# Money, Finance and Growth

**Ping Wang©**
**Department of Economics**
**Washington University in St. Louis**

**April 2017**

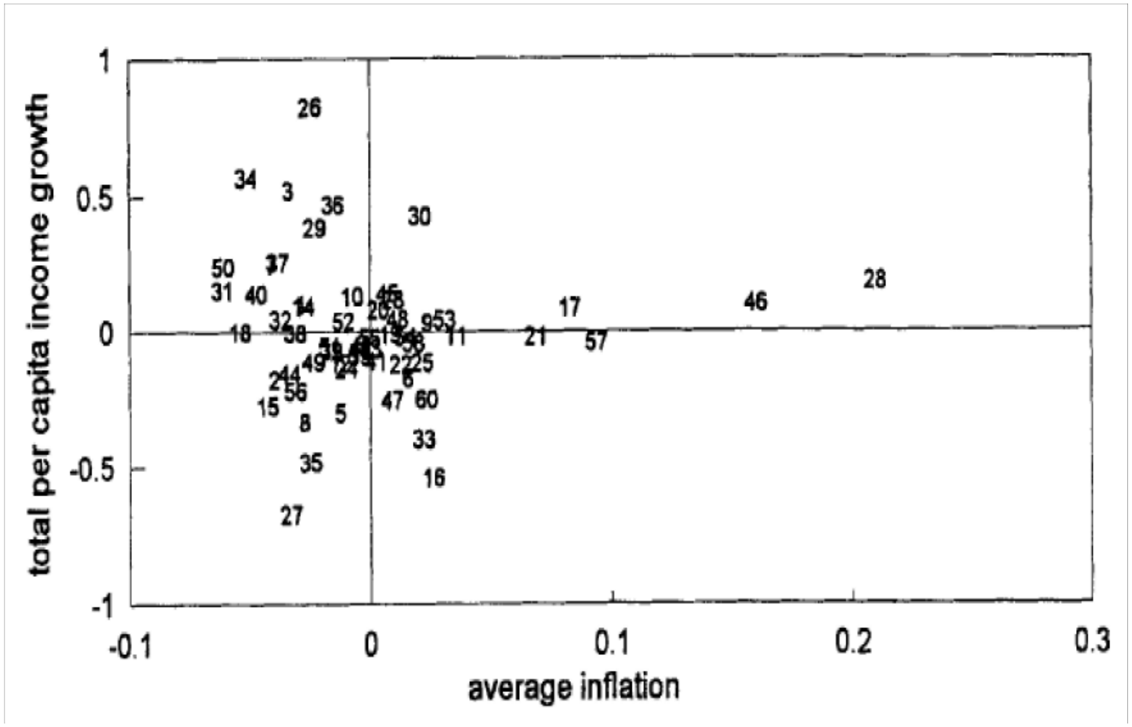
## A. Introduction

## I. Empirical Regularities

### 1. Money and growth (Friedman-Schwartz; Walsh, ch. 1):

**a. positive correlation in levels**
**b. largely negative correlation in growth rates**
**c. presence of a liquidity effect**

**Summer-Heston data (1960-85, excluding poor quality data, 60 countries):**




**Note:** (i) **High inflation countries: 28 = Indonesia, 46 = Iceland, 57 = Turkey, 17 = Mexico, 21 = Columbia;**

(ii) **High growth countries: 26 = Hong Kong, 34 = Singapore, 3 = Morocco, 30 = South Korea, 29 = Japan.**

**2. Finance and growth (Becsi-Wang 1997; Levine 1997):**

- **a. positive correlation in levels (Goldsmith 1969, McKinnon 1973)**
- **b. mixed relation in growth rates:**
    - **i. zero correlation for OECD (Fernandez-Galetovic 1994)**
    - **ii. weakly negative correlation for Latin America (DeGregorio-Guidotti 1995)**
    - **iii. strongly positive correlation for Asia (King-Levine 1993)**

| Financial Deepening | Per Capita Income | | |
|---|---|---|---|
| | High | Middle | Low |
| High | US, France, Italy, Switzerland | Chile, Venezuela | Kenya, Jamaica Honduras |
| Middle | Norway, Germany, Denmark | Malaysia Trinidad & Tobago | Liberia, Uganda |
| Low | | Ireland, Hungary, Yugoslavia | Sri Lanka, Philippines Zimbabwe, Indonesia |

**Notes:**

(i) Per capita income is measured by 1985 real GNP per capita in US$ at 1980 constant prices where high income takes values of $7,500 or above, middle income from $3,000 to $6,000, and low income up to $2,000.

(ii) Financial deepening is measured by the financial intermediation ratio (FIR) defined as in Goldsmith (the ratio of M to GNP) where high deepening takes values of 13% or above, middle deepening from 8% to 12%, and low deepening up to 7%.

(iii) Gaps are allowed to ensure more definitive classification.

## II. Key Literature

- **Use of money:**
  - **money in the utility function (direct value, wealth or transactions time reduction): Samuelson (1947), Patinkin (1965), Sidrauski (1967), Brock (1974), *Wang-Yip (1992a)*, Wang-Yip (1992b)**
  - **cash in advance: Tsiang (1966), Clower (1967), Lucas (1980), Stockman (1981), Lucas-Stokey (1987), Cooley-Hansen (1989), *Wang-Yip (1992a)*, Gomme (1993), Ireland (1994), *Jones-Manuelli (1995)*, Chang-Chang-Tsai-Wang (2017)**
  - **transactions cost: Saving (1973), Drazen (1979), Grossman-Weiss (1983), Rottemberg (1984), , *Wang-Yip (1992a)*, Jha-Wang-Yip (2002)**
  - **medium of intergenerational transactions: Samuelson (1958), Wallace (1980), McCullum (1983), Wang (1993), Van der Ploeg-Alogoskoufis (1994)**
  - **liquidity service: Feenstra (1986), Chang-Chang-Lai-Wang (2008)**

- **money and search: Wicksell (1898), Jones (1976), Wang (1987), Kiyotaki-Wright (1989, 1993), Trejos-Wrigth (1995), Lagos-Wright (2002), Laing-Li-Wang (2007, 2013)**

- **Major Roles of Financial Intermediation:**
  - **liquidity management: Diamond-Dybvig (1993), *Bencinvinga-Smith (1991)***
  - **risk pooling: Townsend (1978), Greenwood-Jovanovic (1990), Bencivenga-Smith (1993)**
  - **productive loan services: *Tssidon (1992)*, *Aghion-Bolton (1997)***
  - **effective monitoring: Williamson (1986), Greenwood-Jovanovic (1990), and the literature on:**
    - **borrowing/collateral/pledgeability constraints**
    - **venture capitalism**
    - **micro finance**
  - **funds pooling: Besley (1994), Becsi-Wang-Wynne (1999), and the literature on micro finance**

## B. Money in Dynamic General Equilibrium: Wang-Yip (1992)

- **Provide a unified framework to study 3 main dynamic general equilibrium models of money**

- **Main issue: is money superneutral?**

    - **Tobin (1965): via asset substitution, higher money growth reduces real balances but encourages capital accumulation and output growth (Tobin effect)**
    - **Sidrauski (1967): even if money is valued directly, money growth has no effect on steady-state output**
    - **Stockman (1981): higher money growth reduces real balances, limits capital investment, and lowers output growth (reversed Tobin effect)**

## a. Money in the Utility Function

$$\max \int_0^{\infty} W(c(t), \ell(t), m(t)) e^{-\rho t} dt$$

$$\text{s.t. } c(t) + \dot{k}(t) + \dot{m}(t) = f(k(t), \ell(t)) - nk(t) - (\pi(t) + n)m(t) + \tau(t)$$

- **FOCs and S-S BC:**

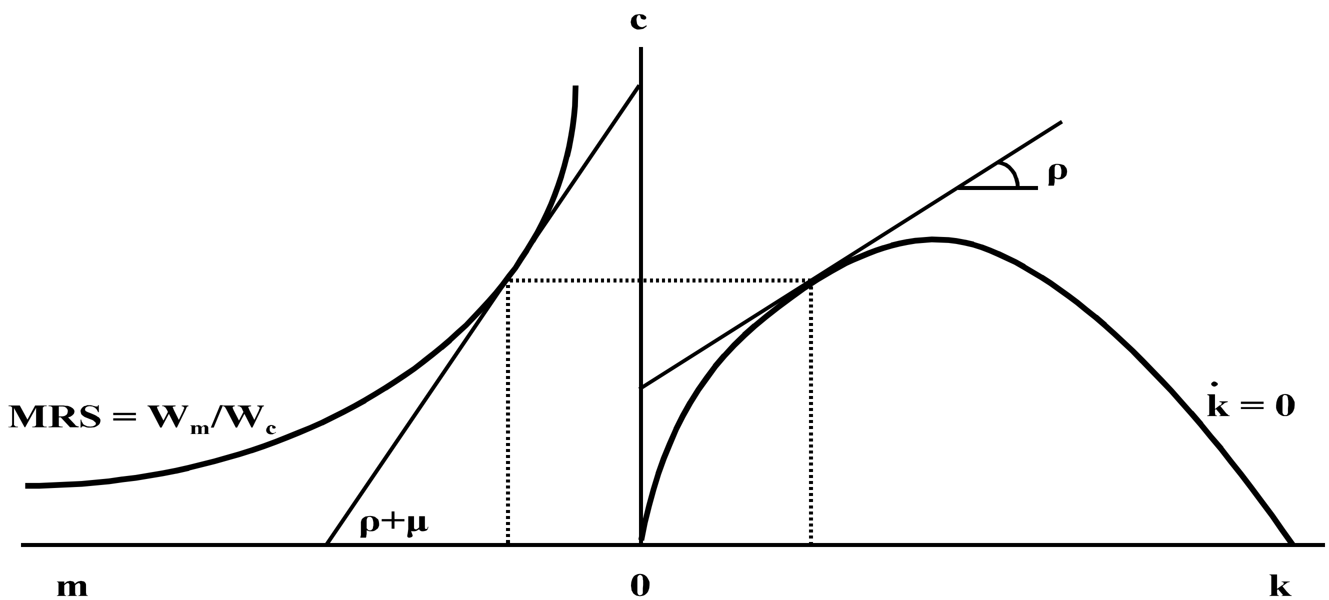
$$f_k(k, \ell) = \rho + n$$

$$W_c(c, \ell, m) f_\ell(k, \ell) = -W_\ell(c, \ell, m)$$

$$W_m(c, \ell, m) = (\rho + \mu) W_c(c, \ell, m) .$$

$$c = f(k, \ell) - nk.$$

- **Equilibrium**




- **Comparative statics**

**higher μ => $W_m/W_c$ increases**
**=> m lowers (if $W_{c\ell} < 0$, i.e., c and x are complem.)**
**=> (W1) $W_{cm} > 0$, $W_{\ell m} > 0$: c, ℓ, k all fall**
**(W2) $W_{cm} < 0$, $W_{\ell m} < 0$: c, ℓ, k all rise**

**b. Cash in Advance**

$$\max \int_0^\infty U(c(t), 1 - \ell(t))e^{-\rho t}dt$$

s.t. $c(t) + \dot{k}(t) + \dot{m}(t) = f(k(t), \ell(t)) - nk(t) - (\pi(t) + n)m(t) + \tau(t)$

$$m(t) \geq c(t) + \Gamma\dot{k}(t)$$

**where $\Gamma = 0$, $U_\ell = 0$ => Lucas; $\Gamma = 1$, $U_\ell = 0$ => Stockman**

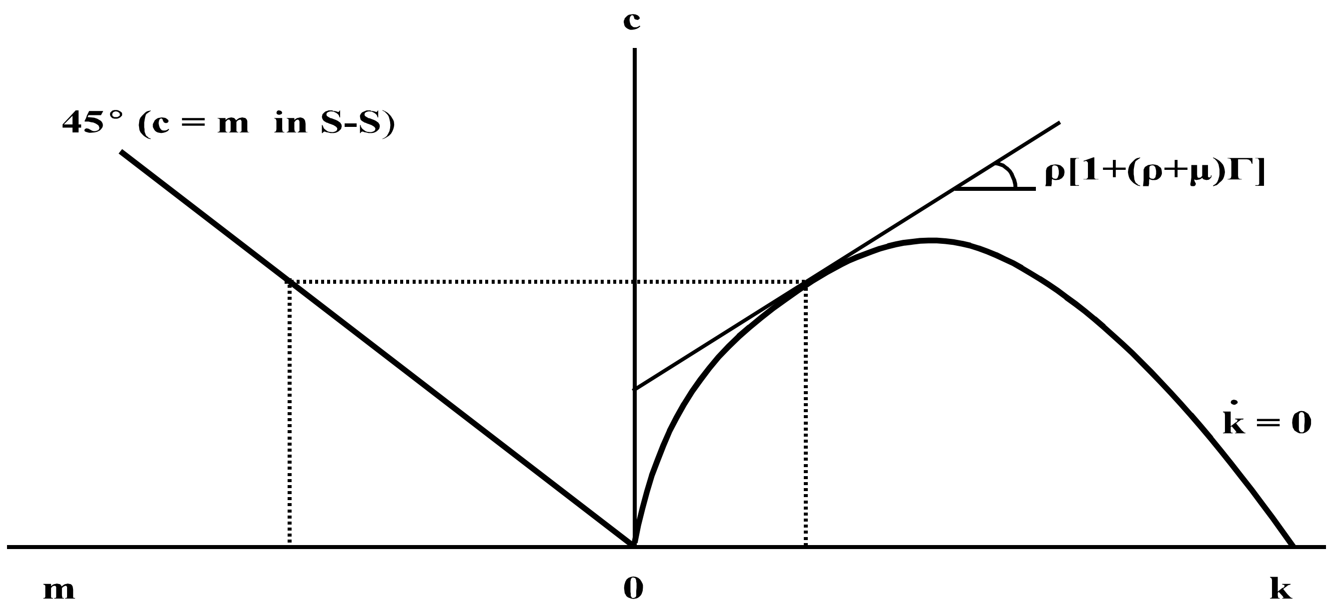
- **FOCs:**

$$f_k = \rho + n + \rho(\rho + \mu)\Gamma$$

$$f_\ell U_c = (1 + \rho + \mu)U_x .$$

**Thus, money growth affects MPK directly**

- **Equilibrium**




- **Comparative statics**

**higher μ => k decreases**
**=> ℓ lowers ($f_{k\ell} > 0$)**
**=> y and c fall**
**=> (1-ℓ) reduces ($U_{cx} > 0$) and so ℓ rises**
**=> net effect on ℓ ambiguous**

## c. Transactions Cost Model

$$\max \int_0^\infty U(c(t), 1 - \ell(t) - T(t))e^{-\rho t}dt$$

**s.t.** $c(t) + \dot{k}(t) + \dot{m}(t) = f(k(t), \ell(t)) - nk(t) - (\pi(t) + n)m(t) + \tau(t)$

**where** $T(t) = T(c(t), m(t))$**, satisfying:**

$T_c > 0, T_m < 0, T_{cc} < 0, T_{mm} > 0, T_{mc} \leq 0$

- **FOCs:**

$$f_k = \rho + n$$

$$f_\ell U_c = (1 + T_c f_\ell)U_x$$

$$-T_m f_\ell = \rho + \mu ,$$

**where money growth affects output via MPL**

- **Comparative statics**

**higher $\mu$ => $k/\ell$ unchanged, as does $f_\ell$**
**=> $(-T_m)$ increases and hence m decreases**
**=> T increases and $\ell$ decreases**
**=> k decreases, as does c**
**=> but the decrease in c lowers T**
**=> net effect on $x = 1- \ell - T$ ambiguous**

**d. Qualitative equivalence between the three models if $W_{c\ell} < 0$, $W_{cm} > 0$, $W_{\ell m} > 0$, $\Gamma = 0$, $T(c,m) = 0$ if $c \leq m$ and $= 1$ otherwise**

**e. Questions: As the three most used "conventional" approaches to money seem to yield qualitatively similar theoretical findings, can "newer" approaches deliver more insights along the lines of money, inflation and growth?**

## C. Money and Endogenous Growth: Jones and Manuelli (1995)

- **Modified Cooley-Hansen (1989) model to permit endogenous growth**

## a. Basic One-Sector Endogenous Growth Model of Money

- **Consumer's optimization:**

$$\max \sum_t \beta^t u(c_{1t}, c_{2t}, 1 - n_t)$$

**s.t.**

$$m_t + b_{t+1} \leq v_t,$$

$$p_{1t} c_{1t} \leq m_t,$$

$$v_{t+1} \leq (v_t - m_t - b_{t+1}) + (m_t - p_{1t} c_{1t}) - p_{2t} c_{2t} - p_{1t} x_t + p_{1t} w_t n_t + p_{1t} r_t k_t + (1 + R_{t+1}) b_{t+1} + T_t,$$

$$k_{t+1} \leq (1 - \delta) k_t + x_t,$$

**where** v = nominal wealth, T = money transfer ($M_{t+1}$ - $M_t$ ),
R = nominal interest rate, w, r = real factor prices
$p_i$ = nominal price of i (1 & 2 are cash & credit good)
$p_i$ = p if both goods are produced

- **Firm's optimization**

  **max** $\pi_t = p_{1t}c_{1t} + p_{2t}c_{2t} + p_{1t}x_t - p_{2t}r_tk_t - p_{1t}w_tn_t$

  **s.t.** $c_{1t} + c_{2t} + x_t \leq F(k_t, n_t)$

- **FOCs**

$$\frac{u_1(t)\,F_2(t)}{u_3(t)} = 1 + R_{t+1},$$

$$\frac{u_1(t)}{u_1(t+1)} = \frac{(1 + R_{t+1})}{(1 + R_{t+2})}\beta\,[1 - \delta + F_1(t+1)]$$

$$\frac{u_1(t)}{u_2(t)} = 1 + R_{t+1},$$

$$1 + R_{t+2} = \frac{p_{t+1}}{p_t}\,[1 - \delta + F_1(t+1)],$$

**and the CIA holds for equality in equilibrium**

- **Asymptotic BGP equilibrium**

**(A1)** $F = Ak + Dk^{\alpha}n^{1-\alpha}$, **with** $\beta(1 - \delta + A) > 1$

**(A2)** $u = [(c_1^{-\lambda} + \eta c_2^{-\lambda})^{-1/\lambda}(1 - n)^{\psi}]^{1-\sigma}/(1 - \sigma)$, **with** $\sigma > 1$, $\lambda \geq -1$

$$\gamma^{\sigma} = \beta[1 - \delta + A]$$

$$\pi\gamma = \mu$$

$$1 + R = \pi[1 - \delta + A]$$

$$c_2/c_1 = [\eta(1 + R)]^{1/(1+\lambda)}$$

**where** $\mu = M_{t+1}/M_t$, $\pi = p_{t+1}/p_t$

**Thus,** $r = A - \delta$ **and** $\gamma = [\beta(1 + r)]^{1/\sigma} = \mu/\pi$ **(growth rate of m/p)**

- **Comparative statics**
  - **money is superneutral in the narrow sense (**$\gamma$ **independent of** $\mu$**)**
  - **higher** $\mu$ **leads to higher** $\pi$**, higher R, and higher** $c_2/c_1$ **(the only source of nonsuperneutrality in the broad sense)**

## b. Two-Sector Endogenous Growth Model of Money

- **Consumer's optimization:**

$$\max \sum_t \beta^t u(c_{1t}, c_{2t}, 1 - n_t)$$

**s.t.** $m_t + b_{t+1} \le v_t,$

$$c_{1t} p_t \le m_t,$$

$$v_{t+1} \le (v_t - m_t - b_{t+1}) + (m_t - p_t c_{1t}) - p_t c_{2t}$$
$$- p_t x_{kt} - p_t x_{ht} + p_t w_t n_t h_t + p_t r_t k_t$$
$$+ (1 + R_{t+1}) b_{t+1} + T_t,$$

$$k_{t+1} \le (1 - \delta_k) k_t + x_{kt},$$

$$h_{t+1} \le (1 - \delta_h) h_t + x_{ht},$$

**where physical/human capital investments are credit goods and the final good production function is given by,** $F(k, nh) = Ak^{\alpha}(nh)^{(1-\alpha)}$**, implying** $k/h = \alpha/(1-\alpha)$ **if** $\delta_k = \delta_h$

- **Firm's optimization stays the same (except modified production)**
- **Asymptotic BGP equilibrium with $\delta_k = \delta_h$**

**2 x 2 system in ($\gamma$,n):**

**(modified GR)** $\gamma^{\sigma} = \beta[1 - \delta + \alpha A n^{1-\alpha}[(1-\alpha)/\alpha]^{1-\alpha}]$

**(labor tradeoff)** $\gamma = 1 - \delta + B n^{1-\alpha}\left(1 - \frac{(1-\alpha)(1-n)}{\psi n f(\mu, \gamma)}\right)$

**where** $f(\mu, \gamma) = 1 + \frac{\mu\beta^{-1}\gamma^{(\sigma-1)} - 1}{1 + \eta^{1/(1+\lambda)}(\mu\beta^{-1}\gamma^{(\sigma-1)})^{1/(1+\lambda)}}$

- **Comparative statics**
- **money is generally nonsuperneutral even in the narrow sense**
- **for $\lambda > 0$ ($c_1$, $c_2$ complements): higher $\mu$ reduces $c_1$ and $c_2$, lowering n and $\gamma$**
- **for $\lambda < 0$ ($c_1$, $c_2$ substitutes): higher $\mu$ reduces $c_1$ but raises $c_2$, lowering n and $\gamma$ if $\eta$ is sufficiently small (CIA binds for almost all purchases)**

## D. Finance and Growth - A First Look: Becsi and Wang (1997)

**(i) Key: Add a banking sector to the AK-model of endogenous growth**

**(ii) A Benchmark AK-model without the Financial Sector:**

**a. optimization:**

**max** $$U = \int_0^\infty \frac{c^{1-\alpha} - 1}{1-\alpha} e^{-\rho t} dt$$

$s.t.\ \dot{k} = Ak - \eta k - c,\ k(0) = k_0 > 0.$

**b. Key relationships without a banking sector:**

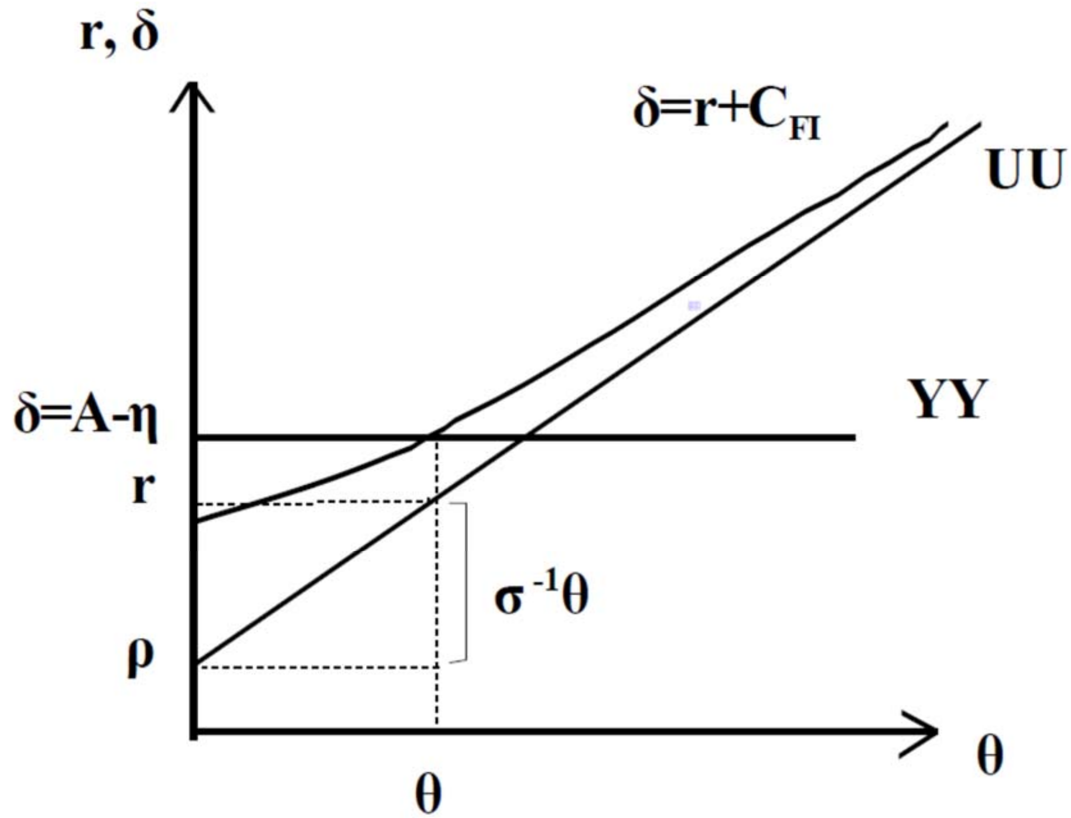
- **Keynes-Ramsey equation:** $\theta = \frac{\dot{c}}{c} = \frac{r - \rho}{\alpha} \Rightarrow r = \rho + \alpha\theta$ (UU)
- **Production efficiency:** $\delta = A - \eta$ (YY)
- **In the absence of an active banking sector:** $r = \delta$

## (iii) Incorporation of the Financial Sector into the AK-Model

- **A key ingredient is to recognize the loan-deposit interest differential. With active banking, deposits are transformed into loans, but such operations are not costless.**
- **In the absence of reserve requirement, loanable funds equilibrium implies that deposits equal to loans, denoted by x (in real values)**
- **Denote the unit financial intermediation cost as $C_{FI}$, which is decreasing as an economy develops (i.e., $\partial C_{FI}/\partial\theta < 0$; see Lehr and Wang 1999 for empirical documentation).**
- **By competitive banking (perfectly competitive or monopolistically competitive), banks must reach zero profit: profit = $\delta x - rx - C_{FI}\, x = 0$, or, $\delta = r + C_{FI}(\theta)$.**
- **The financial markup can be derived as: $\mu = \delta - r = C_{FI}(\theta)$, which depends negatively on the stage of economic development measured by the rate of growth $\theta$.**

## (v) BGP Equilibrium

- **Along a BGP, the endogenous growth rate must be pinned down by the loan rate and the production technology, whereas the preferences determines the deposit rate.**
- **The BGP equilibrium (θ, δ) is determined when the YY locus intersects with the markup locus, which can then be used, in conjunction with UU, to pin down equilibrium r.**



- **Comparative statics:**
  - **production innovation: A ↗ => δ ↗, r ↗, μ ↘, θ ↗**
  - **banking innovation: exog. $C_{FI}$ ↘ => δ unchanged, r ↗, μ ↘, θ ↗**
  - **annuity innovation: ρ ↘ => δ, θ ↗, r ↗, μ ↗**
  - **effective monitoring: A ↗ and $C_{FI}$ ↘ => δ ↗, r ↗, μ ↘, θ ↗**
  - **technological and annuity innovation: A ↗ and ρ ↘ => δ ↗, r ? (↘ if direct effect dominates), μ ↗, θ ↗**
  - **limited bank entry: more local market power => μ ↗, θ ? (Smith vs. Schumpeter)**

## E. Liquidity Management, Financial Intermediation and Growth: Bencivenga-Smith (1991)

- **The role of financial intermediation: liquidity management.**
- **Liquid investment is not as productive as illiquid investment. To accommodate illiquid investment and possible withdrawals, banks hold liquid reserves. However, should there be unexpected withdrawals, banks may face a illiquidity problem.**
- **The model generalizes Diamond-Dybvig (1983) by incorporating liquidity management into an endogenous growth framework.**

### a. The Model

- **3-period overlapping-generations (pop = 1), supplying 1 unit of labor only when young and consuming when middle-aged and old**
- **Production:** $y = \bar{k}^{1-\theta} k^{\theta} L^{1-\theta}$ **(Romer)**
- **Labor demand per entrepreneur: MPL =** $w$

- **Utility:** $U = -(c_2 + \varphi c_3)^{-\gamma} / \gamma$ , **$\gamma > -1$ since $\sigma = 1/(1+\gamma)$**

  **where $\varphi = 0$ with probability $1-\pi$ (early withdrawers)**

  **$\varphi = 1$ with probability $\pi$ (entrepreneurs)**
- **Investment returns:**
  - **liquid investment: return = n > 0 (safe return)**
  - **illiquid investment:**
    - **return after 1 period = x ε [0, n) (liquidated scrap value < n)**
    - **return after 2 period = R > n (LT investment return > n)**
- **Labor market equilibrium: $\pi L = 1$ (young's labor supply)**
- **Factor prices:**
  - $w = (1-\theta)k\pi^{\theta}$
  - $r_k = \theta\pi^{\theta-1}$
- **With financial intermediation, all wages are deposited in banks.**

- **Banks:**
  - **have asset management portfolio of {z, q}, choosing a fraction z in liquid investment and q in illiquid investment, where z + q = 1**
  - **have liabilities, paying**
    - **$r_1$ to 1-period deposits**
    - **$r_2$ to 2-period deposits without withdrawals (capital)**
    - **$r_0$ to liquidated 2-period deposits (scraped for consumption).**
- **Banks' resources constraints (payments = revenues):**
  - **1-period:** $(1-\pi)r_1 = \alpha_1 nz + \alpha_2 xq \qquad (\alpha_1 + \alpha_2 = 1)$
  - **2-period:** $\pi r_2 = (1-\alpha_2)Rq$
  - **2-period scraped:** $\pi r_0 = (1-\alpha_1)nz$
- **Gurley-Shaw's bank (in the interest of the depositors, i.e., banks as coalitions formed by the young): choose {q, z, $\alpha_1$, $\alpha_2$, , $r_1$, $r_2$, $r_0$} to:**

**max** $$EV = (1-\pi)\left[-\frac{(r_1 w)^{-\gamma}}{\gamma}\right] + \pi\left[-\frac{\left(\theta\pi^{\theta-1} r_2 w + r_0 w\right)^{-\gamma}}{\gamma}\right]$$

**s.t. $\alpha_1 + \alpha_2 = 1$, z + q = 1 and 3 bank resources constraints**

**b. Results**

- **Equilibrium decisions: with $r_k R = \theta\pi^{\theta-1}R > n$,**
  - **$\alpha_1 = 1$ (1-period reserves always liquidated)**
  - **$\alpha_2 = 0$ (no pre-mature liquidation of capital)**
  - **$r_0 = 0$ (paying nothing to liquidated consumption)**
- **Financial intermediation emerges with *rate-of-return dominance***
  - **it requires large $\gamma$ (or small intertemporal substitution)**
  - **intuitively, small intertemporal substitution is equivalent to more risk aversion intertemporally, thus giving a stronger role for banks to form.**
- **Key finding: the rate of growth with financial intermediation is higher than without it if $x$ is sufficiently small.**

**Remark:** **Although the current model assumes forced savings (no value of period-1 consumption), main results are robust to such an extension.**

## F. Finance, Human Capital Investment, and Growth: Tssidon (1992)

- **Key: credit market imperfections can cause under-invest in human capital and low-growth trap**

**a. The Model**

- **Introducing Jaffee-Russell (1976)'s credit rationing model into a 3-period OLG model with human-capital based growth**
- **Production (time-to-educate):** $Y_t = F(k_t, E_{t-1})$
- **3-period Individual Decisions:**
  - **period-1: borrow to finance risky investment in human capital with returns at rate $R^j$, with two types** $j \in \{r, s\}$
  - **period-2: work and save**
  - **period-3: consume**

- **Investment in Human Capital:**
  - **investment loan = *L*, with two types** $j \in \{r, s\}$
  - **expected return:** $ER^j = (1 - p^j)(1 + R^j) + p^j \cdot 1$
  - **assumptions:** $ER^r > ER^s$, $p^r > p^s$ **(high risk, high returns)**
  - **individual's choice of *j* is unobservable to banks**

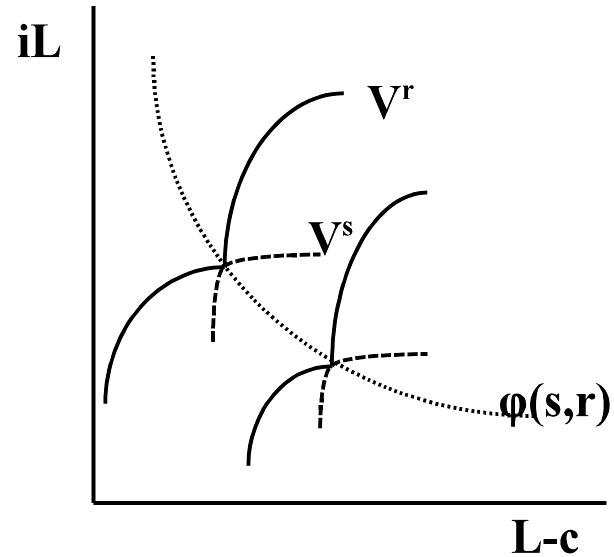
- **Expected Utility** $(j = r, s)$ **:**

$$V_t^j = (1 - p^j)u((1 + \rho_{t+2})[(1 + R^j)W_{t+1} - (1 + i_{t+1})L]) + p^j u((1 + \rho_{t+2})(W_{t+1} - c_{t+1}))$$

  - $\rho$ **= deposit rate**
  - ***c*** **= collateral**
  - **W = wage income**

- **The idea can be best illustrated by a figure in (L-c, iL) space:**
  - **iL = costs of borrowing**
  - ***L-c* = net benefit of borrowing (limited liability)**

**(1) IC upward-sloping:**
**benefit↑ ⇒ cost↑ to keep *V* constant**




**(2) *V*↑ towards southeast**

**(3) *c*↓ (or *L-c*↑) matters more to investment in r**
**⇒ need *iL*↑ more to compensate *c*↓**
**⇒ IC steeper for r**

**(4) *c*↑ (*L-c*↓) ⇒ risk averters prefer *s***
**⇒ upper envelope of $IC^j$**

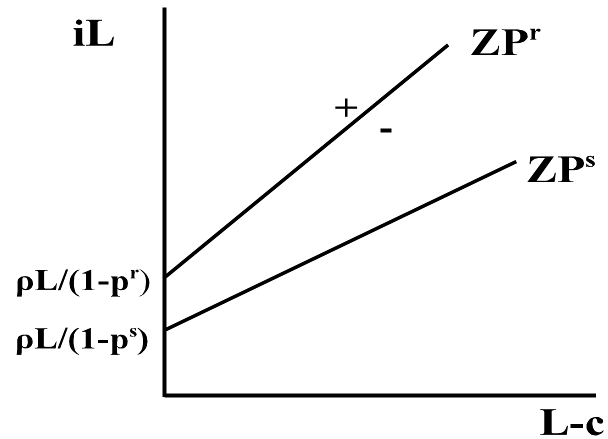
**(5)** $V^s = V^r \Rightarrow \phi$ **(the locus is convex under CRRA utility)**

**(6) Key: non-convexity in IC ⇒ multiple equilibrium**

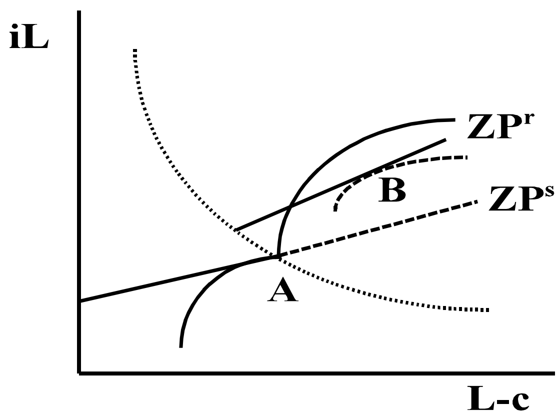
- **Banks' zero profit conditions:**

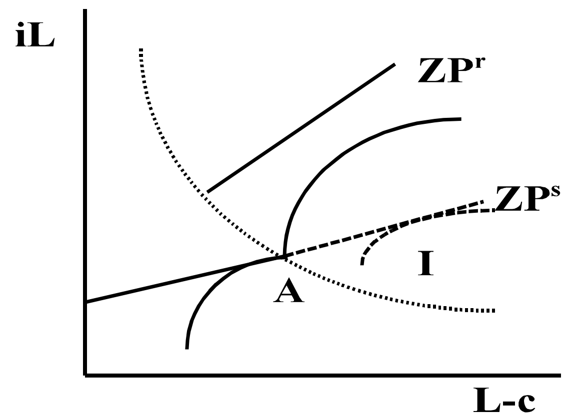
$$(1-p_j)i_t L - p^j(L-c) = \rho_t L$$

  - **upward sloping lines**
  - **risky type with higher failure rate $p^j$ => steeper ZP with higher intercept**




**b. Temporary Equilibrium (given $\rho, W$)**

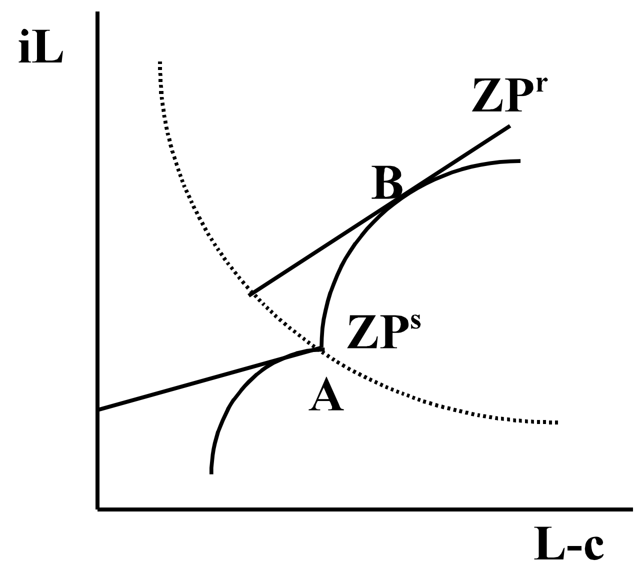



- **A: constrained equilibrium due to moral hazard**
- **B: unconstrained; I: infeasible (negative profit)**

### c. Full Dynamic General Equilibrium (DGE)

- **In DGE, ρ and W can adjust freely s.t.**
  - $WE + F_k K = F$
  - $F_k = 1 + \rho$
- **While W only affects expected utility *V*, ρ affects both *V* and ZP**
- **Adjustment: Look at the benchmark case with B more preferred to A**
  - **all choose B and risk higher**
  - **so expected profit goes down**
  - **funds must be constrained and ZP must shift up**
  - **the full dynamic equilibrium is reached with (ρ, W) adjusted accordingly (lower K, higher ρ and lower W)**



**d. Main Findings**

- **Moral hazard $\Rightarrow$ credit rationing $\Rightarrow$ equilibrium at A:**
  - **low human capital**
  - **low MPK (or $\rho$)**
  - **low-growth trap**

- **Policy Prescription:**
  - **provision of education loans with better monitoring or law enforcement**
  - **this will reduce moral hazard problems, thus promoting investment in high risk but high return higher education**
  - **as a result, it enhances economic growth, avoiding the low human capital-low growth trap**

## G. Finance, Investment, and Growth: Aghion-Bolton (1997)

- **The Aghion-Bolton model can be regarded as an extension of Baneree-Newman (1993) by allowing full dynamics of wealth evolution with:**
  **(i) endogenous occupational choice**
  **(ii) credit market imperfections**
  **(iii) nonstationary distribution (cf. Hopenhyne-Prescott)**

## a. The Model

- **1-period lived agents of with unit mass with bequest motive, one unit of time endowment, and heterogeneous initial wealth w ~ $G_t(w)$**

**Occupational choice:**

- **Home production: return $n$ > 0 (small)**
- **Entrepreneurial activity:**

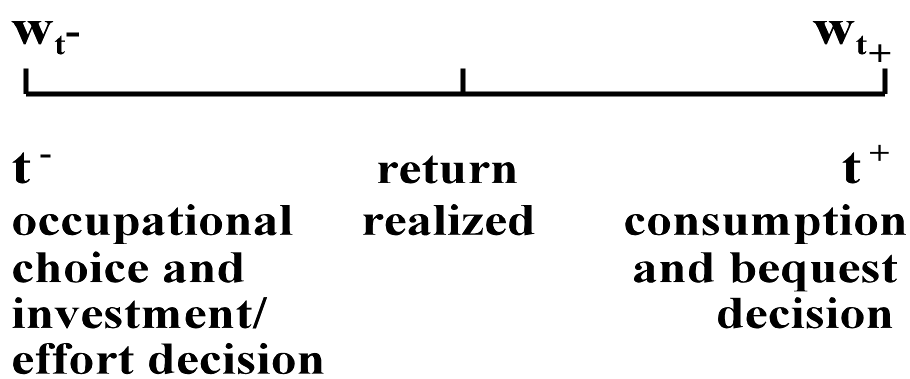
$$F(k,1) = \begin{cases} \text{r} \quad \text{for k} \geq 1 \text{(fixed cost), with prob.} = p \\ 0 \quad \text{for k} \geq 1, \text{ with prob.} = 1 - p \\ 0 \quad \text{for k} < 1 \end{cases}$$

**where $p$ = effort with effort cost** $C(p) = \dfrac{rp^2}{2a}, \ a \in (0,\ 1]$

- **Mutual fund deposit: safe return $A_t w_t$ (no labor input)**

- **Preferences: Leontief in consumption and bequest**
  $U = [\delta(1-\delta)]^{-1} \min\{(1-\delta)c, \delta b\} - C(p)$
- **Budget Constraint: $c + b = w$**

- **Timing**




**b. Optimization**

- **Consumption and bequest:**
  $(1-\boldsymbol{\delta})\boldsymbol{c} = \boldsymbol{\delta b}$ and BC $\Rightarrow \boldsymbol{b} = (1-\boldsymbol{\delta})\boldsymbol{w},\ \boldsymbol{c} = \boldsymbol{\delta w}$

  $$\Rightarrow\ w_{t+1^-} = b_{t+1} = (1-\delta)w_{t^+}$$

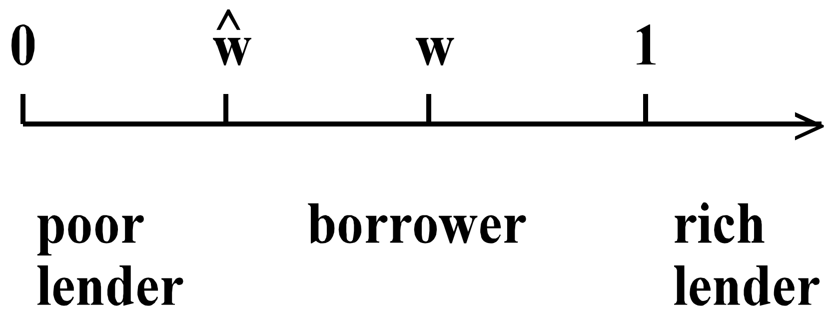
- **Assumptions:**
  - **(A1) (mean wealth)** $\int \boldsymbol{w}d\boldsymbol{G}_0(\boldsymbol{w}) < 1$
  - **(A2) (repayment)** $\forall \boldsymbol{w} < 1$

$$R(w) = \begin{cases} (1-w)\rho(w) & \text{with prob.} = p \\ 0 & \text{with prob.} = 1 - \text{p} \end{cases}$$

- **Occupational choice and investment decision:**
  - **Potential borrower (w <1): $\max_p$** $\quad pr - p(1-w)\rho(w) - C(p)$,

    **implying** $\quad p(w) = a[1 - (\frac{1-w}{r})\rho(w)]$

  - **Rich lender** $(w \geq 1)$**: $\max_p$** $\quad pr - C(p)$, $\quad$ **implying** $\quad p(w) = a$



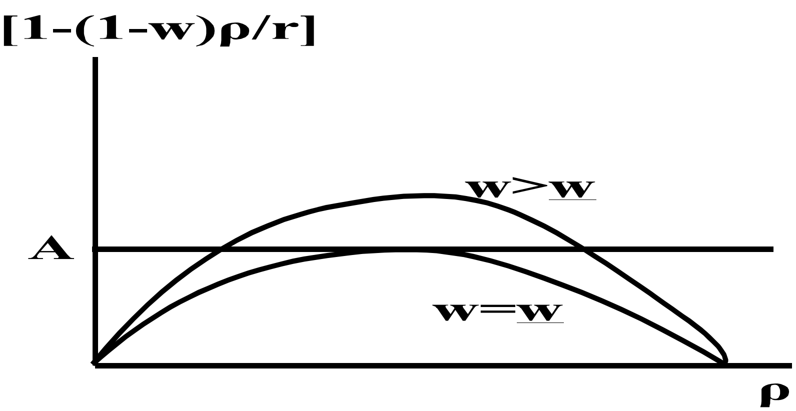

**loan = 1-w**

c. **Atemporal Equilibrium**

- **Rate of return equalization to lenders:**

$$p(w)\rho = A \quad \Rightarrow \quad \rho(w) = \frac{r}{2(1-w)}\left[1-\sqrt{1-\frac{4(1-w)A}{ar}}\right] \quad \forall\ w \in [\underline{w},\ 1]$$

where $\underline{w} = 1 - ar/(4A)$ with $\rho' > 0$ $(p' > 0)$




$w \in [0,\ \underline{w}) \Rightarrow$ no loan supply $\Rightarrow$ unable to borrow

- **Rate of return equalization to borrowers**

$$p(w_C)r - p(w_C)\rho(w_C)(1-w_C) - C(p(w_C)) - 1 = Aw_C + n$$

**=> willing to borrow if $w > w_C$ or $A < (3/8)ar - n - 1$**

- **Condition CR: $\exists\ \underline{w} > 0$ and $w > w_C$, or, $ar/4 < A < (3/8)ar - n - 1$**
- **Under condition CR, we have: $\forall\ w \in (w_C, \underline{w})$, they are willing to borrow but unable to obtain loan => equilibrium credit rationing**

## d. Full Equilibrium

- **Wealth evolution:** $$w_{t+1} = \begin{cases} (1-\delta)(\mathrm{A}w_t + n) & \text{for } w_t \in [0, \hat{w}_t] \\ g_t(w_t, \theta_t) & \text{for } w_t \in [\hat{w}_t, \infty) \end{cases}$$

**where** $\hat{w}_t = \max\{\underline{w}, w_C\}$ **, $\theta_t$ = indicator function for success, and**

$$g(w,1) = \begin{cases} (1-\delta)[r - (1-w)\rho(w)] & w < 1 \\ (1-\delta)[r + (w-1)\mathrm{A}] & w \geq 1 \end{cases} \qquad g(w,0) = \begin{cases} 0 & w < 1 \\ (1-\delta)(w-1)\mathrm{A} & w \geq 1 \end{cases}$$

$$prob.(\theta = 1 | w) = h(w) = \begin{cases} p(w) & w < 1 \\ a & w \geq 1 \end{cases}$$

- **Distribution converges in weak* topology in Polish space**
- **Additional assumptions:**
  - **(A3) (incentive to lend)** $\frac{ar}{4}(1-\delta) > 1$
  - **(A4) (rapid accumulation)** $\frac{3}{8}ar(1-\delta) > 1+n$
- **Safe rate of return:**
  - $A_t \leq \frac{1}{1-\delta} \quad \forall\, t \geq T$ **(ow, unbounded wealth=>excess fund supply)**
  - **under (A3), CR does not exist for** $A_t \in [1,\ 1/(1-\delta)]$
  - **under (A3) and (A4),** $A_t \to 1$ **in finite time**
- **Trickle-down:**
  - **CR exists in early stage of development when** $A_t$ **is high**
  - **As** $w_t \uparrow$ **over time,** $A_t$ **falls in** $[1,\ 1/(1-\delta)] \to 1$ **=> no CR**
  - **Intuition: the rich trickle down increasing supply of loan and enabling the poor to borrow and invest by lowing capital cost** $A_t$