



## Trade, firm selection, and industrial agglomeration

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### ABSTRACT

We develop a model of trade and agglomeration that incorporates trade in both intermediate goods and final goods and allows all firms to choose their locations. There are two types of labor: skilled labor, which is mobile, and unskilled labor, which is immobile. Upon choosing its factory site, a final goods firm that is managed by skilled labor can produce these goods using local unskilled labor and a variety of intermediate goods produced by productivity-heterogeneous producers. We characterize world equilibrium and establish the conditions under which industrial agglomeration arises as a stable equilibrium outcome. We show that when the unskilled labor force is small, the role played by the selection of intermediate firms becomes less important, and trade liberalization induces dispersion. When the unskilled labor force is large and the selection effect becomes influential, trade liberalization can generate non-monotonic effects on industrial agglomeration. The dispersion effect of trade liberalization arises when unskilled labor–intermediate input complementarity matters to firm selection to a greater degree. When this is the case, trade liberalization may induce less selective firm entry and cause average productivity to fall.

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### 1. Introduction

During the past few decades of global economic development, there has been an ongoing increase in production fragmentation and intermediate goods trade.<sup>1</sup> This globalization trend has been accompanied by the rise of several key “world factories” that assemble a variety of final products, including Japan and the smaller, rapidly growing Asian Tigers. More recent years have seen the rapid clustering of manufacturing in China, turning that country into the world’s factory. The overall process has prompted considerable concern over the hollowing-out of domestic industries, starting with the more advanced Japanese economy and followed by the more recently developed Asian Tigers. How does trade liberalization factor into this process? It is generally believed that a reduction in trade costs will promote agglomeration, either international or regional (e.g., see Ottaviano et al., 2002 on international agglomeration and Behrens and Robert-Nicoud, 2011 on regional agglomeration). Yet, if trade liberalization does increase the incentives for firms to agglomerate in China, and if other countries are concerned about industrial hollowing-out, then why would these countries want to sign trade agreements with China, thereby risking the loss of firms to China?<sup>2</sup>

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<sup>1</sup> For example, see Jones (2000) and Hummels et al. (2001).

<sup>2</sup> Such agreements include the pact signed by the ASEAN Plus 2, the Economic Cooperation Framework Agreement (ECFA) with Taiwan, and the Closer Economic Partnership Agreement (CEPA) with Hong Kong.

In this paper, we revisit this important issue employing a model of trade and agglomeration that incorporates trade in both intermediate goods and final goods and allows all firms to choose their locations. There are two types of labor in the model, one skilled and mobile and the other unskilled and immobile. The skilled labor force comprises final goods managers who choose among countries in determining where to locate their firms. When its factory site is determined, each final goods firm produces these goods using local unskilled labor and a variety of intermediate goods produced by heterogeneous producers that differ in terms of productivity. Whereas the intermediate inputs enter production via a quadratic aggregator, as proposed by Peng et al. (2006), intermediate inputs and unskilled labor are complements in assembling the final goods, as in Krugman and Venables (1995) and Fujita et al. (1999b). Importantly, because the unskilled labor force is immobile, this complementarity becomes a dispersion force. To see how it does so, imagine that the complementarity is significant and all final goods firms are located in one of two countries. Because the other country has an unskilled labor force that is completely unemployed due to its immobility, the wage there is nearly zero. Such a low wage is likely to induce some final goods firms to relocate to this country because its production will increase due to the complementarity and the extremely low labor cost.

In this model economy, we establish a world equilibrium in which all final and intermediate goods producers optimize, and the locations of all firms are determined endogenously. We are particularly interested in establishing the conditions under which industrial agglomeration arises as a stable equilibrium outcome. We show that, given the

distribution of final goods firms across countries, trade liberalization renders the selection of intermediate goods firms tougher, and hence improves average productivity. A larger market size yields a similar selection outcome. When the unskilled labor force is small, the selection effect is less important and trade liberalization induces dispersion, as in the intermediate goods trade model proposed by Peng et al. (2006), but in contrast to the final goods trade model posited by Ottaviano et al. (2002). When the unskilled labor force is large, the selection effect is essential and trade liberalization can exert non-monotonic effects on industrial agglomeration. When this is the case, and when unskilled labor–intermediate input complementarity matters to a relatively greater degree, such complementarity can interact with firm selection such that trade liberalization encourages dispersion, which is contrary to general belief. Moreover, in this case, trade liberalization may induce a less selective firm entry and cause average productivity to fall. Using numerical analysis, we illustrate much richer equilibrium configurations than those obtained in the trade and new economic geography literature.

By adopting the firm heterogeneity structure proposed by Melitz and Ottaviano (2008), our model reveals a new and interesting channel to characterize the relationship between firm selection and agglomeration. In particular, the aforementioned non-monotonic effect of trade costs on industrial agglomeration stems from the fact that trade costs, as a parameter determining selection pressure, can be both conducive and detrimental to agglomeration. As we explain this in greater detail in Section 4, we simply offer a quick intuition here. Lower trade costs imply a larger effective market size, and hence tougher selection. As agglomeration induces productivity gains via selection, tougher selection implies greater gains. However, as selection pressure is readily tough with a lower trade cost, a marginal increase in selection pressure via agglomeration is less pronounced. Thus, the marginal gains from agglomeration, i.e., the incentives to agglomerate, are less with a lower trade cost, and trade liberalization may thus lead to dispersion. Which force dominates depends.

### 1.1. Related literature

Our paper is generally related to two strands of economic studies. The first strand embeds firm heterogeneity in the study of international trade (cf. Bernard et al., 2003; Melitz, 2003). The second investigates agglomeration internationally or regionally, and encompasses the new economic geography literature (cf. Fujita et al., 1999b; Fujita et al., 1999a). Several recent works have combined the two strands to investigate spatial sorting (Baldwin and Okubo, 2006; Behrens et al., 2010), economic agglomeration (Behrens and Robert-Nicoud, 2011), and productivity gains across cities (Combes et al., 2009).

The non-monotonic relationship between trade costs and agglomeration lies at the center of our analysis. Although Krugman and Venables (1995) propose an agglomeration model with homogeneous firms that generates a similar non-monotonic relationship, our model of heterogeneous firms provides a new and interesting angle to this relationship, as explained earlier. It is also interesting to compare our model with that in Melitz and Ottaviano (2008). Their model features selection but not agglomeration. Furthermore, trade liberalization always leads to greater productivity, which is not necessarily the case in our model. Particularly in the case in which trade liberalization leads to dispersion, selection pressure is eased, and the average productivity may become lower.

As we employ a quadratic rather than the Dixit–Stiglitz–Ethier aggregator, our paper is also closely related to Ottaviano et al. (2002), Peng et al. (2006), and Behrens and Robert-Nicoud (2011). In contrast to all of these but Peng et al. (2006), we apply the quadratic structure to final goods production rather than to household utility. In contrast to Ottaviano et al. (2002) and Peng et al. (2006), we allow for firm heterogeneity and consider selection of intermediate producers. In contrast to all previous works, we incorporate a new feature, i.e., the dispersion force that results from the complementarity between immobile unskilled workers and tradable intermediate inputs in the final goods

assembly. This dispersion force differs from the oft-used example of agriculture produced by an immobile factor such as land/farmers (e.g., Krugman, 1991; Fujita and Krugman, 1995; Ottaviano et al., 2002) or an immobile consumable such as land or an amenity (e.g., Helpman, 1998; Redding and Sturm, 2008).

The remainder of the paper is organized as follows. Section 2 introduces the model and examines equilibrium given the distribution of skilled labor. Section 3 solves the optimization problems facing final goods producers, intermediate firms, and skilled labor. Section 4 outlines the equilibrium conditions and defines the concept of world equilibrium. In Section 5, we characterize equilibrium outcomes, focusing on how trade liberalization affects industrial agglomeration. In Section 6, we present numerical results that highlight the different agglomeration features from those exhibited by models that employ either agriculture or immobile land as the dispersion forces. Finally, Section 7 concludes the paper and provides possible avenues for extension.

## 2. The model

There are two ex ante identical countries, labeled  $l = A, B$ . In each country, there are  $L$  unskilled workers who are immobile and  $M$  skilled workers who are freely mobile across countries. There are two types of goods: a homogeneous final good and a continuum of differentiated intermediate goods. Both unskilled labor and intermediate goods are used to produce the final good. The market for the final good is perfectly competitive, whereas those for the intermediate goods are monopolistically competitive. These market structure assumptions are analytically friendly benchmarks that are commonly used in the literature, following the lead of Ethier (1982) and Romer (1990). Moreover, as documented in previous studies, the intensity of the intermediate goods trade as measured by the VS index rose from below 2% in the 1960s to over 15% in the 1990s (Hummels et al., 2001) whereas the share of that trade has exceeded two-thirds (Yi, 2003). Thus, by placing emphasis on both intermediate goods production and trade, our model addresses issues of increasing importance.

The production of any intermediate good  $j$  requires a constant marginal cost in terms of the final good, and can vary across firms. After paying a fixed input of  $\kappa$  units of a final good, an entrant obtains a “new” intermediate good blueprint with productivity  $z$ , which is a random realization from a Pareto distribution with lower bound  $z_M$ , i.e.,  $\Pr[Z \geq z] = (z/z_M)^{-k}$  for all  $z \geq z_M$ . This implies that the distribution of marginal cost  $c = 1/z$  is given by

$$G(c) \equiv \Pr\left[\frac{1}{Z} \leq c\right] = \Pr\left[Z \geq \frac{1}{c}\right] = \left(\frac{c}{c_M}\right)^k, \quad \text{for } c \in (0, c_M), \quad (1)$$

where  $c_M \equiv 1/z_M$  is the upper bound of the marginal cost corresponding to the lower bound of productivity  $z_M$ . Intermediate goods are tradable, but subject to an iceberg trade cost if trade occurs across borders. More specifically, to deliver one unit of an intermediate good across borders,  $\tau > 1$  units need to be shipped.

The set of intermediate goods available in country  $l$  is given by  $[0, N^l]$ , where  $N^l$  is endogenously determined. Any skilled worker in  $l$ , as the manager of a final goods firm, can produce a final good of  $Y_1^l$  amount using the following technology:

$$Y_1^l = D_1^l \ln(\ell_1^l), \\ D_1^l = \alpha \int_0^{N^l} q_1^l(j) dj - \frac{\gamma}{2} \int_0^{N^l} [q_1^l(j)]^2 dj - \frac{\eta}{2} \left( \int_0^{N^l} q_1^l(j) dj \right)^2, \quad (2)$$

where  $q_1^l(j)$  is the amount of intermediate good  $j$  employed, and  $\ell_1^l$  is the mass of unskilled labor hired. We assume that  $\alpha > 0$  and  $\gamma > 0$ , where the former is a typical scaling factor and the latter measures the variety effect of the intermediate good inputs. These intermediate goods are Pareto substitutes (resp., complements) if  $\eta > 0$  (resp.,  $< 0$ ). To be compatible with previous studies, inclusive of the new economic geography

literature, we restrict our analysis to the case of  $\eta > 0$ . The final good can be freely traded across countries, and is chosen as the numeraire.

Skilled workers, as managers of final goods firms, choose their respective locations for production. Let  $\mu^l \in (0, 1)$  be the fraction of skilled workers located in  $l$ . Thus,  $M^l = \mu^l M$ , and the population identity requires

$$\mu^A + \mu^B = 1. \tag{3}$$

Let  $w^l$  and  $v^l$  denote the wage of the unskilled labor force and the profit of a final goods firm in country  $l$ , respectively. The utility of an unskilled and skilled worker is thus  $w^l$  and  $v^l$ , respectively.<sup>3</sup>

Note that the production technology for the final good is modified from the quadratic production function used by Peng et al. (2006) by adding a multiplicative term in unskilled labor. This modification allows us to capture the complementarity between unskilled labor and intermediate goods in final goods production. More specifically,  $\frac{\partial^2 Y^l}{\partial D_1^l \partial \ell_1^l} = \frac{1}{\ell_1^l} > 0$ , which implies that unskilled labor and intermediate good inputs are Pareto complements. We will demonstrate that such complementarity plays an important role in the equilibrium outcomes of agglomeration across countries.

### 3. Optimization

We begin by solving a final goods firm's optimization problem in given country  $l$ , followed by an intermediate firm's optimization decision and then a skilled worker's locational choice.

#### 3.1. Final goods sector

Let  $p^l(j)$  be the price of intermediate good  $j$  facing a final goods firm in country  $l$ , whose optimization problem is given by

$$\begin{aligned} \max_{\ell_1^l, q_1^l(j)} \quad & v^l = D_1^l \ln(\ell_1^l) - \int_0^{N^l} p^l(j) q_1^l(j) dj - w^l \ell_1^l \\ \text{s.t.} \quad & D_1^l = \alpha \int_0^{N^l} q_1^l(j) dj - \frac{\gamma}{2} \int_0^{N^l} [q_1^l(j)]^2 dj - \frac{\eta}{2} \left( \int_0^{N^l} q_1^l(j) dj \right)^2. \end{aligned} \tag{4}$$

The first-order condition with respect to the number of unskilled workers hired,  $\ell_1^l$ , is simply

$$\ell_1^l = \frac{D_1^l}{w^l}. \tag{5}$$

The first-order condition with respect to intermediate good  $j$  takes a nice linear form:

$$p^l(j) = b^l [\alpha - \eta \bar{Q}_1^l - \gamma q_1^l(j)], \tag{6}$$

where  $b^l \equiv \ln(\ell_1^l)$  measures the marginal product of the intermediate inputs, and  $\bar{Q}_1^l \equiv \int_0^{N^l} q_1^l(j) dj$  is the aggregate demand for intermediate inputs. The latter condition equates the marginal product of an intermediate input  $j$  with its price.

Before examining the location choices of the final goods firms and the intermediate goods sector, we first observe that, in equilibrium, labor market clearing requires that

$$b^l = \ln(\ell_1^l) = \ln\left(\frac{L}{\mu^l M}\right).$$

To ensure positive output  $Y_1^l$ , we impose the condition that  $L > M$ . Given a fixed mass of skilled workers (final goods firms), the size of the unskilled labor force exerts a positive effect on the marginal product of intermediate inputs and, hence, intermediate goods demand, as a result of Pareto complementarity. When, in contrast, there are more final goods firms competing for a fixed amount of unskilled labor (higher  $\mu^l$ ), each firm hires fewer workers and the marginal product of intermediate inputs is reduced.

Rearranging Eq. (6), we obtain a final goods firm's demand for intermediate good  $j$ :

$$q_1^l(j) = \frac{\alpha - \eta \bar{Q}_1^l - p^l(j)}{\gamma b^l}. \tag{7}$$

From the foregoing equation and the definition of  $\bar{Q}_1^l$ , we can solve the aggregate intermediate goods demand:

$$\bar{Q}_1^l = \frac{\alpha N^l}{\gamma + \eta N^l} - \frac{N^l \bar{p}^l}{\gamma + \eta N^l b^l},$$

which can then be substituted into Eq. (7) to yield

$$q_1^l(j) = \frac{\alpha \gamma + \eta N^l \bar{p}^l / b^l - p^l(j)}{\gamma(\gamma + \eta N^l)},$$

or, equivalently, the inverse demand function for intermediate good  $j$ :

$$p^l(j) = p_{\max}^l - \gamma b^l q_1^l(j), \tag{8}$$

where  $p_{\max}^l \equiv \frac{\alpha \gamma b^l + \eta N^l \bar{p}^l}{\gamma + \eta N^l}$  is the price below which the quantity demanded is nonnegative and  $\bar{p}^l$  is the average price of intermediate goods that prevails. The maximum price increases with both the average price and the number of unskilled workers employed in the final goods sector.

#### 3.2. Intermediate goods sector

Because intermediate goods firms are monopolists in their respective varieties, they can set their own prices. An intermediate goods firm in  $l$  with marginal production cost  $c$  can sell to domestic final goods firms or export to foreign final goods firms in country  $h \neq l$ . Utilizing the inverse demand function (8), its profit from domestic sales (denoted by  $D$ ) is

$$\pi_D^l(c) = \max_{p_D^l(c)} [p_D^l(c) - c] \frac{\mu^l M}{\gamma b^l} (p_{\max}^l - p_D^l(c)), \tag{9}$$

whereas its profit from export sales (denoted by  $X$ ) is

$$\pi_X^l(c) = \max_{p_X^l(c)} [p_X^l(c) - \tau c] \frac{\mu^h M}{\gamma b^h} (p_{\max}^h - p_X^l(c)). \tag{10}$$

The first-order conditions are:

$$\begin{aligned} p_D^l(c) &= \frac{p_{\max}^l + c}{2}, \\ p_X^l(c) &= \frac{p_{\max}^h + \tau c}{2}. \end{aligned}$$

Consider a marginal intermediate goods firm in  $l$  with  $c = c_D^l \equiv p_{\max}^l$ , whose optimal price is  $p_{\max}^l$  and quantity sold is zero. Thus, any firms with  $c > c_D^l$  must exit, whereas those with  $c < c_D^l$  can remain in operation and sell to domestic final goods firms. Similarly, defining  $c_X^l \equiv p_{\max}^h / \tau$ , any firms in  $l$  with  $c > c_X^l$  will not find exporting to be profitable.

<sup>3</sup> For tractability, we assume away the managerial need for intermediate goods firms. Nevertheless, the endogenous entry decisions of the intermediate goods sector allow the agglomeration of intermediate goods firms (as a larger entry) to go hand-in-hand with the agglomeration of final goods firms.

The foregoing conditions can be more intuitively written as

$$p_D^l(c) = \frac{c_D^l + c}{2}, \quad p_X^l(c) = \frac{\tau c_X^l + \tau c}{2}, \quad (11)$$

which can be substituted into Eqs. (8), (9) and (10) to derive

$$q_D^l(c) = \frac{\mu^l M}{2\gamma b^l} (c_D^l - c), \quad q_X^l(c) = \frac{\tau \mu^h M}{2\gamma b^h} (c_X^l - c), \quad (12)$$

$$\pi_D^l(c) = \frac{\mu^l M}{4\gamma b^l} (c_D^l - c)^2, \quad \pi_X^l(c) = \frac{\tau^2 \mu^h M}{4\gamma b^h} (c_X^l - c)^2. \quad (13)$$

Note that  $q_X^l(c)$  is the market demand from final goods firms in country  $h$  for an intermediate good produced in country  $l$  with marginal cost  $c$ . This firm with  $c$  in country  $l$  needs to produce and ship  $\tau q_X^l(c)$  units to country  $h$ , and  $q_X^l(c)$  will actually arrive.

### 3.3. Skilled workers

To derive the profit of a final goods firm, which is the return of a skilled worker, we first solve its value under autarky (i.e., with an infinitely large trade cost  $\tau$  such that there is no trade in intermediate goods). Subsequently, we verify that even with intermediate goods trade, such a functional form continues to hold true. More specifically, in the absence of intermediate goods trade, the intermediate goods aggregator is given by (see Appendix A for a more detailed derivation):

$$D_1^l = \alpha \int_0^{N^l} q_1^l(j) dj - \frac{\gamma}{2} \int_0^{N^l} [q_1^l(j)]^2 dj - \frac{\eta}{2} \left( \int_0^{N^l} q_1^l(j) dj \right)^2 \\ = \frac{\alpha b^l - c_D^l}{\eta b^l} \left[ \frac{\alpha}{2} + \frac{(k+1)c_D^l}{2b^l(k+2)} \right]. \quad (14)$$

A final goods firm's profit  $v^l$  is given by (again, see Appendix A for a more detailed derivation):

$$v^l = b^l D_1^l - \int_0^{N^l} p^l(j) q_1^l(j) dj - w^l \ell_1^l \\ = \frac{\alpha b^l - c_D^l}{2\eta b^l} \left[ \alpha (b^l - 1) - \frac{(k+1)(b^l + 1)c_D^l}{(k+2)b^l} \right]. \quad (15)$$

We now turn to the general case with intermediate goods trade (a finite  $\tau$ ). As Melitz and Ottaviano (2008) show, price distribution of goods sold by foreign firms actually matches those sold by domestic firms. This is the case because of the Pareto assumption, and it also holds true in this paper. Accordingly, inspecting Eq. (8), we can see that the distribution of the quantity demanded  $q_1^l(j)$  is also the same regardless of whether  $j$  is purchased from a domestic or foreign firm, which implies that the formula for  $D_1^l$  in the general case is the same as Eq. (14), except that  $N^l$  and  $c_D^l$  take different values from the case with an infinite  $\tau$ .

Because we consider a homogeneous final good that can be freely traded across countries and is the numeraire, nominal and real profits are identical in our model economy. The locational choice of a final goods firm managed by a skilled worker is therefore given by

$$l^* = \arg \max_{l \in \{A, B\}} \{v^l\}. \quad (16)$$

## 4. Equilibrium

We are now ready to define a competitive world equilibrium. We begin by describing several key equilibrium conditions, followed by market clearing conditions.

### 4.1. Zero cutoff profit

An important implication of the matching price distribution property discussed in the previous section is that, to derive the average price  $\bar{p}^l$ , we need only consider the prices charged by domestic firms:

$$\bar{p}^l = \frac{1}{G(c_D^l)} \int_0^{c_D^l} p_D^l(c) dG(c) = \frac{2k+1}{2k+2} c_D^l. \quad (17)$$

Plugging Eq. (17) into the definition of  $p_{\max}^l$  and utilizing  $b^l = \ln(L/M) - \ln(\mu^l)$ , we obtain the zero cutoff profit (ZCP) condition:

$$N^l = \frac{2(k+1)\gamma \alpha [\ln(L/M) - \ln(\mu^l)] - c_D^l}{\eta c_D^l}. \quad (18)$$

This ZCP condition posits a negative relationship between the cutoff  $c_D^l$  and the number of intermediate firms selling to  $l$ ,  $N^l$ , under a given distribution of skilled workers. In other words, when the measure of intermediate goods competing in the same market is larger, the cutoff is smaller (i.e., tougher selection).

### 4.2. Free entry of intermediate goods firms

We next turn to the free entry conditions faced by intermediate goods firms, which requires the expected operating profit to be equal to the entry cost (see Appendix C):

$$\kappa = G(c_D^l) E(\pi_D^l(c) | c < c_D^l) + G(c_X^l) E(\pi_X^l(c) | c < c_X^l) \\ = \frac{M}{2\gamma(k+1)(k+2)(c_M)^k} \left[ \frac{\mu^l}{b^l} (c_D^l)^{k+2} + \frac{\tau^{-k} \mu^h}{b^h} (c_D^h)^{k+2} \right]. \quad (19)$$

Define  $\theta \equiv \frac{2\gamma \kappa (k+1)(k+2)(c_M)^k}{(1+\tau^{-k})M}$ , which rises with both transport cost  $\tau$  and the strength of variety effect  $\gamma$ . Solving the system of equations in Eq. (19) for  $l=A, B$  and utilizing Eq. (18), we can then determine the cutoff:

$$(c_D^l)^{k+2} = \theta \frac{b^l}{\mu^l} = \theta \frac{\ln(L/M) - \ln(\mu^l)}{\mu^l}. \quad (20)$$

This marginal-cost cutoff is decreasing in  $\mu^l$ , because a larger final goods market (higher  $\mu^l$ ) renders the selection of intermediate goods firms tougher (lower  $c_D^l$ ). Under a given distribution of skilled labor, cutoff  $c_D^l$  grows larger the greater the size of the unskilled labor force (higher  $L$ ), the higher the transport cost  $\tau$  (larger  $\theta$ ), or the stronger the variety effect  $\gamma$  (larger  $\theta$ ).

It should be noted that entry and cutoffs are two sides of the same coin. In a firm heterogeneity model such as this, the cutoffs summarize the key information of the model. The only differences with the cutoff formula in Melitz and Ottaviano (2008) are that they have  $b^l = 1$ , and  $\mu^l$  is exogenous. Let us now examine this formula more carefully. Cutoffs depend on the underlying technology of the intermediate sector, which is captured by  $k$  and  $c_M$ . They also depend on the cost of entry, which is represented by  $\kappa$ . Naturally, the demand side matters, which is why  $\gamma$ ,  $b^l$ , and  $\mu^l M$  enter. If  $\gamma$  is large, then the love of variety effect is stronger, which makes selection easier, all else being equal. With regard to the other parameters,  $\mu^l M$  is simply the “market size effect” emphasized by Melitz and Ottaviano (2008). With a larger market size, the entry becomes larger and hence selection becomes tougher. Now,  $b^l$  is a new element, indicating that the greater the number of workers employed per final goods firm in  $l$ , the larger the marginal product of intermediate goods in producing the final good. Hence, a larger  $b^l$  increases the demand for intermediate goods, thus rendering selection less tough. Evidently, although a high  $\mu^l$  has a direct impact on selection, it also renders intermediate firm entry more selective via a reduction in

unskilled employment per firm. Finally,  $\tau$  matters in a way similar to market size. In fact, the larger  $\tau$  is, the smaller the effective market size. Thus, trade liberalization that reduces  $\tau$  increases the effective market size and renders selection tougher.

Substituting Eq. (20) into Eq. (15), we obtain

$$v^l = \frac{\alpha - (b^l)^{-\frac{k+1}{k+2}} (\theta/\mu^l)^{\frac{1}{k+2}}}{2\eta} \left[ \alpha (b^l - 1) - \frac{(k+1)(b^l + 1)(b^l)^{-\frac{k+1}{k+2}} (\theta/\mu^l)^{\frac{1}{k+2}}}{k+2} \right], \tag{21}$$

where  $b^l = \ln(L/M) - \ln(\mu^l)$  is decreasing in  $\mu^l$ . From Eq. (15), we know that  $v^l$  is strictly decreasing in  $c_b^l$ , and thus tougher selection always generates productivity gains. Because a larger market size induces tougher selection (a higher  $\mu^l$  causes  $c_b^l$  to fall), agglomeration, via selection, can generate productivity gains. Nevertheless, the way in which the key parameters summarized in  $\theta$  (e.g.,  $\tau$  and  $\gamma$ ) determine selection cutoff affects the magnitude of the productivity gains realized by agglomeration via selection. On the one hand, Eq. (20) implies that the larger  $\theta$  is, the larger the productivity gains from increasing  $\mu^l$ . On the other hand, given  $\mu^l$ , a larger  $\theta$  also means less selective firm entry and fewer productivity gains to drive agglomeration. Therefore, there is a potential non-monotonic relationship between  $\theta$  and agglomeration, as we show in Section 5.

**Remark 1.** The foregoing cutoff formula is applicable only to the interior case, where  $c_b^l < c_M$  for both  $l$ . It is possible that  $c_b^l = c_M$  for one country  $l$  even when  $c_M$  is assumed to be large, because when  $\mu^l \rightarrow 0$ ,  $b^l/\mu^l$  goes to infinity, and so  $c_b^l$  must hit the upper bound for a sufficiently small  $\mu^l$ , such that

$$\frac{b^l}{\mu^l} \geq \frac{(1 + \tau^{-k}) M c_M^2}{2\gamma\kappa(k+1)(k+2)}. \tag{22}$$

In other words, when the number of final goods firms in country  $l$  is excessively small, this small market size attracts only a small number of entrants; hence, all entrants can survive. By Eq. (19), when the cutoff in one country  $l$  hits the upper bound ( $c_b^l = c_M$ ), the selection cutoff in the other country  $h \neq l$  is given by

$$(c_D^h)^{k+2} = \frac{b^h \mu^h}{\mu^h b^l} c_M^{k+2}. \tag{23}$$

In general, we assume  $c_M$  to be large to ensure that cases in which the cutoff hits the upper bound are confined to extremely uneven distributions of skilled labor. Note that when survival in one country  $l$  becomes a certainty ( $c_b^l = c_M$ ), the other country's selection pressure becomes more responsive to increases in market size. In other words, the selection cutoff given by Eq. (23) is smaller than that given by Eq. (20) when Eq. (22) holds.

4.3. Labor market and locational equilibrium

Because each skilled worker owns a final goods firm and each final goods firm hires a mass  $\ell_1^l$  of unskilled workers, the total unskilled labor demand is given by  $\mu^l M \ell_1^l$ . The labor market clearing condition is then

$$\mu^l M \ell_1^l = L. \tag{24}$$

This, together with Eqs. (5) and (14), pins down the equilibrium market wage to

$$w^l = \frac{\mu^l M \alpha \ln(L/M) - \ln(\mu^l) - c_D^l}{L \eta b^l} \left[ \frac{\alpha}{2} + \frac{(k+1)c_D^l}{2b^l(k+2)} \right]. \tag{25}$$

Because skilled workers are mobile, the equilibrium location choice of a final goods firm must satisfy the following locational equilibrium condition.

$$v^l = v \text{ for } l = A, B, \text{ whenever } \mu^l \in (0, 1). \tag{26}$$

That is, for an interior distribution, skilled workers' value must be equalized across countries. An extreme distribution,  $\mu^l = 1$  ( $\mu^h = 0$ ), is also an equilibrium if  $v^l \geq v^h$ .

4.4. World equilibrium

We are now ready to define the world equilibrium.

**Definition.** A world equilibrium is a list of quantities  $\{\ell_1^l, q_1^l(j), D^l, Y^l, N^l\}$ , prices  $\{p^l(j), w^l, v^l\}$ , cutoffs  $\{c_b^l, c_M^l\}$ , and population distribution  $\{\mu^l\}$  for  $l = A, B$  and  $j \in [0, N^l]$ , such that both final and intermediate goods firms all optimize (inclusive of the final goods firms' locational choice), the zero profit and free entry conditions are met, labor markets are clear, the locational equilibrium for final goods firms is reached, and population identity (3) holds.

5. Equilibrium characterization

There are two world equilibrium configurations: a *symmetric* configuration with  $\mu^l = 1/2$  and an *agglomeration* configuration with  $\mu^l \neq 1/2$ . Because the two countries in our model are symmetric, we can, without loss of generality, restrict our attention to the case with  $\mu^A \geq \mu^B$ . It is convenient to denote the utility difference of skilled managers between two countries as  $\Delta v \equiv v^A - v^B$  and their distribution as  $\mu \equiv \mu^A$  and  $\mu^B = 1 - \mu$ . Note that if  $\Delta v > 0$ , then  $\mu$  will increase unless  $\mu = 1$ . Similarly, if  $\Delta v < 0$ , then  $\mu$  will decrease unless  $\mu = 0$ . Thus, an equilibrium is  $\Delta v = 0$  at any  $\mu \in [0, 1]$ ,  $\Delta v > 0$  at  $\mu = 1$ , or  $\Delta v < 0$  at  $\mu = 0$ . Following the convention proposed by Benabou (1996), we define the stability of a world equilibrium as follows, focusing on the case with  $\mu \geq \frac{1}{2}$  (i.e.,  $\mu^A \geq \mu^B$ ). Any world equilibrium with  $\mu \in [1/2, 1)$  is said to be *stable* if  $d\Delta v/d\mu \leq 0$ . A world equilibrium with  $\Delta v > 0$  at  $\mu = 1$  is also stable, and a world equilibrium with  $\Delta v = 0$  at  $\mu = 1$  is stable if  $d\Delta v/d\mu \geq 0$ .

Recall that we impose the condition that  $L > M$  and consider a sufficiently large  $c_M$ . We first show the following.

**Theorem 1.** (World equilibrium). *A symmetric world equilibrium always exists, in which the zero profit cutoff is interior (i.e.,  $c_b^l < c_M$  for  $l = A, B$ ). Moreover, if the symmetric world equilibrium is unstable, then at least one agglomeration world equilibrium exists, and almost surely at least one of them is stable.*

**Proof.** All proofs are relegated to the Appendix. ■

This is an important theorem as it reduces analysis of the equilibrium configuration simply to establish the conditions under which the symmetric world equilibrium is unstable. In this case, an agglomeration world equilibrium will arise. *Almost surely*, at least one agglomeration equilibrium is stable; that is, the subset of parameter space under which all agglomeration equilibria are unstable is of measure zero.

To establish such conditions, we observe that  $dv^A/d\mu = dv^A/d\mu^A$  and  $dv^B/d\mu = -dv^B/\mu^B$ , implying that

$$\frac{d\Delta v}{d\mu} = dv^A/d\mu^A + dv^B/\mu^B.$$

Accordingly, we focus on evaluating  $\frac{d\Delta v}{d\mu} \Big|_{\mu=1/2}$ , which depends crucially from the cutoff parameter  $\theta$  and unskilled labor force parameter  $\beta \equiv b^l \Big|_{\mu=1/2} = \ln\left(\frac{2L}{M}\right) > \ln(2) > 0$ . It is convenient to define the following functions.

$$\begin{aligned} \Phi(\beta) &\equiv 2\beta^2 - (k-2)\beta - 2(k+1) \\ \Lambda(\beta) &\equiv -2^{\frac{1}{k+2}} \beta^{-\frac{2k+3}{k+2}} \left[ (2k+3)\beta^2 + (2k+2)\beta + k+1 \right] \\ \Psi(\theta, \beta) &\equiv -\alpha(k+2)^2 \theta^{-\frac{1}{k+2}} - 2^{\frac{2}{k+2}} (k+1) \alpha^{-1} \theta^{\frac{1}{k+2}} \beta^{-\frac{3k+4}{k+2}} \Phi(\beta) \\ \Omega(\beta) &\equiv -2(k+2)(k+1)^{\frac{1}{2}} \beta^{\frac{1}{2}} \Phi(\beta)^{\frac{1}{2}} + (2k+3)\beta^2 + (2k+2)\beta + k+1 \quad \text{for } \Phi(\beta) > 0 \\ \tilde{\theta}(\beta) &\equiv \frac{[\alpha(k+2)]^{k+2}}{2} \left\{ \frac{\beta^{3k+4}}{[(k+1)\Phi(\beta)]^{k+2}} \right\}^{1/2} \quad \text{for } \Phi(\beta) > 0. \end{aligned}$$

The following lemmas are helpful in establishing the aforementioned conditions.

- Lemma 1.**  $\frac{d\Delta v}{d\mu} \Big|_{\mu=1/2} > 0$  if and only if  $\Psi(\theta, \beta) > \Lambda(\beta)$ .
- Lemma 2.** For  $1/2$ , all  $k \geq 1$ , there exists a  $\tilde{\beta} \equiv \frac{1}{4}[(k-2) + (k^2 + 12k + 20)^{1/2}] > \ln(2)$  such that  $\Phi(\beta) < 0$  for all  $\beta < \tilde{\beta}$  and  $\Phi(\beta) > 0$  for all  $\beta > \tilde{\beta}$ .
- Lemma 3.** For all  $\beta < \tilde{\beta}$ ,  $\frac{\partial \Psi(\theta, \beta)}{\partial \theta} > 0$ ; for  $\beta > \tilde{\beta}$ ,  $\frac{\partial \Psi(\theta, \beta)}{\partial \theta} > 0$  if  $\theta < \tilde{\theta}(\beta)$  and  $\frac{\partial \Psi(\theta, \beta)}{\partial \theta} < 0$  if  $\theta > \tilde{\theta}(\beta)$ .
- Lemma 4.**  $\Psi(\tilde{\theta}(\beta), \beta) > \Lambda(\beta)$  if and only if  $\Omega(\beta) > 0$ .

We are now ready to establish the main theorem of the paper.

**Theorem 2. (Agglomeration equilibrium).**

**Case 1.** If  $\beta < \tilde{\beta}$ , then there exists at least one agglomeration world equilibrium with a sufficiently large  $\theta$ .

**Case 2.** If  $\beta > \tilde{\beta}$  and

- (a) if  $\Omega(\beta) > 0$ , then there exists at least one agglomeration world equilibrium with intermediate  $\theta$  in the neighborhood  $I_{\tilde{\theta}(\beta)}$  of  $\tilde{\theta}(\beta)$ ;
- (b) if  $\Omega(\beta) < 0$ , then the symmetric world equilibrium is stable.

Almost surely, at least one of the agglomeration equilibria arising in Cases 1 and 2(a) is stable.

This theorem is important as it states that a stable agglomeration world equilibrium can arise under the conditions given in Cases 1 and 2(a).

**Remark 2.** Notably, when a symmetric equilibrium is unstable, at least one agglomeration equilibrium exists. When a symmetric equilibrium is stable, there may or may not be an agglomeration equilibrium, but there is definitely no agglomeration equilibrium in the neighborhood of  $\mu = 1/2$ .<sup>4</sup> It is in this sense that the conditions established in Theorem 2 are sufficient but not necessary for agglomeration equilibrium.

To understand the conditions established in Theorem 2, recall that  $\beta \equiv \ln\left(\frac{2L}{M}\right)$  is positively dependent on the unskilled labor force  $L$ , and  $\theta \equiv \frac{2\gamma\kappa(k+1)(k+2)(c_M)^k}{(1+\tau^{-k})M}$  on transport cost  $\tau$  and the strength of the variety effect  $\gamma$ . Therefore, for a sufficiently small unskilled labor force such that  $\beta < \tilde{\beta}$ , a stable agglomeration world equilibrium arises when the transport cost and strength of the variety effect are sufficiently high. On the contrary, for a sufficiently large unskilled labor force such that  $\beta > \tilde{\beta}$  and  $\Omega(\beta) > 0$ , a stable agglomeration world equilibrium arises

<sup>4</sup> As we show in Section 6, there is a case in which the symmetric equilibrium is stable, but there also exists a stable agglomeration equilibrium.

when the transport cost and the strength of the variety effect take intermediate values.

To better understand the intuition, we investigate three key channels underlying the aforementioned findings by reexamining  $\frac{d\Delta v}{d\mu} \Big|_{\mu=1/2} = 2 \frac{dv^l}{d\mu^l} \Big|_{\mu=1/2}$ , where

$$\frac{dv^l}{d\mu^l} = \frac{\partial v^l}{\partial c_D^l} \left( \frac{\partial c_D^l}{\partial \mu^l} + \frac{\partial c_D^l}{\partial b^l} \frac{\partial b^l}{\partial \mu^l} \right) + \frac{\partial v^l}{\partial b^l} \frac{db^l}{d\mu^l}. \tag{27}$$

From Eq. (15), we know that the term  $\partial v^l / \partial c_D^l$  is clearly negative, which means that more stringent selection in the intermediate goods sector is always beneficial to final goods firms because the input prices are lower. Therefore, the absolute value of this term measures the *marginal gains of selection*. From Eq. (27), we know that such gains can arise from two channels: one that captures the *direct market size effect on selection* (i.e.,  $\frac{\partial v^l}{\partial c_D^l} \frac{\partial c_D^l}{\partial \mu^l}$ ) and another that represents the (intermediate goods–unskilled labor) *complementarity effect on selection* (i.e.,  $\frac{\partial v^l}{\partial c_D^l} \frac{\partial c_D^l}{\partial b^l} \frac{\partial b^l}{\partial \mu^l}$ ). From Eq. (20), it is clear that an increase in  $\mu^l$  reduces the cutoff and renders the selection tougher, which, in turn, raises the profit accruing to skilled workers, thereby encouraging agglomeration. Focusing now on the complementarity effect, we can see that an increase in  $\mu^l$  reduces the amount of labor hired by each firm,  $b^l$ , and, by the complementarity effect, decreases the demand for intermediate goods and hence the cutoff. Again, as  $c_D^l$  decreases,  $v^l$  increases, which also encourages agglomeration.

There is, however, a *direct labor force effect* via the second term on the right-hand side of Eq. (27). More specifically, as an increase in  $\mu^l$  reduces the amount of labor hired by each firm,  $b^l$ , the final goods production per firm is reduced, and wages rise. That  $\partial v^l / \partial b^l > 0$ , i.e., the profit per firm  $v^l$  is reduced with a decrease in  $b^l$ , is evident from Eq. (15). Thus, the direct labor force effect is a channel that discourages agglomeration. In sum, the sign of  $dv^l/d\mu^l$  obviously constitutes a tug-of-war between the direct labor force effect and the sum of the two selection effects. When the two selection effects dominate, an agglomeration world equilibrium arises.

We now investigate how the magnitude of these three channels depends on the cutoff and unskilled labor force parameters,  $\theta$  and  $\beta$ . As shown in the Appendix, the magnitude of the two selection effects is given by

$$\begin{aligned} &\frac{\partial v^l}{\partial c_D^l} \frac{dc_D^l}{d\mu^l} \\ &= \left\{ \frac{\alpha \left[ (2k+3) - (b^l)^{-1} \right] - 2(k+1) \left[ (b^l)^{-1} + (b^l)^{-2} \right] c_D^l}{2\eta(k+2)} \right\} \left\{ \frac{2}{k+2} \frac{(b^l \theta)^{\frac{1}{k+2}}}{(\mu^l)^{\frac{k-3}{k+2}}} \right\} \end{aligned} \tag{28}$$

$$\begin{aligned} &= \left\{ \frac{\alpha^2 \left[ (2k+3) - \beta^{-1} \right] - 2^{\frac{k+3}{k+2}} \alpha(k+1) \left( \beta^{-\frac{k-1}{k+2}} + \beta^{-\frac{2k-3}{k+2}} \right) \theta^{\frac{1}{k+2}}}{2\eta(k+2)} \right\} \left\{ \frac{2}{k+2} \frac{1}{(\beta \theta)^{\frac{1}{k+2}}} \right\}, \end{aligned} \tag{29}$$

where the first and second terms on the right-hand side are the absolute values of  $\partial v^l / \partial c_D^l$  and  $dc_D^l / d\mu^l$ , respectively.

Moreover,

$$\frac{\partial v^l}{\partial b^l} = \frac{\alpha^2 - \frac{1}{k+2} \left\{ \alpha c_D^l (b^l)^{-2} + (k+1) (c_D^l)^2 \left[ 2(b^l)^{-3} + (b^l)^{-2} \right] \right\}}{2\eta} \tag{30}$$

$$= \frac{\alpha^2 - \frac{1}{\kappa+2} \left[ 2^{\frac{1}{\kappa+2}} \alpha \beta^{-\frac{2\kappa+3}{\kappa+2}} \theta^{\frac{1}{\kappa+2}} + 2^{\frac{2}{\kappa+2}} (k+1) \left( 2\beta^{-\frac{2\kappa+4}{\kappa+2}} + \beta^{-\frac{2\kappa+2}{\kappa+2}} \right) \theta^{\frac{2}{\kappa+2}} \right]}{2\eta} \quad (31)$$

It is clear that the larger  $\theta$  is, the smaller the negative labor force effect and the more likely that agglomeration will be the equilibrium outcome. However, the effect of  $\theta$  on the two selection effects is generally ambiguous, as  $|dc_b/d\mu^l|$  is larger with a larger  $\theta$ , whereas  $|\partial v^l/\partial c_b^l|$  is smaller. The non-monotonic effect of  $\theta$  on agglomeration in **Case 2(a)** of **Theorem 2** is driven mainly by its ambiguous effect via the selection channel, as noted in **Section 4.2**. Further examination of **Eq. (29)** shows that when  $\beta$  is small, the profit is less sensitive to the cutoff (i.e.,  $|\partial v^l/\partial c_b^l|$  is smaller) and the magnitude of the change in the cutoff in response to  $\mu^l$  is smaller (i.e.,  $|dc_b^l/d\mu^l|$  is smaller). In other words, when  $\beta$  is sufficiently small, the selection effects become less important, and the dominant direct labor force effect leads to a positive effect of  $\theta$  on agglomeration. When  $\beta$  is large, in contrast, the selection effects are important, and the effect of  $\theta$  on agglomeration becomes non-monotonic. Intuitively, when  $\beta$  is small, complementarity implies that the marginal product of intermediate goods is small, and hence the selection channel is unimportant.

Drawing on the foregoing arguments and the cutoff pinned down by **Eq. (20)**, we are now ready to evaluate the effects of trade liberalization on agglomeration and firm selection. The results are summarized in the following two propositions.

**Proposition 1.** (*Trade and agglomeration*). *For either a sufficiently small or sufficiently large unskilled labor force with an initially low trade cost, trade liberalization induces dispersion.*

**Proposition 2.** (*Trade and firm selection*). *When trade liberalization induces agglomeration, it leads to tougher firm selection and improves average productivity. When trade liberalization induces dispersion, its effects on firm selection and average productivity become ambiguous.*

Trade liberalization is captured in our model by a decrease in  $\tau$ , which means that  $\theta$  also decreases. With a small unskilled labor force (a small  $\beta$ ), a large and dominant direct labor force effect implies that trade liberalization can create dispersion rather than agglomeration. With a large unskilled labor force, trade liberalization can also lead to dispersion if we have a sufficiently small  $\theta$  to begin with. This result is contrary to the general prediction of core–periphery models. Intermediate goods–unskilled labor complementarity with unskilled labor kept immobile is reminiscent of the model proposed by **Helpman (1998)**, which employs immobile land as the dispersion force. However, the trade cost in **Helpman’s model** has an unambiguous effect on agglomeration: as  $\tau$  decreases, dispersion becomes more likely. Our model, in contrast, features different comparative statics and is dependent on the way in which complementarity kicks in. Another closely related model of agglomeration is that proposed by **Behrens and Robert-Nicoud (2011)**, which likewise adopts the selection structure of **Melitz and Ottaviano (2008)**. However, it does not have the non-monotonic comparative statics on agglomeration that this paper does. In **Behrens and Robert-Nicoud’s paper** a reduction in the trade cost always encourages agglomeration. Moreover, our prediction differs from the findings of **Ottaviano et al. (2002)** and **Peng et al. (2006)**, in which firms are homogeneous, and hence there is no selection effect. The former paper employs a quadratic utility function, where a reduction in the final goods trade cost always encourages agglomeration. The latter uses a quadratic final goods production function, where trade liberalization effected by lowering intermediate goods trade cost discourages agglomeration which resembles **Case 1** in this paper with a sufficiently small unskilled labor force, and hence less influential selection effects.

From cutoff **expression (20)**, it is clear that the direct effect of trade liberalization is to lower the marginal-cost cutoff, thus resulting in a tougher firm selection as in **Melitz and Ottaviano (2008)**. In

contrast with their model, however, we have endogenous entry effects via the intermediate goods–unskilled labor complementarity and final goods producers’ locational choice, which are captured by  $b^l$  and  $\mu^l$ , respectively, in **Eq. (20)**. In the case in which trade liberalization induces agglomeration, both of these endogenous entry effects turn out to be negative, and firm selection becomes tougher, thus reinforcing the direct effect. In the case in which trade liberalization induces dispersion, the two channels of endogenous entry effects ease firm selection. If these channels dominate the direct effect, then trade liberalization can ease firm selection and cause average productivity to fall.

## 6. Numerical analysis

In this section, we examine the agglomeration equilibria in the different cases discussed in the previous section numerically. We show that the set of agglomeration equilibrium configurations in this model is a superset of the canonical models in the literature, e.g., **Krugman (1991)**, **Helpman (1998)**, and **Ottaviano et al. (2002)**.

As noted in **Remark 1**, caution must be exercised in examining the case in which the cutoff hits the upper bound in one of the two countries. Recall that when  $\mu^l$  is close to 0,  $\mu^l/b^l$  goes to infinity. Hence, in the neighborhood of 0,  $c_b^l = c_M$ , and the selection cutoff  $c_b^l$  is determined by **Eq. (23)**. In this case, the selection pressure on country  $h \neq l$  becomes extremely strong because the other location ceases to be a buffer. The absence of selection in country  $l$  brings a non-smooth change to the curve of  $\Delta v$ , the key indicator of the incentive, or lack thereof, to move toward country A. Imagine that  $\mu^B$  is rather small such that the selection cutoff in country B hits the upper bound. The selection pressure then becomes extreme in country A, but this brings out the best in country A’s intermediate goods firms, and the resulting significant productivity gains move the profit difference in favor of country A. However, when  $\mu^B$  moves closer to 0, the super-abundant labor force in country B eventually becomes attractive, which works in country B’s favor. Depending on the parameters, we will occasionally observe huge swings in the  $\Delta v$  curve in this neighborhood, possibly creating additional agglomeration equilibria. Although there may be interesting economics underlying such huge swings, the possibility of them occurring hinges on the fact that there is an upper bound for the cost distribution or, equivalently, a positive lower bound for the productivity distribution. Thus, it does not constitute a general property that would apply to unbounded distributions, such as the log-normal distribution. To rule out these artificial equilibria, we focus on the  $\Delta v$  curve within the range of  $\mu$  such that both countries have interior cutoffs lower than  $c_M$ . More specifically, we show all of the numerical plots of  $\Delta v$  in **Figs. 1 to 8** over  $\mu \in [0.1, 0.9]$ , within which nonrobust artificial equilibria fail to arise.

First look at **Case 1**. Recall that in this case,  $\beta$  is relatively small, and the symmetric equilibrium is unstable when  $\theta$  is sufficiently large, and stable otherwise. By fixing the other parameters,<sup>5</sup> **Fig. 1** shows that the symmetric equilibrium is stable under  $\tau = 1.5$ , and the  $\Delta v$  curve is strictly decreasing, meaning that there is no other equilibrium within the range of  $[0.1, 0.9]$ . In **Fig. 2**, under  $\tau = 10$ , the symmetric equilibrium becomes unstable and  $\Delta v > 0$  at  $\mu = 0.9$  and  $\Delta v < 0$  at  $\mu = 0.1$ , which indicates that both 0.1 and 0.9 are stable agglomeration equilibrium.

We next look at **Case 2(b)**. In **Fig. 3**, the symmetric equilibrium is stable, and no other equilibrium exists, which is similar to **Fig. 1** and consistent with the analytical results.<sup>6</sup> For **Case 2(a)**, with the other parameters fixed,<sup>7</sup> we begin from a rather small  $\gamma$ , with  $\gamma = 0.5$  ( $\Psi(\theta, \beta) < \Lambda(\beta)$ ). In **Fig. 4**, the symmetric equilibrium is stable and is the only equilibrium.

<sup>5</sup> In addition to  $\tau$ , the other parameters are  $\alpha = 10$ ,  $\eta = 1$ ,  $M = 5$ ,  $L = 12$ ,  $\bar{z} = 0.01$  ( $c_M = 100$ ),  $k = 2$ ,  $\gamma = 0.02$ , and  $\kappa = 1$ .

<sup>6</sup> The parameters are  $\alpha = 10$ ,  $\eta = 1$ ,  $M = 5$ ,  $L = 200$ ,  $\bar{z} = 0.01$  ( $c_M = 100$ ),  $k = 2$ ,  $\gamma = 20$ ,  $\kappa = 1$ , and  $\tau = 1.5$ . In this case,  $\beta = 4.38$ .

<sup>7</sup> In addition to  $\gamma$ , the other parameters are  $\alpha = 10$ ,  $\eta = 1$ ,  $M = 5$ ,  $L = 1000$ ,  $\bar{z} = 0.01$  ( $c_M = 100$ ),  $k = 2$ ,  $\tau = 1.5$ , and  $\kappa = 1$ . In this case,  $\beta = 5.99$ .

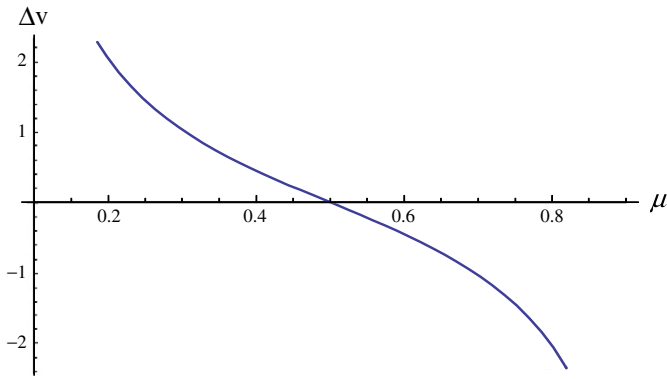


Fig. 1. Case 1 under  $\tau = 1.5$ .

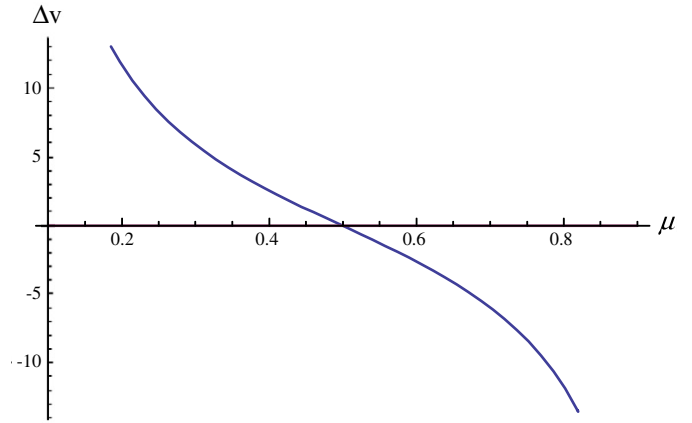


Fig. 3. Case 2(b).

In Fig. 5,  $\gamma$  is increased to 1.5 (still  $\Psi(\theta, \beta) < \Lambda(\beta)$ ), and the symmetric equilibrium remains stable. However, there are two unstable and two stable agglomeration equilibria. In Fig. 6,  $\gamma$  is further increased to 5 ( $\Psi(\theta, \beta) > \Lambda(\beta)$ ), and the symmetric equilibrium becomes unstable, and there are two stable agglomeration equilibria within the range of  $\mu \in (0.1, 0.9)$ . In Fig. 7, where  $\gamma = 25$  (still  $\Psi(\theta, \beta) > \Lambda(\beta)$ ), although the symmetric equilibrium remains unstable, there are also interior agglomeration equilibria. Finally in Fig. 8, where  $\gamma = 30$  (now  $\Psi(\theta, \beta) < \Lambda(\beta)$ ), the symmetric equilibrium becomes stable and is the only equilibrium.

The patterns that appear in Cases 1 and 2(b) confirm our earlier analytical results. These patterns are also seen in Fujita, Krugman and Venables (1999b) and Helpman (1998) when the agglomeration forces are rather weak (Case 1 with a small  $\tau$  and Case 2(b)) or rather strong (Case 1 with a large  $\tau$ ). In Figs. 4 to 8, we observe a transition in the patterns from a small  $\gamma$  to increasingly large  $\gamma$ 's. When  $\gamma$  is either very small or very large (Figs. 4 and 8, respectively), the agglomeration forces are rather weak, and we observe a pattern similar to those in Figs. 1 and 3. Surprisingly, however, for the middle values of  $\gamma$ , the patterns reveal phenomena that cannot be predicted from local stability analysis at  $\mu = 1/2$ . In Fig. 5, although  $\Psi(\theta, \beta) > \Lambda(\beta)$  and the symmetric equilibrium remains stable, agglomeration equilibria show up when the degree of agglomeration is strong. Thus, at a medium range, *agglomeration itself becomes an agglomeration force*. This pattern shows up in Fujita et al. (1999b), but not in Helpman (1998). In Fig. 6 where  $\Psi(\theta, \beta) > \Lambda(\beta)$ , we observe the same pattern as that in Fig. 2. In Fig. 7, however, where we still have  $\Psi(\theta, \beta) > \Lambda(\beta)$ , we observe a pattern opposite to that in Fig. 5. Interestingly, this pattern appears in Helpman (1998), but not in Fujita et al. (1999b).

What causes the difference between Figs. 5 and 7? Case 2(a) is basically an intermediate case in which various scenarios may occur. With a small  $\theta$  and a large  $\beta$ , Theorem 2 indicates that the positive channels of agglomeration cannot be excessively large. Thus, given moderate marginal gains of selection  $|\partial v^l / \partial c_b^l|$ , the symmetric equilibrium is stable at  $\mu = 1/2$ . However, the marginal gains of selection change along with the market size, as it is evident from Eq. (28) that a larger  $\mu^l$  implies tougher selection, which, in turn, enlarges  $|\partial v^l / \partial c_b^l|$  and thus produces the upswing in Fig. 5. With a large  $\theta$ , the selection cutoff is large to begin with. Eq. (30) indicates that the negative impact of tight labor demand due to agglomeration is stronger with a higher selection cutoff  $c_b^l$ . In other words, the lower marginal product of intermediate goods induced by agglomeration (via a lower  $b^l$ ) renders the low productivity of intermediate good inputs (i.e., a high  $\theta$  and  $c_b^l$ ) more detrimental, thus producing the downswing shown in Fig. 7. Although Figs. 5 and 7 have similar counterparts in the extant literature, their underlying economics differ. Moreover, this paper may constitute the first time for both of these patterns to be shown in one model.

7. Concluding remarks

We here construct a model of intermediate goods trade and industrial agglomeration featuring labor–input complementarity and intermediate firm selection. We show that trade liberalization's effect on industrial agglomeration may be non-monotonic. In a highly competitive world market in which intermediate firm selection is important, such as that faced by the newly developed Asian Tigers, will governments always be in favor of liberalizing trade if industrial hollowing-

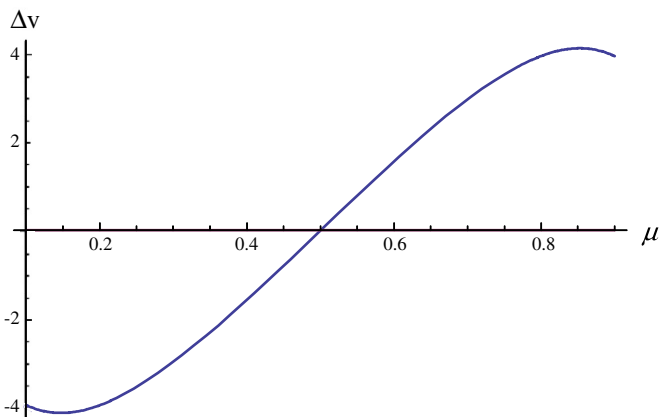


Fig. 2. Case 1 under  $\tau = 10$ .

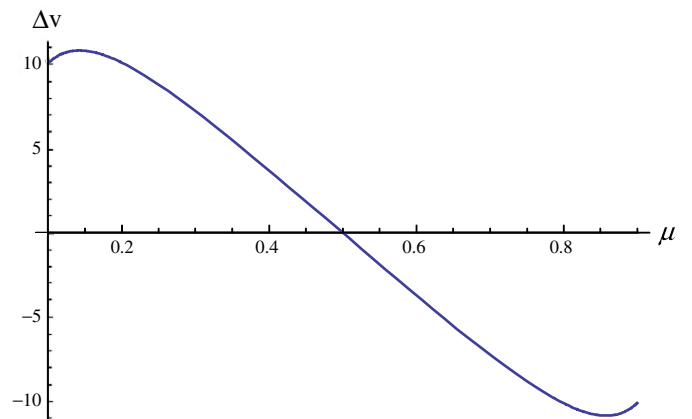


Fig. 4. Case 2(a),  $\gamma = 0.5$ .



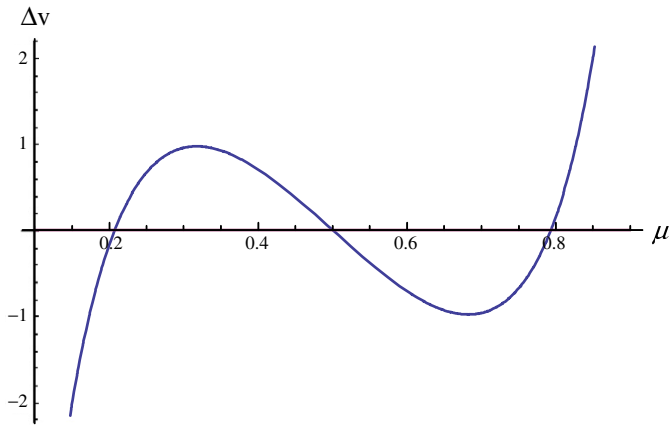


Fig. 5. Case 2(a),  $\gamma = 1:5$ .

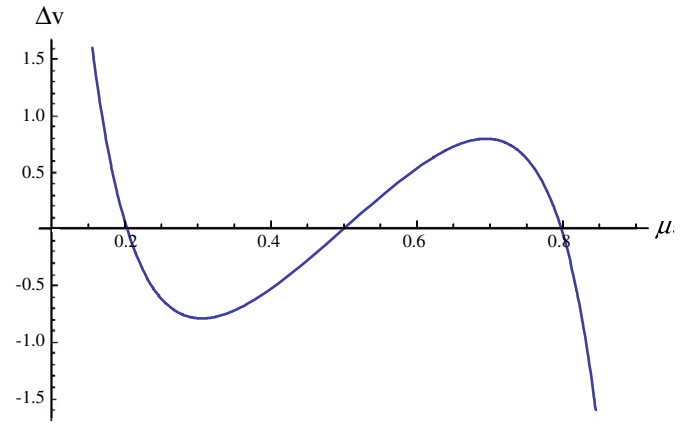


Fig. 7. Case 2(a),  $\gamma = 25$ .

out is a major concern? Notably, all of these economies trade with several much larger economies. Suppose that the trade cost is initially low (as it was in Hong Kong and Taiwan). Then, further trade liberalization can exert a dispersion force that slows down the hollowing-out of domestic final goods firms. Suppose instead that the trade cost is initially moderately high. Then, a strategic slowdown in trade liberalization may be required to prevent hollowing-out.

Along these lines, there are at least two interesting avenues for future work. One would be to calibrate the model presented herein, following the strategies developed by Yi (2003), to two representative economies to explain the different patterns of agglomeration and trade. Another would be to consider firm entry dynamics, following Luttmer (2007). However, the endogenous locational choice of final goods producers in our model means that we must also incorporate migration dynamics, which would lead to additional complexity. Further simplification of the model is thus required before this further analysis can be carried out. Although these extensions are potentially valuable, they are beyond the scope of this work.

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**Appendix A. Derivation of intermediate goods aggregator under autarky**

$$\begin{aligned}
 D_1^l &= \alpha \int_0^{N^l} q_1^l(j) dj - \frac{\gamma}{2} \int_0^{N^l} [q_1^l(j)]^2 dj - \frac{\eta}{2} \left( \int_0^{N^l} q_1^l(j) dj \right)^2 \\
 &= \frac{\alpha N^l}{\mu^l M G(c_D^l)} \int_0^{c_D^l} q_D^l(c) dG(c) - \frac{\gamma N^l}{2(\mu^l M)^2 G(c_D^l)} \int_0^{c_D^l} [q_D^l(c)]^2 dG(c) \\
 &\quad - \frac{\eta}{2} \frac{(N^l)^2}{(\mu^l M)^2 [G(c_D^l)]^2} \left[ \int_0^{c_D^l} q_D^l(c) dG(c) \right]^2 \\
 &= \frac{\alpha N^l}{2\gamma b^l G(c_D^l)} \int_0^{c_D^l} (c_D^l - c) dG(c) - \frac{\gamma N^l}{8(\gamma b^l)^2 G(c_D^l)} \int_0^{c_D^l} (c_D^l - c)^2 dG(c) \\
 &\quad - \frac{\eta (N^l)^2}{8(\gamma b^l)^2 [G(c_D^l)]^2} \left[ \int_0^{c_D^l} (c_D^l - c) dG(c) \right]^2 \\
 &= \frac{k\alpha N^l}{2\gamma b^l (c_D^l)^k k(k+1)} - \frac{k\gamma N^l}{8(\gamma b^l)^2 (c_D^l)^k k(k+1)(k+2)} \frac{2(c_D^l)^{k+2}}{c_D^l} \\
 &\quad - \frac{k^2 \eta (N^l)^2}{8(\gamma b^l)^2 (c_D^l)^{2k} k^2 (k+1)^2} \\
 &= \frac{N^l c_D^l}{2\gamma b^l (k+1)} \left[ \alpha - \frac{c_D^l}{2b^l(k+2)} - \frac{\eta N^l c_D^l}{4\gamma b^l (k+1)} \right] = \frac{\alpha b^l - c_D^l}{\eta b^l} \left[ \frac{\alpha}{2} + \frac{(k+1)c_D^l}{2b^l(k+2)} \right].
 \end{aligned}$$

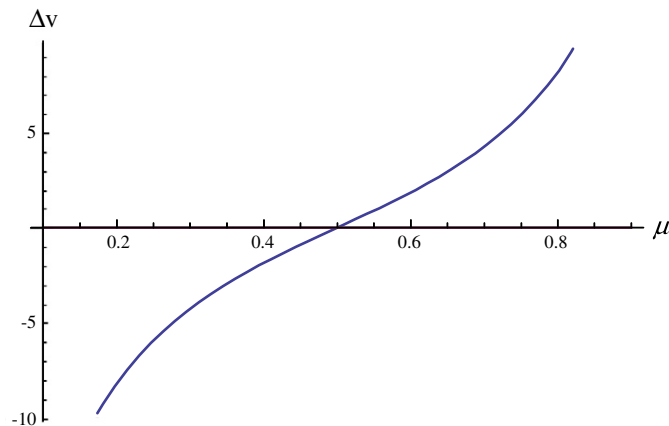


Fig. 6. Case 2(a),  $\gamma = 5$ .

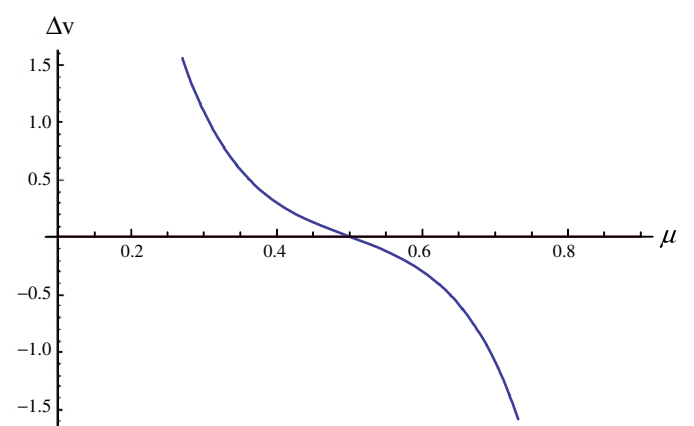


Fig. 8. Case 2(a),  $\gamma = 30$ .

**Appendix B. Derivation of skilled worker value**

$$\begin{aligned}
 v^l &= b^l D_1^l - \int_{\Omega} p^l(j) q_1^l(j) dj - w^l \ell_1^l = (b^l - 1) D_1^l - \int_{\Omega} p^l(j) q_1^l(j) dj = \frac{(b^l - 1)(\alpha b^l - c_D^l)}{\eta b^l} \left[ \frac{\alpha}{2} + \frac{(k+1)c_D^l}{2b^l(k+2)} \right] - \frac{N^l}{4\gamma b^l G(c_D^l)} \int_0^{c_D^l} [(c_D^l)^2 - c^2] dG(c) \\
 &= \frac{(b^l - 1)(\alpha b^l - c_D^l)}{\eta b^l} \left[ \frac{\alpha}{2} + \frac{(k+1)c_D^l}{2b^l(k+2)} \right] - \frac{(k+1)(\alpha b^l - c_D^l)c_D^l}{\eta b^l(k+2)} = \frac{\alpha b^l - c_D^l}{2\eta b^l} \left[ \alpha(b^l - 1) - \frac{(k+1)(b^l + 1)c_D^l}{(k+2)b^l} \right],
 \end{aligned}$$

which is Eq. (15). Upon substituting in the cutoff expression (20), we have Eq. (21). Further manipulation gives us

$$\begin{aligned}
 v^l &= \frac{\alpha - (b^l)^{-\frac{k+1}{k+2}} (\theta/\mu^l)^{\frac{1}{k+2}}}{2\eta} \left[ \alpha(b^l - 1) - \frac{(k+1)(b^l + 1)(b^l)^{-\frac{k+1}{k+2}} (\theta/\mu^l)^{\frac{1}{k+2}}}{k+2} \right] \\
 &= \frac{\alpha^2(b^l - 1) - \frac{(2k+3)b^l - 1}{k+2} \alpha \theta^{\frac{1}{k+2}} (b^l)^{-\frac{k+1}{k+2}} (\mu^l)^{-\frac{1}{k+2}} + \frac{(k+1)\theta^{\frac{2}{k+2}} (b^l + 1)(b^l)^{-\frac{2(k+1)}{k+2}} (\mu^l)^{-\frac{2}{k+2}}}{k+2}}{2\eta},
 \end{aligned}$$

which is used to derive the results in Section 5.

**Appendix C. Derivation of free entry condition**

$$\begin{aligned}
 \kappa &= G(c_D^l) E(\pi_D^l(c) | c < c_D^l) + G(c_X^l) E(\pi_X^l(c) | c < c_X^l) = \int_0^{c_D^l} \pi_D^l(c) dG^l(c) + \int_0^{c_X^l} \pi_X^l(c) dG^l(c) = \frac{M}{4\gamma} \left[ \frac{\mu^l}{b^l} \int_0^{c_D^l} (c_D^l - c)^2 dG^l(c) + \frac{\tau^2 \mu^h}{b^h} \int_0^{c_X^l} (c_X^l - c)^2 dG^l(c) \right] \\
 &= \frac{M}{2\gamma(k+1)(k+2)(c_M)^k} \left[ \frac{\mu^l}{b^l} (c_D^l)^{k+2} + \frac{\tau^{-k} \mu^h}{b^h} (c_D^h)^{k+2} \right].
 \end{aligned}$$

**Appendix D. Proof of Theorem 1**

The first part of the theorem can be easily shown by guess-and-verify. By symmetry,  $\mu^l = \mu^h = \frac{1}{2}$ . Thus, both the locational choice and locational equilibrium conditions are met. From Eq. (20), we know that the cutoff  $c_D^l$  must be the same across countries. It is also clear that, with  $\mu^l = \mu^h = 1/2$ , there must exist a sufficiently large  $c_M$  such that the cutoffs are interior. To show the second part, we apply standard continuity arguments. More specifically,  $d\Delta v/d\mu > 0$  at  $\mu = 1/2$  implies that at least some portion of the  $\Delta v$  curve on  $[1/2, 1]$  is positive. If this portion extends to the entire domain  $[1/2, 1)$ , then  $\Delta v \geq 0$  at  $\mu = 1$ , and we get a full agglomeration equilibrium, which will be stable if  $\Delta v > 0$ . If this portion does not extend to the entire domain, then continuity implies that  $\Delta v = 0$  at least once at an interior  $\mu$ , the smallest of which must be a stable agglomeration equilibrium. The only case in which there is no stable agglomeration equilibrium is that in which the positive portion of the  $\Delta v$  curve extends to the entire domain  $[1/2, 1)$ ,  $\Delta v = 0$ , and  $d\Delta v/d\mu < 0$  at  $\mu = 1$ . However, continuity implies that the subset of the parameter space necessary for this scenario to occur is of zero measure relative to the entire parameter space. ■

**Appendix E. Proof of Lemma 1**

Taking the derivative of  $v^l$  with respect to  $\mu^l$  and using Eq. (21), we obtain

$$\begin{aligned}
 \frac{dv^l}{d\mu^l} &= \frac{\frac{d}{d\mu^l} \alpha^2(b^l - 1) - \frac{d}{d\mu^l} \frac{(2k+3)b^l - 1}{k+2} \alpha \theta^{\frac{1}{k+2}} (b^l)^{-\frac{k+1}{k+2}} (\mu^l)^{-\frac{1}{k+2}} + \frac{d}{d\mu^l} \frac{(k+1)\theta^{\frac{2}{k+2}} (b^l + 1)(b^l)^{-\frac{2(k+1)}{k+2}} (\mu^l)^{-\frac{2}{k+2}}}{(k+2)}}{2\eta} \\
 &= \frac{\alpha^2 \frac{db^l}{d\mu^l} - \frac{\alpha \theta^{\frac{1}{k+2}}}{k+2} d \left\{ \left[ (2k+3)b^l - 1 \right] (b^l)^{-\frac{k+1}{k+2}} (\mu^l)^{-\frac{1}{k+2}} \right\}}{2\eta} + \frac{(k+1)\theta^{\frac{2}{k+2}} d \left\{ \left[ (b^l)^{-\frac{k}{k+2}} + (b^l)^{-\frac{2(k+1)}{k+2}} \right] (\mu^l)^{-\frac{2}{k+2}} \right\}}{2\eta} \\
 &= \frac{-\alpha^2 (\mu^l)^{-1} + \frac{\alpha \theta^{\frac{1}{k+2}}}{(k+2)^2} (\mu^l)^{-\frac{k+3}{k+2}} \left[ (2k+3)(b^l)^{\frac{1}{k+2}} + (2k+2)(b^l)^{-\frac{k+1}{k+2}} + (k+1)(b^l)^{-\frac{2k+3}{k+2}} \right]}{2\eta} \\
 &\quad + \frac{\frac{(k+1)\theta^{\frac{2}{k+2}}}{(k+2)^2} (\mu^l)^{-\frac{k+4}{k+2}} \left[ -2(b^l)^{-\frac{k}{k+2}} + (k-2)(b^l)^{-\frac{2k+2}{k+2}} + 2(k+1)(b^l)^{-\frac{3k+4}{k+2}} \right]}{2\eta}.
 \end{aligned}$$

Evaluating it at  $\mu = 1/2$ , we obtain

$$\begin{aligned} \frac{d\Delta v}{d\mu} \Big|_{\mu=1/2} &= \frac{dv^A}{d\mu^A} \Big|_{\mu^A=1/2} + \frac{dv^B}{d\mu^B} \Big|_{\mu^B=1/2} = 2 \frac{dv^A}{d\mu^A} \Big|_{\mu^A=1/2} \\ &= \frac{-2\alpha^2 (\mu^A)^{-1} + \frac{2\alpha\theta^{\frac{1}{k+2}}}{(k+2)^2} (\mu^A)^{\frac{k+3}{k+2}} \left[ (2k+3)(b^A)^{\frac{1}{k+2}} + (2k+2)(b^A)^{\frac{-k+1}{k+2}} + (k+1)(b^A)^{\frac{2k+3}{k+2}} \right]}{2\eta} \\ &\quad + \frac{\frac{2(k+1)\theta^{\frac{2}{k+2}}}{(k+2)^2} (\mu^A)^{\frac{-k+4}{k+2}} \left[ -2(b^A)^{\frac{-k}{k+2}} + (k-2)(b^A)^{\frac{2k+2}{k+2}} + 2(k+1)(b^A)^{\frac{-3k+4}{k+2}} \right]}{2\eta} \\ &= \frac{-4\alpha^2 + \frac{2k+5}{(k+2)^2} \alpha\theta^{\frac{1}{k+2}} \left[ (2k+3)\beta^{\frac{1}{k+2}} + (2k+2)\beta^{\frac{-k+1}{k+2}} + (k+1)\beta^{\frac{-2k+3}{k+2}} \right]}{2\eta} \\ &\quad + \frac{\frac{2k+6}{(k+2)^2} (k+1)\theta^{\frac{2}{k+2}} \left[ -2\beta^{\frac{-k}{k+2}} + (k-2)\beta^{\frac{2k+2}{k+2}} + 2(k+1)\beta^{\frac{-3k+4}{k+2}} \right]}{2\eta}, \end{aligned}$$

which is positive if and only if  $\Psi(\theta, \beta) > \Lambda(\beta)$ . ■

**Appendix F. Proof of Lemma 2**

If  $\Phi(\ln(2)) < 0$ , then the inequalities are trivial based on the observation that  $\tilde{\beta}$  is the positive root of the quadratic equation  $\Phi(\beta) = 0$ . Note that because  $k \geq 1$ ,  $\Phi(\ln(2)) = 2[\ln(2)]^2 - k[\ln(2) + 2] + 2\ln(2) - 2 \leq 2[\ln(2)]^2 - [\ln(2) + 2] + 2\ln(2) - 2 < 0$ . ■

**Appendix G. Proof of Lemma 3**

Straightforward differentiation yields

$$\frac{\partial \Psi(\theta, \beta)}{\partial \theta} = \theta^{-\frac{k+1}{k+2}} \left[ (k+2)\alpha\theta^{-\frac{2}{k+2}} - 2\frac{2k+1}{k+2}\alpha^{-1}\beta^{-\frac{3k+4}{k+2}}\Phi(\beta) \right],$$

which is positive if  $\Phi(\beta) < 0$ . When  $\Phi(\beta) > 0$ ,  $\Psi(\theta)$  first increases from  $-\infty$  to a peak and then decreases to  $-\infty$ , and so  $\frac{\partial \Psi(\theta, \beta)}{\partial \theta}$  is positive for small  $\theta$  values and negative for large  $\theta$  values. Solving  $\frac{\partial \Psi(\theta, \beta)}{\partial \theta} = 0$  yields the root  $\tilde{\theta}(\beta)$ . The result follows immediately by utilizing Lemma 2. ■

**Appendix H. Proof of Lemma 4**

Plugging  $\tilde{\theta}(\beta)$  into  $\Psi(\theta, \beta)$ , we obtain

$$\begin{aligned} \Psi(\tilde{\theta}(\beta), \beta) &= -\alpha(k+2)^2 (\tilde{\theta}(\beta))^{-\frac{1}{k+2}} - 2\frac{2}{k+2}(k+1)\alpha^{-1} (\tilde{\theta}(\beta))^{\frac{1}{k+2}} \beta^{-\frac{3k+4}{k+2}} \Phi(\beta) \\ &= -\alpha(k+2)^2 \left[ \beta^{-\frac{3k+4}{k+2}} \Phi(\beta) \right]^{-1/2} \frac{1}{2^{\frac{1}{k+2}} \alpha(k+2)} - 2\frac{2}{k+2}(k+1)\alpha^{-1} \left[ \beta^{-\frac{3k+4}{k+2}} \Phi(\beta) \right]^{-1/2} \frac{1}{2^{-\frac{1}{k+2}} \alpha(k+2)} \beta^{-\frac{3k+4}{k+2}} \Phi(\beta) \\ &= -2^{\frac{k+3}{k+2}} (k+2)(k+1)^{1/2} \left[ \beta^{-\frac{3k+4}{k+2}} \Phi(\beta) \right]^{-1/2} < 0. \end{aligned}$$

Straightforward manipulation gives us  $\Psi(\tilde{\theta}(\beta), \beta) > \Lambda$  if and only if  $\Omega(\beta) > 0$ . ■

**Appendix I. Proof of Theorem 2**

For Case 1, if  $\beta < \tilde{\beta}$ , then by Lemma 3,  $\frac{\partial \Psi(\theta, \beta)}{\partial \theta} > 0$  and, for sufficiently large  $\theta$  values,  $\Psi(\theta, \beta) > \Lambda(\beta)$ . Lemma 1 implies that in this case the symmetric world equilibrium must be unstable. By Theorem 1, there must exist an agglomeration world equilibrium that almost surely is stable. For Case 2(a), with  $\Omega(\beta) > 0$ , Lemma 4 implies that  $\Psi(\tilde{\theta}(\beta), \beta) > \Lambda(\beta)$ . Thus, by continuity, there must be a neighborhood  $I_{\tilde{\theta}(\beta)}$  of  $\tilde{\theta}(\beta)$  such that for all  $\theta \in I_{\tilde{\theta}(\beta)}$ ,  $\Psi(\theta, \beta) > \Lambda(\beta)$ . Similar arguments suggest that there must exist a stable agglomeration world equilibrium. However, with  $\Omega(\beta) < 0$  as in Case 2(b),  $\Psi(\tilde{\theta}(\beta), \beta) < \Lambda(\beta)$ . Thus, for all  $\theta$ ,  $\Psi(\theta, \beta) < \Lambda(\beta)$ , which implies that the symmetric world equilibrium is always stable. ■

### Appendix J. Derivation of decomposed channels

$$\begin{aligned} \left| \frac{\partial v^l}{\partial c_D^l} \right| &= -\frac{\partial}{\partial c_D^l} \left( \frac{\alpha b^l - c_D^l}{2\eta b^l} \right) \left[ \alpha (b^l - 1) - \frac{(k+1)(b^l+1)c_D^l}{(k+2)b^l} \right] - \frac{\alpha b^l - c_D^l}{2\eta b^l} \frac{\partial}{\partial c_D^l} \left[ \alpha (b^l - 1) - \frac{(k+1)(b^l+1)c_D^l}{(k+2)b^l} \right] \\ &= \frac{1}{2\eta} \left[ \frac{\alpha \left[ (2k+3) - (b^l)^{-1} \right]}{k+2} - \frac{2(k+1) \left[ (b^l)^{-1} + (b^l)^{-2} \right] c_D^l}{k+2} \right] \\ &= \frac{1}{2\eta} \left[ \frac{\alpha \left[ (2k+3) - (b^l)^{-1} \right]}{k+2} - \frac{2(k+1) \left[ (b^l)^{-\frac{k+1}{k+2}} + (b^l)^{-\frac{2k+3}{k+2}} \right]}{k+2} \left( \frac{\theta}{\mu^l} \right)^{\frac{1}{k+2}} \right], \end{aligned}$$

which can be evaluated at  $\mu^l = 1/2$  to yield

$$\left| \frac{\partial v^l}{\partial c_D^l} \right| = \frac{1}{2\eta(k+2)} \left[ \alpha \left[ (2k+3) - \beta^{-1} \right] - 2^{\frac{k+3}{k+2}} (k+1) \left( \beta^{-\frac{k+1}{k+2}} + \beta^{-\frac{2k+3}{k+2}} \right) \theta^{\frac{1}{k+2}} \right].$$

Moreover, at  $\mu^l = 1/2$ ,

$$\left| \frac{dc_D^l}{d\mu^l} \right| = \frac{2}{k+2} \frac{(b^l \theta)^{\frac{1}{k+2}}}{(\mu^l)^{\frac{k+3}{k+2}}} = \frac{2^{-\frac{1}{k+2}}}{k+2} (\beta \theta)^{\frac{1}{k+2}},$$

which can be substituted into the previous expression to obtain Eq. (29). Furthermore, at  $\mu^l = 1/2$ ,

$$\begin{aligned} \frac{\partial v^l}{\partial b^l} &= \frac{\partial}{\partial b^l} \left( \frac{\alpha b^l - c_D^l}{2\eta b^l} \right) \left[ \alpha (b^l - 1) - \frac{(k+1)(b^l+1)c_D^l}{(k+2)b^l} \right] + \frac{\alpha b^l - c_D^l}{2\eta b^l} \frac{\partial}{\partial b^l} \left[ \alpha (b^l - 1) - \frac{(k+1)(b^l+1)c_D^l}{(k+2)b^l} \right] \\ &= \frac{c_D^l (b^l)^{-2}}{2\eta} \left[ \alpha (b^l - 1) - \frac{(k+1)(1 + (b^l)^{-1})c_D^l}{k+2} \right] + \frac{\alpha - c_D^l (b^l)^{-1}}{2\eta} \left[ \alpha + \frac{(k+1)c_D^l (b^l)^{-2}}{k+2} \right] \\ &= \frac{1}{2\eta} \left\{ \alpha^2 - \frac{\alpha c_D^l (b^l)^{-2} + (k+1)(c_D^l)^2 \left[ 2(b^l)^{-3} + (b^l)^{-2} \right]}{k+2} \right\}, \end{aligned}$$

thus leading to Eq. (31). ■

### References

- Baldwin, R.E., Okubo, T., 2006. Heterogeneous firms, agglomeration and economic geography: spatial selection and sorting. *Journal of Economic Geography* 6, 323–346.
- Behrens, K., Robert-Nicoud, F., 2011. Survival of the Fittest in Cities: Urbanization, Agglomeration, and Inequality, Working Paper. University of Montreal.
- Behrens, K., Duranton, G., Robert-Nicoud, F., 2010. Productive Cities: Sorting, Selection, and Agglomeration, Working Paper. University of Toronto.
- Benabou, R., 1996. Equity and efficiency in human capital investment: the local connection. *Review of Economic Studies* 63, 237–264.
- Bernard, A.B., Eaton, J., Jensen, J.B., Kortum, S., 2003. Plants and productivity in international trade. *American Economic Review* 93, 1268–1290.
- Combes, P.P., Duranton, G., Gobillon, L., Puga, D., Roux, S., 2009. The Productivity Advantages of Large Cities: Distinguishing Agglomeration from Firm Selection, Working Paper. University of Toronto.
- Ethier, W.J., 1982. National and international returns to scale in the modern theory of international trade. *American Economic Review* 72, 389–405.
- Fujita, M., Krugman, P., 1995. When is the economy monocentric: von Thünen and Chamberline unified. *Regional Science and Urban Economics* 25, 505–528.
- Fujita, M., Krugman, P., Mori, T., 1999a. On the evolution of hierarchical urban systems. *European Economic Review* 43, 209–251.
- Fujita, M., Krugman, P., Venables, A.J., 1999b. *The Spatial Economy: Cities, Regions, and International Trade*. MIT Press, Cambridge, MA.
- Helpman, E., 1998. The size of regions. In: Pines, D., Sadka, E., Zilcha, Y. (Eds.), *Topics in Public Economics*. Cambridge University Press, New York, pp. 33–54.
- Hummels, D., Ishii, J., Yi, K., 2001. The nature and growth of vertical specialization in world trade. *Journal of International Economics* 54, 75–96.
- Jones, R.W., 2000. *Globalization and the Theory of Input Trade*. MIT Press, Cambridge MA.
- Krugman, P., 1991. Increasing returns and economic geography. *Journal of Political Economy* 99, 483–499.
- Krugman, P., Venables, A.J., 1995. Globalization and inequality of nations. *Quarterly Journal of Economics* 110, 857–880.
- Luttmer, E., 2007. Selection, growth and the size distribution of firms. *Quarterly Journal of Economics* 122, 1103–1144.
- Melitz, M.J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71, 1695–1725.
- Melitz, M.J., Ottaviano, G., 2008. Market size, trade, and productivity. *Review of Economic Studies* 75, 295–316.
- Ottaviano, G., Tabuchi, T., Thisse, J.F., 2002. Agglomeration and trade revisited. *International Economic Review* 43, 409–435.
- Peng, S.K., Thisse, J.F., Wang, P., 2006. Economic integration and agglomeration in a middle product economy. *Journal of Economic Theory* 131, 1–25.
- Redding, S., Sturm, D., 2008. The costs of remoteness: evidence from German division and reunification. *American Economic Review* 98, 1766–1797.
- Romer, P., 1990. Endogenous technology change. *Journal of Political Economy* 98, 71–102.
- Yi, K., 2003. Can vertical specialization explain the growth of world trade? *Journal of Political Economy* 111, 52–102.