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COMPETITIVE EQUILIBRIUM FORMATION OF MARKETPLACES WITH HETEROGENEOUS CONSUMERS

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This paper studies the formation of marketplaces in a finite linear spatial economy with heterogeneous endowments and preferences in which the residential location of consumers is fixed a priori. Contrary to the standard urban economics literature where the transaction center is exogenously given, this environment allows a continuum of marketplace equilibria. Under proper assumptions, a social welfare maximum exists. Given welfare weights which are inversely related to the ratio of marginal utilities of income, it is supported by a competitive marketplace equilibrium. This establishes the existence of an equilibrium and provides a method of computing the unique welfare-maximizing marketplace equilibrium.

1. Introduction

The aim of this paper is to study the competitive equilibrium formation of marketplaces in a finite linear spatial economy with heterogeneous endowments and preferences in which the residential location of consumers is fixed a priori. Under proper assumptions, a social welfare maximum exists and it can be supported by a competitive marketplace equilibrium, which consequently assures the existence of a competitive marketplace equilibrium. This provides a step toward a general equilibrium analysis for some spatial models of heterogeneous consumers with endogenous determination of marketplaces. Furthermore, the paper offers a method of computing the welfare-maximizing marketplace equilibrium.

In the urban economic literature, little has been done to develop a general equilibrium model with an endogenous formation of marketplaces (or transaction centers).[1] To study the competitive equilibrium formation of marketplaces, we consider a finite (or discrete) economy where the number of

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[1]An exception is Baesemann (1977) who investigated the formation of marketplace in the steady state. A questionable issue in that model is the discontinuity of admissible consumption due to the mismatch of interregional trading with respect to the travel distance for each trader.

agents is finite as in standard general equilibrium models.[2] For simplicity, the residential location of consumers is given a priori and the allocation of land is ignored, since these have been well-established in the literature [see the elaboration by Fujita (1986)]. Notice that in a spatial model, it is rather difficult to prove the existence of a competitive marketplace equilibrium by directly following Arrow and Debreu (1954) in the presence of the location non-convexity of consumption sets and/or the discontinuity of the demand correspondence with respect to location. Alternatively, one may adopt the welfare approach to prove the existence of a Pareto optimum or a social optimum first and then obtain an equilibrium by using the second welfare theorem [see Lucas and Prescott (1971) and Negishi (1960), for example]. Recently, Scotchmer (1985) and Fujita and Smith (1987) applied the welfare approach to the land-use theory under continuum setups. Scotchmer (1985) characterized an efficient allocation of space supported by a hedonic unit price system and established the existence of competitive equilibria by giving a fixed point argument on utilities. Fujita and Smith (1987) proved the existence of a competitive equilibrium for residential land markets by employing a population excess-supply correspondence on a utility space which allowed the use of finite-dimensional fixed point theory. The application of the welfare approach simplifies the existence proof of land-use equilibria substantially.

In this paper, we extend Negishi (1960) to verify the existence of a social welfare maximum and then establish the equivalence theorem (between a social optimum and a marketplace equilibrium) to assure the existence of a competitive marketplace equilibrium.[3] In contrast to Scotchmer (1985) and Fujita and Smith (1987), the equilibrium and optimal location of marketplaces is endogenously determined given a fixed residential location under a finite setup. The main findings of this paper are as follows. Under proper assumptions, a social welfare maximum exists. For any given endowments and preferences, there is a set of social welfare weights such that a social welfare maximum *is* a competitive marketplace equilibrium (i.e., the so-called equivalence theorem). Instead of computing equilibrium solutions, which appears to be very complicated in spatial models, we concentrate on the

[2]In their pivotal work, Schweizer, Varaiya and Hartwick (1976) built up a discrete spatial model in which the existence of a compensated equilibrium and a housing equilibrium was demonstrated. However, the existence of a standard competitive marketplace equilibrium and its welfare properties had not been proved. More recently, Berliant (1982) rectified this deficiency by constructing a pure exchange general equilibrium model of land with finite number of consumers in which all classical general equilibrium properties, such as existence (of equilibrium) and welfare theorems, are restored. In most of the urban models, a continuum of locations together with a continuum of agents are considered.

[3]Negishi (1960) showed that a competitive equilibrium is a social welfare maximum, with the social welfare weights in inverse proportion to the marginal utilities of income. Hence, proving the existence of an equilibrium is equivalent to proving the existence of a social welfare maximum.

social optimization problem. By choosing appropriate social welfare weights, the social optimum is competitively attainable without imposing an endowment redistribution. We find that there is in general a continuum of marketplace equilibria. To uniquely determine the location of marketplace, a soocial planner (or market maker) need be introduced. The unique equilibrium marketplace which maximizes the social welfare is relatively close to the habitation in which the household with a higher taste bias toward spatial factors resides. Moreover, the social welfare weights which provide the equivalence between social welfare maximization and competitive marketplace equilibrium are inversely related to the ratio of marginal utilities of income. Thus, by solving the social planner's optimization problem, one can consequently compute a welfare-maximizing marketplace equilibrium using the aforementioned set of weights.

The reminder of this paper is organized as follows. In section 2, the analogue of simple 2 (types of agents) $\times$ 2 (habitable islands) $\times$ 2 (goods) economy is described, followed by the establishment of the optimization problems of representative consumers. Section 3 constructs a competitive marketplace equilibrium and a utilitarian social welfare maximum. We then prove the equivalence theorem and the existence of a competitive marketplace equilibrium in section 4. Section 5 concludes the paper.

2. The model

This paper deals with a 2 (types of representative agents) $\times$ 2 (habitable islands) $\times$ 2 (goods) deterministic, static, pure exchange economy. The two types of finitely many agents (or households), indexed by $h \in \{1,2\}$, are assumed to reside at two islands, O^1 and O^2 respectively, in which a certain amount of a locational specialized commodity endowment can be collected costlessly.[4] For the sake of simplicity, the population for each type of agents and the distance between the two islands are normalized to be one. Given a strictly positive, finite amount of endowment of commodities (e_h^h),[5] agent h needs to reallocate his/her good endowment by trading with others at the endogenously determined marketplace located on the bridge connecting the two islands.[6] Denote by $x^h \in [0,1]$ agent h's view of the distance between island O^1 and the marketplace to be formed. Let $(p_1(x), p_2(x)) \in R_+$ be a price system of the mobile commodities for distance x and $c_i^h \in R_+$ be

[4]Assume that each agent has better knowledge in collecting the good at his/her own habitable island. Thus agent h has to be an island-O^h resident (or a good-h collector).

[5]Throughout this paper, superscripts and subscripts indicate agent and good, respectively.

[6]The spatial structure is, though it is linear, different from that in the long narrow (or linear) city proposed by Solow and Vickrey (1971) in which there are a continuum of habitable locations in between the two border islands. The reader is also referred to de Palma and Papageorgiou (1988) which consider another type of two-zone spatial structure, representing the center and the suburb, in a continuum economy.

agent h's consumption of good i for $h, i \in \{1,2\}$. Then, for given endowment and prices, the optimization problems of representative consumers can be written as

$$(\mathrm{PC}^1) \qquad \max_{(c_1^1, c_2^1) \in R_+^2,\, x^1 \in [0,1]} \quad U^1 = U^1(c_1^1, c_2^1, 1-x^1),$$

$$\text{s.t.} \quad p_1(x^1)c_1^1 + p_2(1-x^1)c_2^1 = p_1(x^1)e_1^1, \tag{1}$$

$$(\mathrm{PC}^2) \qquad \max_{(c_1^2, c_2^2) \in R_+^2,\, x^2 \in [0,1]} \quad U^2 = U^2(c_1^2, c_2^2, x^2),$$

$$\text{s.t.} \quad p_1(x^2)c_1^2 + p_2(1-x^2)c_2^2 = p_2(1-x^2)e_2^2, \tag{2}$$

where (1) and (2) are budget constraints and preferences are well-behaved, i.e. $U^h: R_+^2 \times [0,1] \mapsto R \cup \{-\infty\}$ is strictly increasing, continuously twice differentiable and strictly concave over the feasible set with the closure of every indifference surface in R_+^3 being contained in the corresponding strictly positive orthant R_{++}^3 (Inada condition). The incorporation of the spatial variable x into both representative agent's utility functions can be interpreted as a general geographical factor or disutility of travel time. The closer the marketplace is to home, the higher will the utility be. The dependence of the commodity pricing on the spatial factor captures the underlying delivery cost.[7] The longer the delivery distance is, the higher will the offering price be. Therefore, all utility and price functions are increasing in the spatial arguments specified above.[8] Further, the use of the Inada condition helps rule out the possible autarchic equilibrium of little interest.[9]

To apply standard concave programming techniques, we need to impose concavity properties on price functions p_1 and p_2 with respect to x^1 and x^2, which, unfortunately, have no theoretical support. For simplicity, this paper takes a linear price system as a benchmark case:

$$p_i(x) = q_i + w^i x, \tag{3}$$

where $(q_i, w^i) \in R_+^2$ for $x \in [0,1]$. If the spatial variable x represents a general geographical factor, w reflects hedonic pricing or unit transportation cost. If

[7]For modelling spatially competitive price systems, see Samuelson (1952) and Takayama and Judge (1964). To construct a real transportation cost function explicitly, one may use the spatial-discounting setup in Smith (1976). In my paper, the consideration of explicit physical transportation cost is not essential and hence omitted (because all importables have to be transported along the entire bridge given fixed residential location a priori).

[8]In fact, x^1 (or $1-x^2$) measures the distance to which agent 1 (or 2) plans to travel. Thus U^1 (or U^2) is decreasing in x^1 (or $1-x^2$) and hence increasing in $1-x^1$ (or x^2).

[9]For alternative conditions, see the necessity assumption (A.2) in Scotchmer (1985) and the essentiality assumption (R.4) in Fujita and Smith (1987).

the spatial variable x measures disutility factor of travel time, w indicates implicit wage rate or marginal utility of leisure in consumption utils.

Let U_j^h be the partial derivative of U^h with respect to the jth argument. Further, denote the Lagrangean multipliers associated with (1) and (2) by λ^1 and λ^2, respectively. Under the above assumptions on preferences, endowments and prices, the application of the Kuhn–Tucker theorem yields the following first-order conditions for interior solutions of (PC^1) and (PC^2):

$$\begin{aligned}
&U_1^1=\lambda_1(q_1+w^1x^1),\\
&U_2^1=\lambda_1[q_2+w^2(1-x^1)],\\
&U_3^1=\lambda_1[w^1(e_1^1-c_1^1)+w^2c_2^1],\\
&U_1^2=\lambda_2(q_1+w^1x^2),\\
&U_2^2=\lambda_2[q_2+w^2(1-x^2)],\\
&U_3^2=\lambda_2[w^2(e_2^2-c_2^2)+w^1c_1^2].
\end{aligned} \tag{4}$$

By eliminating λ_1 and λ_2, one obtains standard 'no-arbitrage' conditions.

3. Competitive marketplace equilibrium and social welfare maximization

We turn next to construct marketplace equilibria and social optima. In the absence of pecuniary transportation costs, material balance conditions can be specified as follows:

$$c_i^1+c_i^2=e_i^i, \quad i=1,2. \tag{5}$$

In the context of a spatial model with endogenously determined market location, the feasibility of transactions between the two types of agents requires that the two types of agents must travel a sufficient distance in order to meet with each other to trade. Since agents behave competitively, each agent should plan to match the transaction by trading a more preferable market location with another good given his/her trade partner's travel distance. Under the assumption of strict monotonicity of preferences in spatial factors, an overlap in travel can never be optimizing in equilibrium. Accordingly, we have the following trip matching condition:

$$x^1=x^2=z\in[0,1]. \tag{6}$$

We first define the market equilibrium. A *competitive marketplace equili-*

brium (*CME*) is a list of non-negative quantities and positive prices $((c_i^{he}), (x^h), (p_i), (q_i), (w^h))$, with $x^h \in [0,1]$ and p_i defined as in (3) $(h, i \in \{1,2\})$, for given space structure, endowments and preferences, such that:

(i) agent h solves (PC^h), for $h \in \{1,2\}$; and,
(ii) material balance conditions (5) and the trip matching condition (6) hold.

Different from conventional locational equilibria, the location of marketplace is endogenously determined, while the residential location is predetermined.[10]

We next construct the social planner's optimization problem. Define the social welfare function as weighted sum of both representative agents' utilities: $SW = \theta^1 U^1 + \theta^2 U^2$, where θ^1, $\theta^2 \in [0,1]$ are social welfare weights satisfying $\theta^1 + \theta^2 = 1$. An allocation $((c_i^h), z)$ defined over $R_+^4 \times [0,1] (h, i \in \{1,2\})$ is *transactively feasible* if it satisfies material balance conditions (5) for each good, given the trip matching condition (6). A *social welfare maximum* (SWM) is a transactively feasible allocation, $((c_i^{h*}), z^*)$, which maximizes the social welfare, $SW = \theta^1 U^1 + \theta^2 U^2$, for a given pair of social welfare weights in the set $\Theta = \{(\theta^1, \theta^2) \mid \theta^1, \theta^2 \in [0,1] \text{ and } \theta^1 + \theta^2 = 1\}$. Using (6) to replace the spatial factors x in both representative agents' utility fuctions by z, the social planner's problem is then governed by

$$\text{(SW1)} \qquad \max_{\{c_i^h\} \in R_+^4,\, z \in [0,1]} \quad SW = \theta^1 U^1(c_1^1, c_2^1, 1-z) + \theta^2 U^2(c_1^2, c_2^2, z),$$

$$\text{s.t.} \quad (5).$$

Let μ_1 and μ_2 be the Lagrangean multipliers associated with (5). We next derive the first-order conditions for interior solutions of (SW1):

$$\begin{aligned} &\theta^1 U_1^1 = \mu_1, \\ &\theta^1 U_2^1 = \mu_2, \\ &\theta^2 U_1^2 = \mu_1, \\ &\theta^2 U_2^2 = \mu_2, \\ &\theta^1 U_3^1 = \theta^2 U_3^2. \end{aligned} \tag{7}$$

First-order conditions (7) together with the material balance conditions (5)

[10]Following Arrow and Debreu (1954), it can be easily shown that a competitive equilibrium with a given market location $z \in (0,1)$ exists. We also note that an equilibrium marketplace cannot be at either island under the Inada condition.

determine the unique socially optimal consumption allocation and marketplace for a given set of social welfare weights.

4. Equivalence and existence theorems

We are now ready to establish the exietence of a social welfare maximum and the equivalence theorem which relates a social welfare maximum to a competitive marketplace equilibrium. Consequently, the existence of a competitive marketplace equilibrium is verified by applying Negishi (1960).

First, we prove the existence of social welfare maximum.

Theorem 1. Under well-behaved preferences, positive, finite endowments and the space structure described in section 2, for every $(\theta^1,\theta^2)\in\Theta$, there is a social welfare maximum.

Proof. Define a^h as agent h's consumption share of his home good, h. Then (SW1) can be rewritten in terms of shares (a^1,a^2):

$$\text{(SW2)}\quad \max_{\{a^h\},z} F_{\text{SW}}=\theta^1U^1[a^1e_1^1,(1-a^2)e_2^2,1-z]+\theta^2U^2[(1-a^1)e_1^1,a^2e_2^2,z]$$
$$+I_{\text{SW}}(a^1,a^2,z),$$

where the indicator function I_{SW} is defined as

$$I_{\text{SW}}(a^1,a^2,z)=\begin{cases}0, & \text{if}\quad (a^1,a^2,z)\in[0,1]\times[0,1]\times[0,1],\\ -\infty, & \text{otherwise.}\end{cases}$$

Then the theorem follows immediately from the fact that (SW2) maximizes a properly continuous (i.e., continuous in the effective domain) function F_{SW} over a compact domain $[0,1]\times[0,1]\times[0,1]$. Q.E.D.

We next construct the equivalence theorem.

Theorem 2. Under the assumptions described in Theorem 1, together with the linear price system specified as in (3), for any given endowments and preferences, there is a set of social welfare weights in Θ such that a social welfare maximum is a competitive marketplace equilibrium.

Proof. By strict monotonicity of preferences, there is no qualitative difference between equality constraints and inequality constraints. Considering inequality constraints, all constraints sets in problems (PC^h) and (SW1) are convex under the linear price setting. Both U^1 and U^2 are strictly increasing, continuously twice differentiable and strictly concave, so is *SW*. Hence standard concave programming techniques can be applied. Since $0<e^h<\infty$ for all h, $c_i^h=0$ (for all h,i) would satisfy the Slater condition for the Kuhn–Tucker theorem. Under the Inada condition, the necessary and sufficient conditions for a CME are (3), (4), (5) and (6); the necessary and sufficient conditions for an SWM are (5) and (7) given (6). Take

$$\theta^h=\delta/\lambda^h, \quad \text{where} \quad \delta=\lambda^1\lambda^2/(\lambda^1+\lambda^2), \tag{8}$$

which makes the sum of social welfare weights one. Further, imposing (3), let the marginal social welfares of income be

$$\mu_1=\delta p_1=\delta(q_1+w^1 z),$$

$$\mu_2=\delta p_2=\delta[q_2+w^2(1-z)].$$

Then, using (5), one can show that (4) and (7) are equivalent. That is, a SWM, associated with a set of social welfare weights defined in (8), is a CME by comparing the necessary and sufficient conditions for a CME and a SWM listed above. Q.E.D.

Technically, solving the social welfare maximization problem is easier than solving the competitive marketplace equilibrium problem. With the equivalence theorem, we can concentrate on the social planner's problem and then get an equilibrium by choosing a proper set of social welfare weights as given in (8). In fact, (8) implies that $\theta^1=1/[1+(\lambda^1/\lambda^2)]$ and $\theta^2=1/[1+(\lambda^2/\lambda^1)]$. That is, the required social welfare weights are inversely related to the ratio of the marginal utilities of income.

Now, using Theorems 1 and 2 together with Negishi's theorem 5 and Kakutani's fixed-point theorem, we conclude the following:

Theorem 3. Under the assumptions described in Theorem 2, there is a competitive marketplace equilibrium.

We have proved the existence of a competitive marketplace equilibrium as well as its welfare properties under a finite setup. There is in general a continuum of marketplace equilibria due to the undefined property right of location. To uniquely determine the location of marketplace, a social planner (or market maker) need be introduced. By the equivalence theorem (Theorem 2), such a social optimum can be supported by competitive prices. In effect, we obtain a unique equilibrium which maximizes the social welfare. Applying Theorem 2 [especially, eq. (8)], the set social welfare weights providing the equivalence between a social optimum and a marketplace equilibrium is a singleton under strict concavity and strict monotonicity of preferences; moreover, it can be easily computed. Finally, the existence of a competitive marketplace equilibrium is assured using Theorem 3.

5. Concluding remarks

This paper has studied the mobile good consumption plan and the endogenous formation of the marketplace given fixed immobile good allocation and residential location a priori. In order to obtain a full competitive spatial equilibrium in which residential location and land consumption are as well determined, one may consider a two-stage choice problem. The optimal plan described in this paper can be treated as the first-stage optimal decision. Given the first-stage decision, the optimal plan for immobile good consumption and residential location can be determined in the second stage, following the standard urban economics literature using a finite setup. Repeating the two-stage decision procedure may produce the full spatial equilibrium allocation under proper assumptions. As long as the utility function is separable in mobile goods and location-dependent variables (immobile goods and spatial factors), one may still expect to obtain a continuum of equilibria unless a market for location is fully specified. Thus, the results of this paper should be applicable to more general models.

For further study along this line, one may establish a well-defined property right of location. For example, the hedonic price setting in Scotchmer (1985) may be of use when immobile goods are introduced. In particular, location may be priced according to its attribute associated with immobile goods or the space that is used to consume.

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