose matching effort by maximizing their output net of a real resource cost. It would be useful to compare the credit market outcomes discussed in Section 5 with those under the middlemen framework developed by Rubinstein and Wolinsky (1987). Second, one could introduce asymmetric information about the firm's type. There are two possibilities. When firms make their investment project selection

(high or low quality) *prior* to bank loan approval, the adverse selection problem may exist as in the middlemen theory developed by Biglaiser (1993). Alternatively, when firms select projects *ex post*, the moral hazard problem may arise. In either case, equilibrium credit rationing may be present with incentive compatible financial contracts. Such additional source of capital unemployment will then compound and enhance the frictional capital unemployment considered in this paper.

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Appendix

This Appendix contains proofs of Lemma 3 and 4 and Propositions 3–5 in the paper.

Proof of Lemma 3.

(i) Differentiating (15) with respect to N^i gives

$$\frac{\partial(p^i R^i)}{\partial N^i} = -\left[\frac{\beta(1-\beta)\mu}{r+\delta+\beta\mu}\right](\bar{a}^i - \bar{a}^i) = \frac{\partial(p^j R^j)}{\partial N^i}.$$

For $i = 1$ we have $\partial p^1 R^1 / \partial N^1 = \partial p^2 R^2 / \partial N^1 < 0$ and for $i = 2$ we have $\partial p^1 R^1 / \partial N^2 = \partial p^2 R^2 / \partial N^2 > 0$.

(ii) Differentiating (15) with respect to μ gives

$$\begin{aligned} \frac{\partial(p^i R^i)}{\partial \mu} &= \frac{\beta(1-\beta)(r+\delta)}{(r+\delta+\beta\mu)^2} [(1-N^i)(\bar{a}^i - \bar{a}^i) - (1-\bar{a}^i)] \\ &= \frac{\partial(p^j R^j)}{\mu} > 0 \text{ for all } i, j. \end{aligned}$$

(iii) Notice that $\partial B / \partial \delta > 0$, and given $\bar{a}^i > 1$, the first term in (15) is strictly decreasing in δ . For $i = 1$, it is clear that the second term in (15) is also strictly decreasing in

δ , implying $\partial p^1 R^1 / \partial \delta < 0$. Thus, manipulating (15), we have $p^2 R^2 = p^1 R^1 + \beta(\bar{a}^2 - \bar{a}^1)$ implying $\partial p^2 R^2 / \partial \delta = \partial p^1 R^1 / \partial \delta$.
 (iv) Since $\partial p^i R^i / \partial v_0^i \propto -\partial p^i R^i / \partial A^i < 0$, the result is immediate. \square

Proof of Proposition 2. From (16), we can derive

$$R^2 - R^1 = \frac{\beta}{p^2}(\bar{a}^2 - \bar{a}^1) + \frac{p^1 - p^2}{p^1 p^2} p^1 R^1.$$

Thus both spreads are positive under Assumption (A4) that ensures $\bar{a}^2 - \bar{a}^1 > 0$. Utilizing Proposition 2, we can see implies that $R^2 - R^1$ rises with μ and \bar{a}^2 , falls with N^1 and δ , may rise or fall with \bar{a}^1 , and is immune to m_0 . Moreover, from (17) we obtain:

$$R^2 - R^1 = \lim_{\mu \rightarrow \infty, v_0^i \rightarrow 0} (R^2 - R^1) - \beta r \left(\frac{v_0^2}{p^2} - \frac{v_0^1}{p^1} \right) - (1 - \beta) \left(\frac{1}{p^2} - \frac{1}{p^1} \right) \Theta,$$

where

$$\Theta = \frac{r + \delta}{r + \delta + \beta\mu} (N^1 p^1 A^1 + N^2 p^2 A^2 - 1) + \frac{\beta\mu}{r + \delta + \beta\mu} (N^1 v_0^1 + N^2 v_0^2)$$

is a weighted sum of aggregate net outputs and aggregate entry costs, which is positive under Assumptions (A4) and (A5). Thus, given (A2), the actual interest rate spread is smaller than that in the absence of search and entry frictions. \square

Proof of Lemma 4. From Proposition 3, it is immediate that ZP^1 and ZP^2 are downward sloping in (η, N^1) space. Differentiating (14) gives

$$\left. \frac{d\eta^*}{dN^1} \right|_{ZP^1} = \frac{d\eta^{ZP^1}}{dp^1 R^1} \frac{dp^1 R^1}{dN^1} = -\frac{(\eta^*)^2}{rv_0^1(r + \delta)} \frac{\beta(1 - \beta)}{r + \delta + \beta\mu} \mu(\bar{a}^2 - \bar{a}^1) < 0,$$

$$\left. \frac{d\eta^*}{dN^1} \right|_{ZP^2} = \frac{d\eta^{ZP^2}}{dp^2 R^2} \frac{dp^2 R^2}{dN^1} = -\frac{(\eta^*)^2}{rv_0^2(r + \delta)} \frac{\beta(1 - \beta)}{r + \delta + \beta\mu} \mu(\bar{a}^2 - \bar{a}^1) < 0,$$

since $v_0^1 < v_0^2$, we have that the pair $\{\eta^*, N^{1*}\}$ satisfying (14) given μ occur where $|d\eta^*/dN^1|_{ZP^1} > |d\eta^*/dN^1|_{ZP^2}$. Since both locus' are downward sloping, this pair is unique. To characterize the (EZ) locus, equate ZP^1 and ZP^2 from (14):

$$v_0^2 p^1 [R^1(N^1, \mu) - 1] - v_0^1 p^2 [R^2(N^1, \mu) - 1] = v_0^2 p^1 (A^1 - 1) - v_0^1 p^2 (A^2 - 1). \tag{A.1}$$

Notice that from Proposition 3, $\partial p^i R^i / \partial \mu = \partial p^i R^i / \partial \mu > 0$ and $\partial p^i R^i / \partial N^1 = \partial p^i R^i / \partial N^1 < 0$. Consider now that μ increases. Since $p^i R^i$ is higher (for $i = 1, 2$), (14) implies η must be higher. However, this changes the LHS of (A.1) away from the RHS: the LHS increases (decreases) iff $v_0^1 p^2 - v_0^2 p^1 < (>) 0$. In either case, N must rise to restore the equality in (A.1), implying $d\eta/dN^1|_{EZ} > 0$.

To characterize the limit points of the EZ locus, consider the case where $\mu \rightarrow \infty$ which implies $\eta \rightarrow 0$ from (7). From (15) $p^i R^i \rightarrow [(1 - \beta)N^i(\bar{a}^j - \bar{a}^i) + \bar{a}^i] > 0$. The

LHS of (A.1) can be written as:

$$\text{LHS} = (v_0^1 p^2 - v_0^2 p^1)(1 - \beta) + \beta\{(1 - \beta)(\bar{a}^2 - \bar{a}^1)[v_0^2 - N^1(v_0^2 - v_0^1)] + (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2) + (v_0^1 p^2 - v_0^2 p^1)\}.$$

Since $\text{RHS} = (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2) + (v_0^1 p^2 - v_0^2 p^1)$ equating LHS with RHS and solving for N^1 yields

$$N^1 = \frac{\beta(\bar{a}^2 - \bar{a}^1)v_0^2 - (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2)}{\beta(\bar{a}^2 - \bar{a}^1)(v_0^2 - v_0^1)}.$$

Thus, a condition for $N^1 \in [0, 1)$ is given by

$$\beta(\bar{a}^2 - \bar{a}^1)v_0^1 < (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2) \leq \beta(\bar{a}^2 - \bar{a}^1)v_0^2. \tag{A.2}$$

Now consider the limiting case where $\mu \rightarrow 0$ which implies $\eta \rightarrow \infty$ from (14). From (15) $p^i R^i \rightarrow 1 + \beta(\bar{a}^i - 1) > 1$. Thus, there exists an upper bound for η such that $\sup_{N^1} \eta(N^1) \ll \infty$. Furthermore, there exists a finite $\eta^{max} \ll \infty$ at $N^1 = 1$. \square

Proof of Propositions 4 and 5. Totally differentiating (7) and (14) yields

$$C \begin{bmatrix} d\eta \\ d\mu \\ dN^1 \end{bmatrix} = \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix} dm_0 + \begin{bmatrix} 0 \\ rv_0^1 + \frac{dp^i R^i}{d\delta} \\ rv_0^2 + \frac{dp^i R^i}{d\delta} \end{bmatrix} d\delta + \begin{bmatrix} 0 \\ -p^1 \left(1 - \frac{dR^1}{dA^1}\right) \\ p^2 \frac{dR^2}{dA^1} \end{bmatrix} dA^1 \\ + \begin{bmatrix} 0 \\ p^1 \frac{dR^1}{dA^2} \\ -p^2 \left(1 - \frac{dR^2}{dA^2}\right) \end{bmatrix} dA^2 + \begin{bmatrix} 0 \\ \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} \\ \frac{dp^2 R^2}{dv_0^1} \end{bmatrix} dv_0^1 \\ + \begin{bmatrix} 0 \\ \frac{dp^1 R^1}{dv_0^2} \\ \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^2 R^2}{dv_0^2} \end{bmatrix} dv_0^2,$$

where

$$C = \begin{bmatrix} 1 - \frac{m_0 M}{\mu} & \frac{m_0 M \eta}{\mu^2} & 0 \\ \frac{r(r+\delta)v_0^1}{\eta^2} & -\frac{d(p^1 R^1)}{d\mu} & -\frac{d(p^1 R^1)}{dN^1} \\ \frac{r(r+\delta)v_0^2}{\eta^2} & -\frac{d(p^2 R^2)}{d\mu} & -\frac{d(p^2 R^2)}{dN^1} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Note that

$$C_{22} = C_{32} = -\frac{dp^i R^i}{d\mu} < 0 < \frac{dp^i R^i}{dN^1}$$

and

$$C_{31} - C_{21} = \frac{r(r + \delta)}{\eta^2}(v_0^2 - v_0^1) > 0.$$

Thus, $|C| = -C_{12}(C_{21} - C_{31})C_{33} > 0$. We also compute comparative static effects:

1. Matching Efficiency:

$$\frac{d\eta^*}{dm_0} = \frac{M}{|C|}(C_{22}C_{23} - C_{23}C_{32}) = 0, \quad \frac{d\mu^*}{dm_0} = \frac{M}{|C|}(C_{31} - C_{21})C_{33} > 0,$$

$$\frac{dN^{1*}}{dm_0} = -\frac{M}{|C|}(C_{31} - C_{21})C_{22} > 0$$

$$\frac{dp^i R^i}{dm_0} = \left(\frac{dp^i R^i}{d\mu}\right) \frac{d\mu}{dm_0} + \left(\frac{dp^i R^i}{dN^1}\right) \frac{dN^1}{dm_0} = C_{i2} \frac{d\mu}{dm_0} + C_{i3} \frac{dN^1}{dm_0} = 0$$

after substituting terms from above. Using this last result and (4) and (5), one sees that $\Pi_m^i - \Pi_u^i$ and $J_m^i - J_u^i$ also are independent of m_0 . Then (1b) implies J_m^i and J_u^i are independent of m_0 .

2. Separation Rate:

$$\begin{aligned} \frac{d\eta^*}{d\delta} &= \frac{C_{12}C_{33}}{|C|}r(v_0^2 - v_0^1) > 0, & \frac{d\mu^*}{d\delta} &= -\frac{C_{11}C_{33}}{|C|}r(v_0^2 - v_0^1) < 0, \\ \frac{dN^{1*}}{d\delta} &= \frac{1}{|C|} \frac{r(v_0^2 - v_0^1)}{\eta^2} \left((r + \delta)C_{12} \frac{dp^i R^i}{d\delta} - \eta^2 C_{11} \frac{dp^i R^i}{d\mu} \right) < 0. \end{aligned}$$

3. Productivity:

$$\begin{aligned} \frac{d\eta^*}{dA^1} &= \frac{C_{12}C_{33}}{|C|} \left(p^1 \left(1 - \frac{dR^1}{dA^1} \right) + p^2 \frac{dR^2}{dA^1} \right) > 0, \\ \frac{d\mu^*}{dA^1} &= \frac{C_{11}C_{33}}{|C|} \left(p^1 \left(1 - \frac{dR^1}{dA^1} \right) + p^2 \frac{dR^2}{dA^1} \right) < 0, \\ \frac{dN^{1*}}{dA^1} &= \frac{C_{11}C_{22}}{|C|} \left(p^1 \left(1 - \frac{dR^1}{dA^1} \right) + p^2 \frac{dR^2}{dA^1} \right) \\ &\quad - \frac{C_{12}}{|C|} \left(C_{31}p^1 \left(1 - \frac{dR^1}{dA^1} \right) + C_{21}p^2 \frac{dR^2}{dA^1} \right) < 0. \end{aligned}$$

Also, the impact of A^2 is inversely related to that of A^1 :

$$\frac{d\eta^*}{dA^2} = -\frac{C_{12}C_{33}}{|C|} \left(p^2 \left(1 - \frac{dR^2}{dA^2} \right) + p^1 \frac{dR^1}{dA^2} \right) < 0,$$

$$\begin{aligned} \frac{d\mu^*}{dA^2} &= \frac{C_{11}C_{33}}{|C|} \left(p^2 \left(1 - \frac{dR^2}{dA^2} \right) + p^1 \frac{dR^1}{dA^2} \right) > 0, \\ \frac{dN^{1*}}{dA^2} &= \frac{C_{11}C_{22}}{|C|} \left(p^2 \left(1 - \frac{dR^2}{dA^2} \right) + p^1 \frac{dR^1}{dA^2} \right) \\ &\quad - \frac{C_{12}}{|C|} \left(C_{21}p^2 \left(1 - \frac{dR^2}{dA^2} \right) + C_{31}p^1 \frac{dR^1}{dA^2} \right) > 0. \end{aligned}$$

4. Entry costs: the effects are more difficult to sign:

$$\begin{aligned} \frac{d\eta^*}{dv_0^1} &= -\frac{C_{12}C_{33}}{|C|} \left(\frac{r(r + \delta + \eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} - \frac{dp^2 R^2}{dv_0^1} \right), \\ \frac{d\mu^*}{dv_0^1} &= -\frac{C_{11}C_{33}}{|C|} \left(\frac{r(r + \delta + \eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} - \frac{dp^2 R^2}{dv_0^1} \right), \\ \frac{dN^{1*}}{dv_0^1} &= -\frac{C_{11}C_{22}}{|C|} \left(\frac{r(r + \delta + \eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} - \frac{dp^2 R^2}{dv_0^1} \right) \\ &\quad + \frac{C_{12}}{|C|} \left(-C_{21} \frac{dp^2 R^2}{dv_0^1} + C_{31} \left[\frac{r(r + \delta + \eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} \right] \right). \end{aligned}$$

From (16) one obtains

$$\frac{dp^i R^i}{dv_0^i} - \frac{dp^j R^j}{dv_0^i} = r\beta < 0$$

and

$$\frac{dp^j R^j}{dv_0^i} = -r \frac{\beta\mu(1 - \beta)N^i}{r + \delta + \beta\mu} < 0$$

which can be substituted into the above relationships. By defining

$$Q \equiv r \left[\frac{r + \delta + (1 - \beta)\eta}{\eta} \right] > 0,$$

we obtain:

$$\begin{aligned} \frac{d\eta^*}{dv_0^1} &= -\frac{C_{12}C_{33}}{|C|} Q, \quad \frac{d\mu^*}{dv_0^1} = -\frac{C_{11}C_{33}}{|C|} Q, \\ \frac{dN^{1*}}{dv_0^1} &= -\frac{C_{12}C_{31} - C_{11}C_{22}}{|C|} Q + \frac{C_{12}(C_{31} - C_{21})}{|C|} \frac{dp^2 R^2}{dv_0^1}. \end{aligned}$$

Thus, it follows that

$$\frac{d\eta^*}{dv_0^1} < 0 < \frac{d\mu^*}{dv_0^1}.$$

With regard to the sign of dN^{1*}/dv_0^1 , let us assume the positive direct effect (via Q) dominates the negative indirect effect (via $p^2 R^2$) such that $(dN^{1*}/dv_0^1) > 0$.

Using the same approach as above yields:

$$\frac{d\eta^*}{dv_0^2} = -\frac{C_{12}C_{33}}{|C|}Q, \quad \frac{d\mu^*}{dv_0^2} = -\frac{C_{11}C_{33}}{|C|}Q,$$

$$\frac{dN^{1*}}{dv_0^2} = -\frac{C_{11}C_{22} - C_{12}C_{21}}{|C|}Q + \frac{C_{12}(C_{31} - C_{21})}{|C|} \frac{dp^1 R^1}{dv_0^2}.$$

Thus,

$$\frac{d\eta^*}{dv_0^2} > 0 > \frac{d\mu^*}{dv_0^2}$$

Also, since, $C_{22} < 0$, $(dN^{1*}/dv_0^2) < 0$ without having to make any further assumptions (in contrast to the assumptions needed to establish $(dN^{1*}/dv_0^1) > 0$). \square

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