# Spatial Mismatch in Search Equilibrium

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We construct a search equilibrium model for a city with central and suburban labor markets that is consistent with the set of empirical regularities commonly associated with the spatial mismatch hypothesis: a higher rate of unemployment for central city residents than suburban residents, a higher job vacancy rate for suburban firms, and reverse commuting and higher suburban wages. The effectiveness and welfare implications of public policy programs that might be used to remedy the underlying mismatch are examined.

#### I. Introduction

Inner-city unemployment, in the midst of an otherwise robust metropolitan economy, is often referred to as spatial mismatch (Kain 1968). The phrase refers to the spatial separation of unemployed central city workers and the low-skilled job vacancies that are increasingly concentrated in the suburbs of major U.S. metropolitan areas. Wilson (1987, 1996) has done much to intensify recent interest in the mismatch phe-

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nomenon and to recommend its alleviation as a partial remedy for the unfortunate pathologies of central cities.

With regard to the labor market, spatial mismatch is characterized by three salient empirical features: (1) the unemployment rate for low-skilled central city residents exceeds that of comparable suburban dwellers, (2) the low-skilled job vacancy rate is higher for suburban firms than for central city firms, and (3) wages for low-skilled workers are higher at suburban firms than at central city firms. Evidence for these phenomena is provided by Holzer (1991), Ihlanfeldt (1992), and Zax and Kain (1996), among others.

On the face of it, mismatch is a puzzle to labor economists. Why do not unemployed workers in one location simply fill the free vacancies in another location? In an excellent survey of the empirical evidence, Ihlanfeldt (1994) offers three possible reasons. One is racial discrimination.<sup>1</sup> To the extent that inner-city workers are black and employers and/or suburban workers are white, discrimination by employers might suffice to generate the different labor market outcomes for inner-city and suburban residents. Yet, whatever its merits as an explanation for urban unemployment, this cannot explain spatial mismatch. As Ihlanfeldt notes, this would require that suburban employers engage in greater discrimination than central city employers, and the only study that touches on this issue (Turner, Fix, and Struyk 1991) fails to find this differential. A second explanation, emphasized in the urban economics literature, focuses on differential transportation costs (particularly the difficulties faced in "reverse commuting" from the central city to the suburbs). Again, this cannot be the whole story. First, commuting costs simply represent reductions in workers' net incomes, and empirically the wage elasticity of labor force participation is low. Second, firms often provide dedicated transportation from central city locations to their suburban sites, suggesting that transport costs per se are not the issue.<sup>2</sup> A third explanation—the focus of this article—involves the difficulties that inner-city workers face in gathering information about suburban jobs. Casual evidence suggests that this cost can be substantial: Wilson (1996), for example, notes the paucity of job announcements in locations that central city jobseekers are likely to frequent, such as unemployment offices or even metropolitan newspapers.

In this article, following the approach of Diamond (1982), Mortenson (1982), and Pissarides (1985), we model the process of gathering information regarding available jobs as one entailing costly and time-consum-

<sup>&</sup>lt;sup>1</sup> For instance, Ellwood (1986), emphasizes this explanation, claiming "it's race, not space" that is behind black unemployment in the central city.

<sup>&</sup>lt;sup>2</sup> The evidence regarding this transportation arrangement abounds. See, e.g., Roberts (1990), Ibata (1991), Wolf (1996), and Phillips (1997).

ing search. More specifically, we construct a general search equilibrium model with two spatially separated sectors, wherein workers and firms are brought together at random points in time in accordance with a matching technology. It is important that, although workers' residencies are—in our basic model—predetermined, workers are free to search for work in either (or both) locations and to commute to work if employment is found outside the zone in which they are domiciled (of course, commuting costs are greatest if this latter option is selected). We assume free entry by firms, in which the costs of establishing a vacancy are lowest for firms that enter the suburbs.<sup>3</sup> We are able to fully delineate the conditions under which spatial mismatch does and does not arise, something that eluded earlier efforts to model this phenomenon. We show that costly search and differential entry costs are sufficient to generate an equilibrium in which all of the stylized facts of spatial mismatch emerge. Although we extend the model in a number of directions (including differential transport costs between the two zones, network effects, and worker migration), we find that none of them is sufficient in itself to create mismatch without search frictions and asymmetric setup costs. It is in this sense that we conclude that these two conditions are also necessary for spatial mismatch. Therefore, we draw the natural conclusion that policy attempting to alleviate mismatch requires both a reduction in entry costs and the mitigation of search frictions.

Although this article is, to our knowledge, the first to adopt a search-equilibrium approach, related literature includes papers by Arnott (1997) and Brueckner and Martin (1997). Arnott (1997) proposes a model with costly commuting and housing discrimination, which leads to an equi-

<sup>3</sup> Innovations in transportation (e.g., larger trucks) and transportation infrastructure (especially the radial and suburban beltway pattern of the U.S. Interstate Highway System) are seen as "first causes" of the suburbanization of jobs, particularly in the manufacturing sector. The Route 128 corridor in the Boston Metropolitan Statistical Area is the classic example of suburban growth putatively generated by transport- (i.e., beltway-) oriented development. McMillen and McDonald (1995) find for Chicago that suburban employment centers are centered around transportation hubs, including O'Hare Airport. This is certainly consistent with Wolf's (1996) findings, which indicated that 90% of new metropolitan-area blue-collar jobs created in Chicago are located in the suburbs. More generally, suburban setup costs are lower for a wide variety of reasons: transport, taxation, labor costs, and land prices are all examined in the literature on intraurban firm location (see, e.g., Erickson and Wasylenko 1980; Wasylenko 1984), although many of these latter cost differentials may have arisen endogenously.

<sup>4</sup> Simpson (1992) considers search frictions in a location model of labor markets, but his model is a partial equilibrium one in which firms play no effective role; neither spatial mismatch nor the equilibrium unemployment and vacancy rates can be analyzed. Another recent study by Wasmer and Zenous (1997) uses location-dependent search intensity to explain high unemployment in Parisian

suburbs.

librium with reverse commuting by low-skilled black workers to the suburbs. However, in his paper, cross-location differences in unemployment and vacancy rates are not studied. Brueckner and Martin (1997) present two models of spatial mismatch, emphasizing housing discrimination and transportation costs. However, their analysis does not address suburban–central city differentials in unemployment and vacancy rates.

The next section presents the economic environment of our model, particularly the assumptions we make about firm costs, worker commuting, and the search environment. Section III discusses the existence of a search equilibrium, while Section IV characterizes the equilibrium, with special regard to the comparison of suburban and central city wage, unemployment, and vacancy rates. Section V discusses the welfare implications of potential public policy measures designed to alleviate spatial mismatch. Section VI briefly discusses some extensions of the model, none of which overturn our basic conclusions. Section VII concludes.

#### II. The Basic Environment

We consider a closed city consisting of a central business district (CBD), indexed j=1, and a suburban district (SBD), indexed j=2.5 Each zone is characterized by a distinct labor market in which (1) workers are employed and (possibly) commute between zones, and (2) unemployed workers and unfilled vacancies search for suitable partners. In anticipation, we are particularly interested in circumscribing the conditions under which CBD residents search for employment in the SBD and (on finding work) reverse commute from the CBD to the SBD.

Time is continuous. There are two distinct risk-neutral economic units corresponding to unskilled workers and to firms.<sup>6</sup> All agents are infinitely lived and discount the future at the common rate r > 0. Workers are endowed with a unit of labor that may be supplied inelastically without disutility from effort. We assume that each worker's residence, indexed i = 1, 2, is predetermined.<sup>7</sup> However, workers are free to search for and to obtain employment in either of the two districts. For simplicity, we normalize the population of workers in each zone to unity. In addition to their initial residential assignment, workers differ according to their commuting ability, indexed by  $a \in A \equiv [0,1]$ . This heterogeneity reflects differences in the utility costs of traveling (e.g., those based on the worker's precise geographic location within each of the two regions). We

<sup>&</sup>lt;sup>5</sup> For the moment, the two zones are modeled so as to provide the minimal location-specific attributes necessary for creating spatial mismatch (other relevant geographic features of CBDs as compared with SBDs are incorporated in Sec. VI below).

<sup>&</sup>lt;sup>6</sup> Given that the spatial mismatch phenomenon applies mainly to unskilled labor, we do not model the skilled labor market explicitly.

<sup>&</sup>lt;sup>7</sup> In Sec. VI, we allow workers to choose their residential location.

assume, through a suitable choice of units, that  $a \in A$  possesses a uniform distribution and g(a) = 1 for all  $a \in A$ . Assumption 1 describes the commuting technology.

Assumption 1. Commuting technology: The effective commuting cost is  $\theta_{ij}$ , where  $\theta_{ii} = 0$  and  $\theta_{ij}(a) = \theta_0 - \Theta(a)$  for  $a \in A$ , i = 1, 2 and  $j \neq i$ , where  $\theta_0 > 0$ ,  $\Theta' > 0$ ,  $\Theta(0) = 0$ , and  $\Theta(1) = \theta_0$ .

In assumption 1, it is costless to commute within one's own labor market (this is simply a normalization), but it is costly to commute from one location to another.<sup>8</sup> (The maximum commuting cost for any individual is consequently  $\theta_0$ .)

We consider a fixed-coefficient technology in which each vacancy is filled by only one worker. The ex ante cost of establishing a vacancy in location j is  $\nu_{0j}$ . There is unrestricted entry, so that any number of vacancies can enter zone j on paying the fixed cost  $\nu_{0j}$ . Once filled, the technology allows a worker-vacancy pair to produce a flow output  $\nu_j$ . Vacancies are identical in every respect ex post and differ only in the ex ante location-specific fixed-entry cost,  $\nu_{0j}$ . This feature captures differences in the transportation, rents, and production costs across the two zones. Thus, we have the following assumption.

Assumption 2. Heterogeneous firm entry costs:  $v_{01} > v_{02}$ .

We assume that worker-firm matches dissolve at the rate  $\delta > 0$ , which is independent of the location j (allowing the job separation rate to depend on j leads to obvious and straightforward modifications of the results). In the event that the match dissolves, firms and workers search for new partners.

In order to further develop the properties of the model, it is important to introduce notation describing the participants in each of the two labor markets. Let  $U_{ij}(E_{ij})$  denote the measure of resident i workers who search for work (are employed) in zone j. This gives a total measure of the labor force at j as  $N_j = N_{1j} + N_{2j}$ , where  $N_{ij} = U_{ij} + E_{ij}$ . The vacancy rate at j is simply  $R_j = V_j/N_j$ , and the measure of workers searching for a job in location j is  $S_j = U_{1j} + U_{2j}$ . We denote the measure of employed and unemployed workers with residence i by  $E^i$  and  $U^i$ , respectively. This gives  $E^i = E_{i1} + E_{i2}$  and  $U^i = U_{i1} + U_{i2}$ . Since the population of workers in each location is unity, we have:

$$E^{i} + U^{i} = E_{i1} + E_{i2} + U_{i1} + U_{i1} = 1, i = 1, 2.$$
 (1)

Our earlier normalization implies that the level and rate of unemployment

<sup>8</sup> More precisely, it is costly to commute between locations for all but the most able person (a = 1), whose commuting cost is normalized to zero.

 $<sup>^{9}</sup>$  In Coulson, Laing, and Wang (1999) we show, in a precise sense, that this particular type of asymmetry is crucial in generating spatial mismatch. Moreover, there we also endogenize the entry cost by allowing the mass of firms to affect land values and thus  $\nu_{0j}$ .

are identical. The flow probability that a location j firm locates an unemployed worker is denoted by  $\eta_j$ . Likewise, the flow rate with which an unemployed worker searching in location j locates work is  $\mu_j$ .<sup>10</sup> Let  $e_i$  and  $(1 - e_i)$  denote the effort a resident i worker devotes to search in the CBD and the SBD, respectively.<sup>11</sup>

On a successful match, each firm-worker pair produces a flow output  $y_j > r \nu_{0j}$ . We assume that the flow wage,  $w_{ij}$ , offered to resident i workers by zone j firms divides the surplus accruing to a symmetric Nash bargain. Each worker's income, net of commuting costs, is  $w_{ij} - \theta(a)_{ij}$ . Let  $\Pi^{ij}_{r}$  denote the present value of a location j vacancy filled by a worker from zone i, and let  $\Pi^{ij}_{v}$  denote the (expected) value of opening a vacancy in location j. The value functions are

$$r\Pi_{\mathfrak{r}}^{ij} = (\gamma_i - w_{ii}) + \delta(\Pi_{\mathfrak{r}}^{j} - \Pi_{\mathfrak{r}}^{ij}), \tag{2a}$$

$$r\Pi_{\nu}^{j} = \eta_{i}[Q_{i}\Pi_{\nu}^{1j} - (1 - Q_{i})\Pi_{\nu}^{2j} - \Pi_{\nu}^{j}],$$
 (2b)

where  $Q_j \equiv U_{1j}/(U_{1j}+U_{2j})$  measures the fraction of workers seeking jobs at location j who come from the CBD (Q is endogenously determined). These asset values have, by now, a familiar interpretation. Equation (2a) states that the (flow) value of a firm located at j that hires a resident i worker is the sum of the firm's share of the surplus that accrues from a successful match (the first term on the right-hand side) and the expected capital loss from the break-up (the second term on the right-hand side). In equation (2b), the bracketed term is the expected capital gain from filling the vacancy. Notice that the terms Q and (1-Q) appear, reflecting that vacancies may meet workers from either location and that the surplus,  $\Pi_{rr}^{ij}$ , may vary according to the residence of the worker.

Turning now to the asset values for workers, let  $J_{\nu}(a)^{ij}$  denote the net (of commuting cost) value of a resident i worker employed by a firm in zone j. Likewise, let  $\bar{J}_{\nu}(a)^{ij}$  represent the value of search for a resident i worker looking for employment in zone j. Workers use their labor endowment optimally to search for work in the two zones. Obviously, if  $\bar{J}_{\nu}(a)^{i1} > (<)\bar{J}_{\nu}(a)^{i2}$ , then  $e_i = 1$  ( $e_i = 0$ ). In the event of equality, let  $e_i = 1$ . With this in mind, denote the expected present value of a resident i worker who optimally searches for work in zone j by  $J(a)^{ij}_{\nu} = \max_{e^i \in [0,1)} [e_i \bar{J}_{\nu}(a)^{i1} + (1-e_i) \bar{J}_{\nu}(a)^{i2}]$ . The value functions are

<sup>&</sup>lt;sup>10</sup> In Sec. VI, we allow this flow rate to differ according to the worker's residence i, giving  $\mu_{ij}$ .

Although unemployed workers are free to use their unit labor endowment to search for employment in either or both locations, simple arguments show that search effort is completely specialized, i.e.,  $e(a)_i \in \{0,1\}$ .

$$r J_{\text{E}}(a)^{ij} = [w_{ij} - \theta(a)_{ij}] + \delta[J_{\text{U}}(a)^{ij} - J_{\text{E}}(a)^{ij}],$$
 (3a)

$$rJ_{r}(a)^{ij} = -\theta(a)_{ii} + \mu_{i}[J_{r}(a)^{ij} - J_{r}(a)^{ij}].$$
 (3b)

Equation (3a) says that the flow value of employment at location j for a resident i worker equals the flow wage plus the flow probability that the worker suffers a capital loss (bracketed portion of the second term on the right-hand side) as a result of the break-up of the match. Equation (3b) says that the flow value from job search consists of the cost of commuting to find work  $(\theta_{ij})$  as well as the flow probability of obtaining employment in  $j(\mu_j)$  multiplied by the capital gain from that event (bracketed portion of the second term on the right-hand side).

# III. Steady-State Locational Search Equilibrium

In this section we establish the existence of a steady-state equilibrium and circumscribe the conditions under which it entails spatial mismatch. Our analysis proceeds through a process akin to backward induction. First, we use the asset values to determine the wage in each of the two locations. We then derive two key steady-state relationships pertaining to the equilibrium entry of vacancies and to the steady-state matching of vacancies to workers. These relationships are then used to determine the allocation of time by unemployed workers to search activity in each location. Once this is done, we derive the distribution of workers in each location.

#### A. Wage Bargaining

Given a continuum of firms and workers, each agent takes the matching rates  $\eta_i$  and  $\mu_i$  as given.

Assumption 3. Bargaining: The wage bargain follows a symmetric Nash rule:

$$\Pi_{\scriptscriptstyle F}^{ij} - \Pi_{\scriptscriptstyle V}^{j} = J_{\scriptscriptstyle E}(a)^{ij} - J_{\scriptscriptstyle U}(a)^{ij} \ge 0, \quad \forall a \in A.$$
 (4)

From the value functions (2a), (2b), (3a), and (3b), we have the following lemma.

LEMMA 1. Wage offer: Under a symmetric Nash bargain, the flow wage offer is

$$w_{j} \equiv w_{ij} = \left(\frac{r + \delta + \mu_{j}}{2(r + \delta) + \mu_{j}}\right)(y_{j} - r\Pi_{V}^{j}) = w_{j}(\mu_{j}, \Pi_{V}^{j}; y_{j}, r, \delta),$$
 (5)

where  $\partial w_i/\partial \mu_i > 0$ ,  $\partial w_i/\partial \Pi_i^i < 0$ ,  $\partial w_i/\partial y_i > 0$ ,  $\partial w_i/\partial r < 0$ , and  $\partial w_i/\partial \delta < 0$ .

The first thing to note in equation (5) is that the agreed wage  $w_j$  is independent of each worker's residence i (note, however, that net income,  $w_j - \theta_{ij}$ , does depend on workers' commuting costs). The reason is simple and follows from a consideration of the equilibrium bargaining threat-

point (e.g., Binmore, Rubinstein, and Wolinsky 1986). More precisely, if it is optimal for a type  $a \in A$  resident i worker to search for employment in zone j, then it remains optimal for him to do so in the event that the bargain breaks down, forcing him to search for an alternative trading partner. This implies that the commuting cost terms,  $\theta_{ij}$ , in effect "net out" from the surplus accruing to workers on securing employment (the right-hand side of (4). The comparative-static properties reported in lemma 1 are intuitive. An increase in the (effective) contact rate,  $\mu_j$ , enhances the bargaining power of workers (by making it easier for them to locate an alternative vacancy), which raises the wage. Alternatively, an increase in the market value of unfilled vacancies,  $\Pi_{\nu}^{i}$ , improves each firm's threat point and lowers the wage. The other results follow in a similar manner.

# B. Equilibrium entry

The unrestricted-entry assumption implies that firms attain zero ex ante profits in steady-state equilibrium.

Assumption 4. Equilibrium Entry: Firms at each location J enter until the associated value of search equals the fixed entry cost:

$$\Pi_V^j = \left[\frac{\eta_j(r+\delta)}{r(r+\delta+\eta_i)}\right] [y_j - w_j(\mu_j;.)] = \nu_{0j}.$$
 (6)

Given the exogenous parameters and the wage offer function obtained in (5), equation (6) determines a unique relationship between the two contact rates,  $\mu_j$  and  $\eta_j$ , which, for convenience, we refer to as the  $EE_j$  (equilibrium entry) locus:

$$\eta_j = \eta_j^{\text{ee}}(\mu_j, y_j, \nu_{0j}, r, \delta). \tag{7}$$

# C. Steady-State Matching

Since a vacancy is filled by exactly one worker, steady-state matching in each of the two labor markets (i = 1, 2) implies

$$\eta_j V_j = \mu_j (U_{1j} + U_{2j}) = \mu_j S_j = m_0 M(S_j, V_j),$$
 (8)

where  $m_0 > 0$  and M(.) is the random matching technology. Consider the following assumption.

Assumption 5. The matching technology: M is a strictly increasing, concave, and constant-returns-to-scale function of  $S_j$  and  $V_j$ , satisfying the Inada( $\lim_{\xi\to 0} M_{\xi} = \infty$ ,  $\lim_{\xi\to \infty} M_{\xi} = 0$ ,  $\xi = U, V$ ) and the boundary conditions M(0, V) = M(S, 0) = 0.

The matching technology parameter  $m_0$  captures the efficacy of the matching process. For example, an improvement in communication and transportation infrastructure increases the flow-matching rate for any

given mass of searching workers  $(S_j)$  and vacancies  $(V_j)$ . The properties of the matching function M ensure a well-behaved, hyperbolic Beveridge curve in which the absence of either side of the market results in zero matches. To facilitate the later discussion, we assume that the matching technology is the same in each location. Given constant returns to scale, the two equalities in (8) reduce to

$$\eta_i = m_0 M(\eta_i/\mu_i; 1) \equiv \eta^{ss}(\mu_i; m_0). \tag{9}$$

The locus of points  $(\mu_j, \eta_j)$  satisfying (9) is referred to as the SS (steady-state matching) locus. The steady-state population of job searchers is derived by equating the relevant inflows into and outflows from employment:

$$\delta E_{ii} = \mu_i U_{ii}, \tag{10}$$

where it will be recalled that worker-firm matches dissolve at the rate  $\delta > 0$ . Using (1), the definition of  $N_{ij}$ , in conjunction with equation (10) yields

$$U_{ij} = \delta N_{ij} / (\delta + \mu_i). \tag{11}$$

In (11), the population of job searchers in a given zone depends negatively on the rate at which workers find jobs,  $\mu_j$ . From (11), the mass of households searching for work in location j is  $S_j = \delta N_j/(\delta + \mu_j)$ , which with (8) gives the mass of vacancies in zone j:

$$V_j = \delta \mu_j N_j / [\eta_j (\delta + \mu_j)]. \tag{12}$$

This indicates that, for given  $N_j$ , the measure of vacancies is negatively related to the vacancy contact rate,  $\eta_j$ , but positively related to the worker contact rate  $\mu_j$ .

## D. Steady-State Distribution of Labor Force

We now determine the steady-state distribution of the labor force across the two zones, which depends crucially on whether workers search for employment within/outside their residential locations. From (3a), (3b), and lemma 1, the net present value accruing to a resident i worker who searches for work in location j is

<sup>12</sup> Two comments are in order regarding the matching technology. First, our results also hold for any well-behaved, quasi-concave matching technology, including ones exhibiting increasing returns to scale. Second, our results hold a fortiori if the relative matching efficacy of the CBD is inferior to that of the SBD or, alternatively, provided the matching process in the CBD is not too much faster than that of the SBD.

$$J_{v}(a)^{ij} = \left[\frac{\mu_{j}}{(\delta + r + \mu_{i})}\right] \left[\frac{w_{j} - \theta_{ij}(a)}{r}\right]. \tag{13}$$

Recall from assumption 2 that  $\theta_{ii} = 0$  (it is less costly for the worker to commute within his zone of residence) and that  $d\theta_{ij}/da < 0$  for  $i \neq j$ . It follows from (13) that  $J_{v}(a)^{ij}$  is independent of  $a \in A$  if i = j and that  $J_{v}(a)^{ij}$  is increasing in  $a \in A$  otherwise. This implies—quite naturally—that if commuting occurs in equilibrium, then it is done by those with the lowest commuting cost (i.e., those with the greatest value of  $a \in A$ ). It follows from (13) that there is a unique (possibly noninterior) critical value,  $a_{ci}$ , such that every worker at location i of characteristic  $a \in [0,a_{ci}]$  optimally searches for a job at j = i, whereas every worker at location i of characteristic  $a \in (a_{ci}, 1]$  searches for a job at  $j \neq i$ . The reservation value,  $a_{ci}$ , partitions workers into "stayers" and "movers." The critical values  $a_{ci}$  solve

$$J_{u}^{i1} = \left[\frac{\mu_{1}}{(\delta + r + \mu_{1})}\right] \left[\frac{w_{1} - \theta_{i1}(a_{ci})}{r}\right]$$

$$= \left[\frac{\mu_{2}}{(\delta + r + \mu_{2})}\right] \left[\frac{(w_{2} - \theta_{i2}(a_{ci}))}{r}\right]$$

$$= J_{u}^{i2}, i = 1, 2.$$
(14)

Thus, the marginal individual is just indifferent between staying (and incurring zero transport costs) and searching for employment in the other location. Given the values of a, the labor force participating in each zone is

$$N_{ij} = (\chi_{ij}) \int_{0}^{a_{ci}} g(a)da + (1 - \chi_{ij}) \int_{a_{ci}}^{1} g(a)da \equiv \Gamma_{j}(a_{ci}),$$
 (15)

where  $\chi_{ij} = 1$  if i = j and  $\chi_{ij} = 0$  otherwise. Since  $a \in A$  is uniformly distributed on  $a \in A \equiv [0,1]$ , it follows from (15) that  $\Gamma_j(a_{ci}) = a_{ci}$  as j = i and  $\Gamma_j(a_{ci}) = 1 - a_{ci}$  as  $j \neq i$ .

DEFINITION 1. Steady-state locational search equilibrium: A tuple  $\{U_{ij}^*, N_{ij}^*, a_{ci}^*\}_{i=1,2}; \mu_j^*, \eta_i^*, V_j^*, w_j^*\}$  satisfying the following conditions:

- 1) Symmetric Nash bargain, (4),
- 2) Equilibrium entry, (7),
- 3) Steady-state matching, (8-11),
- 4) Optimal locational job search (14),
- 5) A steady-state labor force (15).

In order to characterize the steady-state equilibrium of the model, it is necessary to examine the properties of the  $EE_j$  and SS loci. At this

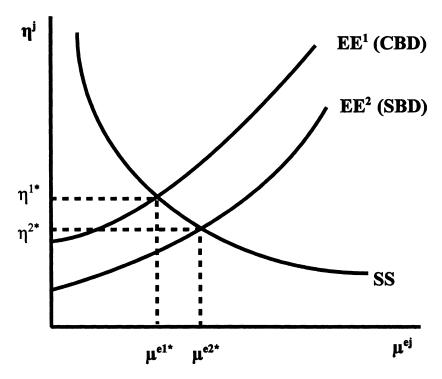


Fig. 1.—Determination of contact rates and spatial mismatch

juncture, it is helpful to note, by inspecting equations (7) and (9), that the  $EE_j$  and SS loci are alone sufficient to yield solutions for the contact rate pairs  $(\mu_j^*, \eta_j^*)$ . This feature greatly facilitates our later analysis and characterization of the equilibrium. Moreover, it implies that our model is amenable to a simple graphical analysis (see fig. 1). Thus, we have the following lemma.

LEMMA 2. The EE and SS loci: For each location, we have

- 1) the equilibrium entry  $(EE_j)$  locus is upward sloping and convex in  $(\mu, \eta)$  space; it shifts rightward as  $\nu_{0j}$  decreases; and
- 2) the steady-state matching (SS) locus is downward sloping and convex in  $(\mu, \eta)$  space and asymptotes at each axis; it shifts outward as  $m_0$  increases.

Since both the *EE* and *SS* loci are monotonic and the *SS* locus satisfies the Inada property, the four equilibrium contact rates,  $\mu_j^*$ , and  $\eta_j^*(j=$ 

1, 2) are unique and nondegenerate.<sup>13</sup> The equilibrium values  $(\mu_i^*, \eta_j^*)$  are used to determine, from (5) and (6), the equilibrium wage  $w_j^*$  and, from (12), the equilibrium vacancy rate  $R_j^* (= V_j^* / N_j^*)$ . Finally, equations (11), (12), (14), and (15) are used to derive the equilibrium: labor force,  $N_{ij}^*$ , mass of unemployed workers,  $U_{ij}^*$ , mass of vacancies,  $V_{ij}^*$ , and the critical reservation values,  $a_{ci}^*$ . We are interested in a particular SSLSE in which none of the unemployed workers in the SBD searches for a job in the CBD. Specifically, we focus our attention on the potential for "reverse commuting," wherein CBD residents commute to the SBD. We deemphasize "forward commuting" as largely being characteristic of high-skilled labor markets.<sup>14</sup> In this particular candidate equilibrium, there is an interior critical value  $a_{ci}^* \in (0, 1)$ , but a corner solution for  $a_{c2}^* = 1$ . We show that, for suitable parameter values, this is the only nondegenerate equilibrium. The existence of such an equilibrium requires

$$J_{v}^{11} = \left[\frac{\mu_{1}^{*}}{(\delta + r + \mu_{1}^{*})}\right] \left[\frac{(w_{1}^{*})}{r}\right]$$

$$= \left[\frac{\mu_{2}^{*}}{(\delta + r + \mu_{2}^{*})}\right] \left[\frac{w_{2}^{*} - \theta_{12}(a_{c1}^{*})}{r}\right] = J_{v}(a_{c1}^{*})^{12}; \quad a_{c1}^{*} < 1$$

$$J_{v}^{21} = \left[\frac{\mu_{1}^{*}}{(\delta + r + \mu_{1}^{*})}\right] \left[\frac{w_{1}^{*} - \theta_{21}(a)}{r}\right]$$

$$= \left[\frac{\mu_{2}^{*}}{(\delta + r + \mu_{2}^{*})}\right] \left[\frac{(w_{2}^{*})}{r}\right] = J_{v}(a_{c2}^{*})^{22}; \quad a_{c2}^{*} \in [0, 1].$$

$$(16)$$

An interior  $a_{c1}^*$  is (uniquely) determined for a generic set of parameter values as the right-hand side of (16) is monotonic (increasing) in  $a_{c1}$ . Hence, equation (16) ensures that some CBD residents commute to the SBD. The strict inequality in (17) holds for all  $a \in [0,1]$ , ensuring a corner solution in which SBD residents do not seek employment in the CBD. Consider the following lemma.

LEMMA 3. Critical value for search ability: The critical value,  $a_{ci}^*$ , is uniquely determined by (16).

While the properties of the Beveridge (SS) curve are standard, those of the EE locus deserve further comment. From (5)–(7),  $\mu_j=0$  implies  $\eta_j=2r(r+\delta)\nu_{0j}/(y_j-3r\nu_{0j})>0$ ;  $d\eta_i/d\mu_i=\eta_i[1+2\eta_i/(r+\delta)]/[2(r+\delta)+\mu_i]>0$ , which decreases in  $(r+\delta)$ ; and  $d^2\eta_i/d\mu_i^2=4[\eta_i/(r+\delta)](d\eta_i/d\mu_i)/[2(r+\delta)+\mu_i]>0$ .

<sup>14</sup> If SBD workers also search for jobs in the CBD, the spatial mismatch problem is further intensified. Thus, our focus on this regime is made only for simplicity and, obviously, is not essential to our results. Moreover, it is consistent with empirical observations in post-World War II U.S. cities. See the discussion by Sassen (1991, p. 260) on the sparsity of blue-collar and office-worker commuting from New York City suburbs into Manhattan.

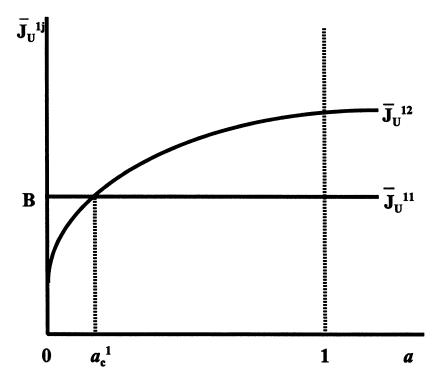


Fig. 2.—Determination of search activity

This property is easily seen from figure 2, which plots the asset values for unemployed workers,  $J_{\rm u}(a)^{11}$  and  $J_{\rm u}(a)^{12}$ , against  $a \in A$ . By assumption 2, there are no within-location commuting costs, and thus  $J_{\rm u}^{12}$  is independent of individual worker's characteristics. Moreover, since  $\theta_{12}(a)$  is a decreasing function of ability,  $a \in A$ , it is trivial to see that  $J_{\rm u}^{12}$  is monotonically increasing in  $a \in A$ . It follows that for a generic range of values of  $\{y_1,y_2\}$  (provided  $y_1-y_2$  is not too large, i.e., the traditional urban advantage resulting from agglomerative economies is not too large), a unique interior solution for  $a_{\rm cl}^*$  is guaranteed in (16). Furthermore, the inequality in (17) holds for a sufficiently high  $\theta_0$  or a sufficiently large entry cost differential  $\nu_{01}-\nu_{02}$  (which implies that  $w_2^*$  is "large" when compared with  $w_1^*$ ). These requirements of the parameter values can be summarized as the following condition.

<sup>&</sup>lt;sup>15</sup> For instance, if  $\Theta(a) = \theta_0 a$ , it is always true that  $J_v(a)^{12} > J_v(a)^{11}$ . The fixed point is thus ensured for values of  $\{y_1, y_2\}$  such that  $J_v(0)^{12} < J_v(a)^{11}$ .

<sup>&</sup>lt;sup>16</sup> An example can be easily constructed in which the entry cost differential is so high that the resulting gross wage satisfies  $w_1^*/r < J_v^{22}$ , in which case, even with instantaneous CBD employment, SBD residents prefer to search for work in the SBD.

CONDITION E. Existence of unique critical value for job search: The parameters of entry costs and production and transportation technologies satisfy

- 1)  $\theta_0 > 0$ ;
- 2)  $\nu_{01} \nu_{02} > \Delta_{\nu}$ ;
- $3) \quad y_1 y_2 < \Delta_{\gamma}$

for some positive constants,  $\Delta_{\theta}$  and  $\Delta_{\theta}$ , such that (16) and (17) hold.

Conditions (1) and (2) above guarantee the inequality (17), that is, it is never profitable for SBD workers to commute to the CBD. As illustrated in figure 2, conditions (1) and (3) ensure that  $J_v(0)^{12} < J_v^{11} < J_v^{12}(1)$ , implying the existence of an interior  $a_{c1}^*$  that solves (16). Thus, from the above arguments and lemmas 1–3, we can conclude the following theorem.

THEOREM 1. Existence of the SSLSE: Under assumptions 1–5 and condition E, there exists a nondegenerate SSLSE in which none of unemployed workers in the SBD search for jobs at the CBD and in which a positive mass of CBD residents search for work in the SBD.

Notably, as in other papers in the "coordination failure" literature (e.g., Cooper and John 1988), there are degenerate SSLSE's, corresponding to one of the locations being inactive, in which either all firms enter the SBD market with no workers searching in the CBD or all firms enter the CBD market with no workers searching in the SBD.<sup>17</sup> These degenerate equilibria are consequences of self-fulfilling prophecies and depend on workers' and firms' beliefs. They result in unused land in either the CBD or SBD, which is not interesting in the current context. Under assumptions 1–5, the nondegenerate SSLSE is unique.

## IV. Characterization of Steady-State Equilibrium

In this section, we characterize the SSLSE described in theorem 1 above. The recursive nature of the problem provides a simple analytical solution of the equilibrium. We first characterize the contact rates,  $\mu_j^*$  and  $\eta_j^*$  (j=1,2) using lemma 3 (see fig. 1),

PROPOSITION 1. Equilibrium contact rates: In each labor market *j*, we have

- 1)  $\mu_1^* < \mu_2^*$  and  $\eta_1^* > \eta_2^*$ ;
- 2)  $d\mu_i^*/d\nu_{0i} < 0$ ;  $d\mu_i^*/dm_0 > 0$ ; and  $d\eta_i^*/dm_0 > 0$ .

It is clear from lemma 2 that a higher entry cost for firms in the CBD (assumption 2) implies the CBD's  $EE_1$  locus lies above the SBD's  $EE_2$  locus on the SS steady-state locus common to both zones (see fig. 1).

<sup>&</sup>lt;sup>17</sup> We can rule out these degenerate equilibria by imposing (17) (ensuring that the SBD market is active) and by assuming  $\theta_0 > y_1$ , implying that a positive measure of CBD residents remain there.

This proves part 1 of proposition 1. Part 2 of proposition 1 follows from the total differentiation of equations (7) and (9). An increase in the fixed entry cost  $\nu_{0j}$  reduces the representative firm's net value of entry. Heuristically, this requires an increase in the firm's contact rate,  $\eta_j$ , or a decrease in the worker contact rate  $\mu_j$  (and, hence, the wage) to restore the zero ex ante profit condition. An increase in matching efficacy makes it easier for firms to find workers and for workers to locate vacancies, which raises the contact rates for both firms and workers.

The results in lemma 1 and proposition 1 are now used to prove the following proposition.

PROPOSITION 2. Equilibrium wages: In each location j,, the equilibrium wage possesses the properties  $dw_j^*/d\nu_{0j} < 0$ ;  $dw_j^*/dm_0 > 0$ ; and  $dw_j^*/dy_j > 0$ .

As the entry cost  $(\nu_{0j})$  rises, there is a direct negative effect on the wage because of the enhanced bargaining power of firms. In addition, the increase in entry cost shifts up the  $EE_j$  locus and lowers the equilibrium worker-contact rate  $\mu_j$ , which also suppresses the wage (by weakening each worker's bargaining strength). These two effects work in the same direction, implying an unambiguous reduction in the wage. The effect of a productivity increase  $(y_j)$  is exactly opposite to the effect of an increase in the entry cost. Finally, the effect of an increase in matching efficacy,  $m_0$ , works purely through its enhancement of each worker's equilibrium contact rate  $\mu_j$ .

Equations (5) and (6) yield (evaluated in the SSLSE)

$$w_{2}^{*} - w_{1}^{*} = \left[1 - \frac{(r+\delta)}{2(r+\delta) + \mu_{2}^{*}}\right] (y_{2} - r\nu_{02})$$
$$-\left[1 - \frac{r+\delta}{2(r+\delta) + \mu_{1}^{*}}\right] (y - r\nu_{01}). \tag{18}$$

which determines the locational wage differential. More specifically, suppose that the productivity of workers is the same for each job in each location (i.e.,  $y_1 = y_2$ ). Then, since  $\mu_1^* < \mu_2^*$  and  $\nu_{01} > \nu_{02}$ , the gross SBD wage exceeds the gross CBD wage. This result follows not only because of the entry-cost differential but also because of a general equilibrium effect that operates through the endogenous contact rate for workers  $(\mu_j^*)$ . The same result holds if  $y_1 < y_2$  or provided  $y_1$  is not too much greater than  $y_2$ .

We next turn to the characterization of the unemployment and vacancy rates in each location. After solving for the unique interior (steady-state) equilibrium contact rates ( $\mu_j^*$  and  $\eta_j^*$ ), equations (11) and (15) determine the equilibrium mass of unemployed workers residing at location  $i(U_i^*)$ :

$$U^{1*} = U_{11}^* + U_{12}^* = [\delta/(\delta + \mu_1^*)]a_{c1}^* + [\delta/(\delta + \mu_2^*)](1 - a_{c1}^*), \quad (19a)$$

$$U^{2*} = U_{21}^* + U_{22}^* = [\delta/(\delta + \mu_2^*)].$$
 (19b)

where the critical values,  $a_{ci}^*$ , solve (16) and (17). (Our earlier population normalization implies that  $U^{i*}$  also equals the unemployment rate in i.) From equations (11) and (12), the level of employment in zone j is  $E_{1j}^* + E_{2j}^* = (\mu_j^* N_j^*)/(\delta + \mu_j^*)$ . Using this expression in  $R_j^* \equiv V_j^*/(V_j^* + E_{1j}^* + E_{2j}^*)$  in conjunction with (12) gives the equilibrium job-vacancy rate at location j ( $R_j^*$ ):

$$R_i^* = \delta/(\delta + \eta_i), \tag{20}$$

which depends inversely on the firm-contact rate,  $\eta_i^*$  alone.

Equations (19a), (19b), and part 1 of proposition 1 (i.e.,  $\mu_1^* < \mu_2^*$ ) implies that (1)  $U_1^*$  is increasing in  $a_{c1}^*$  and (2) if  $a_{c1}^* = 0$ , then  $U_1^* = U_2^*$ . It follows immediately, since  $a_{c1}^* > 0$ , that the unemployment rate in the CBD is higher than that in the SBD:  $U_1^* > U_2^*$ . Similarly, equation (20) implies a higher vacancy rate in the SBD:  $R_1^* < R_2^*$ . The above arguments lead to theorem 2.

THEOREM 2. Spatial mismatch: In the SSLSE described in theorem 1:

- 1) the unemployment rate in the CBD is higher than in the SBD (i.e.,  $U_1^* > U_2^*$ );
- 2) the vacancy rate in the CBD is lower than in the SBD (i.e.,  $R_1^* < R_2^*$ ); and,
- 3) provided that  $y_1$  is not too large relative to  $y_2$ , there is reverse commuting (i.e.,  $e_1^* = 0$  for all  $a \in (a_{c1}^*, 1]$ ) and gross wages earned in the CBD are lower than in the SBD (i.e.,  $w_1^* < w_2^*$ ).

The cross-location differences in labor market activity described in theorem 2 support the empirical observations associated with spatial mismatch. It is obvious from the above arguments that the unemployment and vacancy rate differentials depend crucially on the entry-cost heterogeneity. However, nontrivial search frictions are crucial for the emergence of spatial mismatch.

PROPOSITION 3. The role of search frictions: Let output exceed the commuting cost for all workers  $a \in A$  (i.e.,  $y_j - \theta_0 > 0$ ). As search frictions vanish (i.e.,  $m_0 \to \infty$ ), all firms enter the SBD market, and no workers are employed in the CBD (i.e.,  $N_1^* = V_1^* = e_1^* = 0$ ).

Theorem 2 and proposition 3 indicate that differential entry costs, together with nontrivial search frictions, are sufficient to generate spatial mismatch, even in the absence of other locational asymmetries in either search or commuting costs (although, for completeness, these are considered in Sec. VI).

# V. Spatial Mismatch: Policy and Welfare Analysis

In this section, we study the implications of several different policies that have been suggested as means for alleviating problems of spatial mismatch. To facilitate the discussion, we first examine the effects of parametric changes in  $\nu_{0j}$ ,  $m_0$ , and  $\theta_0$  on unemployment and vacancy rates in the SSLSE. Consider the following proposition.

Proposition 4. Unemployment and vacancy rates: In the SSLSE

- 1) for each location an increase in the entry cost (higher  $v_{0j}$ ) raises the unemployment rate and lowers the job vacancy rates at both locations;
- 2) an increase in matching efficacy (higher  $m_0$ ) decreases the unemployment and job vacancy rates in each location; and
- 3) an improvement in the transportation technology (lower  $\theta_0$ ) lowers the unemployment rate for CBD residents but does not affect the vacancy rate in any location.

Parts 1 and 2 of proposition 4 follow from the fact that an increase in the entry cost,  $\nu_{0j}$ , raises  $\eta_j^*$  and lowers  $\mu_j^*$ , while an increase in matching efficacy,  $m_0$ , raises both workers' and firms' contact rates (see fig. 1). Part 3 of proposition 4 is explained as follows. In response to an improvement in the transport technology (a decrease in  $\theta_0$ ), the net value of search (wage net of commuting cost) increases, inducing greater numbers of CBD residents to search for employment in the SBD (also lowering the critical value,  $a_{c1}^*$ , for CBD residents). However, from equation (2a) and part 1 of proposition 1 (specifically  $\mu_2^* > \mu_1^*$ ), the net effect of the reallocation of labor to the SBD is to lower the unemployment rate of CBD residents, as job offers are received at a faster rate in the former zone (the SBD) than in the latter. Also, from proposition 1 and equation (12), it is clear that neither the worker/firm contact rates nor the equilibrium vacancy rates depend on the commuting parameters.

Proposition 4 is used to study the effect of four policies that have been proposed to ameliorate the problems associated with spatial mismatch. First, consider an improvement in the communication and market infrastructure that increases  $m_0$ . The effect of this is to reduce the unemployment and vacancy rates in both locations. However, this may not eliminate mismatch, since a gap remains between the unemployment and vacancy rates in the CBD and SBD. Second, improvements in the transportation infrastructure or other policies that simultaneously reduce  $\nu_{01}$  and  $\nu_{02}$  help lower the unemployment rates in both locations. However, provided assumption 2 continues to hold, this policy does not resolve the spatial mismatch problem (theorem 2). Third, a subsidy or a tax incentive to CBD firms that lowers  $\nu_{01}$  reduces unemployment in the CBD by stimulating the entry of vacancies and narrows the (gross) wage differential

between the CBD and SBD.<sup>18</sup> Nevertheless, it does not decrease the vacancy rate in the SBD. Fourth, a reduction in commuting costs,  $\theta_0$ , induces more search in the SBD by CBD residents (i.e., it encourages reverse commuting). This lowers the unemployment rate in the CBD; however, it does not reduce the cross-location wage differential. In view of these findings, we have the following proposition.

Proposition 5. Public policy: The elimination of spatial mismatch requires a policy mix consisting of a subsidy or a tax incentive to vacancies in the CBD accompanied by an improvement in the city infrastructure so as to reduce transportation and search costs.

Finally, we analyze the welfare properties of the model, which is easily accomplished in this framework. First, note that the unrestricted entry of firms pins down their equilibrium values at  $\Pi_v^j = \nu_{0j}$ , which implies that the (ex ante) steady-state equilibrium net value of firms is exactly zero. Thus, the relevant endogenous components of aggregate welfare are workers' utilities,  $J_v^{ij}$ . It can be seen from propositions 1 and 2 that  $J_v^{ij}$  is increasing in matching efficacy,  $m_0$ , but decreasing in the entry and commuting costs,  $\nu_{i0}$  and  $\theta_0$ . As a consequence, one can conclude the following.

Proposition 6. Welfare analysis: An increase in matching efficacy or a reduction in entry/commuting costs is welfare-improving in the Pareto sense.

The results concerning  $m_0$ ,  $\nu_{02}$ , and  $\theta_0$  are trivial, while that concerning  $\nu_{01}$  deserves further discussion. Obviously a reduction in  $\nu_{01}$ , by increasing  $w_1^*$ , raises  $J_v^{i1}$  for every worker who maintains the same search strategy  $e_i^*$  (for i=1,2). However, in evaluating the welfare effect of the reduction in  $\nu_{01}$ , there is a potential ambiguity, since some CBD residents might switch their search strategy and remain in the CBD. This is easily resolved through a revealed preference argument. Specifically, in response to a reduction in  $\nu_{01}$ , the asset value  $J_v(a)^{11}$  increases for all  $a \in A$ , while  $J_v^{12}$  is unchanged. This leads to a larger critical value,  $a_{c1}^*I > a_{c1}^*$ , indicating that a greater number of CBD residents remain in the CBD and seek employment there. Yet, the reason members of this group switch their search strategies is that their welfare is unambiguously higher by doing so. This verifies proposition 6.

#### VI. Extensions

Our basic framework can be extended in a number of directions. We focus on those that are of most interest because they incorporate realistic labor-market features.

<sup>&</sup>lt;sup>18</sup> Hence, we have Wilson's (1987) stress on "tight labor markets" in the CBD as the (partial) remedy for inner-city difficulties (through enterprise zones or other incentives).

# A. Locational Asymmetry in Commuting

We first consider asymmetric commuting technologies. Wilson's (1996) interviews with central city Chicago residents emphasize the frustrations of "reverse commuting" to jobs within suburban firms. His evidence suggests that there are costs associated with morning travel to suburban locations on a mass transit system seemingly designed to bring high-wage suburban workers to the center. Thus, one may consider the following assumption.

Assumption 1a. Commuting technology: The effective commuting cost is  $\theta_{ij}$ :  $\theta_{ii} = 0$  and  $\theta_0 - \theta_1 \Theta(a) = \theta_{12} > \theta_{21} = \theta_0 - \Theta(a)$ , where  $\theta_1 \in (0, 1)$  and  $\Theta(a)$  satisfies the conditions of assumption 1.

The parameter  $\theta_1 < 1$  captures the relative difficulty of commuting from the CBD to the SBD (larger values of  $\theta_1$  reduce the gap between the costs of commuting from the CBD to the SBD). Spatial mismatch (theorem 2) continues to arise as an equilibrium outcome, provided  $\theta_1$  is not too high (otherwise, reverse commuting ceases). An interesting policy implication is that an increase in the availability of public transport for reverse commuting (financed by a lump-sum tax on all workers) raises  $\theta_1$  and thus reduces the severity of spatial mismatch by encouraging the CBD residents to search and work in the SBD. However, in the presence of differential entry costs and nontrivial search frictions, it fails to alleviate the spatial mismatch problem entirely; moreover, it does not resolve the cross-location wage disparity (since this depends only on the differential firm entry cost).

## B. Asymmetric Contact Rates

We next relax the assumption of homogeneous job search and allow for asymmetries in workers' job-contact rates according to their residence. This might stem from differential costs of information gathering: suburban employers may have less access to the central city sources of job postings, such as the city newspapers or unemployment/welfare offices.

Assumption 5a. Workers' contact rates: The effective contact rate at which a resident i worker locates a vacancy at j is  $\mu_{ij}$ :  $\mu_{11} = \mu_1 e_1$ ,  $\mu_{12} = \phi \gamma (1 - e_1) \mu_2$ ,  $\mu_{21} = \mu_1 \gamma e_2$ , and  $\mu_{22} = \mu_2 (1 - e_2)$ , where  $\gamma, \phi \in (0, 1)$ .

In assumption 5a, we admit location-specific contact rates, capturing intrinsic heterogeneity in the rate at which workers locate employment opportunities within and across the two zones. In particular, notice that  $\phi \gamma < \gamma < 1$ . Intuitively,  $\gamma \in (0, 1)$  captures the presence of spatial search frictions across locations, while  $\phi \in (0,1)$  reflects the additional difficulty that CBD residents might have in locating a job in the SBD.<sup>19</sup> With this

<sup>&</sup>lt;sup>19</sup> Wilson (1996) finds that only 40% of central city employers advertised openings in metropolitan newspapers, only 33% used unemployment offices, and only

emendation, not only is the spatial mismatch property preserved in equilibrium, but it is also reinforced: it is even more difficult for CBD residents to locate a job in the SBD.

A government-operated job placement center (financed by a lump-sum tax) that facilitates job search (increasing  $\gamma$  and  $\phi$ ) can partially reduce spatial mismatch, but, once again, such a policy fails to eliminate completely the mismatch problem and the concomitant wage differential between locations (theorem 2 holds even if  $\phi = \gamma = 1$ ).

# C. Externalities in Networking

As noted by Holzer (1987) and Montgomery (1991), informal networks are more important in generating job offers and acceptances than other formalized methods (such as job placement center or newspaper advertisements). This is particularly so for job search in low-skilled labor markets. There is a natural self-reinforcing mechanism through "neighborhood externalities," wherein job search relies on personal contacts and search is more difficult (easier) if personal contacts are themselves without (with) jobs. To capture this network externality effect, we propose the following.

Assumption 6. Networking: The effective contact rate with which a resident i worker locates j-vacancies is  $\mu_{ij} = \mu_j \Lambda(E_{ij})$ , where  $\Lambda' > 0$  and  $\Lambda(1) < \infty$ .

The force of assumption 6 is that a higher level of employment of i residents in location j facilitates job search by unemployed i residents for jobs in j. In order to ensure nontrivial search frictions, the finite upper bound is imposed on  $\Lambda$ . It is no surprise that spatial mismatch still emerges and is indeed more severe than in the basic setting (because of the network externality). One major difference is that the equilibrium entry (EE) locus need not be monotonic and multiple nondegenerate steady-state equilibria may arise, as a result of self-fulfilling prophecies working through neighborhood spillovers. In particular, there may exist a low-contact-rate "equilibrium trap" in a nondegenerate spatial equilibrium, accompanied by high unemployment, low income, and low welfare.<sup>20</sup>

<sup>16%</sup> used welfare offices for job postings. The portion of suburban employers who did so is almost surely less.

<sup>&</sup>lt;sup>20</sup> Laing, Palivos, and Wang (1995) illustrate the possibility of multiple non-degenerate steady-state equilibria in a search model with endogenous learning. See Sawhill (1988) for a comprehensive survey of the economics of poverty and O'Regan and Quigley (1993) and references cited therein on access to networks and job search. Models of multiple equilibria with so-called poverty traps include Bond and Coulson (1989).

# D. Worker Migration

We next examine whether or not spatial mismatch arises if workers are free to select their residential location ex ante (up to this point workers are exogenously domiciled in a given residence but can commute between locations). Let us label workers by their initial residence in location i and denote the cost of moving from the CBD to the SBD by  $z_0 > 0$  (in the SSLSE equilibrium considered in theorem 1 SBD residents have no incentive to move to the CBD, which allows us to focus exclusively on the behavior of CBD residents). Furthermore, denote by  $x_i$  the correspondent migration choice variable (taking values in  $\{0,1\}$ ) of a worker residing initially at i: when  $x_i = 0$ , the worker stays; otherwise, he or she moves. The asset values for CBD residents are

$$rJ_{E}^{1j} = (w_{j} - \theta_{ij}) + \delta \left[ \max_{e_{1}, x_{1}} \left[ J_{U}(e_{1}, x_{1})^{i} - J_{E}^{1j} \right] \right], \tag{21a}$$

$$rJ_{\mathbf{v}}^{1j} = -\theta_{ij} + x_1 \mu_j [J_{\mathbf{E}}^{1j} - J_{\mathbf{v}}^{1j}] + (1 - x) \mu_j [J_{\mathbf{E}}^{2j} - J_{\mathbf{v}}^{2j}]. \tag{21b}$$

In addition to the locational equilibrium conditions, there is a migration equilibrium condition for CBD residents:

$$rJ_{\sigma}^{1j}|_{x=1} - rJ_{\sigma}^{1j}|_{x=0} = \left[\frac{\mu_{1}^{*}}{(\delta + r + \mu_{1}^{*})}\right] \left[\frac{(w_{j}^{*} - \theta_{1j}(a_{c1}^{*}))}{r}\right] - \left[\frac{\mu_{2}^{*}}{(\delta + r + \mu_{2}^{*})}\right] \left[\frac{(w_{j}^{*} - \theta_{2j}(a_{c1}^{*}))}{r}\right] - z_{0} = 0.$$
(22)

That is, the incremental values of a mover net of migration costs must be zero in equilibrium. Of course, for stayers, the incremental values of migrating are strictly less than the migration costs.

Yet, even if workers are free to choose their residential locations, spatial mismatch persists, provided the migration cost is strictly positive. The reasons are simple. First, even though migration implies that the ex post populations of workers in each location differ from their normalized initial values of unity, none of our earlier results depend on population levels per se. Second, regarding the composition of the postmigration population, nonmigrants are precisely those individuals with the highest commuting ability  $a \in A$ . In equilibrium, these individuals continue to reverse commute from the CBD to the SBD. Unfortunately, since the optimal migration choice depends on the equilibrium critical value  $a_{c1}^*$ , the equilibrium contact rates are no longer determined recursively. As a consequence, although a (nondegenerate) steady-state locational search equilibrium exists, it may not be unique. Owing to the complexity of the solution, we have been unable to characterize the welfare properties of the model with migration.

#### VII. Conclusion

The model developed in this article offers a parsimonious explanation for the three stylized empirical regularities associated with spatial mismatch. The two key assumptions underpinning our model are that search is costly and that the cost of setting up a firm in the suburbs is lower than in the central business district. We prove the existence of a steady-state locational search equilibrium in which SBD residents only search for home jobs and CBD workers search for jobs in both locations, reverse commuting if employment is secured in the SBD. Our results suggest that an urban policy that provides a subsidy or a tax incentive to firms entering the CBD in conjunction with an improvement in the city's infrastructure (lowering transportation and search costs) may ameliorate problems associated with mismatch.

We believe that our two-sector search equilibrium analysis can usefully be applied to address other important issues in settings where individuals choose among multiple labor markets. Most prominent among these extensions might be to the very well known Harris-Todaro (1970) model for analyzing the formidable migration-driven population growth in major cities of the developing world. In Harris and Todaro's framework, manufacturing job locations are exogenously set in the urban area, and rural residents maximize their well-being by moving to urban areas, even though the resulting unemployment rate is very high (as long as the high urban wages more than offset the high, exogenous probability of being unemployed). A variant of our model could be used to determine and characterize the general equilibrium patterns of rural-urban migration. Some progress has already been made in this direction (see Park 1999).

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