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Inflation and productive activity in a multiple-matching model of money[☆]

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Abstract

This paper investigates the relationship between money growth, inflation, and productive activity in a dynamic general-equilibrium, multiple-matching framework where trade frictions are manifested by limited consumption variety. Productive activity and matching in the goods market are endogenized by a time allocation decision of work and search effort. We find that a high degree of complementarity between participation in the labor and goods markets creates a channel by which inflation can positively influence production and output. This feature arises when household preferences for consumption variety is sufficiently large and it can also lead to the multiplicity of monetary equilibria.

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1. Introduction

The relationship between money growth, inflation, and real activity continues to be a central and much debated issue in monetary economics. The majority of neoclassical dynamic general equilibrium models suggest that steady inflation is disruptive to real economic activity. For example, money growth in cash-in-advance models with production generates a pure inflation tax effect which discourages market activity, capital accumulation, consumption and output. While such frameworks have been widely used for monetary policy analysis their theoretical foundations and predictions have been problematic.¹

This paper evaluates the consequences of money growth and inflation on economic activity in the context of a *multiple-matching model* of money wherein trade frictions lead to *limited consumption variety*. It builds upon a growing literature emphasizing the importance of incomplete markets and frictions in monetary economies. The multiple-matching approach, developed by Laing et al. (2005), is characterized by several key features. First, instead of a sequential search structure highlighted by recent monetary search models (e.g., Kiyotaki and Wright, 1993), (i) buyers contact multiple numbers of sellers in a given period, each selling a differentiated good, and (ii) households have a preference for consumption variety and consume a basket of goods. An analogy of this process is a consumer who shops in a marketplace and encounters many different products but not all of those desired in the economy. Monetary exchange expands trading opportunities by increasing the number of sellers willing to trade and hence improves welfare from its ability to procure greater consumption variety.²

Second, instead of pursuing a bilateral bargaining approach, we employ a Dixit–Stiglitz style monopolistic competition structure where firms set prices. Finally, the model emphasizes time allocation where the representative household optimally chooses between leisure, labor supply, and exchange (search) effort. In particular, households must exert time consuming effort to encounter sellers in the goods market and such effort is viewed as an input into the matching technology. Hence the time allocation feature of our model provides an important linkage between labor market activity and exchange activity.

In the context of our model we establish the following relationships between money growth, exchange activity, labor allocation, and aggregate output. If the level of search effort is exogenously determined we find that the impact of steady money growth is very similar to those in traditional competitive equilibrium modes, such as cash-in-advance. That is, inflation taxes participation in both the goods and labor markets and reduces consumption and real output. However, if the level of search effort is optimally chosen these implications may be very different. When product variety is sufficiently important money growth and inflation can encourage both work and search effort, leading to higher output. Moreover, this mechanism may also give rise to multiple steady states. While the positive effect of inflation on output applies more generally than the multiplicity of

¹These frameworks have been questioned on both theoretical and empirical grounds. Their Walrasian foundations preclude an accounting of the impact of monetary policy on the very frictions that create the need for money. Empirical work has not only shown an absence of a negative correlation between steady inflation and economic activity (Bullard and Keating, 1995) but also a positive long-run impact in stable price industrialized countries (Ahmed and Rogers, 2000).

²In contrast to the money search literature, the benefits of monetary exchange over barter in our approach stresses its ability to enhance consumption variety rather than its quantity or frequency.

equilibria, both of these findings arise directly from the link between the labor and product markets provided by time allocation towards exchange activity. The intuition of these findings can be seen as follows. If product variety is sufficiently valued by consumers, the inflation tax leads to a substitution towards search effort as households augment their consumption basket with additional variety. When consumption and leisure are good substitutes, additional consumption variety enhances the incentives for work effort and production. It is precisely this complementarity between work and search effort that leads to a positive feedback between production and exchange activities. Such a feedback explains a positive money-output correlation as well as the possible emergence of multiple steady states. Furthermore, as these results are conditional upon *how* valuable consumption variety is to consumers, our paper suggests that how important money is as a medium of exchange matters for its impact on economic activity.

2. The model

The multiple-matching approach to money is unique and hence distinct from other friction-based monetary models in the literature, and we begin by elaborating on how the various features of our model are interrelated. First, as a transactions model of money, the co-existence of multiple matching and product variety is important. Since buyers contact many sellers in a given period there is always a subset of goods among those contacted which a household will find desirable. The friction associated with the absence of the double coincidence of wants is not limited consumption quantity but rather limitations in the density of sellers that buyers encounter and the variety of goods consumed. The use of money overcomes these frictions by expanding the opportunities to trade for greater consumption variety. In this setting, Laing et al. (2005) establishes the existence of a pure monetary equilibrium where cash is used in every transaction as an endogenous outcome of equilibrium trading strategies.³

Second, given that product differentiation is central to our multiple-matching structure, monopolistic competition among sellers is the most natural approach to pricing and it is consistent with the idea that most transactions in modern economies occur at prices set by firms rather than determined via bilateral bargaining. The simplicity and plausibility of monopolistic competition is precisely why it has attracted so much attention in the macroeconomics literature (e.g., Blanchard and Kiyotaki, 1987; Ohanian et al., 1995). However, its role in monetary economies with trade frictions has so far remained largely unexplored.

Finally, we extend a simplified version of Laing et al. (2005) by (i) focusing on the pure monetary equilibrium outcome, rather than barter exchange, and (ii) incorporating an endogenous household time allocation decision across leisure, work and exchange effort. This latter feature provides an important link between the product and labor markets and plays a central role in our findings regarding the interaction between money, inflation, and productive activity.

³A multiple-matching mechanism also keeps the steady-state distribution of cash/goods trivial, an issue that has been problematic in the earlier generation of search-theoretic monetary models where the random nature of sequential search generally leads to an analytically complex endogenous distribution of cash and inventory holdings. Other approaches that circumvent these distributional issues in money-search models include Shi (1997) who uses the concept of large risk-sharing households and Lagos and Wright (2005) who have a Walrasian competitive market co-existing with the search market.

We now delineate the basic framework. Time is discrete and the economy is populated by a continuum of infinitely lived households (indexed by h) and firms, with each of their masses normalized to unity. There is a large number of differentiated commodities of mass one, indexed by $\omega \in \Omega$. Each firm produces a particular good using labor as the sole input, so firms can also be indexed by ω . A household of type h desires a variety of goods over a subset $\Omega(h) \subseteq \Omega$. The commodity space is ordered in such a way that a household of type h , employed by a particular firm, produces a good outside of her preference domain, $\Omega(h)$, and thus there is no double coincidence of wants between them.⁴ In this way, we rule out the uninteresting case of autarky as well as any possible exchanges between a worker and her employer. As in [Diamond and Yellen \(1990\)](#), we assume that associated with each firm ω is an infinitely lived owner who desires good ω and acts as the residual claimant of the firm's output. Both goods and money are perfectly divisible and exchanges only occur between households and unaffiliated firms.

2.1. Preferences and production

The lifetime utility for household h is given by,

$$V = \sum_{t=0}^{\infty} \beta^t U(D_t, L_t), \quad (1)$$

where $U(\cdot)$ is strictly increasing and quasi-concave in its arguments and $\beta \in (0, 1)$ is the subjective time discount factor. In addition to leisure L_t , households value a composite consumption good D_t , which is a [Dixit–Stiglitz \(1977\)](#) aggregator of the basket of goods consumed in each period t ,

$$D_t = \left[\int_{\omega \in \Omega(h)} c_t(\omega)^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}, \quad (2)$$

where $c_t(\omega)$ is h 's consumption of good ω and $\gamma > 1$ denotes the elasticity of substitution across varieties. A lower γ implies that variety is important and hence it can be regarded as an indicator of the significance of money in overcoming the trade frictions associated with limited consumption variety.

The owner of firm $\omega \in \Omega$ has a lifetime utility given by,

$$\hat{V} = \sum_{t=0}^{\infty} \beta^t \hat{c}(\omega), \quad (3)$$

where $\hat{c}(\omega)$ is the owner's consumption of his own production good, and corresponds to the residual output of the firm.

⁴[Laing et al. \(2005\)](#) carefully specifies preferences to support the possibility of a double coincidence of wants and barter between households and firms and establishes the existence of both barter and pure monetary equilibria as well as mixed trading regimes. Since the current paper focuses strictly on the pure monetary equilibrium, the detailed structure to support fiat currency will not be elaborated.

Firm ω manufactures output by employing labor $n_t(\omega)$ according to the following production technology:

$$y_t(\omega) = f[n_t(\omega)], \quad (4)$$

where f satisfies: $f' > 0, f'' < 0$, and $f(0) = 0$.⁵

2.2. Labor and product markets

At the beginning of each period households allocate their time endowment (normalized to one) to work effort, n_t , search effort, s_t , and leisure, $L_t = 1 - n_t - s_t$. Households possess the ability to produce many types of goods but are only productive at a single firm per period. Firm $\omega \in \Omega$ offers a competitive labor contract which pays nominal cash wage W_t in exchange for the household's labor services. The firm then produces output $y_t(\omega)$ according to the production technology given by (4).

Once households receive wages from the competitive labor market, they travel to the goods market where they are randomly matched with a set of $\chi_t \subset \Omega(h)$ firms with measure $\alpha_t = \alpha(s_t)$, where $\alpha'(s_t) \geq 0$ and $\alpha(0) \geq 0$. This matching technology uses household search effort as an input to determine the matching rate α_t and hence it provides a measurement of the severity of trade frictions in the goods market. After matching takes place, trades occur at prices $P_t(\omega)$ set by monopolistically competitive firms. Households consume $c_t(\omega)$ for each $\omega \in \chi_t$, and the firms' owners consume their residual output $\hat{c}_t(\omega)$.

2.3. The money supply process

The monetary authority injects money into the economy via a lump sum transfer received by firms in each period. Firms do not have access to receipts from current sales of goods and must finance current wage payments from cash accumulated from the previous period.⁶ Letting X_t denote this cash transfer, the money supply process is $M_{t+1}^s = M_t^s + X_t = (1 + \mu)M_t^s$ where μ is the money growth rate. This transfer occurs after the labor market closes but before the goods market opens. As will be noted formally below, our steady-state results will not be sensitive to this particular timing convention for the cash transfer.

2.4. Optimization by households

In each period, each household is matched with a set of χ_t firms/products with measure α_t in their desirable consumption set and faces a monetary price $P_t(\omega)$ set by each firm ω . Letting M_t denote the beginning-of-period household money holdings, the representative household chooses $\{c_t(\omega), n_t, s_t, M_{t+1}\}$ to maximize (1), subject to the composite good specification, (2), the time endowment constraint, $L_t = 1 - n_t - s_t$, and the budget

⁵We generally impose the Inada conditions, $\lim_{n \rightarrow 0} f'(n) = \infty$ and $\lim_{n \rightarrow \infty} f(n) = 0$, except in the special case of a linear production function.

⁶This finance constraint should not be interpreted as an ex-ante cash-in-advance constraint. It is the endogenous ex-post outcome of the richer environment detailed in Laing et al. (2005) where firms have the option of accumulating goods for the payment of wages. In a different context, wage finance constraints have been used in limited participation models (e.g., Fuerst, 1992). Also, as households in our model do not face a finance constraint, cash transfers to them will not alter our principal findings.

constraint,

$$M_t + W_t n_t - \int_{\lambda_t} P_t(\omega) c_t(\omega) d\omega - M_{t+1} \geq 0. \quad (5)$$

Since optimal consumption involves a pairwise decision relative to other differentiated goods encountered in the market let bold face notation represent market variables. Consider consumption demand $c_t(\omega)$ at a particular firm setting price $P_t(\omega)$ given market consumption choices c_t at firms setting prices P_t . Optimization equates the marginal rate of substitution between these two differentiated goods with their relative price:

$$\left(\frac{c_t}{c_t(\omega)} \right)^{-1/\gamma} = \frac{P_t}{P_t(\omega)}, \quad (6)$$

where consumption demand for a particular product ω is given by

$$c_t(\omega) = \frac{W_t n_t}{\alpha(s_t) P_t^{1-\gamma} P_t(\omega)^\gamma}. \quad (7)$$

The remaining efficiency conditions for work effort and search effort are

$$U_L(D_t, L_t) = U_D(D_t, L_t) \frac{W_t}{P_t} \alpha(s_t)^{1/(\gamma-1)}, \quad (8)$$

$$\frac{W_t}{P_t} = \frac{\gamma}{\gamma-1} \alpha'(s_t) c_t. \quad (9)$$

Eq. (8) equates the marginal disutility of work effort with its marginal benefit and (9) equates the opportunity cost of search effort with its marginal benefit of additional variety. The latter is strictly increasing in the preference for variety (i.e., decreasing in γ). Finally, as standard in the literature, we consider the situation where the opportunity cost of holding cash (the implicit nominal rate) is strictly positive. This corresponds to the steady-state restriction that $\mu > \beta - 1$. Hence, as households do not face a finance constraint, this restriction implies that they end each period with $M_{t+1} = 0 \forall t$. The household's optimal choices are captured by a sequence $\{c_t, c_t, n_t, s_t\}$ that solves Eqs. (6)–(9) given $\{P_t, P_t, W_t\}$.

2.5. Optimization by firms

We now consider the optimal price setting behavior of a generic firm ω that takes the prices of other firms P_t as given, and sets a best response price $P_t(\omega)$. Denoting the firm's labor and money demand as \hat{n}_t and \hat{M}_{t+1} , respectively, the representative firm takes household consumption demands as given by (7) and chooses $\{\hat{c}(\omega), \hat{n}_t(\omega), P_t(\omega), \hat{M}_{t+1}\}$ to maximize the discounted value of the firm's owner consumption, $\sum_{t=0}^{\infty} \beta^t \hat{c}(\omega)$ subject to

$$f[\hat{n}_t(\omega)] - \alpha(s_t) c_t(\omega) - \hat{c}(\omega) \geq 0, \quad (10)$$

$$\hat{M}_t + \alpha(s_t) P_t(\omega) c_t(\omega) + X_t - W_t \hat{n}_t(\omega) - \hat{M}_{t+1} \geq 0, \quad (11)$$

$$\hat{M}_t \geq W_t \hat{n}_t(\omega). \quad (12)$$

Eq. (10) is the firm's resource constraint, (11) is its flow budget constraint—total cash balances at the beginning of next period cannot exceed the sum of current period balances,

receipts from sales, and the monetary transfer net of wage payments, and (12) is the firm's wage finance constraint.

To characterize the stationary equilibrium we scale all nominal variables by the beginning-of-period money stock: $\hat{m}_t = \hat{M}_t/M_t^s$, $w_t = W_t/M_t^s$, and $p_t = P_t/M_t^s$. With strictly binding resource, budget, and cash constraints, we can rewrite (11) and (12) as

$$\hat{m}_{t+1} = \frac{1}{1 + \mu} [\hat{m}_t + \alpha(s_t)p_t(\omega)c_t(\omega) - w_t\hat{n}_t(\omega) + \mu], \quad (11')$$

$$\hat{m}_t = w_t\hat{n}_t(\omega). \quad (12')$$

The Nash equilibrium in the price-setting game yields the familiar markup of price over marginal cost:

$$p_t = \left(\frac{\gamma}{\gamma - 1} \right) \frac{w_{t+1}(1 + \mu)}{\beta f'(\hat{n}_{t+1})}. \quad (13)$$

The monopolistic markup of price over next period wages depends negatively on γ and next period's marginal product of labor. Since firms must finance wage payments with cash carried over from the previous period, this markup is also increasing in the inflation rate μ .⁷ The firm chooses an optimal sequence $\{p_t, \hat{n}_t\}$ given $\{w_t, \hat{c}_t\}$, where $\{\hat{c}_t\}$ is then determined by (10) with equality.

2.6. Equilibrium

We focus on a *stationary symmetric monetary equilibrium* that is a sequence of quantities: $\{c_t, n_t, s_t\}$ and prices $\{p_t, w_t\}$ that satisfy $p_t = \mathbf{p}_t$, (7)–(9), (12') and (13), and market clearing for the labor and money markets: $n_t = \hat{n}_t$ and $\hat{m}_t = \hat{m}_{t+1} = 1$. From (12') we have $w_t = 1/n_t$ and from (7) and (2) we have

$$c_t = \frac{1}{p_t \alpha(s_t)}, \quad (14)$$

$$D_t = \alpha(s_t)^{\gamma/(\gamma-1)} c_t = \alpha(s_t)^{1/(\gamma-1)} \left(\frac{1}{1 + \mu} \right) \left(\frac{\gamma - 1}{\gamma} \right) \beta f'(n_t) n_t. \quad (15)$$

A *steady-state equilibrium* is hence a symmetric monetary equilibrium with time invariant quantities and prices, denoted by: $\{c^*, n^*, s^*\}$ and $\{p^*, w^*\}$, respectively.⁸

To make our analysis more concrete, we will adopt some specific functional forms for preferences and technology. Consider: $f(n) = n^\varphi$, where $\varphi \leq 1$, a linear matching technology $\alpha(s) = a_0 + a_1 s$, where $a_0, a_1 \geq 0$ and $U(D, L) = [\eta D^\rho + (1 - \eta)L^\rho]^{1/\rho}$, where $\eta \in (0, 1)$ and $\rho \in [0, 1]$. These forms allow us to consider the scale effects of labor demand elasticity (φ) and pure wealth effects from the matching technology (a_0). The CES specification embodies both the linear case ($\rho \rightarrow 1$) where goods and leisure are perfect

⁷As $\mu \rightarrow (\beta - 1)$ and $\gamma \rightarrow \infty$, the conventional competitive equilibrium result emerges, as the product price converges to the marginal cost of production.

⁸At this point note that the timing convention of when firms receive the cash transfer is inconsequential. If X_t is received before the labor market opens then in a market clearing steady-state equilibrium (12') and (14) become $(1 + \mu) = wn$ and $c = (1 + \mu)/\alpha p$. Substituting these into (13) and simplifying yields (15). Eqs. (8) and (9) are unchanged and we have identical conditions.

substitutes and the Cobb–Douglas case ($\rho \rightarrow 0$) where goods and leisure are Pareto complements.⁹ In the next two sections, we begin by considering a benchmark specification with fixed search effort and then the general case where search effort is chosen optimally.

3. A benchmark case: exogenous search and matching

In order to highlight the essential role of endogenous search effort and matching, we begin by considering the case where the matching rate is fixed at $\alpha(s) = a_0 > 0$ and hence $s^* = 0$.¹⁰

Proposition 1. *Given $\alpha_t = a_0$, there exists a unique steady-state equilibrium n^* solving the system (8), (13)–(15). In this monetary equilibrium: (i) an improvement in matching (a_0) increases work effort, n^* , consumption, D^* , and real output y^* ; (ii) a higher money growth rate (μ) reduces work effort, n^* , consumption, D^* , and real output, y^* .*

These results are verified by substituting (15) into (8) to yield an equilibrium locus determining n^* .

Intuitively, an increase in the exogenous matching rate a_0 increases the marginal benefit of work effort expressed in terms of greater consumption variety and this leads to higher overall consumption and steady-state output. Money growth, on the other hand, creates an inflation tax effect which, for a given matching rate, decreases labor demand and equilibrium work effort. Real money balances used by firms to finance labor declines as well as composite consumption and output. This inflation tax effect on work effort is consistent with many general equilibrium monetary models which predict a negative relationship between inflation and market activity. However, as we shall see below, endogenizing the household's choice of search effort and the extent that search frictions bind can drastically change the characterization of the steady state and the real effects of inflation.

4. The general case: endogenous search

Having characterized the properties of an economy with exogenous search effort and a fixed matching rate, we now return to the more interesting general model outlined in Section 2, where $\alpha(s) = a_0 + a_1s$. For a given search time allocation s , (8) corresponds to an efficiency condition for optimal work effort. Substituting (13)–(15) into (8) gives the *LL locus*:

$$\frac{1 - \eta}{\eta} (1 + \mu)^\rho = \left[\alpha(s)^{1/(\gamma-1)} \left(\frac{\gamma - 1}{\gamma} \right) \beta f'(n) \right]^\rho \left(\frac{1 - n - s}{n} \right)^{1-\rho}. \quad (16)$$

⁹For balanced growth preferences: (i) with separability in consumption and leisure and exhibiting constant elasticity of intertemporal substitution (as in the endogenous growth literature), the qualitative results resemble those in the linear case and (ii) when the form is multiplicative (as in the real business cycle literature), the qualitative results resemble those in the Cobb–Douglas case.

¹⁰To isolate the pure wealth effects of the matching technology we are essentially assuming that it is costless to contact a fixed measure a_0 of firms in the goods market. Alternatively we could specify a fixed time cost \bar{s} of contacting these firms and re-normalize the time endowment to $1 = L + n + \bar{s}$. This would not alter the marginal conditions which drive our basic findings.

The LL locus denotes the optimal response of work effort to a change in search effort and its shape in the (s, n) space depends on the substitutability of consumption and leisure. All else being equal, when ρ is sufficiently large (high consumption-leisure substitutability), an exogenous increase in s reduces leisure time but provides additional consumption. Since consumption and leisure are highly substitutable, optimal work effort increases implying an upward sloping LL locus. Conversely, when ρ is sufficiently small (low consumption-leisure substitutability), an exogenous increase in s leads to substitution towards leisure and away from work effort. This gives a downward sloping LL locus. There can also exist values of ρ where an increase in search effort raises work effort for low values of s and vice versa, leading to a non-monotonic LL locus.

For a given level of work effort allocation n , (9) corresponds to an efficiency condition for optimal search effort. Substituting (14) and $w = 1/n$ into (9) gives the *SS locus*:

$$n = \left(\frac{\alpha(s)}{\alpha'(s)} \right) \left[\frac{\gamma - 1}{\gamma} \right]. \quad (17)$$

The SS locus denotes the optimal response of search effort to a change in work effort in the (s, n) space. An exogenous increase in work effort lowers the marginal benefit of labor supply and this must be equated with the marginal benefit of search effort. As search effort must rise, the SS locus is always upward sloping.

A steady state can be characterized by $\{s^*, n^*\}$ satisfying (16) and (17) and hence the intersection of LL and SS in the (s, n) space.¹¹ The impact of exogenous changes in the matching rate a_0 are similar to Section 3—an increase in a_0 (decrease in search frictions) generally promotes productive real activity. The additional feature is that the positive wealth effect from the matching technology decreases optimal search effort s^* . In the propositions below we characterize equilibria for several alternative cases, and focus specifically on the effects of the money growth rate μ .

Proposition 2. *Given $\rho > 0$ sufficiently small, there exists a unique steady-state equilibrium $\{s^*, n^*\}$ where money growth discourages work and search effort, n^* and s^* , and decreases aggregate output, y^* .*

Graphically, the uniqueness of the steady state is verified by the intersection of a strictly upward sloping SS with the downward sloping LL. Money growth shifts LL downwards as the inflation tax causes households to substitute away from work effort towards leisure. This leads to a steady-state reduction in both work and search effort (see Fig. 1). The predicted decline in labor and product market activities and steady-state output is again consistent with conventional monetary models.¹² However, as we shall see below, these predictions are not robust to variations in the substitutability between consumption and leisure.

Proposition 3. *For $\rho = 1$, there exists a unique steady state $\{s^*, n^*\}$ in which money growth affects work, search effort, and output (n^*, s^*, y^*) negatively for $(\gamma - 1) > 1/(1 - \varphi)$, and positively for $1 < (\gamma - 1) < 1/(1 - \varphi)$.*

¹¹Formal verification of our analysis comes directly from (16) and (17) and is omitted here. The reader is referred to an earlier working paper, Laing et al. (2003), for the technical details.

¹²Also, in the case of Cobb–Douglas utility where $\rho = 0$, money is superneutral.

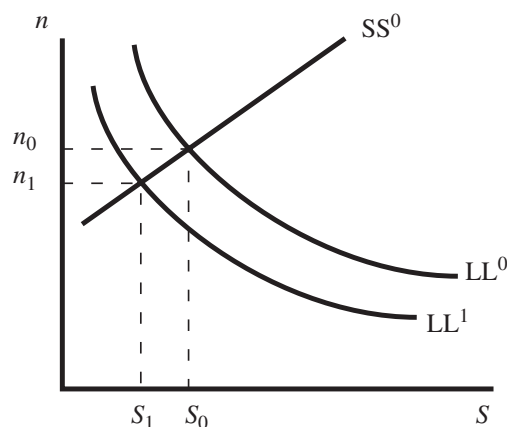


Fig. 1. Unique steady state with money growth, $\rho > 0$ small.

Recall that with ρ sufficiently large, both SS and LL are upward sloping. As before, a greater money growth rate, all else equal, taxes work effort, shifting LL downwards. For γ sufficiently large, equilibria occurs where SS is steeper than LL and the decline in work effort lowers the incentives to invest in search effort (see Fig. 2B). However, for γ sufficiently small, steady state occurs where LL is steeper than SS and the decline in work effort leads to a substitution towards search effort. Consequently, the resulting increase in consumption variety increases the marginal incentives of work effort. Such a positive feedback caused by money growth leads to an overall increase in employment and real output (see Fig. 2A).

This latter result is in stark contrast to the outcome of Section 3 with a fixed matching rate. The intuition is as follows. As inflation taxes real balances there is a substitution towards augmenting the consumption basket with greater variety if the preference for variety is sufficiently large (γ small). Hence, households increase the number of firms contacted via search effort. This “hot potato” effect of inflation was first identified in money search models by Li (1994) and further elaborated upon by subsequent work in the literature. In our multiple-matching framework a greater matching rate enlarges consumption variety and increases optimal work effort when consumption and leisure are highly substitutable.¹³ It is this complementarity between work and search effort that leads to a positive correlation between money, inflation, and output. It suggests that if trade frictions are severe and money is important as a medium of exchange, increasing money growth can actually encourage productivity through its impact on trade and market participation.¹⁴ Furthermore, this market participation complementarity leads to the following proposition regarding the non-uniqueness of steady states:

¹³The size of the inflation tax effect on real wages depends upon the elasticity of labor demand in the competitive labor market. A more elastic labor demand (larger φ) leads to a greater decline in the market clearing real wage (given labor supply). This strengthens the value of additional variety and the substitution towards search effort for ρ large. Hence, a linear production function ($\varphi = 1$) provides the strongest case for $\partial n^*/\partial \mu$ and $\partial s^*/\partial \mu > 0$.

¹⁴These results hold for inflation rates that are consistent with the pure monetary equilibrium. In Laing et al. (2005) we show that sufficiently large values of μ will lead to outcomes where the monetary equilibrium unravels and barter transactions emerge.

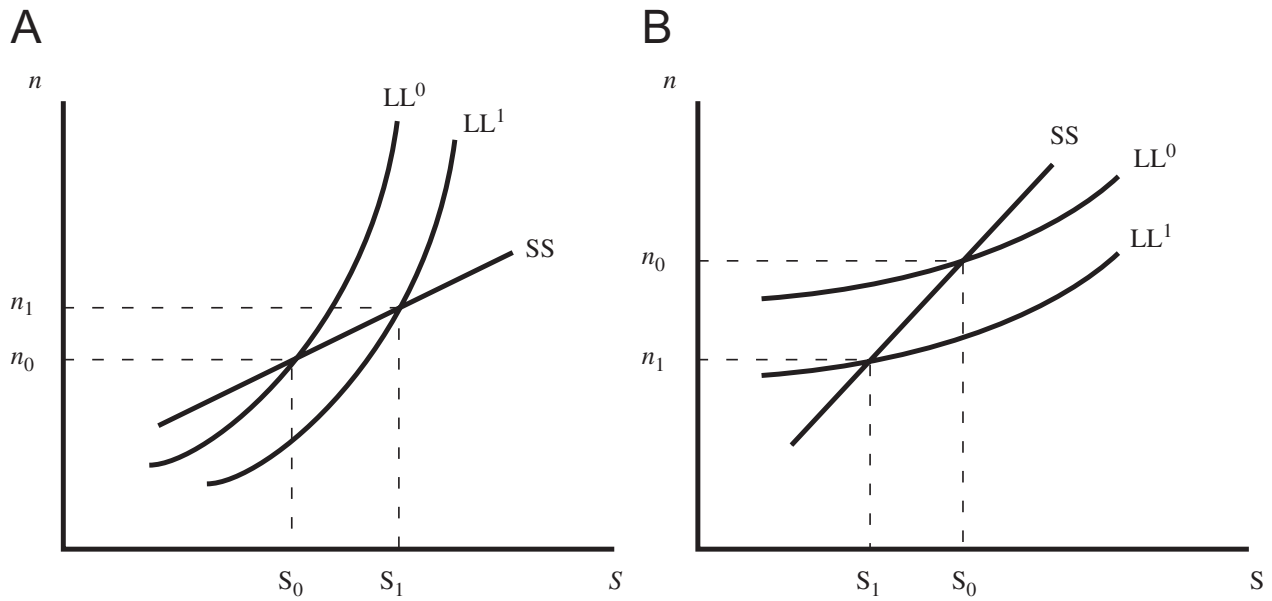


Fig. 2. Unique steady state with money growth, $\rho = 1$: (A) $\gamma - 1 < 1/(1 - \phi)$; (B) $\gamma - 1 > 1/(1 - \phi)$.

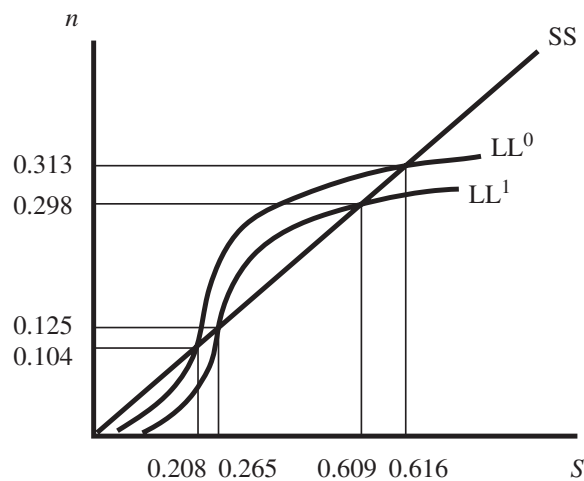


Fig. 3. Multiple equilibria and money growth

Proposition 4. For $0 < \rho < 1$, multiple (non-degenerate) steady states may emerge where: (i) equilibria can be ranked by a monotone increasing relationship between n^* and s^* ; (ii) there will be at least one equilibria where work and search effort, n^* and s^* , and output y^* respond positively to the money growth rate.

The existence of multiple equilibria can be verified graphically given a non-monotonic LL locus as described above. Since SS is strictly upward sloping part (i) of the proposition is immediate. Hence there may be an even or odd number of steady states. To see part (ii), Fig. 3 provides a numerical illustration where there are two steady states: a low output equilibria, where $(s^*, n^*) = (0.208, 0.104)$ and a high output equilibria, $(s^*, n^*) = (0.626, 0.313)$.¹⁵ Increasing the money growth rate μ from 0 to 0.10 shifts LL

¹⁵Parameters used for this illustration are $\gamma = 2$, $\eta = 0.4$, $\rho = 0.8$, $\phi = 0.8$, $a_0 = 0$, $a_1 = 8$, $\beta = 0.99$ and initially $\mu = 0$.

downwards, increasing work and search effort in the low output equilibria to (0.256,0.125) and decreasing work and search effort in the high output equilibria to (0.609,0.298). Hence the properties of the low output steady state, where LL intersects SS from below are the same as in Proposition 3 when γ is small and money growth increases n^* and s^* .

The existence of multiple equilibria arises once again from the complementarity between the optimal choices of work and search effort. When the elasticity of substitution between goods and leisure is moderately high and the preference for consumption variety is sufficiently great, nonlinear positive interactions between work and search effort can generate a multiplicity of non-degenerate steady states where at least one of the equilibria will display a positive money-output correlation.

5. Summary and conclusion

This paper has investigated the effects of monetary growth and inflation on economic activity in a multiple-matching model of money where trade frictions are characterized by limited consumption variety. Our principle finding is that the allocation of time towards costly exchange activity (i) provides a channel wherein money growth is positively correlated with real output and (ii) may lead to the existence of multiple steady states. This finding is driven by several features of our framework that we believe provide a novel way of modeling monetary economies. First, as the benefits of monetary exchange stem from the utility generated from additional consumption variety, the preference for variety provides a useful metric to think about the importance of search frictions. This feature adds an additional dimension of the exchange process not addressed in the monetary search and matching literature. Second, time allocation allows us to link productive activity in the labor market with exchange activity in the goods market, which distinguishes our approach from conventional monetary models. This linkage implies that a strong preference for consumption variety leads to a high degree of complementarity between participation in both markets. It is this positive feedback between production and exchange decisions that explains our findings. If money as a medium of exchange is valued in overcoming such search frictions, then a positive relationship between inflation and productive activity and/or the multiplicity of monetary equilibria can arise as an equilibrium outcome.

We believe that a multiple-matching approach is well suited to study issues of interest in monetary economics. It highlights the trade frictions which leads to the use of money in a tractable manner and permits a more transparent comparison with traditional Walrasian approaches. By doing so it helps bridge a gap in the literature between monetary theory, which was the focus of the first generation search models of money, and monetary policy, which was the emphasis of the competitive equilibrium frameworks. The multiple-matching framework is also amenable to quantitative applications, such as studying the effects of monetary shocks on labor market and output variability over the business cycle and it can be easily modified to consider the economic and welfare implications of alternative monetary policy rules.

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