

Preference Bias and Outsourcing to Market: A Steady-State Analysis*

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Abstract

We analyze a model that focuses on the export/outsourcing decision. Outsourcing has the advantage of providing better information about local preferences. The disadvantage is that producing in the host country also means using the inferior technology embodied in the local capital. The decision of whether to offer an outsourcing contract weighs these two effects against each other. The host country accepts the outsourcing contract if the higher price they pay for the outsourced good is worth the benefit of consuming a manufactured good closer to their ideal variety. These results suggest that as low-income countries develop they become a more attractive destination for outsourcing because their capital grows to meet production needs and the local market is more lucrative. In addition, the developing low-income country finds the outsourcing contract more attractive since their increased demand for the correct variety of the manufactured good increases. This suggests that preference-based outsourcing is more likely to occur with higher-income host countries.

1. Introduction

The study of the causes and consequences of outsourcing of production by firms has received increasing attention since the turn of the century (e.g. Jones, 2000; Grossman and Helpman, 2002, 2005; Antras, 2005). This literature focuses primarily on the outsourcing decision based on the theory of the firm, regarding the make-or-buy outcome as an equilibrium phenomenon.¹ Our paper develops a preference-based theory of outsourcing in which outsourcing is beneficial because it mitigates the information problem with respect to the identification of local tastes for the manufactured good—that is, the local economy is engaged in “design for manufacturing” to suit local preferences.²

We consider a simple, dynamic framework with the following specific features:

- There are two countries, a low-income developing local country and a high-income developed source country.
- There are two types of goods, a homogeneous good (food, for example) produced only in the local country, and a particular variety of a manufactured good (laptop computer, for example) that may be produced in the source or the local country.
- Only the source country does R&D to improve the production technology of the manufactured good.
- The source country may outsource production of the manufactured good to the local economy.

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- Only through outsourcing can the source country firm correctly identify the local economy's variety-specific preferences.
- Each country employs a specific factor. The source country utilizes general capital for production and technology advancement, whereas the local country uses physical capital for production only.
- The manufacturers in the source country maximize its value by choosing (i) whether to outsource the production of their good and (ii) the allocation of capital between production and research.
- In the local country, an integrated representative consumer-producer decides (i) whether to accept the outsourcer's contract (if offered) and (ii) how to allocate consumption and capital intertemporally (and intersectorally if an outsourcing contract is offered and accepted).

We assume initially the source country exports the manufactured good to the local country, referred to as *the export regime*. We then identify under what circumstances the equilibrium switches from the status quo to *the outsourcing regime*. We show that with a sufficiently high initial capital stock, the preference of residents in the local country for the manufactured good grows over time to a level that is sufficiently high to shift preferences away from the necessity toward the manufactured good. Such shifts enhance the importance of variety-specific preferences. As a result, the local country agents find the better matched, outsourced manufactured good more desirable and are more willing to pay for it. The larger price markup over the non-ideal product makes outsourcing more profitable. Thus, our model features "cross-country complementarity" that triggers the outsourcing equilibrium when local country income is sufficiently high.

It is possible that outsourcing may never arise. In particular, this can happen if local country income is sufficiently low, capital is scarce, the discount rate high, and its consumers spend a major portion of their income on the necessity. In this case, consumers care less about the variety of the manufactured good and hence have lower demand. This lower demand drives down the price markup for outsourced goods, thereby giving the source country less incentive to outsource. Hence, for very low-income countries the export regime can be the steady state.

2. The Basic Model

We begin with a very simple model. There are two countries (local and source), two production inputs (general capital and land), and two goods N (necessity) and M (manufactured good). There are J varieties, $j \in \{1, 2, \dots, J\}$, $J > 2$ of the manufactured good. Denote the j th variety as M^j . General capital can be thought of as a composite stock of human, physical, and knowledge capital. The necessity is produced in the local country using both local capital and land. However, the manufactured good requires the input of the source country's research capital. Hence, by assumption the manufactured good can only be produced in the local country if it is outsourced.³ We focus on two sets of decisions. In the source country, the manufacturer decides (i) whether to export good M^j or whether to outsource the production of good M^j and (ii) how to allocate capital to production and research. In the local country, an integrated representative consumer-producer decides (i) whether to accept the outsourcing contract (if offered) and (ii) how to allocate consumption and capital intertemporally (and intersectorally if an outsourcing contract is offered and accepted).

Source Country

The source country owns the “high-quality” general capital that can be used for either manufacturing or research. Though the allocation of this capital is essential, its accumulation is not the focus of this paper but rather the accumulation of knowledge for production technology is the key. Thus, denoting this capital stock at the beginning of time t as H_t , we assume for simplicity that there is no depreciation or accumulation.

The aggregate general capital stock at the beginning of time t is divided into manufacturing capital (H_t^M) and research capital (H_t^R):

$$H_t = H_t^M + H_t^R, \tag{1}$$

and the stock of capital remains constant over time:

$$H_t = H_0 = 1, \text{ for all } t = 1, 2, \dots \tag{2}$$

It is convenient to denote $s_t = H_t^R / H_t$ as the share of research capital, which under (2) also represents the stock of research-use capital.

In addition to research capital we assume that source country firms can invest in R&D directly. The timing of decisions is as follows. At the beginning of period t , the existing capital stock $H_t = 1$ is divided into research uses (s_t) and manufacturing uses ($1 - s_t$). Next, production of the manufactured good occurs based on the current technology (A_t^j). Then the R&D investment (z_t) is implemented, which is added to the research-use capital (s_t) to determine the next-period production technology (A_{t+1}^j). This structure allows us to reduce the dimensionality of the state variables to one (A_t^j only).

By assuming for simplicity that the R&D investment and the research-use capital are perfect substitutes, the evolution of manufacturing technology is then governed by:

$$A_{t+1}^j = \psi (A_t^j)^\mu (H_t^R + z_t)^{1-\mu} = \psi (A_t^j)^\mu (s_t + z_t)^{1-\mu}, \tag{3}$$

where $\psi > 0$ is a technology advancement scaling factor and $\mu \in (0, 1)$.

Exporting the manufactured good Now consider the case in which the source country exports. We assume that the source country does not know the *ideal variety* of the manufactured good desired by the local country (call this variety i). So, we assume the exporter chooses some variety $j \neq i$. Manufacturing capital enters the production of M_t^j immediately, but research capital only enhances future production of M_{t+1}^j .

Assume that the technologies of producing any varieties require identical resources, i.e. there are only horizontal differentiations among different varieties. The production of a manufacturing product of variety j is therefore given by,

$$M_t^j = A_t^j (H_t^M)^\gamma = A_t^j (1 - s_t)^\gamma, \tag{4}$$

where $\gamma \in (0, 1)$.

Given a constant discount rate r and the exogenous unit cost of R&D investment q_t , the Bellman equation facing the representative firm in the source country producing and exporting the manufactured good to the local country (denoted by superscript EX) can therefore be specified as:

$$W^{EX}(A_t^j) = \max_{s_t, z_t} [p_t^j A_t^j (1 - s_t)^\gamma - q_t z_t] + \frac{1}{1+r} W^{EX}(A_{t+1}^j), \tag{5}$$

subject to

$$A_{t+1}^i = \psi(A_t^i)^\mu (s_t + z_t)^{1-\mu},$$

where we have excluded variables taken as given by individuals in the arguments of the value function. That is, the source firm’s value is equal to its current profit (revenue from producing and exporting the manufactured good, net of the R&D investment cost) plus its discounted future value.

The first-order conditions with respect to s_t and z_t are:

$$\frac{1}{1+r} \frac{\partial W^{EX}(A_{t+1}^i)}{\partial A_{t+1}^i} (1-\mu) \psi(A_t^i)^\mu (s_t + z_t)^{-\mu} = \gamma p_t^i A_t^i (1-s_t)^{\gamma-1}, \tag{6}$$

$$\frac{1}{1+r} \frac{\partial W^{EX}(A_{t+1}^i)}{\partial A_{t+1}^i} (1-\mu) \psi(A_t^i)^\mu (s_t + z_t)^{-\mu} = q_t, \tag{7}$$

which equate the marginal benefit of each of the two choice variables with the marginal cost. The Benveniste–Scheinkman condition that governs the optimal path of A_t^i is:

$$\frac{\partial W^{EX}(A_t^i)}{\partial A_t^i} = p_t^i (1-s_t)^\gamma + \frac{1}{1+r} \frac{\partial W^{EX}(A_{t+1}^i)}{\partial A_{t+1}^i} \mu \psi(A_t^i)^{\mu-1} (s_t + z_t)^{1-\mu}. \tag{8}$$

Outsourcing With production outsourcing, the source country still maintains full control of the technology. Since it does not produce the manufactured good it uses all of the general capital H_t for research purposes. The local country’s subcontracting firm determines the employment of the local capital K_t^M to manufacture the contracted good. The advantage of outsourcing is that involvement of the local country enables an exact identification of the local *ideal variety* i .

It is necessary to modify the production of a manufacturing product of the *ideal variety* i and the evolution of manufacturing technology A_t^i as follows:

$$M_t^i = A_t^i (K_t^M)^\gamma, \tag{9}$$

$$A_{t+1}^i = \psi(A_t^i)^\mu (H_t + z_t)^{1-\mu} = \psi(A_t^i)^\mu (1+z_t)^{1-\mu}. \tag{10}$$

Assume that the outsourcing contract is one featuring *revenue-sharing*. More specifically, the value of the contracted manufacturing output is so divided that a fraction ϕ goes to the local country with the remaining fraction to the source country.

The Bellman equation facing the representative manufacturer is now given by,

$$W^{OS}(A_t^i) = \max_{z_t} \left[(1-\phi) p_t^i A_t^i (K_t^M)^\gamma - q_t z_t \right] + \frac{1}{1+r} W^{OS}(A_{t+1}^i), \tag{11}$$

subject to

$$A_{t+1}^i = \psi(A_t^i)^\mu (1+z_t)^{1-\mu}.$$

Compared to the exporting regime, the production technology under the outsourcing regime is unambiguously higher due to complete specialization by devoting the entirety of the general capital to research ($s = 1$). Yet, the associated profit can be higher (as a

result of using a better technology) or lower (if the local country’s supply of capital is scarce or if the cost of revenue-sharing outweighs the technology gain).

The first-order condition and the Benveniste–Scheinkman condition are:

$$\frac{1}{1+r} \frac{\partial W^{OS}(A_{t+1}^i)}{\partial A_{t+1}^i} (1-\mu)\psi(A_t^i)^\mu (1+z_t)^{-\mu} = q_t, \tag{12}$$

$$\frac{\partial W^{OS}(A_t^i)}{\partial A_t^i} = (1-\phi)p_t^i(K_t^M)^\gamma + \frac{1}{1+r} \frac{\partial W^{OS}(A_{t+1}^i)}{\partial A_{t+1}^i} \mu\psi(A_t^i)^{\mu-1} (1+z_t)^{1-\mu}. \tag{13}$$

Local Country

We assume that capital in the less-developed local economy is of “low quality” in the sense that it can only be used for manufacturing purposes. So, without research capital or spending on R&D (which we rule out) $A_t^i = 0$ and the manufactured good cannot be produced by the local company without help from the source country.⁴ Denote the aggregate capital stock in the local country at the beginning of time t as K_t . Then, capital may be devoted to production of the necessity or production of the *outsourced* manufactured good:

$$K_t = K_t^N + K_t^M. \tag{14}$$

The entire stock depreciates at a rate $\delta \in (0, 1)$ and is augmented by gross investment v_t , thus evolving according to:

$$K_{t+1} = (1-\delta)K_t + v_t. \tag{15}$$

This evolution process is modeled explicitly because it is the only force of growth to the local economy in transition to the steady state.

The production of the necessity takes the simple Cobb–Douglas form:

$$N_t = (K_t^N)^\beta, \tag{16}$$

where $\beta \in (0, 1)$. The output of the necessity will be partly consumed by the local economy (call this fraction η) and partly exported to the world market (fraction $1 - \eta$) at a fixed price, normalized to one.

Denote the subjective time discount rate as $\rho > 0$. The periodic utility of the representative consumer-producer (of a given ideal taste type i) purchasing manufactured goods of variety j is specified as (index i dropped for notational convenience whenever it does not create any confusion):

$$U^i(N_t^d, M_t^j) = \ln(N_t^d) + \ln(\theta + \Gamma^j M_t^j), \tag{17}$$

where N_t^d measures the local demand for the necessity, $\theta > 0$ indicates that the manufactured good is not a necessity, and $\Gamma^j \leq 1$ captures preference bias toward the ideal variety. Obviously, when the purchased variety j is the ideal variety i (which occurs only when the manufactured good is outsourced), we have $\Gamma^i = 1$. Otherwise, there is “variety-specific preference discounting” at a factor $\Gamma^j < 1$ ($j \neq i$). The lower is Γ^j , the greater the variety-specific preference discount.

Let Y_t denote the periodic income in units of the necessity good (to be specified later). The representative consumer-producer’s intertemporal budget constraint is given by,

$$N_t^d + p_t^j M_t^j + v_t = Y_t,$$

which together with (15) implies,

$$K_{t+1} - K_t = Y_t - \delta K_t - N_t^d - p_t^j M_t^j. \tag{18}$$

Importing the manufactured good When the manufactured good is produced in the source country, the optimization problem facing the local country’s representative agent is simple: maximize lifetime utility by allocating intertemporally available resources to consumption of one (the necessity) or both goods and capital investment. In this case, the variety j of the manufactured good provided to the local economy is given in the absence of outsourcing. Thus, we have: $K_t^N = K_t$, $K_t^M = 0$, and $Y_t = (K_t)^\beta$.

Given the discount rate $\rho > 0$, the Bellman equation facing the representative agent in the case where the manufactured good is produced in the source country and exported is given by,

$$V^{EX}(K_t) = \max_{N_t^d, M_t^j} \{ \ln(N_t^d) + \ln(\theta + \Gamma^j M_t^j) \} + \frac{1}{1 + \rho} V^{EX}(K_{t+1}), \tag{19}$$

subject to

$$K_{t+1} = (K_t)^\beta + (1 - \delta)K_t - N_t^d - p_t^j M_t^j.$$

The first-order conditions for N_t^d and M_t^j equate the respective marginal utility with the associated shadow price,

$$\frac{1}{N_t^d} = \frac{1}{1 + \rho} \frac{dV^{EX}(K_{t+1})}{dK_{t+1}}, \tag{20}$$

$$\frac{\Gamma^j}{\theta + \Gamma^j M_t^j} = \frac{1}{1 + \rho} \frac{dV^{EX}(K_{t+1})}{dK_{t+1}} p_t^j. \tag{21}$$

Given $\theta > 0$, the lower the variety-specific preference discounting (higher Γ^j), the greater the marginal utility of consuming the manufactured good (the left-hand side of (21)). Should the luxurious nature of the manufactured good disappear ($\theta = 0$), the variety-specific preference discounting will no longer influence the consumption behavior.

The Benveniste–Scheinkman condition is:

$$\frac{dV^{EX}(K_t)}{dK_t} = \frac{1}{1 + \rho} \frac{dV^{EX}(K_{t+1})}{dK_{t+1}} [\beta(K_t)^{\beta-1} + (1 - \delta)]. \tag{22}$$

Producing the outsourced manufactured good When the manufactured good is produced in the local economy via outsourcing, the optimization problem facing the local country’s representative agent has to include the allocation of capital between the two sectors (provided that accepting the outsourcing contract is profitable). With the local economy’s involvement, it is assumed that the ideal variety i of the manufactured good can be identified. In this case, $K_t^M = K_t - K_t^N$, $N_t = (K_t^N)^\beta$, $M_t^i = A^i(K_t - K_t^N)^\gamma$, and

$$Y_t = (K_t^N)^\beta + \phi p_t^i A_t^i (K_t - K_t^N)^\gamma. \tag{23}$$

That is, real income accrued to the local country now depends on two sources, both from producing the necessity and from revenue sharing by producing the manufactured good.

The Bellman equation facing the representative agent in this outsourcing case now becomes:

$$V^{OS}(K_t) = \max_{N_t^d, M_t^i, v_t} \{ \ln(N_t^d) + \ln(\theta + M_t^i) \} + \frac{1}{1+\rho} V^{OS}(K_{t+1}), \tag{24}$$

subject to

$$K_{t+1} = (K_t^N)^\beta + \phi p_t^i A_t^i (K_t - K_t^N)^\gamma + (1-\delta)K_t - N_t^d - p_t^i M_t^i.$$

The first-order conditions are:

$$\frac{1}{N_t^d} = \frac{1}{1+\rho} \frac{dV^{OS}(K_{t+1})}{dK_{t+1}}, \tag{25}$$

$$\frac{1}{\theta + M_t^i} = \frac{1}{1+\rho} \frac{dV^{OS}(K_{t+1})}{dK_{t+1}} p_t^i, \tag{26}$$

$$\beta(K_t^N)^{\beta-1} = \gamma \phi p_t^i A_t^i (K_t - K_t^N)^{\gamma-1}. \tag{27}$$

We note that the first-order condition with respect to M_t^i (26) differs from (21) under the exporting regime because there is no variety-specific preference discounting under outsourcing. Also in contrast with the previous case, there is an additional first-order condition (27) that governs optimal capital allocation between the two sectors.

The Benveniste–Scheinkman condition is:

$$\frac{dV^{OS}(K_t)}{dK_t} = \frac{1}{1+\rho} \frac{dV^{OS}(K_{t+1})}{dK_{t+1}} [\gamma \phi p_t^i A_t^i (K_t - K_t^N)^{\gamma-1} + (1-\delta)]. \tag{28}$$

3. Equilibrium

We next turn to the determination of equilibrium. Consider the following timing of events:

1. The source country determines whether to outsource production of the manufactured good to the local country with a revenue-sharing contract that rewards the local country with a fraction ϕ of the surplus accrued.
2. The local country decides whether to accept this outsourcing contract, if offered.
3. Under the exporting or the outsourcing regime, the source country and the local country determine optimal allocations and the equilibrium relative price of the manufactured good is determined as follows.

$$(EX) \quad p_t^j = \frac{\Gamma^j N_t^d}{\theta + \Gamma^j M_t^j} = \frac{(K_t)^{\beta} \eta_t}{(\theta/\Gamma^j) + A_t^j (1-s_t)^\gamma}, \tag{29}$$

$$(OS) \quad p_t^i = \frac{N_t^d}{\theta + M_t^i} = \frac{(K_t^N)^\beta \eta_t}{\theta + A_t^i (K_t - K_t^N)^\gamma}, \tag{30}$$

where η_t is the share of N_t that is consumed in the local country.

Next, we solve for the equilibrium. To satisfy subgame perfection, we solve the problem backward. Before solving the model, however, it is useful to provide a brief comparison between the two market prices of the manufactured goods in units of the necessity. From (29) and (30), there are four channels through which the two manufactured good prices may differ:

	p^j vs p^i
(1) $\Gamma^j < \Gamma^i$	<
(2) $(K^N)^{EX} < (K^N)^{OS}$	>
(3) $A^j < A^i$	>
(4) H^M vs K^M	?

The first channel is through variety-specific preference bias. Since the imported manufactured good is less preferred than the locally produced one (under the outsourcing regime) there is less demand and a lower market price, thus $\Gamma^j < 1$ implies $p^j < p^i$. Secondly, by importing the manufactured good, the local economy can focus exclusively on producing the necessity, and greater supply of the necessity leads to a higher relative price of the manufactured good. Thirdly, by exporting the manufactured good, the source country’s economy must allocate a portion of capital to its production. Thus, less capital is allocated to research leading to a slower rate of technological advancement. This in turn lowers the productivity and hence the supply of manufactured good, leading to a higher relative price of the exported manufactured good. Finally, the supply of the manufactured good also depends on the factor input: $H^M = 1 - s$ versus $K^M = K - K^N$. Even by assuming that the more advanced source country is capital abundant, it is not clear its production-use capital is larger than the counterpart in the less-developed local country. Thus, the effect on the market prices is ambiguous.

In summary, while variety-specific preference bias tends to make the relative price of the manufactured good under the exporting regime lower than under the outsourcing regime, outsourcing-driven production specialization both in the local economy ($K^N < K$) and in the source country ($A^j < A^i$) leads to the opposite outcome. Thus, even without accounting for relative capital abundance (H^M vs K^M), the offsetting demand and supply forces yields inconclusive results regarding the relative price of the manufactured good under the two regimes. We now proceed to solving for the equilibrium.

Stage 3

We will first derive the steady-state solutions in each of the two regimes: exporting and outsourcing.

The exporting regime Under the exporting regime, we can manipulate the first-order conditions and the Benveniste–Scheinkman condition, (6), (7), and (8), to obtain:

$$\gamma p_t^j A_t^j (1 - s_t)^{\gamma-1} = q_t, \tag{31}$$

$$\frac{p_t^j}{p_{t-1}^j} = \frac{(1+r)\gamma(A_{t-1}^j)^{1-\mu}(s_t+z_t)^\mu}{(1-\mu)\psi(1-s_{t-1})^{1-\gamma}(1-s_t)^\gamma\left(1+\gamma\frac{\mu}{1-\mu}\frac{s_t+z_t}{1-s_t}\right)}. \tag{32}$$

Moreover, from (20) and (22), we have:

$$p_t^j(\theta + \Gamma^j M_t^j) = \Gamma^j \eta_t N_t, \tag{33}$$

$$\eta_t N_t = \frac{\eta_{t-1} N_{t-1}}{1+\rho} [\beta(K_t)^{\beta-1} + (1-\delta)]. \tag{34}$$

Imposing the steady state, (31), (32), and (34) become:

$$p^j A^j (1-s)^\gamma = \frac{q}{\gamma} (1-s), \tag{35}$$

$$(1+r)\gamma(A^j)^{1-\mu}(s+z)^\mu = \psi[(1-\mu)(1-s) + \gamma\mu(s+z)], \tag{36}$$

$$\beta(K)^{\beta-1} = \delta + \rho. \tag{37}$$

The expression in (37) is the standard modified golden rule in discrete-time neoclassical growth models, which alone pins down the steady-state value of capital in the local economy:

$$\bar{K} = \left(\frac{\beta}{\delta + \rho}\right)^{\frac{1}{1-\beta}}$$

From (15) and (18), $v = \delta\bar{K}$ and $p^j M^j + \delta\bar{K} = (1-\eta)Y$. This latter relationship, together with the production functions, (4) and (16), implies: $p^j A^j (1-s)^\gamma + \delta\bar{K} = (1-\eta)(\bar{K})^\beta$, which can be further combined with (35) to yield:

$$\frac{q}{\gamma}(1-s) = (\bar{K})^\beta(1-\eta) - \delta\bar{K} = \left[\frac{\delta + \rho}{\beta}(1-\eta) - \delta\right]\bar{K}. \tag{38}$$

or

$$s = 1 - \frac{\gamma}{q} \left[\frac{\delta + \rho}{\beta}(1-\eta) - \delta\right]\bar{K}. \tag{39}$$

That is, a higher export share of the necessity in the local economy $(1-\eta)$ makes the local economy richer and demand higher for the manufactured good, which in turn encourages the source country to allocate general capital to production $(1-s)$. In other words, the share of production-use general capital in the source country and the export share of the necessity in the local country are positively related. This relationship is referred to as the *exportable production (XP) locus*; see the bottom panel of Figure 1.

From (3), $A^j = \psi^{1/(1-\mu)}(s+z)$, which can be substituted into (36) to derive $\gamma(1-\mu+r)(s+z) = (1-\mu)(1-s)$, or

$$z = \frac{(1-\mu) - [(1-\mu) + \gamma(1-\mu+r)]s}{\gamma(1-\mu+r)}. \tag{40}$$

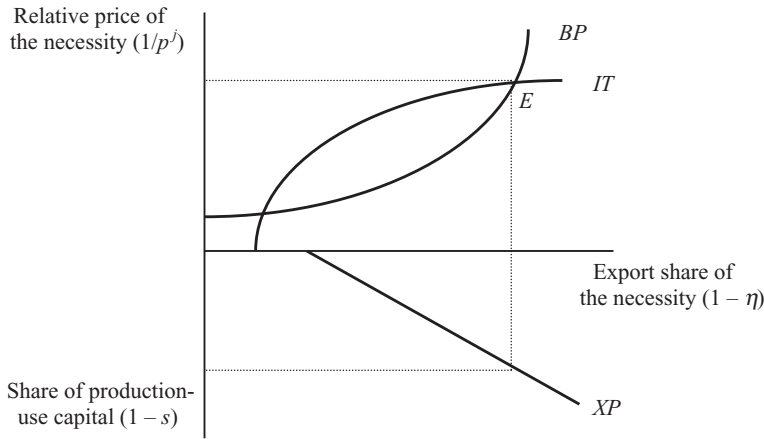


Figure 1. Allocation and Pricing in the Exporting Regime (EX)

Thus, the steady-state level of technology is given by,

$$A^j = \frac{\psi^{1-\mu}(1-\mu)}{\gamma(1-\mu+r)}(1-s). \tag{41}$$

Intuitively, more allocation of general capital to R&D substitutes for R&D investment. Because an increase in s decreases z more than proportionately, it results in a net reduction in the steady-state level of technology.⁵

We next combine (35), (38), and (41) to obtain:

$$\frac{1}{p^j} = B_0(\bar{K})^\gamma \left[\frac{\delta+\rho}{\beta}(1-\eta) - \delta \right]^\gamma, \tag{42}$$

where

$$B_0 = \frac{1-\mu}{1-\mu+r} \frac{\gamma^\gamma \psi^{1-\mu}}{q^{1+\gamma}} > 0.$$

This gives a positive relationship between the relative price of the necessity ($1/p^j$) and the export share of the necessity. This relationship is based on the budget constraint and the intertemporal tradeoff between current production and technological advancement. We therefore refer to it as the *intertemporal trade (IT) locus*. It is straightforward to show that the *IT* locus is not only increasing but strictly concave in $(1-\eta, 1/p^j)$ space, with a horizontal intercept, $\delta\beta/(\rho+\delta)$.

Moreover, we can substitute (35) into (33) (imposing the steady state) and utilize (38) to get:

$$\frac{1}{p^j} = \frac{\theta}{\Gamma^j \left[\delta \bar{K} + (\bar{K})^\beta - 2(\bar{K})^\beta (1-\eta) \right]} = \frac{\theta}{\Gamma^j \bar{K} \left[\delta + \frac{\delta+\rho}{\beta} - 2 \frac{\delta+\rho}{\beta} (1-\eta) \right]}. \tag{43}$$

This gives another positive relationship between the relative price of the necessity and the export share of the necessity. This relationship follows from the budget constraint

and the marginal rate of substitution conditions. We can think of this as the balance-of-payments condition and is hence referred to as the *balance-of-payments (BP)* locus. The *BP* locus is strictly increasing and strictly convex in $(1 - \eta, 1/p^j)$ space, with an asymptote,

$$1 - \eta = \frac{\delta + \frac{\delta + \rho}{\beta}}{2 \frac{\delta + \rho}{\beta}} = \frac{\delta\beta}{\rho + \delta} + \frac{\rho + \delta(1 - \beta)}{2(\rho + \delta)} > \frac{\delta\beta}{\rho + \delta}.$$

Intuitively, the *BP* locus may simply be viewed as the local country’s supply curve of the necessity embedded with optimal consumption allocation and trade balance conditions. Thus, it is not surprising that the *BP* locus slopes upward. The *IT* locus is somewhat harder to explain. Precisely, it is a combination of the intertemporal consumption–capital choice by the local country and the capital allocation decision between production and R&D investment by the source country. Thus, the *IT* locus does not represent the source country’s demand for the necessity and need not be downward-sloping.

Under some regularity conditions, the *IT* and the *BP* loci intersect twice, yielding two steady-state solutions (see the top panel of Figure 1). By numerical exercises (see the Numerical Analysis subsection, below), we find that the low stationary point never arises in equilibrium as it violates some feasibility conditions (particularly, the non-negativity constraint of the production-use general capital). Thus, we will focus exclusively on the high stationary point (point *E* in the top panel of Figure 1) around which we now perform comparative statics.

Changes in	<i>BP</i> locus	<i>IT</i> locus	$1 - \eta$	p^j	s	A^j	z
Decrease in Γ^j or increase in θ	Upward shift	No change	–	+	+	–	–
Increase in ψ	No change	Upward shift	+	–	–	+	+

In response to a greater variety-specific preference bias (due to a reduction in Γ_0 or d^j) or a greater preference bias away from the manufactured good (due to an increase in θ), the *BP* locus shifts up whereas the *IT* locus remains unchanged. As a result, the demand for the manufactured good in the local economy falls, demand for the necessity increases and the export share of the necessity decreases. Owing to a reduced valuation of the manufactured good, the value of marginal product of general capital in production decreases and the general capital stock is reallocated from manufacturing to research. However, the value of technological advancements is also lower, which results in less R&D investment. On balance, the steady-state level of technology is lower (the negative effect via z outweighs the positive effect via s). Both lower technology and lower production-use general capital lead to a lower supply of the manufactured good and hence less export this good to the local country. This supply effect turns out to dominate the demand effect (because the local country lacks the option to produce the manufactured good) and the equilibrium price of the manufactured good rises.

In response to more productive R&D (represented by an increase in ψ), the *IT* locus shifts up whereas the *BP* locus remains unchanged. Thus, the level of technology is higher and the value of technological advancements also rises, thereby increasing the

R&D investment. By substitution, the latter enables more general capital to be re-allocated to production. As a result of both better technology and more general capital input, the supply of the manufactured good by the source firm increases, which subsequently lowers the relative price of the manufactured good and encourages the local economy to substitute the manufactured good for the necessity which leads to a higher export share for the necessity.

The outsourcing regime Analogously, under the outsourcing regime, we can use (12) and (13) to derive:

$$p_t^i = \frac{\mu}{1-\mu} \left[\frac{1+r}{\mu\psi} \left(\frac{1+z_{t-1}}{A_{t-1}^i} \right)^\mu q_{t-1} - \frac{1+z_t}{A_t^i} q_t \right] \frac{(K_t - K_t^N)^\gamma}{1-\phi}, \tag{44}$$

and use (25), (26), (27), and (28) to get:

$$p_t^i(\theta + M_t^i) = \eta_t N_t, \tag{45}$$

$$\eta_t N_t = \frac{\eta_{t-1} N_{t-1}}{1+\rho} \left[\beta(K_t^N)^{\beta-1} + (1-\delta) \right], \tag{46}$$

$$\beta(K_t^N)^{\beta-1} = \gamma\phi p_t^i A_t^i (K_t - K_t^N)^{\gamma-1}. \tag{47}$$

Note that (45) is a special case of (33) with no variety preference discount (as $\Gamma^i = 1$).

Imposing the steady-state condition, we repeat the same steps, using (46) to obtain the modified golden rule:

$$\beta(K_t^N)^{\beta-1} = \rho + \delta, \tag{48}$$

which yield the steady-state value of capital allocated to the production of the necessity:

$$\bar{K}^N = \left(\frac{\beta}{\delta + \rho} \right)^{\frac{1}{1-\beta}}.$$

We then utilize (48) together with the production functions, (4) and (16), to write (45) and (47) as:

$$p^i \theta + p^i A^i (K - \bar{K}^N)^\gamma = \eta (\bar{K}^N)^\beta = \frac{\rho + \delta}{\beta} \bar{K}^N \eta, \tag{49}$$

$$p^i A^i (K - \bar{K}^N)^{\gamma-1} = \frac{\rho + \delta}{\gamma\phi}, \tag{50}$$

which can be combined to yield:

$$p^i \theta + \frac{\rho + \delta}{\gamma\phi} (K - \bar{K}^N) = \frac{\rho + \delta}{\beta} \bar{K}^N \eta. \tag{51}$$

From (15) and (18), we have: $v = \delta K$ and $(1 - \phi)p^i M^i + \delta K = (1 - \eta)N$. This latter expression, together with (4), (16), (23), (48), and (50), implies:

$$K - \bar{K}^N = \bar{K}^N \left[\frac{\frac{\rho + \delta}{\beta}(1 - \eta) - \delta}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right] \quad \text{and} \quad K = \frac{\rho + \delta}{\beta} \bar{K}^N \left[\frac{\frac{1 - \phi}{\phi} \frac{\beta}{\gamma} + (1 - \eta)}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right]. \quad (52)$$

Substituting (52) into (51), we obtain a fundamental steady-state relationship, which is the *BP* locus under outsourcing:

$$\frac{1}{p^i} = \frac{\beta\theta}{(\rho + \delta)\bar{K}^N} [B_1 - B_2(1 - \eta)]^{-1}, \quad (53)$$

where

$$B_1 = 1 + \frac{\beta}{\gamma\phi} \frac{\delta}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} > 1 \quad \text{and} \quad B_2 = 1 + \frac{\frac{\rho + \delta}{\gamma\phi}}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} > 1.$$

This *BP* locus has a similar shape to that under the export regime, which is strictly increasing and strictly convex in $(1 - \eta, 1/p^i)$ space, with an asymptote,

$$1 - \eta = \frac{B_1}{B_2} = \frac{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta + \frac{\delta\beta}{\gamma\phi}}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta + \frac{\rho + \delta}{\gamma\phi}} > \frac{\delta\beta}{\rho + \delta}.$$

Now, from (10) and imposing the steady state, we have:

$$A^i = \psi^{1-\mu}(1+z). \quad (54)$$

By applying (48), (53), and (54), (44) in the steady state can be rewritten as:

$$\frac{1}{p^i} = \frac{(1 - \mu)(1 - \phi)\psi^{\frac{1}{1-\mu}}}{(1 - \mu + r)q(\bar{K}^N)^\gamma} \left[\frac{\frac{\rho + \delta}{\beta}(1 - \eta) - \delta}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right]^{-\gamma}, \quad (55)$$

which is the *IT* locus under outsourcing. Unlike the exporting case, the *IT* locus for outsourcing is downward-sloping and strictly convex in $(1 - \eta, 1/p^i)$ space, with an asymptote, $1 - \eta = \delta\beta/(\rho + \delta)$. The outsourcing equilibrium is illustrated in Figure 2.

Next, we obtain:

$$A^i = \frac{\rho + \delta}{\gamma\phi} (\bar{K}^N)^{1-\gamma} \left[\frac{\frac{\rho + \delta}{\beta}(1 - \eta) - \delta}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right]^{1-\gamma} \frac{1}{p^i}, \quad (56)$$

$$1 + z = \psi^{\frac{-1}{1-\mu}} \frac{\rho + \delta}{\gamma\phi} (\bar{K}^N)^{1-\gamma} \left[\frac{\frac{\rho + \delta}{\beta}(1 - \eta) - \delta}{\frac{1 - \phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right]^{1-\gamma} \frac{1}{p^i}. \quad (57)$$

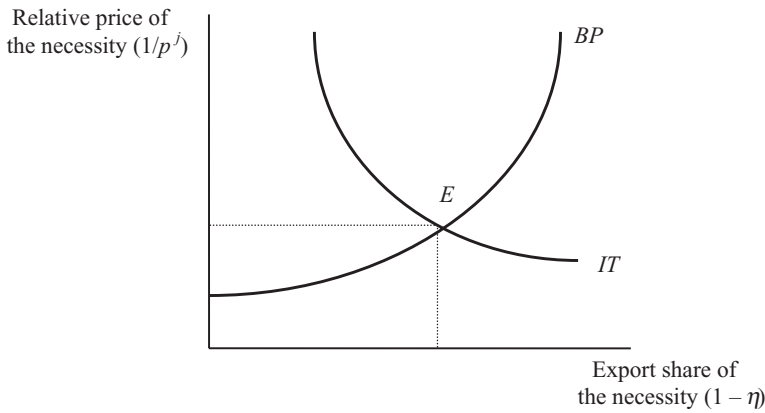


Figure 2. Allocation and Pricing in the Outsourcing Regime (OS)

The comparative statics can be summarized as follows:

(OS)	BP locus	IT locus	$1 - \eta$	p^j	K	A^j	z
Increase in θ	Upward shift	No change	-	-	-	?	?
Increase in Ψ	No change	Upward shift	+	-	+	+	?

In response to a preference shift away from the manufactured good (due to an increase in θ), the BP locus shifts up whereas the IT locus remains unchanged. As a result, the demand for the manufactured good in the local economy is lower, the export share of the necessity is lower, and the relative price of the manufactured good falls. While the total stock of capital falls due to reduced input into manufacturing the outsourced good (the capital input in the traditional sector remains unchanged), the opposing forces cause ambiguous effects on A^j and z .

In response to more productive R&D (represented by an increase in ψ), the IT locus shifts up whereas the BP locus remains unchanged. Thus, local capital is reallocated from the production of the necessity to the production of the outsourced manufactured good, causing the relative price of the manufactured good to fall, the local consumption of the necessity to decrease, and hence the export share of the necessity increases. The total stock of capital increases as a result of more induced demand to manufacture the outsourced good. The steady-state level of technology is unambiguously higher in the absence of sectoral allocation effects in the source country ($s = 1$). The effect on the investment in R&D is ambiguous as higher productivity leads to more investment in R&D via the substitution effect, but less investment due to the factor-saving effect of the better technology.

Stage 2

We turn next to examining whether the representative consumer-producer in the local economy would accept the outsourcing contract. This decision depends on the comparison of the respective value function facing the representative local agent under each regime, which in the steady state can be expressed as:

$$W^{EX} = \frac{1+\rho}{\rho} [\ln(N_t^d) + \ln(\theta + \Gamma^j M^j)],$$

$$W^{OS} = \frac{1+\rho}{\rho} [\ln(N_t^d) + \ln(\theta + M^i)].$$

Using results derived in Stage 3, we have:

$$\begin{aligned} W^{EX} &= \frac{1+\rho}{\rho} \left[\ln(\eta^{EX}) + \frac{\beta}{1-\beta} \ln\left(\frac{\beta}{\delta+\rho}\right) + \ln(\theta + \Gamma^j (A^j)^{EX} (1-s^{EX})^\gamma) \right] \\ &= \frac{1+\rho}{\rho} \left[\ln(\eta^{EX}) + \frac{\beta}{1-\beta} \ln\left(\frac{\beta}{\delta+\rho}\right) \right. \\ &\quad \left. + \ln\left\{ \theta + \frac{\Gamma^j \bar{K}}{(p^j)^{EX}} \left[\frac{\delta+\rho}{\beta} (1-\eta^{EX}) - \delta \right] \right\} \right], \end{aligned}$$

$$\begin{aligned} W^{OS} &= \frac{1+\rho}{\rho} \left[\ln(\eta^{OS}) + \frac{\beta}{1-\beta} \ln\left(\frac{\beta}{\delta+\rho}\right) + \ln(\theta + (A^j)^{OS} (K^{OS} - \bar{K}^N)^\gamma) \right] \\ &= \frac{1+\rho}{\rho} \left[\ln(\eta^{OS}) + \frac{\beta}{1-\beta} \ln\left(\frac{\beta}{\delta+\rho}\right) \right. \\ &\quad \left. + \ln\left\{ \theta + \frac{(\rho+\delta)\bar{K}^N}{\gamma\phi(p^j)^{OS}} \left[\frac{\rho+\delta}{\beta} (1-\eta^{OS}) - \delta \right] \right\} \right]. \end{aligned}$$

When the source country deals with a low-income country, the local economy's export share of the necessity is low and, thus, the associated consumption share (η^{EX}) is high. Owing to high discounting (ρ), the steady-state value of capital (\bar{K}) is low. Thus, the direct consumption effect via the first term of the value function dominates the indirect trade effect via the third term, making the local agent more likely to import the manufactured good (M^i) than accepting an outsourcing contract. Moreover, the less desirable the manufactured good is, the smaller the gap between the willingness to pay for an ideal variety and the reference variety j . That is, $(p^j)^{OS} - (p^j)^{EX}$ is low. This will be crucial for the next stage to which we now turn.

Stage 1

Finally, we determine whether the representative firm in the source country would choose outsourcing of the manufactured good over exporting. This is determined by comparing the respective value function facing the representative source country agent under each regime, which in the steady state can be expressed as:

$$V^{EX} = \frac{1+r}{r} [p^j A^j (1-s)^\gamma - qz],$$

$$V^{OS} = \frac{1+r}{r} [(1-\phi) p^i A^i (K - \bar{K}^N)^\gamma - qz].$$

Again, we can utilize results derived in Stage 3 to obtain:

$$\begin{aligned}
 V^{EX} &= \frac{1+r}{r} \left\{ \left[\frac{\delta + \rho}{\beta} (1-\eta) - \delta \right] \bar{K} - qz \right\} \\
 &= \frac{1+r}{r} \left\{ q + \left[\frac{r - \gamma(1-\mu + r)}{1-\mu + r} \right] R(1-\eta) \right\},
 \end{aligned}$$

$$\begin{aligned}
 V^{OS} &= \frac{1+r}{r} \left[(1-\phi) p^i A^i (K - \bar{K}^N)^\gamma - qz \right] \\
 &= \frac{1+r}{r} \left\{ (1-\phi) \frac{\rho + \delta}{\gamma\phi} \bar{K}^N \left[\frac{\frac{\rho + \delta}{\beta} (1-\eta) - \delta}{\frac{1-\phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right] \right. \\
 &\quad \left. + q - q\psi^{\frac{-1}{1-\mu}} \frac{\rho + \delta}{\gamma\phi} \frac{1}{p^i} \left[\bar{K}^N \frac{\frac{\rho + \delta}{\beta} (1-\eta) - \delta}{\frac{1-\phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right]^{1-\gamma} \right\} \\
 &= \frac{1+r}{r} \left\{ q + (1-\phi) R^{OS}(1-\eta) - q\psi^{\frac{-1}{1-\mu}} \left(\frac{\rho + \delta}{\gamma\phi} \right)^\gamma \frac{1}{p^i} [R^{OS}(1-\eta)]^{1-\gamma} \right\},
 \end{aligned}$$

where

$$R^{EX}(1-\eta) = \bar{K} \left[\frac{\delta + \rho}{\beta} (1-\eta) - \delta \right]$$

and

$$R^{OS}(1-\eta) = \frac{\rho + \delta}{\gamma\phi} \bar{K}^N \left[\frac{\frac{\rho + \delta}{\beta} (1-\eta) - \delta}{\frac{1-\phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right],$$

measuring steady-state revenue gained by the source country firm (both are increasing in the local firm’s export share of the necessity, $1 - \eta$).

Based on the results from Stage 2, $(p^i)^{OS} - (p^i)^{EX}$ is low in the case when the source country is dealing with a low-income country. In this case, the source country has less incentive to outsource. Thus, the source country will choose to produce and export the manufacture good, so the low-income local country imports the manufactured good.

Without further specification of the bargaining between the source and the local countries, we can conclude:

<i>Equilibrium outcome</i>	<i>Conditions</i>
Exporting (EX)	$W^{EX} - W^{OS} > 0$ and $V^{EX} - V^{OS} > 0$
Outsourcing (OS)	$W^{EX} - W^{OS} < 0$ and $V^{EX} - V^{OS} < 0$
Indeterminate	$(W^{EX} - W^{OS})(V^{EX} - V^{OS}) < 0$

Numerical Analysis

Since we cannot obtain an analytic solution for the equilibrium prices and allocations we turn to numerical analysis. Our benchmark parametrization is to set both the depreciation rate and the time discount rate at $\delta = \rho = 5\%$, the variety-specific preference discounting at $\Gamma^j = 0.8$, and the real interest rate at $r = 10\%$. The output elasticities of capital are chosen to be $\beta = 0.3$ and $\gamma = 0.35$, implying the production of the manufactured good is more capital intensive than the production of the necessity. The technology parameters are set at $\psi = 0.85$ and $\mu = 0.9$, the real cost of R&D investment at $q = 0.3$, and the preference bias parameter at $\theta = 0.07$.

Then, if the equilibrium results in exporting of the manufactured good, the relative price of the manufactured good is $p^i = 3.63$. The comparable figure under the outsourcing regime with $\phi = 0.25$ is $p^i = 3.30$. Thus, under the benchmark case, the supply effect dominates the demand effect.

The allocation under each regime is provided as follows:

<i>Regime</i>	$1 - \eta$	s	z	A	K^M/K	N^d	M	N^d/Y
EX	0.476	0.391	0.479	0.171	0.0%	0.839	0.144	52.4%
OS	0.462	1	0.195	0.235	11.4%	0.861	0.198	48.8%

Thus, under the outsourcing regime, the technology in the source country is far better, which generates a “trickle-down” effect via revenue-sharing, enabling the local country to consume more of both goods and to export more of its own products.

Next, we ask which regime arises in equilibrium? Using the benchmark parametrization, our numerical results suggest outsourcing occurs if (but not only if) $\phi \in (0.21143, 0.33179)$. When ϕ (the fraction of output that goes to the host country under outsourcing) is too low, the local country desires to import the manufactured good despite the fact that the source country prefers to outsource. When ϕ is too high, the source country no longer offers outsourcing as an option but simply produces and exports the manufactured good. Should the local country feature a relatively high initial capital stock, there always exists a non-empty range of revenue-sharing schemes under our benchmark parametrization such that outsourcing can arise in the steady state. Of course, outsourcing may never arise if (i) local country capital is scarce (due to a sufficiently low initial capital stock or an insufficient accumulation of the capital stock as a result of a high time discount rate), (ii) local country income is low (due to a sufficiently low capital return associated with low β), or (iii) its consumers spend a major portion of their income on the necessity (due to a sufficiently high value of θ).

4. Conclusion

We consider a model that focuses on the export/outsource decision. For the source country the advantage of outsourcing is that the host country partners have better information about local preferences, so the outsourcing outcome involves “design for manufacturing” to suit local preferences. The tradeoff is that producing in the host country also means using the inferior technology embodied in the local capital. The decision of whether to offer an outsourcing contract weighs these two effects against each other. For the host country, they accept the outsourcing contract if the higher price they pay for the outsourced good is worth the benefit of consuming a manufactured

good closer to their ideal variety. These results suggest that as the low-income country develops it becomes a more attractive destination for outsourcing because their capital grows to meet production needs and the local market is more lucrative. In addition, the developing low-income country finds the outsourcing contract more attractive since their increased demand for the correct variety of the manufactured good increases. This suggests that preference-based outsourcing is more likely to occur with higher-income host countries.

Appendix

We list some useful expressions for computing values.

1. (EX):

$$M^i = A^i(1-s)^\gamma,$$

$$N = (\bar{K})^\beta,$$

$$Y = (\bar{K})^\beta,$$

$$\bar{K} = \left(\frac{\beta}{\delta + \rho} \right)^{\frac{1}{1-\beta}},$$

$$s = 1 - \frac{\gamma}{q} \left[\frac{\delta + \rho}{\beta} (1-\eta) - \delta \right] \bar{K},$$

$$z = \frac{(1-\mu)}{\gamma(1-\mu+r)} - \frac{(1+\gamma)(1-\mu) + \gamma r}{\gamma(1-\mu+r)} \left[1 - \frac{\gamma}{q} R(1-\eta) \right].$$

2. (OS):

$$M^i = A^i(K - \bar{K}^N)^\gamma,$$

$$N = (\bar{K}^N)^\beta,$$

$$Y = (\bar{K}^N)^\beta + \phi p^i A^i (K - \bar{K}^N)^\gamma,$$

$$\bar{K}^N = \left(\frac{\beta}{\delta + \rho} \right)^{\frac{1}{1-\beta}},$$

$$p^i A^i (K - \bar{K}^N)^\gamma = \frac{\rho + \delta}{\gamma \phi} \bar{K}^N \left[\frac{\frac{\rho + \delta}{\beta} (1-\eta) - \delta}{\frac{1-\phi}{\phi} \frac{\rho + \delta}{\gamma} + \delta} \right],$$

$$1+z = \psi^{\frac{-1}{1-\mu}} \frac{\rho+\delta}{\gamma\phi} \frac{1}{p^i} (\bar{K}^N)^{1-\gamma} \left[\frac{\frac{\rho+\delta}{\beta}(1-\eta)-\delta}{\frac{1-\phi}{\phi} \frac{\rho+\delta}{\gamma} + \delta} \right]^{1-\gamma}.$$

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Notes

1. Ethier (1986) argues that production outsourcing will occur when information exchanges between the source and local countries are not too costly to prevent arm's-length contracting.
2. In a very different context, Kemp et al. (2001) investigate gains from trade when trade can change preferences.
3. We are ruling out the possibility that the local manufacturer can purchase the research capital from the source country. For a paper that looks at this issue see Spulber (2008).
4. A natural and interesting extension is to follow Chen and Shimomura (1998) by allowing the local economy to adopt the technology developed in the source country.
5. More precisely,

$$\frac{dz}{ds} = -\frac{(1-\mu)+\gamma(1-\mu+r)}{\gamma(1-\mu+r)} < -1.$$