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RESEARCH ARTICLE

Flying or trapped?

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Abstract
We develop a unified theory with endogenous technology choice in human/knowledge capital accumulation to obtain a rich array of development paradigms. The transition from a less productive technology to a more productive one occurs via endogenous technology choice without requiring any force of big push. However, technologies that are more productive face higher scale barriers and thus multiple steady states may appear with each of which being associated with a different technology. We characterize global dynamics and establish conditions under which middle-income trap or flying geese growth may arise in equilibrium. By calibrating the general model to the data from several representative economies at different development stages with different growth patterns, we identify various prolonged flying geese episodes and middle-income traps. The analysis shows that while improving the efficacy of human capital accumulation is more crucial for advanced and fast-growing economies, mitigating barriers to human capital accumulation is more rewarding for emerging growing economies and development laggards. Our growth accounting results indicate that once the efficacy of and the barriers to human capital accumulation are incorporated, the residual TFP component no longer plays a substantial quantitative role.

Keywords Flying geese development · Middle-income trap · Technology choice · Human capital upgrading

JEL Classification O40 · E20 · D20

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1 Introduction

During the post-WWII era, one witnessed widening world income disparities, with the per capita real income ratio of the richest to the poorest 10% rising from below 20 in 1960 to above 50 at the turn of the new millennium. This further encourages the study of poverty traps to better understand why the poorest countries have failed to advance. Not until recently have more development economists recognized the presence of a broadly defined "middle-income trap" in which many previously fast-growing middle-income countries and even some more advanced countries have suffered periods of sluggish growth at various points in time. This is not only a concern for countries mired in the trap but also a worldwide issue because many of them have been important forces for advancing global growth.

To motivate our study, we present cross-country income mobilities over the past four decades. In Table 1, we report the proportion of countries transiting from the ith quintile (denoted Qi) in year t to the jth quintile (Qj) in year $t + 20$ over two 30-year windows (1971–2001 and 1981–2011). The high persistence in the Q1–Q1 transition is consistent with the conventional poverty trap argument. Moreover, countries in Q3 and Q4 face high probabilities, ranging from 15 to 30%, of falling back by a quintile. During the period 1981–2011, in particular, the probability of countries in Q4 falling back is particularly high at 28%; even for those in Q3, this probability also exceeds 15%. This observation is consistent with the possibility of a middle-income trap. More systematic empirical evidence for the existence of the middle-income trap has been provided by Eichengreen et al. (2013) among others. In their work, the middle-

Table 1 World-income mobility matrix

	Window: 1971–2001					
				$t + 20$		
		Q1	Q2	Q3	Q4	Q5
	Q1	0.814	0.156	0.030	0	0
	Q2	0.114	0.650	0.191	0.045	0
t	Q3	0.082	0.164	0.559	0.168	0.027
	Q4	0	0.027	0.205	0.432	0.336
	Q5	0	0	0.048	0.290	0.662
	Window: 1981–2011					
				$t + 20$		
		Q1	Q2	Q3	Q4	Q5
	Q1	0.784	0.182	0.009	0.019	0.006
	Q2	0.201	0.649	0.146	0.003	0
t	Q3	0.023	0.159	0.571	0.247	0
	Q4	0	0	0.282	0.529	0.188
	Q5	0	0	0	0.188	0.812

Note: The proportion of countries transiting i th quintile in year t to the j th quintile in year t 20 is presented over two 30-year windows income trap is identified as a substantive fall (2% or more) in the per capita real income growth of a previously fast-growing (3.5% or higher) middle-income country (with per capita real income exceeding US$10,000 in 2005 constant PPP prices) for a considerable duration (7 years before and after the structural break). Under these criteria in conjunction with Chow tests, they find many such traps in Asian, European and Latin American countries in various years depending on a country's stage of development.[1] More interestingly, upon checking a number of possible drivers, they find that having a population *more educated at secondary and higher education levels* is a robust factor leading to a lower likelihood for a country falling into a middle-income trap. This motivates our study.

Two natural questions arise. First, can one establish a unified theory under which a middle-income trap may emerge beyond the conventional poverty trap? Second, to reconcile with the aforementioned empirical findings, how and how much would human capital, or more generally knowledge capital, play a key role in the likelihood of middle-income traps, and how important would it be quantitatively if one were to put the question to the data? In this paper, we will address both questions within an optimal growth framework with *endogenous technology choice*. Specifically, we consider a representative agent who maximizes her lifetime utility subject to periodic budget constraints that allocate income to consumption and investments in physical and human or knowledge capital. While investment in physical capital directly contributes to its stock one for one, investment in human capital depends crucially on the knowledge accumulation technology. Knowledge accumulation technology exhibits an important feature: while better technology is more productive, it is associated with a higher scale barrier. As a result of this tradeoff and the dynamic process of technology choice, multiple steady states may arise, each associated with a different technology choice outcome. Moreover, the transition from one steady state to another occurs via endogenous technology choice without requiring any force of big push. This differs sharply from the literature that relies critically on threshold externalities, increasing returns, imperfect market structures, or complex functional forms. Our paper thus contributes methodologically to the literature on growth and economic dynamics.

Specifically, in this otherwise simple growth model, we are able to establish conditions under which at least one middle-income trap coexists with the conventionally studied poverty trap. We find that a middle-income trap is more likely to arise when the productivity of the prevailing technology is not too large, the scale barrier of the prevailing technology is sufficiently high, and the productivity jump to the better technology is sufficiently large. However, what happens if an economy is not mired in a middle-income trap? We establish conditions under which the economy features a "flying geese" paradigm à la Akamatsu (1962), where countries with continual technology upgrading in human/knowledge accumulation tend to experience more sustained growth to stay ahead of others in the development process. We also show that with negligible productivity upgrading, an economy can be permanently trapped in poverty

[1] Cases identified include (i) in the 1970s, Greece (1972), Finland (1974), Japan (1974), Venezuela (1974), and Ireland (1978); (ii) in the 1980s, Singapore (1980), Mexico (1981), Puerto Rico (1988), Cyprus (1989), and Korea (1989); and (iii) in the 1990s, Portugal (1990), Hong Kong (1993), and Taiwan (1995).

and that with a negligible increase in the technology scale barrier, an economy can advance rapidly into a high-income society featuring sustained growth.

We then generalize our baseline model to permit employment variations over time and to relax the ranking assumption on human capital accumulation technology.[2] By applying our modified conditions for the flying geese paradigm and the middle-income trap to a representative set of 14 countries/economies using data from the Penn World Table (PWT9.0, Feenstra et al. 2015), we find that even in four advanced countries and four fast-growing Asian Tigers that experienced flying geese growth for prolonged periods, traps still arise at various periods of time. In three emerging growing economies, such traps occur more often but are still less frequent than flying geese growth. Three development laggards featuring chopped and slower growth paths, on the contrary, are trapped as often as flying. We are able to compare our findings with those in the literature by focusing on common samples. We find that several, but not all, traps identified in our paper are in line with those in previous studies.

Overall, the results suggest that large declines in the efficacy of human capital technology are overwhelmingly the primary force driving a country to fall into a middle-income trap and are particularly important in advanced and fast-growing countries. Large increases in barriers to human capital accumulation are also important, playing a particularly critical role in emerging growing countries and development laggards. By contrast, total factor productivity (TFP) slowdowns are only occasionally essential. Thus, an important policy implication obtained from our quantitative analysis is that public policy directed toward improving the efficacy of human or knowledge capital accumulation is more rewarding for advanced and fast-growing countries, whereas mitigating barriers to human or knowledge capital accumulation is more rewarding for emerging growing countries and development laggards. This suggests differential policy designs to advance an economy. By performing growth accounting, we find that human capital technology upgrading and human capital barrier reduction are crucial, contributing to 37–51% of economic growth on average. In contrast to conventional growth accounting studies, TFP turns out to be largely inconsequential. Notably, in more than half of the episodes considered in our study (15 out of 28 episodes in 14 countries), human capital technology upgrading and human capital barrier reduction jointly account for more than half of economic growth.

The main message delivered by this paper is that human capital upgrading and barrier reduction can play a key role in economic development, determining whether a country may be flying or trapped. We thus offer a novel endogenous TFP story: once the efficacy of and the barriers to human capital accumulation are incorporated via endogenous technology choice, the residual TFP component no longer plays a substantial role.

[2] The human capital accumulation process in an economy may change over time. For instance, Hsieh et al. (2019) demonstrate that in the U.S. economy, the frictions in human capital accumulation have been mitigated, and the improvement of the allocation of talent has promoted economic growth since the 1960s. The generalization of our baseline model can capture changes in the human capital accumulation process in an economy.

Related literature

There is a sizable body of literature demonstrating the existence of poverty traps. The argument is based on multiple steady states: the coexistence of a degenerate low steady state (trap) and a high steady state (see the comprehensive survey by Azariadis and Stachurski 2005). When limiting our attention to human capital-related studies, poverty traps may arise due to human capital threshold externalities (Azariadis and Drazen 1990; Redding 1996), moral hazard problems associated with human capital investment (Tsiddon 1992), or barriers to investment in children's human capital (Galor and Weil 2000). However, these frameworks cannot generate middle-income traps or the flying geese paradigm.

There is only a small body of research on the flying geese paradigm with micro-founded theory. In particular, the flying geese paradigm may arise as a result of product upgrading (Matsuyama 2002), industry upgrading (Wang and Xie 2004; Ju et al. 2015) or the staged assimilation of global technologies (Wang et al. 2018). In Wang et al. (2018), the possibility of a middle-income trap via the assimilation of global technologies is also established. Specifically, technology assimilation in a country is more effective if its factor endowment gap from the technology source country is small, and a country may overaccumulate an originally disadvantageous factor to cause a serious mismatch and a middle-income trap. In Marsiglio and Tolotti (2018), when the impact of research externalities is high but the unit profit from research is low, there may exist two locally saddle-path stable balanced growth equilibria. They further note that one of the two locally stable equilibria with lower growth can be thought of as a middle-income trap. Generally speaking, the work by Matsuyama (2002) depends crucially on demand-induced scale economies that reduce product prices to lead to better but originally unaffordable products to be consumed by the less rich. All other studies are based on factor endowment. In Wang and Xie (2004) and Ju et al. (2015), rising physical capital or human skills may advance industries with better technologies and hence promote growth. In Marsiglio and Tolotti (2018), research intensity and the externality within the research sector play a key role in determining growth outcomes. In Wang et al. (2018), accumulating disadvantageous factor to avoid mismatch may enhance the effectiveness of technology assimilation and lead to a growing paradigm.

By contrast, we propose a novel mechanism via human capital technology choice that does not rely directly on factor endowment. Moreover, we are able to fully characterize the global dynamics to establish precise conditions for both a middle-income trap and the flying geese paradigm to arise within a unified framework.

2 The model

Consider a discrete-time model with time indexed by t. The economy consists of identical infinitely lived agents. There is a single general good that can be used for consumption or investment purposes. In addition to labor, there are two capital inputs: physical and human capital. The key feature of the model is that the human capital upgrading technology is endogenously determined by discrete choices with costly barriers.

2.1 The environment

A representative agent in period t produces general goods Y_t using a constant-returns Cobb-Douglas production technology $Y_t = AH_{t-1}^{\alpha} K_{t-1}^{\beta} L_t^{1-\alpha-\beta}$ where $\alpha, \beta \in (0, 1)$. The inputs of the production technology are physical capital K_{t-1}, labor L_t, and human capital H_{t-1}—which should be interpreted more generally to include knowledge capital and know-how. Note that the production of general goods takes one gestation period in which the human capital, H_{t-1}, and the physical capital, K_{t-1}, are prepared one period before the production occurs (time-to-build). A is the TFP of the production technology.[3] General goods are consumed or used for investments by the representative agent. The aggregate budget constraint is given by $AH_{t-1}^{\alpha} K_{t-1}^{\beta} L_t^{1-\alpha-\beta} = C_t + I_t^h + I_t^k$, where C_t is aggregate consumption and I_t^h and I_t^k are investments in the production of human capital and physical capital, respectively. Then, the agent's budget constraint is given as follows:

$$y_t := Ah_{t-1}^{\alpha} k_{t-1}^{\beta} = c_t + i_t^h + i_t^k, \tag{1}$$

where $h_{t-1} := H_{t-1}/L_t$, $k_{t-1} := K_{t-1}/L_t$, $c_t := C_t/L_t$, $i_t^h := I_t^h/L_t$, and $i_t^k := I_t^k/L_t$. Although labor force growth is introduced in the quantitative analysis in Sect. 5, the population of the labor force is normalized to $L_t = 1$ for all $t \geq 0$ in the baseline setting to simplify the theoretical analysis.

2.2 Optimization problem

Let us first specify a representative agent's optimization problem by maximizing her lifetime utility, $\sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln c_\tau$, subject to three constraints in addition to the typical nonnegativity constraints on investments, which are given by

$$y_\tau \ : = Ah_{\tau-1}^{\alpha} k_{\tau-1}^{\beta} = c_\tau + i_\tau^h + i_\tau^k \tag{2}$$

$$k_\tau = g_0(i_\tau^k) \tag{3}$$

$$h_\tau = \max_{m=1,2,\ldots,M} \{g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)\}, \tag{4}$$

for $\tau \geq t$, where $\delta \in (0, 1)$ is the subjective discount factor. In Eqs. (2)–(4), both physical and human capital depreciate entirely in one period. The representative agent is endowed with two types of investment projects: one is to produce physical capital, and another is to produce human capital. In the first investment project, a one-for-one simple linear technology produces capital from general goods. That is, in Eq. (3), the production function for physical capital, $g(i_\tau^k)$, is given by

$$g_0(i_\tau^k) = i_\tau^k. \tag{5}$$

[3] In the quantitative analysis in Sect. 5, where we generalize the basic model, A is allowed to be time-varying.

In Eq. (4), $g_m(i^h_\tau; \bar{h}_{\tau-1}, \bar{y}_\tau)$ is the production function for human capital when the representative agent applies the mth technology. Human capital is produced from general goods, and human capital formation is subject to technology choice: the representative agent chooses the best technology among a number of M technologies for human capital formation.

In each period, the agent chooses the best technology for human (or knowledge) capital formation and solves her maximization problem, taking as given the (average) past human capital, $\bar{h}_{\tau-1}$, and the (average) current-period output, $\bar{y}_\tau$, in the economy. We specify the production function for human capital as:

$$g_m(i^h_\tau; \bar{h}_{\tau-1}, \bar{y}_\tau) := \frac{\theta_m(\bar{h}_{\tau-1} - \eta_m)^\sigma}{\bar{y}_\tau} i^h_\tau, \tag{6}$$

for $\bar{h}_{\tau-1} \geq \eta_m$, with $\alpha < \sigma \in (0, 1)$, $\theta_m \in [1, \infty)$, and $\eta_m \in [0, \infty)$.[4] Equation (6) possesses several important governing properties.

- $\partial g_m(i^h; \bar{h}, \bar{y})/\partial i^h > 0$: The marginal product of human capital investment is positive.
- $\partial^2 g_m(i^h; \bar{h}, \bar{y})/\partial i^h \partial \bar{h} > 0$: Past knowledge accumulation, which is condensed into past human capital formation $\bar{h}_{\tau-1}$, has a positive external effect on the marginal product, thereby promoting human capital formation. Thus, human or knowledge capital accumulation depends on society's existing stock in the spirit of the knowledge spillovers in Romer (1986) and human capital spillovers in Lucas (1988).
- $\partial^3 g_m(i^h; \bar{h}, \bar{y})/\partial i^h \partial \bar{h}^2 < 0$: As past knowledge accumulates further, new knowledge (ideas) becomes more difficult to find, and hence the effect of the positive externality diminishes (see Bloom et al. 2020).
- $\partial^2 g_m(i^h; \bar{h}, \bar{y})/\partial \bar{h} \partial \bar{y} < 0$: This is to mitigate the scale effect of human capital externalities. As the economy develops, $\bar{h}_{\tau-1}$ and $\bar{y}_\tau$ continue to rise, so this property limits the scale of knowledge spillovers. More concretely, the inclusion of the externality term of $\bar{y}_\tau$ is to remove the ad hoc scale effect—without it, i^h_τ and $\bar{h}_{\tau-1}$ would jointly lead to social increasing returns, which we find it hard to justify. Jones (1995) uses a similar method to remove such an artificial scale effect in an R&D model. As elaborated below, the consideration of these two forms of externalities is only to capture reality; they are inconsequential for the existence of multiple steady states or the dynamic patterns of economic development.

The representative agent chooses the best technology depending on human capital accumulation. The set of global technologies available to the economy of interest is denoted as $\Theta := \{\theta_1, \theta_2, \cdots, \theta_M\}$, with θ_M representing the world frontier technology à la Caselli and Coleman (2006). If the productivity with respect to a certain technology becomes small, another suitable technology may be chosen, as noted from the max operator on the right-hand side of Eq. (4). While human or knowledge capital accumulation is more effective when θ_m is higher, the presence of η_m captures a scale barrier in human or knowledge capital accumulation.

[4] The assumption of $\alpha < \sigma$ guarantees that the net effect of $\bar{h}_{\tau-1}$ is always positive.

We impose a parameter condition in Assumption 1 below on the technology efficacy ranking with a tradeoff between productivity, θ_m, and the scale barrier, η_m.

Assumption 1 (i) $1 = \theta_1 < \theta_2 < \cdots < \theta_M$ and (ii) $0 = \eta_1 < \eta_2 < \cdots < \eta_M$.

Assumption 1 implies that human capital accumulation needs to exceed a higher barrier for an economy to utilize higher productivity technology. The service upgrading model by Buera and Kaboski (2012) motivates Assumption 1. In their model, larger scale barriers in skills are associated with the production of better services with greater complexity. Furthermore, in the appropriate technology model of Caselli and Coleman (2006), skilled-labor-abundant rich countries tend to choose technologies that are more efficient for skilled workers.[5] Our scale barrier setup in human capital resembles the service upgrading proposed by Buera and Kaboski (2012) and generates similar implications to the appropriate technology model of Caselli and Coleman (2006). In our model, human capital abundance enables the representative agent to choose better technologies. Two real world examples are provided in Remark 1 below.

Remark 1 Artificial intelligence (AI) technology is an illustrative example that supports Assumption 1. Clearly, past accumulation of knowledge capital is reflected in AI technology as, say, machine learning algorithms. Whereas AI technology increases productivity in knowledge capital production, even experts in machine learning sometimes face a high barrier to developing new AI technology. Another example is genome editing technology, which can be regarded as knowledge capital stock. Recently, many genome editing technologies have been developed. However, had genome analysis not succeeded, to which there had long been a high barrier, genome editing technology would not have arrived.

We are now ready to formalize the conventional poverty trap and long-run balanced growth equilibrium.

- A poverty trap is associated with primitive technology ($\theta_1 = 1$), which can be accessed with no obstructions ($\eta_1 = 0$). This is a trap if there exists $t_c \gg 0$ such that $1 = \arg\max_{m=1,2,\ldots,M}\{\theta_m(\bar{h}_{\tau-1} - \eta_m)^\sigma\}$ for all $t \leq t_c$.
- After reaching the highest technology M, the economy will be on a perpetual balanced growth path. Along such a balanced growth path, human capital production is simply linear with $g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau) := \left(\theta_{\max}\bar{h}_{\tau-1}/\bar{y}_\tau\right) i_\tau^h$ for $\bar{h}_{\tau-1} > \eta_{\max}$, where $\theta_{\max} > \theta_M$ and $\eta_{\max} > \eta_M$. This highest stage of development corresponds to the Rostovian state of Mass Consumption. Because this equilibrium is trivial, for the sake of brevity, we will not characterize it explicitly but note that this state resembles one in Bruno et al. (2009) under a very different setting in which an economy with sufficiently high human capital would always adopt new technologies and achieve unbounded growth.
- Our focus is therefore on the "interior" technologies to examine the associated patterns of the development process between the initial poverty trap and the final balanced growth equilibrium (see Sects. 4.2 and 4.3).

[5] To explain productivity differences across countries, Acemoglu et al. (2007) also consider firms' technology choice, in which less advanced technologies are chosen under greater contractual incompleteness between firms and suppliers of intermediate inputs.

The first-order conditions of the representative agent's maximization problem are given by

$$\lambda_t = \frac{1}{c_t} \tag{7}$$

$$\lambda_t = \left(\frac{\delta\alpha b(\bar{h}_{t-1}, \bar{y}_t) y_{t+1}}{h_t}\right) \lambda_{t+1} \tag{8}$$

$$\lambda_t = \left(\frac{\delta\beta y_{t+1}}{k_t}\right) \lambda_{t+1} \tag{9}$$

$$\lambda_t = p_t^k = b(\bar{h}_{t-1}, \bar{y}_t) p_t^h, \tag{10}$$

where $b(\bar{h}_{t-1}, \bar{y}_t) := \max_m\{\theta_m(\bar{h}_{t-1} - \eta_m)^\sigma\}/\bar{y}_t$, and λ_t, p_t^h, and p_t^k are the shadow prices of general goods, human capital, and physical capital, respectively. The necessary and sufficient conditions for the optimality of this maximization problem consist of Eqs. (7)–(10) and the transversality conditions, $\lim_{t\to\infty} \delta^t p_t^k k_t = \lim_{t\to\infty} \delta^t p_t^h h_t = 0$.

3 Equilibrium

We are now prepared to define and establish the dynamic competitive equilibrium and to characterize the dynamics. A key strategy is to reduce the dynamical system to two states—physical and human capital—and the values of these two types of capital. This strategy greatly simplifies the analysis because the two capital values are proportional and their limits are governed by the transversality conditions. From Eqs. (8) and (9), it follows that $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$. From this equation and Eqs. (3)–(6), we obtain $i_t^k + i_t^h = k_t(\alpha + \beta)/\beta$. These two equations allow us to rewrite the periodic budget constraint (2) as

$$c_t + \frac{\alpha+\beta}{\beta} k_t = A\left(\frac{\alpha b(\bar{h}_{t-2}, \bar{y}_{t-1})}{\beta}\right)^\alpha k_{t-1}^{\alpha+\beta}. \tag{11}$$

Additionally, substituting $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$ into Eq. (8) yields

$$\lambda_{t-1} = \lambda_t \delta\alpha A b(\bar{h}_{t-2}, \bar{y}_{t-1})^\alpha \left(\frac{\alpha}{\beta}\right)^{\alpha-1} k_{t-1}^{-(1-\alpha-\beta)}. \tag{12}$$

Multiplying both sides of Eq. (11) by λ_t leads to

$$\lambda_t c_t + \frac{\alpha+\beta}{\beta} \lambda_t k_t = \lambda_t A\left(\frac{\alpha b(\bar{h}_{t-2}, \bar{y}_{t-1})}{\beta}\right)^\alpha k_{t-1}^{\alpha+\beta}. \tag{13}$$

Applying Eqs. (7) and (12) to the left-hand and right-hand sides of Eq. (13) respectively, we obtain

$$q_t^k = \frac{1}{\delta(\alpha+\beta)} q_{t-1}^k - \frac{\beta}{\alpha+\beta}, \tag{14}$$

where $q_t^k := p_t^k k_t = \lambda_t k_t$, which is the value of physical capital in period t. It follows from Eq. (10) and $k_t = (\beta/(\alpha b(\bar{h}_{t-1}, \bar{y}_t)))h_t$ that $q_t^k = p_t^k k_t = (\beta/\alpha)p_t^h h_t$. Substituting $q_t^k = p_t^k k_t = (\beta/\alpha)p_t^h h_t$ into Eq. (14) yields

$$q_t^h = \frac{1}{\delta(\alpha+\beta)} q_{t-1}^h - \frac{\alpha}{\alpha+\beta}, \tag{15}$$

where $q_t^h := p_t^h h_t$, which is the value of human capital in period t. Equations (14) and (15) govern the dynamics of the values of the two types of capital, recursively, independent of other states or controls or technology choice. From Eqs. (9) and (10), it follows that $p_t^k y_t = q_{t-1}^k/(\delta\beta)$. By using this equation, Eqs. (4), (6) and (14) with $i_t^h = (\alpha/\beta)k_t$ yield

$$h_t = \alpha \max_m \{\theta_m (h_{t-1} - \eta_m)^\sigma\} \left[\frac{1}{\alpha+\beta} - \frac{\beta\delta}{\alpha+\beta}\left(\frac{1}{q_{t-1}^k}\right)\right], \tag{16}$$

where we have used the fact that $\bar{h}_{t-1} = h_{t-1}$ in equilibrium. Additionally, from $h_t = (\alpha b(\bar{h}_{t-1}, \bar{y}_t)/\beta)k_t$ and Eq. (16), we obtain

$$k_t = \beta A h_{t-1}^\alpha k_{t-1}^\beta \left[\frac{1}{\alpha+\beta} - \frac{\beta\delta}{\alpha+\beta}\left(\frac{1}{q_{t-1}^k}\right)\right]. \tag{17}$$

We can now define a *dynamic competitive equilibrium* as a sequence, $\{q_t^h, q_t^k, h_t, k_t\}$, for $t \geq 0$ that satisfies Eqs. (14)–(17) and the transversality condition, given $h_0 \geq 0$ and $k_0 \geq 0$. However, we have not yet analyzed the technology choice, to which we now turn.

Remark 2 Because the representative agent solves the lifetime utility maximization problem with the externalities exogenously given, the accumulation of physical and human capital in the competitive equilibrium may depart from their efficient accumulation. In particular, when individuals ignore the contribution to a positive human capital externality, $\bar{h}_{t-1}$, they underinvest in human capital production (a free-rider effect similar to those in Romer 1986 and Lucas 1988). However, when individuals ignore a negative output externality used to remove artificial scale effects, they tend to overinvest in human capital production. On balance, it is ambiguous whether human capital accumulation in equilibrium is faster or slower than the social optimum.

3.1 Technology choice

We now consider technology choice $\max_m\{B_m(h_{t-1})\}$ in Eq. (16) where $B_m(h_{t-1}) := \theta_m(h_{t-1} - \eta_m)^\sigma$. For illustrative purposes, we focus on three technologies ($M = 3$), subsequently followed by generalization with $M \geq 3$ technologies.

Given the tradeoff under Assumption 1, the key is to determine the cutoff human capital scales between each pair of technologies. Define such cutoffs as v_1 and v_2 so that $B_1(v_1) = B_2(v_1)$ and $B_2(v_2) = B_3(v_2)$. Note that v_1 is the cutoff of h between the first and second technologies, whereas v_2 is the cutoff between the second and third technologies. It is straightforward to show that v_1 and v_2 satisfy

$$v_1^{\sigma} = \theta_2(v_1 - \eta_2)^{\sigma} \tag{18}$$

$$\theta_2(v_2 - \eta_2)^{\sigma} = \theta_3(v_2 - \eta_3)^{\sigma}, \tag{19}$$

from which v_1 and v_2 are uniquely determined as $v_1 = \theta_2^{\frac{1}{\sigma}}\eta_2/(\theta_2^{\frac{1}{\sigma}} - 1)$ and $v_2 = (\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2)/(\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}})$.

Lemma 1 *Under Assumption 1, the following hold.*

- *If $\eta_2 \le h_{t-1} < v_1$ (resp. $h_{t-1} > v_1$), it holds that $B_1(h_{t-1}) > B_2(h_{t-1})$ (resp. $B_1(h_{t-1}) < B_2(h_{t-1})$).*
- *If $\eta_3 \le h_{t-1} < v_2$ (resp. $h_{t-1} > v_2$), it holds that $B_2(h_{t-1}) > B_3(h_{t-1})$ (resp. $B_2(h_{t-1}) < B_3(h_{t-1})$).*

Proof See Appendix A. □

Proposition 1 *Suppose that Assumption 1 holds. Suppose also that $v_1 < v_2$. Then, in $\max_m\{B_m(h_{t-1})\}$, the representative agent optimally chooses the first technology ($m = 1$) if $0 \le h_{t-1} < v_1$, the second technology ($m = 2$) if $v_1 < h_{t-1} < v_2$, and the third technology ($m = 3$) if $v_2 < h_{t-1}$.*

Proof See Appendix A. □

Note that if $h_{t-1} = v_1$, the representative agent is indifferent between the first and second technologies. In this case, it is assumed that the second technology is chosen over the first technology. Analogously, if $h_{t-1} = v_2$, the representative agent is indifferent between the second and third technologies, and the third technology is chosen over the second technology in this case. It is clear that Proposition 1 can be readily generalized to M technologies. Suppose that $v_1 < v_2 < \cdots < v_{m-1}$ where $v_i = (\theta_{i+1}^{\frac{1}{\sigma}}\eta_{i+1} - \theta_i^{\frac{1}{\sigma}}\eta_i)/(\theta_{i+1}^{\frac{1}{\sigma}} - \theta_i^{\frac{1}{\sigma}})$. Then, the representative agent optimally chooses the first technology if $0 \le h_{t-1} < v_1$, the mth technology if $v_{m-1} \le h_{t-1} < v_m$ ($m = 2, ...M - 1$), and the Mth technology if $v_{M-1} \le h_{t-1}$.

3.2 Steady states

Equations (14)–(17) form the dynamical system in equilibrium, and the steady states can be solved for in a recursive manner by deriving the capital values first. The values of human and physical capital in steady state, q^{h*} and q^{k*}, are independent of human and physical capital and the technology choice. Equations (14) and (15) yield

$$q^{h*} = \frac{\alpha\delta}{1 - \delta(\alpha + \beta)} \text{ and } q^{k*} = \frac{\beta\delta}{1 - \delta(\alpha + \beta)}. \tag{20}$$

In contrast, physical capital and human capital in steady state depend crucially on the technology choice. Suppose that the jth technology in human capital production is optimally chosen, i.e., $\max_m\{B_m(h_{t-1})\} = B_j(h_{t-1})$ ($j = 1, 2$ or 3). Since $B_j(h_{t-1})$ is concave, we can "potentially" obtain at most two steady states, say, $h^*_{j,1}$ and $h^*_{j,2}$. From Eqs. (16) and (17), it follows that

$$h^*_{j,s} = \alpha\delta\theta_j(h^*_{j,s} - \eta_j)^\sigma \tag{21}$$

$$k^*_{j,s} = \beta\delta A(h^*_{j,s})^\alpha(k^*_{j,s})^\beta, \tag{22}$$

for $s = 1, 2$. Assuming that $h^*_{j,1} < h^*_{j,2}$, we call $(q^{h*}, q^{k*}, h^*_{j,1}, k^*_{j,1})$ and $(q^{h*}, q^{k*}, h^*_{j,2}, k^*_{j,2})$ the low and high steady states, respectively.

From Eq. (21), the two potential steady states of human capital, $h^*_{j,1}$ and $h^*_{j,2}$, satisfy the following equation:

$$\Pi_j(h) := h^{\frac{1}{\sigma}} - (\delta\alpha\theta_j)^{\frac{1}{\sigma}}h + (\delta\alpha\theta_j)^{\frac{1}{\sigma}}\eta_j = 0. \tag{23}$$

Because $\Pi_j(h)$ is convex and $\hat{h}_j := \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}}$ yields a minimum of $\Pi_j(h)$, there exist two distinct real number solutions of Eq. (23) if and only if $\Pi_j(\hat{h}_j) < 0$. Formally, we have Lemma 2 below.

Lemma 2 *There exist two distinct real number solutions of Eq. (23) if and only if*

$$\eta_j - \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}}(1-\sigma) < 0. \tag{24}$$

Proof The claim of Lemma 2 follows from the fact that $\Pi_j(h)$ is convex, $\Pi_j(0) > 0$, and $\Pi_j(\hat{h}) < 0 \iff$ Eq. (24). □

In what follows, we derive conditions under which each technology actually has two steady states (including the trivial one) in the dynamical system.

First technology: $j = 1$.—Note that under Assumption 1, Eq. (23) with $j = 1$ has two solutions, $h^*_{1,1} = 0$ and

$$h^*_{1,2} := (\delta\alpha)^{\frac{1}{1-\sigma}}. \tag{25}$$

From Eq. (22), we have

$$k^*_{1,2} := (\delta\beta A)^{\frac{1}{1-\beta}}(h^*_{1,2})^{\frac{\alpha}{1-\beta}}. \tag{26}$$

Proposition 2 *Under Assumption 1, suppose that the following parameter condition holds:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2. \tag{27}$$

Then, a nontrivial steady state, $(q^{h*}, q^{k*}, h^*_{1,2}, k^*_{1,2})$, associated with the first technology exists in the dynamical system.

Proof See Appendix A. □

Second technology: $j = 2$.—We next turn to the case with $j = 2$ in which two steady states still prevail.

Proposition 3 *Under Assumption 1, suppose that the following parameter conditions hold:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \tag{28}$$

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3. \tag{29}$$

Then, two steady states, $(q^{h*}, q^{k*}, h^*_{2,1}, k^*_{2,1})$ and $(q^{h*}, q^{k*}, h^*_{2,2}, k^*_{2,2})$, associated with the second technology exist in the dynamical system.

Proof See Appendix A. □

Third technology: $j = 3$.—When the highest technology is chosen ($j = 3$), it once again features two steady states.

Proposition 4 *Under Assumption 1, suppose that the following parameter condition holds:*

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}. \tag{30}$$

Then, two steady states, $(q^{h*}, q^{k*}, h^*_{3,1}, k^*_{3,1})$ and $(q^{h*}, q^{k*}, h^*_{3,2}, k^*_{3,2})$, associated with the third technology exist in the dynamical system.

Proof See Appendix A. □

3.2.1 Summary

Propositions 2–4 imply that six steady states (including a trivial one) exist in the dynamical system with $M = 3$ if the following inequalities hold:

$$(\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \tag{31}$$

$$(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}}, \tag{32}$$

in which each technology yields two steady states.

It is straightforward to extend the analysis to the case in which the number of technologies is M with $1 = \theta_1 < \cdots < \theta_M$ and $0 = \eta_1 < \cdots < \eta_M$. In this case, $2M$ steady states appear if the following inequalities hold:

$$\begin{cases} (\delta\alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} \\ (\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}} \\ \qquad\vdots \\ (\delta\alpha\theta_{M-1})^{\frac{1}{1-\sigma}} < \eta_M < (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_M)^{\frac{1}{1-\sigma}}. \end{cases} \tag{33}$$

3.3 Local dynamics

Suppose that the jth technology has two steady states: a lower one, $(q^{h*}, q^{k*}, h^*_{j,1}, k^*_{j,1})$, and a higher one, $(q^{h*}, q^{k*}, h^*_{j,2}, k^*_{j,2})$, where $h^*_{j,1} < h^*_{j,2}$. Linearization of the dynamical system formed by Eqs. (14)–(17) with the jth technology around one of the steady states, $(q^{h*}, q^{k*}, h^*_{j,s}, k^*_{j,s})$ $(s = 1$ or $2)$, implies $\mathbf{X_t} = \tilde{\mathbf{J}}\mathbf{X_{t-1}}$, where $\mathbf{X_t} = (q^h_t - q^{h*}, q^k_t - q^{k*}, h_t - h^*_{j,s}, k_t - k^*_{j,s})'$ and

$$\tilde{\mathbf{J}} = \begin{pmatrix} \frac{1}{\delta(\alpha+\beta)} & 0 & 0 & 0 \\ 0 & \frac{1}{\delta(\alpha+\beta)} & 0 & 0 \\ 0 & J_{1,q^k}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \alpha\delta B'_j(h^*_{j,s}) & 0 \\ 0 & J_{2,q^k}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & J_{2,h}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \beta \end{pmatrix} \tag{34}$$

with $J_{n,q^k}(q^k, h, k) := \partial J_n(q^k, h, k)/\partial q^k$ and $J_{n,h}(\lambda, h, k) := \partial J_n(\lambda, h, k)/\partial h$ for $n = 1, 2$. The eigenvalues of this dynamical system are given by $\rho_1 := 1/(\delta(\alpha + \beta))$, $\rho_2 := 1/(\delta(\alpha+\beta))$, $\rho_3 := \alpha\delta B'_j(h^*_{j,s})$, and $\rho_4 = \beta$. In the dynamical system, whereas h_t and k_t are state variables that cannot jump because they are predetermined, q^h_t and q^k_t can jump, which are determined by expectations. Due to the recursive property, the dynamical system can be fully characterized. In particular, since $\rho_1, \rho_2 > 1$ and $0 < \rho_4 < 1$, the property of the local dynamics depends entirely on $\alpha\delta B'_j(h^*_{j,s})$.

Lemma 3 *Under Assumption 1, suppose that inequalities in (33) are satisfied. Then, for any j, it holds that $\alpha\delta B'_j(h^*_{j,1}) > 1$ and $0 < \alpha\delta B'_j(h^*_{j,2}) < 1$.*

Proof See Appendix A. □

Lemma 3 shows that in the linearized dynamical system around the lower steady state, $(q^{h*}, q^{k*}, h^*_{j,1}, k^*_{j,1})$, the three eigenvalues are greater than one and the absolute value of one eigenvalue is less than one. Therefore, no equilibrium sequence, $\{q^h_t, q^k_t, h_t, k_t\}$, exists that converges to the lower steady state. Additionally from Lemma 3, one can see that in the linearized dynamical system around the higher steady state, $(q^h_t, q^k_t, h^*_{j,2}, k^*_{j,2})$, the two eigenvalues are greater than one and the absolute values of the two eigenvalues are less than one. Therefore, a unique equilibrium sequence, $\{q^h_t, q^k_t, h_t, k_t\}$, exists around the higher steady state that converges to this steady state. We summarize these results in Proposition 5 below.

Proposition 5 *Under Assumption 1, suppose that the inequalities in (33) are satisfied. Consider the jth technology. Then, the following hold:*

- *There exists no equilibrium sequence, $\{q^h_t, q^k_t, h_t, k_t\}$, around the lower steady state, $(q^{h*}, q^{k*}, h^*_{j,1}, k^*_{j,1})$, that converges to this steady state.*
- *There exists a unique equilibrium sequence, $\{q^h_t, q^k_t, h_t, k_t\}$, around the higher steady state, $(q^{h*}, q^{k*}, h^*_{j,2}, k^*_{j,2})$, that converges to this steady state.*

Proof The discussion just before Proposition 5 proves the claims. □

4 Global analysis

In the previous section, we analyzed the local dynamic properties. In the analysis, it remains unclear whether an equilibrium sequence exists around the lower steady state. Moreover, even if an equilibrium sequence exists around the lower steady state, the analysis of local dynamics does not clarify where it goes. In this section, we address these questions.

4.1 Phase diagrams

In the dynamical system, the difference equations with respect to q_t^h and q_t^k, i.e., Eqs. (14) and (15), are independent of h_t and k_t, and they are solvable analytically. Solving Eqs. (14) and (15) forward, we obtain

$$q_t^k = [\delta(\alpha+\beta)]^u q_{t+u}^k + \beta\delta + \beta\delta[\delta(\alpha+\beta)] + \cdots + \beta\delta[\delta(\alpha+\beta)]^{u-1} \quad (35)$$

$$q_t^h = [\delta(\alpha+\beta)]^u q_{t+u}^h + \alpha\delta + \alpha\delta[\delta(\alpha+\beta)] + \cdots + \alpha\delta[\delta(\alpha+\beta)]^{u-1}. \quad (36)$$

It follows from the transversality condition, $\lim_{u\to\infty} \delta^u q_{t+u}^k = \lim_{u\to\infty} \delta^u q_{t+u}^h = 0$, that $\lim_{u\to\infty}[\delta(\alpha+\beta)]^u q_{t+u}^k = \lim_{u\to\infty}[\delta(\alpha+\beta)]^u q_{t+u}^h = 0$. Therefore, Eqs. (35) and (36) yield $q_t^k = \beta\delta/(1-\delta(\alpha+\beta))$ and $q_t^h = \alpha\delta/(1-\delta(\alpha+\beta))$, respectively, which implies that the equilibrium sequences of $\{q_t^k, q_t^h\}$ are uniquely determined, being equal to $\{q_t^k, q_t^h\} = \{q^{k*}, q^{h*}\}$.

Suppose that $v_1 < v_2$ under Assumption 1 with $M = 3$. Then, from Proposition 1, Eqs. (16) and (17) become

$$h_t = \begin{cases} \alpha\delta h_{t-1}^{\sigma} = \alpha\delta B_1(h_{t-1}) & \text{if } 0 \le h_{t-1} < v_1 \\ \alpha\delta\theta_2(h_{t-1}-\eta_2)^{\sigma} = \alpha\delta B_2(h_{t-1}) & \text{if } v_1 \le h_{t-1} < v_2 \\ \alpha\delta\theta_3(h_{t-1}-\eta_3)^{\sigma} = \alpha\delta B_3(h_{t-1}) & \text{if } v_2 \le h_{t-1} \end{cases} \quad (37)$$

and

$$k_t = \beta\delta A h_{t-1}^{\alpha} k_{t-1}^{\beta}. \quad (38)$$

Figure 1 provides the phase diagram of Eq. (37) when inequalities (31) and (32) hold under Assumption 1.

Remark 3 The presence of multiple steady states and the associated development patterns of flying geese or middle-income traps are driven not by externalities but by technology choice. To see this, consider the case in which $M = 1$ and hence technology choice is fully eliminated. From Fig. 1, even with the externalities via $\left(\bar{h}_{\tau-1}, \bar{y}_{\tau}\right)$ in determining human capital in period τ, there are not multiple steady states, and a unique steady state arises at $h_t = h_{1,2}^*$ as in conventional optimal growth models. These externalities do not play any essential role in the occurrence of multiple steady states. Our paper thus provides a novel channel of rich global dynamics driven

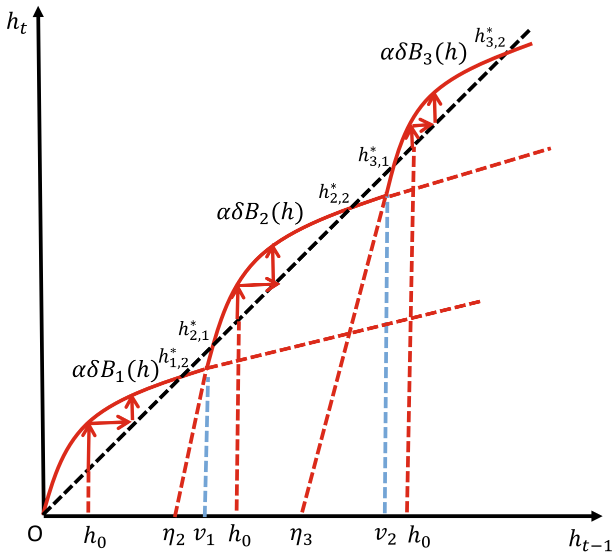

Fig. 1 Phase diagram of h_t

by endogenous technology choice, contributing methodologically to the literature on growth and economic dynamics.

4.2 Takeoff and flying geese

If the productivity of human capital formation for each technology, $B_m(\bar{h}_{t-1})/\bar{y}_t$, is high, the economy does not fall into the low- or middle-income traps. Proposition 6 below provides a condition under which the economy does not fall into a low- or middle-income trap but converges to the high steady state of the third technology even if the initial human capital is very low.

Proposition 6 *Suppose that Assumption 1 with $M = 3$ holds. Then, there exist only two steady states in the dynamical system, which are a trivial one and the high steady state of the third technology, if the following parameter condition holds:*

$$(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \Phi_2\}, \tag{39}$$

where

$$\Phi_i := \frac{\left(\theta_{i+1}^{\frac{1}{\sigma}}\eta_{i+1} - \theta_i^{\frac{1}{\sigma}}\eta_i\right)^{\frac{1}{1-\sigma}}}{\theta_i^{\frac{1}{1-\sigma}}\theta_{i+1}^{\frac{1}{1-\sigma}}\left(\theta_{i+1}^{\frac{1}{\sigma}} - \theta_i^{\frac{1}{\sigma}}\right)(\eta_{i+1} - \eta_i)^{\frac{\sigma}{1-\sigma}}}.$$

Proof See Appendix A. □

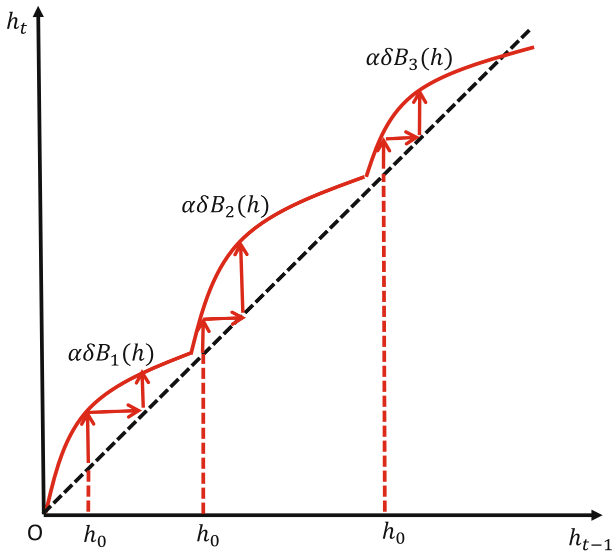

Fig. 2 Flying geese pattern

Figure 2 provides the phase diagram of Eq. (37) when inequality (39) holds under Assumption 1 with $M = 3$.

The outcome of Proposition 6 can be extended to the case in which the number of technologies is M. Under Assumption 1, there exist two steady states: one is a trivial one and the other is the high steady state of the Mth technology if and only if $(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{M-1}\}$. When this inequality holds, the economy develops from a low human capital state to a high human capital state proceeding along the M technologies.

By examining this inequality, one can see that if changes in either θ_i or η_i become negligible (but not both), then Φ_i tends to be larger, and the flying geese paradigm is less likely to arise. The two polar cases are actually different. When the barrier to human capital technology upgrading is essentially unchanged, the development pattern exhibits immediate upgrading to the highest technology M as soon as the level of human capital exceeds the barrier. In contrast, when the productivity gain from upgrading is nil, the economy is trapped in the lowest technology.

4.3 Middle-income trap

What if a certain Φ_j at an intermediate technology j breaks the inequality $(\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{M-1}\}$ stated in the previous subsection? In this case, the economy can fall into a trap with the jth technology. Specifically, if there is a Φ_j such that $\Phi_j > (\delta\alpha)^{\frac{1}{1-\sigma}} > \max\{\Phi_1, \ldots, \Phi_{j-1}, \Phi_{j+1}, \ldots, \Phi_{M-1}\}$ and if $v_{j-1} < v_j < v_{j+1}$, then the economy converges to the high steady state of the jth technology if the economy starts with low human capital. This phenomenon is depicted in Fig. 3.

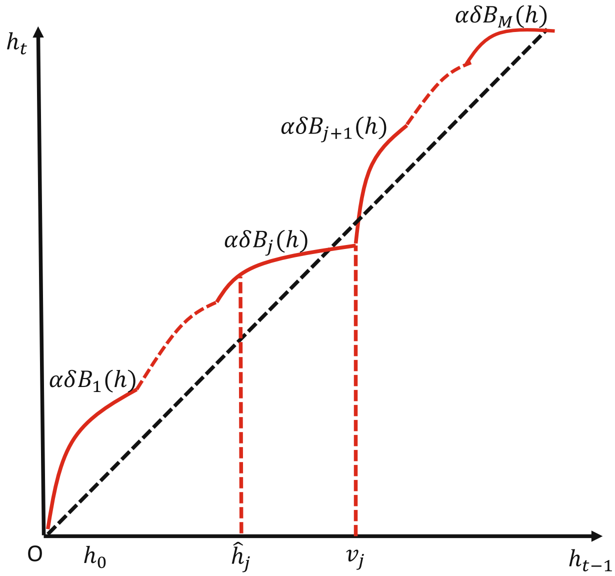

Fig. 3 Trap in the jth technology

While the trivial steady state is frequently referred to as the low-income trap, the steady state at the jth technology can be called a middle-income trap. In general, there can be more than one technology at which a middle-income country may be trapped. To establish this main result, define the set of all globally available technologies as $T := \{1, \ldots, M\}$ and the set of "interior" technologies as $I := \{2, \ldots, M-1\}$. Generically, it is conveniently summarized below.

Proposition 7 *Suppose that Assumption 1 holds. Then, there can exist nontrivial steady states in the dynamical system, featuring middle-income traps at various technologies in $J \subseteq I$ if the following parameter conditions hold:*

$$\min_{j \in J}\{\Phi_j\} > (\delta\alpha)^{\frac{1}{1-\sigma}} > \max_{m \in T \setminus J}\{\Phi_m\}, \tag{40}$$

$$v_{i-1} < v_i \ \ \textit{for all } i \in I. \tag{41}$$

Proof See Appendix A. □

We next turn to characterizing the circumstances in which a middle-income trap is more likely to arise. From the first inequality of (40), Φ_j of the jth technology in J satisfies the following inequality:

$$\Phi_j := \frac{\left(\theta_{j+1}^{\frac{1}{\sigma}}\eta_{j+1} - \theta_j^{\frac{1}{\sigma}}\eta_j\right)^{\frac{1}{1-\sigma}}}{\theta_j^{\frac{1}{1-\sigma}}\theta_{j+1}^{\frac{1}{1-\sigma}}\left(\theta_{j+1}^{\frac{1}{\sigma}} - \theta_j^{\frac{1}{\sigma}}\right)\left(\eta_{j+1} - \eta_j\right)^{\frac{\sigma}{1-\sigma}}} > (\delta\alpha)^{\frac{1}{1-\sigma}}. \tag{42}$$

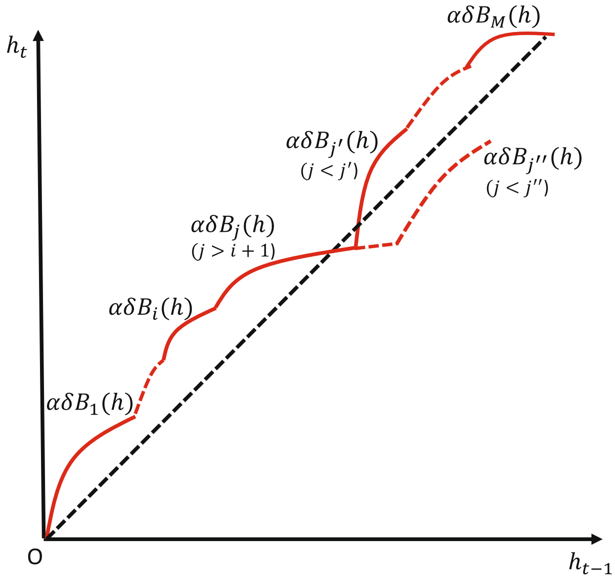

Fig. 4 Restricted middle-income trap

It is straightforward to show that inequality (41) holds for $i = j, j + 1$ if

$$\theta_j^{\frac{1}{\sigma}}(\eta_{j+1} - \eta_j) > \theta_{j-1}^{\frac{1}{\sigma}}\eta_{j+1} \tag{43}$$

$$\theta_{j+1}^{\frac{1}{\sigma}}(\eta_{j+2} - \eta_{j+1}) > \theta_j^{\frac{1}{\sigma}}\eta_{j+2}. \tag{44}$$

Let $g_{\theta_j} = \theta_{j+1}/\theta_j$ and $g_{\eta_j} = \eta_{j+1}/\eta_j$ capture the productivity gap and the barrier gap between the j+1th and the jth technologies, respectively, under inequalities (43) and (44). Then, inequality (42) reduces to

$$\frac{(\eta_j)^{1-\sigma}\left[(g_{\theta_j})^{\frac{1}{\sigma}}g_{\eta_j} - 1\right]}{\theta_j g_{\theta_j}\left[(g_{\theta_j})^{\frac{1}{\sigma}} - 1\right]^{1-\sigma}\left(g_{\eta_j} - 1\right)^{\sigma}} > \delta\alpha. \tag{45}$$

It is straightforward to show that the left-hand side of this inequality is strictly decreasing in θ_j and strictly increasing in η_j. Thus, other things being equal, a middle-income trap of the jth technology for $1 < j < M$ is likely to arise if the productivity of the jth technology is not too large and the scale barrier of the jth technology is sufficiently high.

While we have established a sufficient condition given by (40) and (41) for broadly defined middle-income traps to arise, one may inquire whether our model may support a more restricted middle-income trap, such as to satisfy the conditions outlined by Eichengreen et al. (2013). We will elaborate on this issue using Fig. 4. First, our technology choice allows for a sustained flying geese paradigm that may even feature jumps from, for example, the ith technology to the jth technology ($j > i + 1$). Continual technology upgrading in conjunction with some technology leap frogging (jumps) would ensure that the country under consideration experiences fast growth, thus satisfying the growth condition (say, at least 3.5% annually). However, when the conditions stated by (40) and (41) hold at the jth technology, a country is stuck therein. When barriers to the next generations of technologies are high and the corresponding productivity gaps are large, there could be several generations of technologies not worth adopting (such as $B_{j''}$, $j < j''$, j'). As a result, the country may stay at the jth technology for years before shifting (to the $j+1$th technology), thus causing relatively low growth (say, at least 2% lower than the pre-trap era). Of course, given appropriate values of B_j and the country's factor endowments (k, h) prevailing, it is not difficult to satisfy the middle-income condition (say, at least US$10,000 in 2005 constant PPP prices). That is, our model may support more narrowly defined middle-income traps.

Finally, we note that it is possible for a country to experience multiple traps at different points in time and that technology downgrading may arise when TFP falls and physical capital decumulates. We illustrate all such possibilities in the quantitative analysis to which we now turn.

4.4 Flying geese paradigm and technology transfer

The flying geese paradigm of Akamatsu (1962) explains technological development through technology transfer from advanced countries to developing countries in a region, particularly in East Asia with Japan as the leading country. As investigated by Keller (2002) and Alviarez et al. (2019) among others, trade and/or multinational firm activities play a crucial role in international technology transfer. Along the same lines as these notions, our theory can be readily extended and reinterpreted in a context of international technology transfer.

The main strategy is to extend the baseline framework to allow for differentiation between the usage of domestic human/knowledge capital technology and the adoption of foreign technology. Briefly, if a technology m is domestic, its barrier is η_m; if it is acquired from abroad, it is more costly, and hence the associated barrier is $\breve{\eta}_m = (1 + \mathbf{1}_m \cdot \gamma)\, \eta_m$, where γ is a foreign technology cost markup and $\mathbf{1}_m$ is an indicator function (= 1 if foreign, 0 if domestic). Thus, other than replacing η_m with $\breve{\eta}_m$, all the analyses in our theory remain unchanged. Moreover, an effective technology transfer via intermediate trade or multinationals would lower the cost markup, thereby encouraging technology upgrading and better development. Countries with better local technologies or stronger human capital to access better technologies from world leaders may thus undertake continual technology upgrading in human/knowledge accumulation and tend to experience more sustained growth to stay ahead of others in the development process.[6] This captures the flying geese paradigm described by Akamatsu (1962).

[6] Although we focus on the role played by technology transfer, this extended structure can also capture the "learning from exporting" story developed by Bond et al. (2005), where openness to trade helps exporters to accumulate knowledge for producing modern exportables, thus serving to explain partly why East Asian economies have a comparative edge over many Latin American countries.

For illustrative purposes, consider the three-technology case with $m = 1, 2, 3$ representing domestic low technology (low tech), domestic high technology (middle tech), and foreign high technology from a regionally leading country (high tech), respectively. One may consider that the third technology is transferred by multinational firms. As demonstrated in the previous section, given the productivity gap, $g_{\theta_j} = \theta_{j+1}/\theta_j$, and the barrier gap, $g_{\breve{\eta}_j} = \breve{\eta}_{j+1}/\breve{\eta}_j$, an economy with a larger θ_j and smaller $\breve{\eta}_j$ is more likely to avoid middle-income traps and experience flying geese growth. Even though the economy initially employs the low tech, it can accumulate sufficiently high human capital to adopt the middle tech if θ_2 is sufficiently high and η_2 is sufficiently low. Furthermore, even though the economy currently uses the middle tech, it can accumulate sufficiently high human capital to adopt the high tech transferred from advanced countries if θ_3 is sufficiently high and $\breve{\eta}_3$ is sufficiently low, which is particularly more likely when the cost markup γ from technology transfer is sufficiently low. To understand why the flying geese phenomenon has prevailed particularly in East Asia, one can guess that in addition to the presence of Japan as a technological leader, the domestic technologies for the development of human capital in East Asia have outperformed those in other regions. Kunieda et al. (2021) provide evidence indicating that human capital formation in East Asia has more strongly promoted the adoption of technology from abroad than in the rest of the world over the past five decades.

5 Generalization and applications

Before conducting the quantitative analysis, we note that adequate generalization is needed to apply the bare-bones theoretical model to the real world. In this section, we will discuss how to generalize the setup, modifying the conditions for flying geese and middle-income trap paradigms and then perform quantitative analyses.

5.1 Modified conditions for flying geese and middle-income trap

Whereas in the theoretical analysis, we set the size of the labor force L to one, labor force growth is observed in the actual data. By introducing this into the model, we generically obtain the law of motion of human/knowledge capital when the jth technology is adopted as follows:

$$h_t = \frac{\alpha\delta\theta_j}{n_{t+1}}(h_{t-1} - \eta_j)^{\sigma} \ \text{ if } \ v_{j-1} \leq h_{t-1} < v_j, \tag{46}$$

where $n_{t+1} := L_{t+1}/L_t$. The derivation of Eq. (46) is demonstrated in Appendix B.

In our theory, the technology set, Θ, satisfying Assumption 1 is time-invariant. However, it would evolve continually over time in reality. This is because, for instance, offshoring activities in advanced countries might cause technology innovations as demonstrated by Goel (2017), and the newly available technology set can be consistently added to the old set. Therefore, the calibrated pairs of θ and η do not necessarily satisfy Assumption 1. If Assumption 1 is actually met and there is no worker population growth, the condition for being trapped at the jth technology is given by Eqs. (40) and (41). However, there are various patterns of the calibrated θ and η that lead a country to traps. Technically speaking, if the condition for flying geese growth does not hold in switching technologies, it is highly likely that a country falls into a trap. Suppose that a country switches technologies from technology j to technology $j+1$ in period $T+1$. In this case, from Eq. (46), the transition of h_t from T to $T+2$ is given by

$$\begin{cases} h_T = \frac{\alpha\delta\theta_j}{n_{T+1}}(h_{T-1} - \eta_j)^\sigma \\ h_{T+1} = \frac{\alpha\delta\theta_{j+1}}{n_{T+2}}(h_T - \eta_{j+1})^\sigma. \end{cases} \tag{47}$$

The flying geese condition is categorized into the following three patterns, the derivations of which we discuss in Appendix B.

First case:

$$\begin{cases} \eta_j < \eta_{j+1}, \\ \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}}, \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j \\ < \alpha\delta\left(\frac{\theta_j}{n_{T+1}}\right)\left(\frac{\theta_{j+1}}{n_{T+2}}\right)(\eta_{j+1} - \eta_j)^\sigma \left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}. \end{cases} \tag{48}$$

Second case:

$$\begin{cases} \eta_{j+1} < \eta_j, \\ \frac{\theta_{j+1}}{n_{T+2}} < \frac{\theta_j}{n_{T+1}}, \\ \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} \\ > \alpha\delta\left(\frac{\theta_j}{n_{T+1}}\right)\left(\frac{\theta_{j+1}}{n_{T+2}}\right)(\eta_j - \eta_{j+1})^\sigma \left[\left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}, \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma)\left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \end{cases} \tag{49}$$

Third case:

$$\begin{cases} \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}}, \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}(\eta_{j+1} - \eta_j) < \left(\frac{\alpha\delta\sigma\theta_j}{n_{T+1}}\right)^{\frac{1}{1-\sigma}}\left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right], \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma)\left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}. \end{cases} \tag{50}$$

5.1.1 Summary

Thus, it is said that a country experiences flying geese growth if and only if inequalities (48), (49), or (50) hold. If inequalities (48), (49), and (50) all fail to hold and if human capital technology fails to advance (θ stagnating and/or η rising too much relative to the technology prevailed) and/or TFP is stalled (TFP falling), it is said that a country falls into a middle-income trap.[7] Again, the terminology of middle income is used more generally for any level of development before reaching a perpetually growing balanced growth path (that is, for all $m < M$).

5.2 Quantitative analysis

To begin, we illustrate the basis for our country selection. First, we consider four groups based on development patterns: (i) advanced countries, (ii) fast-growing economies, (iii) emerging economies with decent development speed and (iv) development laggards with frequent growth slowdowns despite earlier development. Second, the selected countries have long time series starting no later than the early 1960s. Third, all countries are qualified as middle-low, middle-high or high income countries by the mid-1980s (the middle year of our sample). Finally, we attempt to achieve balance with respect to continents, namely, Asia, Europe, North America and South America. The 14 representative countries from the four groups are as follows (see Table 4 in Appendix C for a summary of key average growth rates of each country):

- (i) advanced countries: the U.S., the U.K., Germany, and Japan, with relatively stable growth ranging from 1.7% per year (the U.S.) to 3.6% (Japan);
- (ii) fast-growing economies: Hong Kong, S. Korea, Singapore, Taiwan, the so-called Asian Tigers, all growing at more than double the rate of the U.S., ranging from 3.6% (Singapore) to 4.9% (Taiwan);
- (iii) emerging growing economies: China, Greece, and Malaysia, all growing faster than 3.1% but slower than fast-growing economies;
- (iv) development laggards: Argentina, Mexico, and the Philippines, featuring chopped growth paths with coefficients of variation all exceeding one and at slower speed than emerging growing economies.

As observed by Wang et al. (2018), most countries have decent physical capital growth but sluggish year-of-schooling-based human capital growth. Moreover, regarding the "standard" TFP, while advanced and fast-growing countries have experienced decent TFP growth no lower than 1.1% annually, the TFP growth in the three development laggards has been mediocre at rates below 0.87%.

5.2.1 Calibrating human capital technology parameters

Suppose that the jth technology is adopted from period t to period $t+1$. Then, we can update Eq. (46) by one period and obtain

$$h_{t+1} = \frac{\alpha\delta\theta_j}{n_{t+2}} \left(h_t - \eta_j\right)^{\sigma}. \tag{51}$$

It is standard in growth models to set $\delta = 0.95$. To determine the value of σ, note that we have imposed an assumption of $\sigma > \alpha$ in Sect. 2.2, which guarantees that

[7] We relegate the detailed criteria for a middle-income trap to section 5.3.2.

$\partial g_m / h_{t-1} > 0$ in equilibrium. With $\alpha = 1/3$, we thus select a country-specific σ from $[0.34, 1)$ to yield the best fit to actual output. For a given value of σ selected, we then use Eqs. (46) and (51) to derive θ_j and η_j by applying the data on human/knowledge capital constructed in Appendix C. Although we have two equations with respect to two unknowns, θ_j and η_j, two regularity conditions must be imposed: (i) a nonnegativity constraint $\eta_j \geq 0$ and (ii) a positivity constraint on $h_{t-1} - \eta_{j-1} > 0$. More concretely, when computing η_j by applying the data on human/knowledge capital, one must set its minimum at 0. Additionally, if the computed value of η_j leads to $h_{t-1} - \eta_{j-1} \leq 0$ when computing θ_j, we set $\theta_j = \theta_{j-1}$, i.e., no upgrading takes place. Refer to Appendix C for the detailed calibration procedure and the data to be used.

Three remarks are in order. First, although we have assumed that physical capital entirely depreciates in one period in our theory, the time series data on physical capital quoted from PWT9.0 are directly used in the production function to produce the "standard" TFP as explained in Appendix C, which means that in the quantitative analysis, the full depreciation of physical capital does not play any role. Second, although in calibrating the values of $\{\theta_j, \eta_j\}$, we have incorporated the information from the TFP and human capital, we have not accounted for any sources of slowdown due to capital or trade barriers. As a result, we, on the one hand, may miss some capital or trade-led traps and, on the other hand, may find that in some of our human capital-based traps, output may not grow slowly. Third, in practice, outputs fluctuate, partly driven by short-run shocks that are unrelated to the consideration of middle-income traps. Thus, some smoothing strategies must be adopted. To do so, as explained in Appendix C, we first take the 3-year moving average of each time series to remove uninteresting short-run movements. We then compute the values of $\{\theta_j, \eta_j\}$ in nonoverlapping 5-year intervals (for brevity, called episodes) for each of the 14 economies, assuming that an economy employs a certain technology for each 5-year interval. Because the data on human/knowledge capital for three sequential years are necessary to compute one set of $\{\theta_j, \eta_j\}$, we can have three sets of $\{\theta_j, \eta_j\}$ for each nonoverlapping 5-year episode. Hence, we choose the best fit to the time series of human/knowledge capital among the three sets of $\{\theta_j, \eta_j\}$ for each 5-year episode by solving the minimization problem, $\left\{\theta_j^*, \eta_j^*\right\} = \arg\min_{\{\theta_j, \eta_j\}} \sum_{s=0}^{4} \left| h_{t'+s+1} - \frac{\alpha\delta\theta_j}{n_{t'+s+2}} \left(h_{t'+s} - \eta_j\right)^{\sigma} \right|$, where t' is the starting year of a certain episode.

From Eq. (46) and $y_t = A_t h_{t-1}^{\alpha} k_{t-1}^{\beta}$, it follows that

$$\ln y_t = \alpha \ln(\alpha\delta) + \beta \ln k_{t-1} - \alpha \ln n_t + \ln A_t + \alpha \ln \theta_j^* + \alpha\sigma \ln(h_{t-2} - \eta_j^*). \quad (52)$$

By using the calibrated A, h, θ_j, and η_j and actual k and n, the fitted value of the right-hand side of Eq. (52) can be obtained. The selected σ and the associated coefficients of determination are reported in Table 6 in Appendix D. All the coefficients of determination except that of Argentina are greater than 0.99 (even that of Argentina is still high at 0.97). Thus, the fitness of the simulated time series of $\ln y$ is viewed as very good.

The calibrated θ_j and η_j are reported in Table 7 in Appendix E. The values of θ_j and η_j can be compared year by year within a country but cannot be compared across countries. This is because we use the data for the real GDP and capital stock at constant 2011 national prices. Whereas in our calibration, we assume that the standard TFP consists of human capital and institutional quality and construct the data of human capital with a semiparametric method by controlling for institutional quality, the calibrated θ_j and η_j would not only capture the knowledge capital fraction of the economy. This is a limitation in the quantitative insights obtained in what follows.

5.2.2 Flying or trapped: a unified quantitative framework

Our primary task is to identify potential traps and prolonged flying geese periods in the first three categories: advanced countries, fast-growing economies, and emerging growing economies. For the cases of traps, we will also provide potential underlying drivers, possibly due to upgrading technology slowdown, technology barriers rising, or TFP slowdown. This enables us to compare our findings with those in Eichengreen et al. (2013), where traps are identified by empirical structural breaks, and with those in Wang et al. (2018), where traps are caused by factor endowment reversal under mismatch in technology assimilation. Of course, due to different samples of countries, such comparisons would be restricted to common samples only. Table 2 summarizes our quantitative results for advanced countries, fast-growing economies, emerging growing economies, and development laggards, where we assign an indicator, 1, -1, or 0 for each period. Specifically, if any one of inequalities (48), (49), or (50) is satisfied in a period, we assign "1 " to that period and judge that the country experiences flying (denoted by $\Nearrow$). If inequalities (48), (49), and (50) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then "-1" is assigned, and we diagnose such a country as falling into a middle-income trap (denoted by $\Searrow$). Otherwise, "0" is assigned, which means that the country may not experience flying but not necessarily be trapped (denoted by $\Leftrightarrow$).

Table 2 clearly shows that most of the advanced countries and fast-growing economies have experienced prolonged periods of the flying geese paradigm with lower frequencies of traps, with the exception of Japan. In these two groups of countries, Eichengreen et al. (2013) identify several middle-income traps in the cases of Hong Kong in 1993, Japan in 1974, Korea in 1989, Singapore in 1980, Taiwan in 1995, and the U.K. in 1988 and 2002. Wang et al. (2018) find traps in the cases of Hong Kong in 1984 and Taiwan in 1999. Different from their works, we do not restrict the sample to fall into the World Bank ranges of the middle-income group. By comparison, we learn the following:

- Hong Kong: flying in the early 1980s and early 1990s (inconsistent with Eichengreen et al. and Wang et al.) but trapped in the late 1990s and early 2000s around the Asian financial crisis;
- Japan: trapped in the early 1970s (consistent with Eichengreen et al.) and most certainly trapped in the early 1980s during the second oil crisis;
- Korea: trapped in the early 1990s (not far from 1989 identified by Eichengreen et al.) and early 2000s around the Asian financial crisis;
- Singapore: trapped in the early 1980s (consistent with Eichengreen et al.) and certainly trapped in the early 2000s around the Asian financial crises;

Table 2 Flying or trapped

		late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
(i) Advanced countries												
Germany	$\sigma = 0.50$	1	1	1	1	1	1	1	−1	1	1	−1
	Flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Searrow$
Japan	$\sigma = 0.50$	1	1	1	−1	−1	−1	1	−1	−1	1	−1
	Flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Nearrow$	$\Searrow$
U.K.	$\sigma = 0.47$	1	1	1	1	1	1	−1	1	1	1	−1
	Flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$
U.S.	$\sigma = 0.65$	1	1	−1	1	1	1	1	1	0	1	1
	Flying or trapped	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Leftrightarrow$	$\Nearrow$	$\Nearrow$
(ii) Fast-growing economies												
Hong Kong	$\sigma = 0.39$			−1	1	1	1	1	1	−1	−1	1
	Flying or trapped			$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Searrow$	$\Nearrow$
S. Korea	$\sigma = 0.34$		1	1	1	1	1	1	−1	1	−1	1
	Flying or trapped		$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Searrow$	$\Nearrow$
Singapore	$\sigma = 0.50$			−1	1	1	−1	1	1	1	−1	1
	Flying or trapped			$\Searrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$	$\Nearrow$
Taiwan	$\sigma = 0.75$	1	1	1	1	1	1	1	1	1	1	−1
	Flying or trapped	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Nearrow$	$\Searrow$

Table 2 continued

		late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
(iii) Emerging growing economies												
China	$\sigma = 0.49$		−1	−1	1	−1	−1	−1	1	1	1	1
	Flying or trapped		⇘	⇘	⇗	⇘	⇘	⇘	⇗	⇗	⇗	⇗
Greece	$\sigma = 0.90$	1	1	1	1	−1	−1	−1	1	1	1	−1
	Flying or trapped	⇗	⇗	⇗	⇗	⇘	⇘	⇘	⇗	⇗	⇗	⇘
Malaysia	$\sigma = 0.52$		1	1	1	1	−1	1	1	1	−1	−1
	Flying or trapped		⇗	⇗	⇗	⇗	⇘	⇗	⇗	⇗	⇘	⇘
(iv) Development laggards												
Argentina	$\sigma = 0.44$	1	1	−1	−1	1	1	−1	1	1	−1	1
	Flying or trapped	⇗	⇗	⇘	⇘	⇗	⇗	⇘	⇗	⇗	⇘	⇗
Mexico	$\sigma = 0.36$	1	1	−1	1	1	1	−1	−1	1	−1	−1
	Flying or trapped	⇗	⇗	⇘	⇗	⇗	⇗	⇘	⇘	⇗	⇘	⇘
Philippines	$\sigma = 0.73$	−1	−1	1	−1	−1	−1	1	−1	−1	0	−1
	Flying or trapped	⇘	⇘	⇗	⇘	⇘	⇘	⇗	⇘	⇘	⇔	⇘

Note: If any one of inequalities (48), (49), or (50) is satisfied in a period, we assign "1 " to that period, judging that the country experiences flying with ⇗ denoting it. If inequalities (48), (49), and (50) all fail and if large technology downgrading with θ falling by more than 5%, large increases in barriers with η rising by more than 5%, and/or TFP slowdowns with negative TFP growth are observed, then we assign "−1," diagnosing that the country falls in a middle-income trap with ⇘ denoting it. Otherwise, we assign "0," meaning that the country may not experience flying but not necessarily be trapped with ⇔ denoting it

- Taiwan: flying in the 1990s (inconsistent with Eichengreen et al. and Wang et al.) but trapped in the late 2000s during the Great Recession (which is beyond the sample period used by Eichengreen et al.);
- U.K.: trapped in the late 1980s (consistent with Eichengreen et al.) and late 2000s during the Great Recession (again, which is beyond the sample period used by Eichengreen et al.).

Japan was trapped during the second oil crisis in the early 1980s and Korea was trapped during the Asian financial crisis in the early 2000s. Although it is difficult to find the causal linkages between specific historical events and technological choice, one may attempt to understand these outcomes in terms of economic incentives and larger barriers during severe crises. That is, when an economy faces a severe crisis, it is highly likely that individuals in the economy are not incentivized to update technologies and that lager barriers stand in front of them. Thus, the economy cannot choose better technologies even though such technologies are available.

Turning to the third group where Greece constitutes the only sample in common, both Eichengreen et al. and Wang et al. identify a single trap in 1972, while traps from the late 1970s to late 1980s are observed in our case. Relative to the first two groups, more traps are identified in the third group. For example, there are earlier traps in China through the 1960s and in the late 1970s when its human capital upgrading was stalled by the Great Leap Forward policy and the Cultural Revolution. Finally, we turn to the last group with Mexico constituting the only common sample, and five traps are identified in our paper. Although a trap in 1981 is identified by Eichengreen et al., a trap in the late 1980s is found in our paper. Additionally, we find four traps in Argentina and eight traps in the Philippines.

We summarize the appearance of flying, inconclusive and trapped episodes in each country in Table 8 in Appendix F. We find that advanced and fast-growing economies experienced the flying paradigm more than 70% of the time whereas development laggards were trapped 50% of the time. For illustrative purposes, we construct a development score by assigning 1 to each of the flying episodes, 0 to inconclusive episodes and −1 to trapped episodes as indicated in Table 8. We can thus compute the average development scores for each country and the group average: by definition, the score must fall between −1 and 1. Our results suggest that advanced and fast-growing economies have a decent average score greater than 0.477, whereas development laggards feature an average score of −0.059.

One may then inquire what are the main reasons behind the above results. In particular, in Table 9 in Appendix F, we summarize whether (i) large technology downgrading with θ falling by more than 5%, (ii) a large increase in barriers with η rising by more than 5%, or (iii) TFP slowdowns with negative TFP growth are observed. These are significant downward deviations from their respective trends in the 14-country panel. Overall, the results suggest that large drops in the efficacy of human capital technology are overwhelmingly the primary force driving a country into a middle-income trap. Large increases in barriers to human capital accumulation are also important, playing a larger role in emerging growing countries. In all countries, TFP slowdowns are only occasionally important in leading to traps. A more detailed investigation leads to further insights summarized below:

- In advanced countries, slowdown in TFP plays a small role relative to large technology downgrading in the occasional traps identified.
- In fast-growing economies, large drops in the efficacy of human capital technology are important for explaining their occasional traps.
- In emerging growing economies, all three factors play nonnegligible roles.
- In the three development laggards, all three factors also play nonnegligible roles. While their roles are comparable in the cases of Argentina and the Philippines, slowdown in TFP plays a lesser role in the case of Mexico.

5.2.3 Growth accounting

To this end, we conduct a growth accounting analysis to examine the long-run quantitative importance of human capital technology and barriers in economic growth. Taking the time difference of Eq. (52) yields

$$1 = \beta \frac{\triangle \ln k_{t-1}}{\triangle \ln y_t} - \alpha \frac{\triangle \ln n_t}{\triangle \ln y_t} + \frac{\triangle \ln A_t}{\triangle \ln y_t} + \alpha \frac{\triangle \ln \theta_j^*}{\triangle \ln y_t} + \alpha\sigma \frac{\triangle \ln \left(h_{t-2} - \eta_j^*\right)}{\triangle \ln y_t}. \quad (53)$$

This is our growth accounting basis: the five components represent the growth effects of physical capital accumulation, employment growth, TFP advancement, human capital technology upgrading, and human capital barrier reduction (conditional on human capital accumulation). All logged differences can be measured by the growth rates of the variables. In our growth accounting, we consider the growth rate for thirty years to investigate a relatively longer structural growth effect of each variable. Note from Eq. (53) that each term on the right-hand side measures the percentage contribution of each variable corresponding to a percentage change in y_t. Table 3 presents the growth accounting results of the two 30-year episodes.[8]

In advanced countries, with the exception of Japan, human capital technology and human capital barriers are overwhelmingly more important drivers than physical capital accumulation (and other variables). The second 30-year period in Japan represents an unusual case in which human capital technology and human capital barriers have negative contributions to economic growth, whereas the contribution of physical capital accumulation exceeds 100%. This outcome might reflect the stagnation in the Japanese economy after an era of high economic growth. Japan experiences a growth process similar to those of the other three advanced countries; however, the main drivers of growth in Japan appear quite different from those of the other three countries. On average, in advanced countries, human capital technology upgrading and human capital barrier reduction account for 51% of economic growth during the first 30-year window and for 39% during the second 30-year window.

In the fast-growing economies except Korea, human capital technology and human capital barriers are more important drivers than the other variables, as in advanced countries. Whereas Korea experiences a high rate of growth in both periods, physical

[8] In conducting the growth accounting with Eq. (53), we have an error term as seen in the last column of Table 3. In Table 3, we choose the results that have minimum errors among the initial and last five sets of the 30-year growth accounting.

Table 3 Growth accounting

Growth accounting (%)	Physical capital	Labor	TFP	Human capital technology	Human capital Barriers	Error
(i) Advaned countries						
Germany $\sigma = 0.50$						
$\Delta \ln(y) = 1.071$ (1954–1984)	44%	1%	3%	17%	34%	1%
$\Delta \ln(y) = 0.478$ (1979–2009)	44%	0%	9%	15%	32%	0%
Japan $\sigma = 0.50$						
$\Delta \ln(y) = 1.684$ (1954–1984)	62%	0%	2%	13%	23%	0%
$\Delta \ln(y) = 0.590$ (1978–2008)	114%	1%	9%	−23%	−1%	0%
UK $\sigma = 0.47$						
$\Delta \ln(y) = 0.633$ (1952–1982)	44%	2%	2%	37%	15%	0%
$\Delta \ln(y) = 0.526$ (1979–2009)	27%	0%	6%	33%	34%	0%
U.S. $\sigma = 0.65$						
$\Delta \ln(y) = 0.525$ (1954–1984)	35%	−1%	1%	26%	40%	−1%
$\Delta \ln(y) = 0.490$ (1979–2009)	32%	3%	1%	46%	18%	0%
(ii) Fast-growing economies						
Hong Kong $\sigma = 0.39$						
$\Delta \ln(y) = 1.348$ (1963–1993)	32%	0%	3%	35%	29%	1%
$\Delta \ln(y) = 1.010$ (1980–2010)	39%	2%	0%	47%	12%	0%
S. Korea $\sigma = 0.34$						
$\Delta \ln(y) = 1.456$ (1957–1987)	47%	−1%	12%	29%	13%	0%
$\Delta \ln(y) = 1.451$ (1978–2008)	49%	0%	27%	16%	8%	0%
Singapore $\sigma = 0.50$						
$\Delta \ln(y) = 1.369$ (1963-1993)	33%	0%	8%	38%	20%	1%
$\Delta \ln(y) = 0.969$ (1979–2009)	39%	1%	15%	32%	14%	−1%

Table 3 continued

Growth accounting (%)	Physical capital	Labor	TFP	Human capital technology	Human capital Barriers	Error
Taiwan $\sigma = 0.75$						
$\Delta \ln(y) = 1.720$ (1953-1983)	36%	0%	10%	15%	39%	0%
$\Delta \ln(y) = 1.356$ (1979-2009)	43%	1%	7%	11%	39%	−1%
(iii) Emerging growing economies						
China $\sigma = 0.49$						
$\Delta \ln(y) = 0.454$ (1957–1987)	82%	1%	20%	−5%	2%	0%
$\Delta \ln(y) = 1.730$ (1981–2011)	45%	1%	11%	14%	29%	0%
Greece $\sigma = 0.90$						
$\Delta \ln(y) = 1.350$ (1955–1985)	41%	0%	−1%	1%	58%	1%
$\Delta \ln(y) = 0.329$ (1976–2006)	51%	−1%	30%	0%	22%	−2%
Malaysia $\sigma = 0.52$						
$\Delta \ln(y) = 1.068$ (1957–1987)	32%	0%	2%	26%	40%	0%
$\Delta \ln(y) = 0.826$ (1980–2010)	49%	0%	0%	15%	35%	1%
(iv) Development laggards						
Argentina $\sigma = 0.44$						
$\Delta \ln(y) = 0.362$ (1955–1985)	64%	−1%	−14%	−1%	51%	1%
$\Delta \ln(y) = 0.060$ (1978–2008)	98%	−4%	134%	−198%	71%	−1%
Mexico $\sigma = 0.36$						
$\Delta \ln(y) = 0.776$ (1953–1983)	27%	−1%	−3%	31%	46%	0%
$\Delta \ln(y) = -0.088$ (1979–2009)	113%	10%	36%	−268%	9%	0%
Philippines $\sigma = 0.73$						
$\Delta \ln(y) = 0.402$ (1954–1984)	61%	0%	−2%	−20%	61%	0%
$\Delta \ln(y) = 0.153$ (1979–2009)	86%	8%	10%	9%	−13%	0%

Note: The percentage contribution of each variable (%) is provided corresponding to $\Delta \ln(y)$. In conducting the growth accounting with Eq. (53), we have an error term; accordingly, we show two results among the initial and last five sets of the 30-year growth accounting that have the minimum errors for each country capital accumulation is the main driver of growth, consistent with its reliance on large business conglomerates (chaebols). In the fast-growing economies, human capital technology upgrading and human capital barrier reduction, on average, contribute to 55% and 45% over the two 30-year windows, respectively.

Regarding emerging growing economies, human capital technology and human capital barriers continue to play relatively important roles in the second 30-year window in China and are more important than physical capital (and other variables) in the first 30-year window in Greece and in both 30-year windows in Malaysia. On average, human capital technology upgrading and human capital barrier reduction account for 41% and 38% of economic growth over the two 30-year windows, respectively.

Among the development laggards, in the first 30-year window, human capital technology and human capital barriers are more important drivers than the other variables in Mexico and remain quite important in the other two countries, accounting for more than 40% of their economic growth. Over the second 30-year window when growth rates in Argentina and Mexico turn out to be too close to zero, growth accounting results in large percentage contributions. By excluding these extreme accounting estimates, human capital technology upgrading and human capital barrier reduction on average contribute to 56% and 25% of economic growth over the two 30-year windows, respectively.

On the whole, in more than half of the episodes (15 out of 28 30-year episodes in 14 countries), human capital technology upgrading and human capital barrier reduction account for more than half of economic growth. When eliminating the two extreme accounting estimates and averaging over the remaining 26 episodes, human capital technology upgrading and human capital barrier reduction contribute to 51% during the first 30-year window and 37% during the second 30-year window. This indicates that omitting the roles played by human capital technology upgrading and human capital barrier reduction could lead to biased outcomes by a noticeable margin. Whether to improve human capital efficacy or to mitigate the associated technological barriers depends critically on the stage of the development. The results also offer a novel endogenous TFP story: once the efficacy of and the barriers to human capital accumulation are incorporated via endogenous technology choice, the residual TFP component no longer plays as large a role.

6 Concluding remarks

We have constructed a simple growth model with endogenous technology choice in human/knowledge capital accumulation in which we have been able to establish a rich array of equilibrium development paradigms, including middle-income traps and flying geese growth. We have identified the productivity of the prevailing technology, the productivity jump of technology upgrading and the hike in technology scale barriers as the key drivers of the emergence of different paradigms in equilibrium. The different combinations of these factors in conjunction with TFP help us understand the different development patterns facing different countries at different development stages.

What are the policy implications? We have highlighted the nexus between the incentives for and the obstruction of technology choice along the dynamic production process and the transitional growth of the economy. We have identified that in fast-growing economies the efficacy of human capital accumulation plays a more crucial role, but in emerging economies, reducing the barriers to human capital accumulation is more important for growth outcomes. For example, beyond education, policies that facilitate better knowledge exchange or adult training are valuable for stimulating better technology choices and thereby promoting growth. In contrast, protective policies are generally harmful in the longer run because they reward the continual use of current technology and discourage individuals from selecting a more advanced technology.

Along these lines, an interesting avenue for future research is to introduce limitations to knowledge formation. This would allow for growing over cycles rooted in human capital technology choice. As such, it would complement the R&D-based theory of growth and cycles pioneered by Matsuyama (1999) and Jovanovic (2009) and the threshold externality-based theory by Kaas and Zink (2007). Another interesting line of extension is to incorporate trade into the current framework to examine how the interactions between trade and global talent flows may result in different development patterns, an issue recently explored in Jung (2019) using a Roy model. Finally, it would also be interesting to consider multiple dimensions of technology choice with physical and human capital upgrading and barriers. In so doing, one may separate different sources of traps into those based on physical and human capital. Of course, to accomplish any of these would require further simplification of the basic structure and more restrictive assumptions. These are beyond the scope of the present paper and left for future research.

Appendix A. Proofs of Lemmata and Propositions

In Appendix A, we provide proofs of various lemmata and propositions.

Proof of Lemma 1

To prove the first claim, define $\Psi_1(h) := B_1(h) - B_2(h)$ for $h \geq \eta_2$. Then, we have

$$\Psi_1(h) = h^\sigma - \theta_2(h - \eta_2)^\sigma \tag{A.1}$$

and

$$\Psi_1'(h) = \sigma[h^{\sigma-1} - \theta_2(h - \eta_2)^{\sigma-1}]. \tag{A.2}$$

Since $\Psi_1'(h) < \sigma[(h - \eta_2)^{\sigma-1} - \theta_2(h - \eta_2)^{\sigma-1}] < 0$ under Assumption 1, $\Psi_1(h)$ is a decreasing function for $h \geq \eta_2$. Additionally, it follows that $\Psi_1(\eta_2) > 0$ and $\lim_{h\to\infty} \Psi_1(h) = \lim_{h\to\infty} h^\sigma(1 - \theta_2(1 - \eta_2/h)^\sigma) = -\infty$. Therefore, the first claim of Lemma 1 holds. To prove the second claim, define $\Psi_2(h) := B_2(h) - B_3(h)$ for $h \geq \eta_3$. Then, we have

$$\Psi_2(h) = \theta_2(h - \eta_2)^\sigma - \theta_3(h - \eta_3)^\sigma \tag{A.3}$$

and

$$\Psi_2'(h) = \theta_2\sigma(h-\eta_2)^{\sigma-1} - \theta_3\sigma(h-\eta_3)^{\sigma-1}. \tag{A.4}$$

Since $\Psi_2'(h) < \sigma[\theta_2(h-\eta_3)^{\sigma-1} - \theta_3(h-\eta_3)^{\sigma-1}] < 0$ under Assumption 1, $\Psi_2(h)$ is a decreasing function for $h \geq \eta_3$. Additionally, it follows that $\Psi_2(\eta_3) > 0$ and $\lim_{h\to\infty}\Psi_2(h) = \lim_{h\to\infty}\theta_2(h-\theta_2)^{\sigma}[1-(\theta_3/\theta_2)[(h-\eta_3)/(h-\eta_2)]^{\sigma}] = -\infty$.Therefore, the second claim of Lemma 1 holds. □

Proof of Proposition 1

From Lemma 1, it follows that $B_3(h_{t-1}) > B_2(h_{t-1})$ if $h_{t-1} > v_2$. Thus, if $h_{t-1} > v_2$, the third technology is preferred to the second technology. From Lemma 1, it follows that $B_2(h_{t-1}) > B_3(h_{t-1})$ if $\eta_3 \leq h_{t-1} < v_2$. Moreover, if $\eta_2 \leq h_{t-1} < \eta_3$, the third technology is not applicable. Thus, if $\eta_2 \leq h_{t-1} < v_2$, the second technology is preferred to the third technology. Analogously, from Lemma 1, it follows that $B_2(h_{t-1}) > B_1(h_{t-1})$ if $h_{t-1} > v_1$. Thus, if $h_{t-1} > v_1$, the second technology is preferred to the first technology. From Lemma 1, it follows that $B_1(h_{t-1}) > B_2(h_{t-1})$ if $\eta_2 \leq h_{t-1} < v_1$. Moreover, if $0 \leq h_{t-1} < \eta_2$, the second technology is not applicable. Thus, if $0 \leq h_{t-1} < v_1$, the first technology is preferred to the second technology. Then, we have a desired conclusion. □

Proof of Proposition 2

It suffices to show that $h^*_{1,2} < v_1$. From Eq. (27), it holds that $h^*_{1,2} < \eta_2 < v_1$, and Eq. (26) gives $k^*_{1,2}$. Then, the desired conclusion holds. □

Proof of Proposition 3

From the convexity of $\Pi_2(h)$ and since $k^*_{2,s}$ is obtained from Eq. (22), it suffices to show the following three claims: *Claim 1*: Eq. (23) with $j = 2$ has two distinct real number solutions, *Claim 2*: $v_1 < \hat{h}_2 < v_2$, and *Claim 3*: $\Pi_2(v_1) > 0$ and $\Pi_2(v_2) > 0$.

Claim 1

The second inequality of (28) is equivalent to inequality (24) with $j = 2$. From Lemma 2, Claim 1 is proven.

Claim 2

Under Assumption 1, it follows from inequalities in (28) that $\theta_2 > 1/\sigma^{\sigma}$, or equivalently,

$$\frac{\theta_2^{\frac{1}{\sigma}}}{\theta_2^{\frac{1}{\sigma}}-1} < \frac{1}{1-\sigma}. \tag{A.5}$$

From inequality (A.5) and the second inequality of (28), we obtain

$$v_1 = \frac{\theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_2^{\frac{1}{\sigma}}-1} < \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} = \hat{h}_2. \tag{A.6}$$

Under Assumption 1, inequality (29) yields

$$\hat{h}_2 = \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < (\delta\alpha\theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < \frac{\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_3^{\frac{1}{\sigma}}-\theta_2^{\frac{1}{\sigma}}} = v_2. \tag{A.7}$$

Claim 2 is proven by inequalities (A.6) and (A.7).

Claim 3

From Eq. (18), it holds that $v_1 = \theta_2^{\frac{1}{\sigma}}(v_1 - \eta_2)$. Therefore, it follows that

$$\Pi_2(v_1) = v_1(v_1^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}). \tag{A.8}$$

Since $\eta_2 < v_1$, from Eq. (A.8) and the first inequality of (28), we obtain

$$\Pi_2(v_1) = v_1(v_1^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}) > v_1(\eta_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha)^{\frac{1}{\sigma}}) > 0. \tag{A.9}$$

From Claim 1, it follows that

$$\Pi_2(\hat{h}_2) < 0. \tag{A.10}$$

From inequality (29), we have $\hat{h}_2 < \eta_3$ and

$$\Pi_2(\eta_3) := \eta_3(\eta_3^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + (\delta\alpha\theta_2)^{\frac{1}{\sigma}}\eta_2 > 0. \tag{A.11}$$

From inequalities (A.10), (A.11), and $\eta_3 < v_2$ with the convexity of $\Pi_2(h)$, it holds that

$$\Pi_2(v_2) > 0. \tag{A.12}$$

Claim 3 is proven by inequalities (A.9) and (A.12). □

Proof of Proposition 4

From the convexity of $\Pi_3(h)$ and since $k^*_{3,s}$ is obtained from Eq. (22), it suffices to show the following three claims: *Claim 1*: Eq. (23) with $j = 3$ has two distinct real number solutions, *Claim 2*: $v_2 < \hat{h}_3$, and *Claim 3*: $\Pi_3(v_2) > 0$.

Claim 1

The second inequality of (30) is equivalent to inequality (24) with $j = 3$. From Lemma 2, Claim 1 is proven.

Claim 2

Under Assumption 1, it follows from inequalities (30) that $\theta_2^{\frac{1}{\sigma}}/\theta_3^{\frac{1}{\sigma}} < \sigma$, or equivalently,

$$\frac{\theta_3^{\frac{1}{\sigma}}}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \frac{1}{1-\sigma}. \tag{A.13}$$

From inequality (A.13) and the second inequality of (30), we obtain

$$v_2 = \frac{\theta_3^{\frac{1}{\sigma}}\eta_3 - \theta_2^{\frac{1}{\sigma}}\eta_2}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \frac{\theta_3^{\frac{1}{\sigma}}\eta_3}{\theta_3^{\frac{1}{\sigma}} - \theta_2^{\frac{1}{\sigma}}} < \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_3)^{\frac{1}{1-\sigma}} = \hat{h}_3, \tag{A.14}$$

which is Claim 2.

Claim 3

From Eq. (19), it holds that $\theta_2^{\frac{1}{\sigma}}(v_2 - \eta_2) = \theta_3^{\frac{1}{\sigma}}(v_2 - \eta_3)$. Therefore, it follows that

$$\Pi_3(v_2) = v_2(v_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}}. \tag{A.15}$$

Since $\eta_3 < v_2$, from Eq. (A.15) and the first inequality of (30), we obtain

$$\begin{aligned}\Pi_3(v_2) &= v_2(v_2^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}} > v_2(\eta_3^{\frac{1-\sigma}{\sigma}} - (\delta\alpha\theta_2)^{\frac{1}{\sigma}}) \\ &\quad + \eta_2(\delta\alpha\theta_2)^{\frac{1}{\sigma}} > 0,\end{aligned} \tag{A.16}$$

which is Claim 3. □

Proof of Lemma 3

Eq. (21) rewrites $\alpha\delta B_j'(h_{j,s}^*)$ as

$$\alpha\delta B_j'(h_{j,s}^*) = \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(h_{j,s}^*)^{\frac{1-\sigma}{\sigma}}}. \tag{A.17}$$

Since $h_{j,1}^* < \hat{h}_j = \sigma^{\frac{\sigma}{1-\sigma}}(\delta\alpha\theta_j)^{\frac{1}{1-\sigma}} < h_{j,2}^*$, the use of Eq. (A.17) yields

$$\alpha\delta B_j'(h_{j,1}^*) > \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(\hat{h}_j)^{\frac{1-\sigma}{\sigma}}} = 1 \tag{A.18}$$

and

$$0 < \alpha\delta B_j'(h_{j,2}^*) < \frac{\sigma(\alpha\delta\theta_j)^{\frac{1}{\sigma}}}{(\hat{h}_j)^{\frac{1-\sigma}{\sigma}}} = 1 \tag{A.19}$$

Inequalities (A.18) and (A.19) are the desired conclusions. □

Proof of Proposition 6

From Eq. (21) and the definitions of v_1 and v_2, it suffices to show that $v_i < \alpha\delta\theta_{i+1}(v_i - \eta_{i+1})^\sigma$ for $i = 1, 2$. $v_i < \alpha\delta\theta_{i+1}(v_i - \eta_{i+1})^\sigma$ is equivalent to $(\alpha\delta)^{\frac{1}{1-\sigma}} > \Phi_i$. From the last inequality, we obtain the desired conclusion. □

Proof of Proposition 7

For any $j \in J$, from the first inequality of (40), it follows that $\alpha\delta B_j(v_j) < v_j$. Additionally, from the second inequality of (40), it follows that $v_1 < \alpha\delta B_1(v_1)$. By technology choice, the equation for the transitional dynamics with respect to h_t (i.e., Eq. (37) extended to the case of M technologies) is continuous. Therefore, there can be more than one steady state with multiple technologies in J that feature a middle-income trap. □

Appendix B. Derivation of the flying geese condition

Derivation of Eq. (46)

When the growth in the labor force is introduced into the model, Eqs. (3) and (4) become

$$k_\tau = g_0(i_\tau^k)/n_{\tau+1} \tag{B.1}$$

and

$$h_\tau = \max_{m=1,2,\ldots,M} \{g_m(i_\tau^h; \bar{h}_{\tau-1}, \bar{y}_\tau)\}/n_{\tau+1}, \tag{B.2}$$

where $n_{\tau+1} = L_{\tau+1}/L_\tau$ whereas Eq. (2) remains the same. Then, the first-order conditions are obtained as follows:

$$\lambda_t = \frac{1}{c_t}, \tag{B.3}$$

$$\lambda_t = \left(\frac{\delta\alpha b(\bar{h}_{t-1}, \bar{y}_t) y_{t+1}}{n_{t+1} h_t} \right) \lambda_{t+1}, \tag{B.4}$$

$$\lambda_t = \left(\frac{\delta\beta y_{t+1}}{n_{t+1} k_t} \right) \lambda_{t+1}, \tag{B.5}$$

$$n_{t+1}\lambda_t = p_t^k = b(\bar{h}_{t-1}, \bar{y}_t) p_t^h. \tag{B.6}$$

The same manipulations as those in Sect. 3 yield exactly the same equations with respect to $q_t^k = p_t^k k_t$ and $q_t^h = p_t^h h_t$ as Eqs. (14) and (15), respectively. Additionally, with respect to h_t and k_t, it follows that

$$h_t = \frac{\alpha \max_m \{B_m(h_{t-1})\}}{n_{t+1}} \left[\frac{1}{\alpha+\beta} - \frac{\beta\delta}{\alpha+\beta} \left(\frac{1}{q_{t-1}^k} \right) \right], \tag{B.7}$$

and

$$k_t = \frac{\beta A h_{t-1}^\alpha k_{t-1}^\beta}{n_{t+1}} \left[\frac{1}{\alpha+\beta} - \frac{\beta\delta}{\alpha+\beta} \left(\frac{1}{q_{t-1}^k} \right) \right]. \tag{B.8}$$

Again, from the same manipulations as those in Sect. 4, we obtain the law of motion of human/knowledge capital when $j = \arg\max_m \{B_m(h_{t-1})\}$ as follows:

$$h_t = \frac{\alpha\delta\theta_j}{n_{t+1}} (h_{t-1} - \eta_j)^\sigma \ \text{ if } \ v_{j-1} \le h_{t-1} < v_j$$

which is Eq. (46).

Flying geese condition

To investigate the flying geese condition, we define functions such that $f_j(x) := (\alpha\delta\theta_j/n_{T+1})(x-\eta_j)^\sigma$ and $f_{j+1}(x) := (\alpha\delta\theta_{j+1}/n_{T+2})(x-\eta_{j+1})^\sigma$, which are the right-hand sides of (47). We also define z that denotes a potential intersection of the transition equations in (47) such that $f_j(z) = f_{j+1}(z)$, or equivalently,

$$z := \frac{\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j}{\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}}. \tag{B.9}$$

Furthermore, we define z_j and z_{j+1} such that $f'_j(z_j) = f'_{j+1}(z_{j+1}) = 1$, or equivalently,

$$z_{j+s} := \eta_{j+s} + \left(\frac{\alpha\delta\sigma\theta_{j+s}}{n_{T+1+s}}\right)^{\frac{1}{1-\sigma}} \quad (s = 1, 2). \tag{B.10}$$

The flying geese condition is categorized into three patterns, as illustrated in Fig. 5. In the first case, whereas it holds that $\eta_{j+1} > \eta_j$ as in Assumption 1, it may not necessarily hold that $\theta_{j+1} > \theta_j$. In the second (third) case, it holds (may hold) that $\eta_{j+1} < \eta_j$, which contradicts Assumption 1.

First case.—It follows from Fig. 5 that the parameter condition for the first case is given by $\eta_j < \eta_{j+1} < z$ and $f_j(z) > z$. From Eq. (B.9), these inequalities can be transformed into

$$\begin{cases} \eta_j < \eta_{j+1}, \\ \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}}, \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j \\ < \alpha\delta \left(\frac{\theta_j}{n_{T+1}}\right)\left(\frac{\theta_{j+1}}{n_{T+2}}\right)(\eta_{j+1} - \eta_j)^{\sigma} \left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}, \end{cases} \tag{B.11}$$

which is Eq. (48).

Second case.—As illustrated in Fig. 5, the second flying geese condition is given by $\eta_{j+1} < \eta_j < z$, $f_j(z) < z$, and $f_{j+1}(z_{j+1}) > z_{j+1}$. Eqs. (B.9) and (B.10) rewrite these inequalities as follows:

$$\begin{cases} \eta_{j+1} < \eta_j, \\ \frac{\theta_{j+1}}{n_{T+2}} < \frac{\theta_j}{n_{T+1}}, \\ \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} \eta_j - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} \eta_{j+1} \\ > \alpha\delta \left(\frac{\theta_j}{n_{T+1}}\right)\left(\frac{\theta_{j+1}}{n_{T+2}}\right)(\eta_j - \eta_{j+1})^{\sigma} \left[\left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}, \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma)\left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}, \end{cases} \tag{B.12}$$

which is Eq. (49).

Third case.—We obtain the necessary condition for the third case such that (i) $\eta_{j+1} < \eta_j$ and $z < \eta_j$, which is consistent with $f_{j+1}(x)$'s solid line in Fig. 5, or (ii)

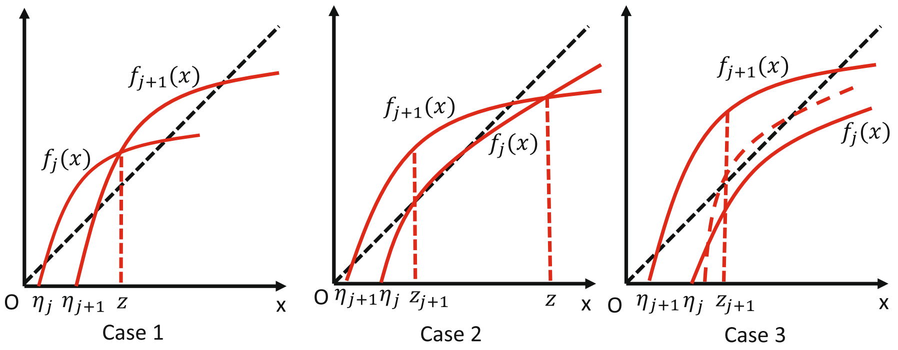

Fig. 5 Flying geese condition

$\eta_{j+1} > \eta_j$ and $z > \eta_j$, which is consistent with $f_{j+1}(x)$'s dotted line, ignoring the case in which $\eta_{j+1} = \eta_j$. Conditions (i) and (ii) are unified as $(\eta_{j+1} - \eta_j)(z - \eta_j) > 0$. In addition to this condition, $z < z_j$, and $z_{j+1} < f(z_{j+1})$ are essential for the third case. Eqs. (B.9) and (B.10) rewrite these inequalities as follows:

$$\begin{cases} \frac{\theta_{j+1}}{n_{T+2}} > \frac{\theta_j}{n_{T+1}}, \\ \left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} (\eta_{j+1} - \eta_j) < \left(\frac{\alpha\delta\sigma\theta_j}{n_{T+1}}\right)^{\frac{1}{1-\sigma}} \left[\left(\frac{\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{\sigma}} - \left(\frac{\theta_j}{n_{T+1}}\right)^{\frac{1}{\sigma}}\right], \\ \eta_{j+1} < \sigma^{\frac{\sigma}{1-\sigma}} (1-\sigma) \left(\frac{\alpha\delta\theta_{j+1}}{n_{T+2}}\right)^{\frac{1}{1-\sigma}}, \end{cases} \tag{B.13}$$

which is Eq. (50).

Appendix C. Calibration procedure

To apply the theoretical model to the real world by means of calibration, we allow TFP, A, in Eq. (1) to be time-varying such that $y_t = A_t h_{t-1}^{\alpha} k_{t-1}^{\beta}$. For convenience, we define an "endogenous" TFP as $\tilde{A}_t := A_t h_{t-1}^{\alpha}$ and then rewrite the production function as $y_t = \tilde{A}_t k_{t-1}^{\beta}$. Once we obtain the data for y_t and k_{t-1} with β being fixed, $\tilde{A}_t = y_t / k_{t-1}^{\beta}$ can be computed. Note that $\tilde{A}_t$ is the TFP in the "standard" growth accounting in the absence of human capital. $\tilde{A}_t$ is composed of TFP, A_t, and human/knowledge capital, h_{t-1}. Then, it is convenient to rewrite endogenous TFP as

$$\ln \tilde{A}_t^{1/\alpha} = \ln A_t^{1/\alpha} + \ln h_{t-1},$$

so we need to estimate a transformed TFP function $\ln A_t^{1/\alpha}$ and a human capital function h_{t-1}. The transformed TFP function is assumed to be parametric, being linear in the institutional quality index, d_t: $\ln A_t^{1/\alpha} = \lambda d_t$, where the data for the institutional quality index are assembled from the Economic Freedom of the World 2019, as explained below. The human capital function is a function of the measured (per

Table 4 Average growth rates

Average growth (%)	y	k	h	A	$\tilde{A}$
Advanced countries					
Germany	2.7	3.2	4.5	0.1	1.6
Japan	3.6	7.4	3.2	0.1	1.1
U.K.	1.8	1.9	3.3	0.05	1.2
U.S.	1.7	1.7	3.4	0.01	1.1
Fast growing economies					
Hong Kong	3.7	3.6	7.3	0.1	2.5
S. Korea	4.3	5.6	4.1	1.0	2.4
Singapore	3.6	3.6	6.7	0.2	2.4
Taiwan	4.9	5.4	8.4	0.36	3.1
Emerging growing economies					
China	3.5	5.6	3.5	0.4	1.6
Greece	2.7	3.2	4.3	0.18	1.6
Malaysia	3.1	3.3	6.0	0.03	2.0
Development laggards					
Argentina	1.1	1.4	1.8	-3.9×10^{-5}	0.63
Mexico	1.2	1.3	2.0	0.05	0.77
Philippines	1.4	1.6	2.6	0.01	0.87

Note: Output per worker (y) and physical capital per worker (k) are directly computed by using the data of PWT9.0. Human/knowledge capital per worker (h) and TFP (A) are obtained as indicated above. $\tilde{A}$ is the TFP in the standard growth accounting, which is computed from y and k

capita) human capital index, $\hat{h}_{t-1}$: $h_{t-1} = \vartheta(\hat{h}_{t-1})$, the data of which are taken from the Penn World Table, version 9.0 (PWT9.0), again as explained below. Accordingly, we have

$$\ln \tilde{A}_t^{\frac{1}{\alpha}} = \tilde{\vartheta}(\hat{h}_{t-1}) + \lambda d_t, \tag{C.1}$$

where $\tilde{\vartheta}(\hat{h}_{t-1}) = \ln \vartheta(\hat{h}_{t-1})$. We estimate Eq. (C.1) by a semiparametric method with $\alpha = \beta = 1/3$, as frequently used in the growth accounting literature with both physical and human capital, following Mankiw et al. (1992). Overall, other than α and β, all technology parameters, including the TFP function and human capital technology, are country-specific.

Data

We draw all the data from PWT9.0 (Feenstra et al. 2015), except the institutional quality index. We assemble the annual data for the 14 countries over the period 1950-2014, although the starting year varies slightly based on data availability. To obtain the per worker output, y, and the per worker physical capital, k, we use the real GDP

Table 5 Estimation of λ

Countries	λ	Standard error	P-value
(i) Advaned countries			
Germany	0.1502	0.0332	0.000
Japan	0.1120	0.0501	0.029
U.K.	0.0451	0.0224	0.049
U.S.	0.0483	0.0166	0.005
(ii) Fast growing economics			
Hong Kong	0.1487	0.0718	0.043
S. Korea	0.5511	0.1275	0.000
Singapore	0.3373	0.1586	0.038
Taiwan	0.1606	0.0340	0.000
(iii) Emerging growing economies			
China	0.2437	0.0721	0.001
Greece	0.2157	0.0536	0.000
Malaysia	0.0696	0.0333	0.041
(iv) Development laggards			
Argentina	0.0716	0.0347	0.043
Mexico	0.1226	0.0439	0.007
Philippines	0.0292	0.0149	0.063

Note: λ is estimated from Eq. (C.1) with a semi-parametric method for each country

at constant 2011 national prices (rgdpna), the capital stock at constant 2011 national prices (rkna), and the number of persons engaged (emp) in PWT9.0. We compute $\tilde{A}_t = y_t/k_{t-1}^{\beta}$ from the per worker output and the per worker physical capital. For $\hat{h}$, we use the human capital index, based on years of schooling and returns to education (hc) in PWT9.0. The human capital index in PWT9.0 is based on the average years of schooling data in Barro and Lee (2013). To eliminate short-run movements, we take the 3-year moving average of each variable.

The data on the index of institutional quality, d, are collected from the Economic Freedom of the World 2019 (Gwartney et al. 2019). Specifically, we use the Economic Freedom of the World (EFW) index, which is a comprehensive measure of the consistency of institutions and policies with economic freedom in a country. The EFW index consists of five dimensions: (i) Size of Government, (ii) Legal System and Property Rights, (iii) Sound Money, (iv) Freedom to Trade Internationally, and (v) Regulation of Credit, Labor and Business. The index data are available in 5-year intervals from 1950 to 2000 and at an annual frequency from 2000 to 2017. The index from 1950 to 1965 does not reflect the regulation dimension. See Gwartney et al. (2019) for details.

Table 5 reports λ estimated from Eq. (C.1) for each country. Since we apply a semiparametric method, the information of the human capital index is reflected in the estimation of λ (see Robinson 1988). In all cases, λ is positive and significant at the conventional level, which means that an improvement in institutional quality enhances TFP. Once λ is estimated, $A_t = \exp(\alpha\lambda d_t)$ is obtained, and accordingly, we have human/knowledge capital such that $h_{t-1} = [\tilde{A}_t/\exp(\alpha\lambda d_t)]^{1/\alpha}$, which is then used for the calibration of human capital technology parameters.

Data summary and estimation of λ

The average growth rates of the output per worker (y), physical capital per worker (k), human/knowledge capital per worker (h), and TFP (A) of each country are summarized in Table 4. Additionally, Table 5 indicates λ estimated for each country.

Calibration procedure for θ_j and η_j

We use Eqs. (46) and (51) to derive θ_j and η_j by applying the data on human/knowledge capital constructed above. As explained in Sect. 5.2, however, we impose two natural constraints: (i) a nonnegativity constraint on η_j and (ii) a positivity constraint on $h_{t-1} - \eta_j$. The detailed calibration procedure is given in the following.

- By using Eqs. (46) and (51) and taking into account η_j's minimum setting, we compute

$$\tilde{\eta}_j = \max\left\{ \frac{n_{t+2}^{\frac{1}{\sigma}} h_{t+1}^{\frac{1}{\sigma}} h_{t-1} - n_{t+1}^{\frac{1}{\sigma}} h_t^{\frac{1+\sigma}{\sigma}}}{n_{t+2}^{\frac{1}{\sigma}} h_{t+1}^{\frac{1}{\sigma}} - n_{t+1}^{\frac{1}{\sigma}} h_t^{\frac{1}{\sigma}}}, 0 \right\}. \tag{C.2}$$

- If $h_{t-1} - \tilde{\eta}_j > 0$, we set $\eta_j = \tilde{\eta}_j$ and compute θ_j by using Eq. (46) as follows:

$$\theta_j = \frac{n_{t+1} h_t}{\alpha\delta \left(h_{t-1} - \tilde{\eta}_j\right)^{\sigma}}. \tag{C.3}$$

- If $h_{t-1} - \tilde{\eta}_j \leq 0$, we set $\theta_j = \theta_{j-1}$ and compute η_j by using Eq. (46) and taking into account η_j's minimum setting as follows:

$$\eta_j = \max\left\{ h_{t-1} - \left(\frac{n_{t+1} h_t}{\alpha\delta\theta_{j-1}}\right)^{\frac{1}{\sigma}}, 0 \right\}. \tag{C.4}$$

Appendix D. Determination of σ and fitness of the time series of ln y

Table 6 provides the coefficients of determination for the selected σ.

Table 6 Determination of σ

Countries	σ	R^2	Countries	σ	R^2
(i) Advaned countries			*(ii) Fast growing economies*		
Germany	$\sigma = 0.50$	0.99925	Hong Kong	$\sigma = 0.39$	0.99883
Japan	$\sigma = 0.50$	0.99972	S. Korea	$\sigma = 0.34$	0.99854
U.K.	$\sigma = 0.47$	0.99944	Singapore	$\sigma = 0.50$	0.99832
U.S.	$\sigma = 0.65$	0.99921	Taiwan	$\sigma = 0.75$	0.99959
(iii) Emerging growing economies			*(iv) Development laggards*		
China	$\sigma = 0.49$	0.99567	Argentina	$\sigma = 0.44$	0.97063
Greece	$\sigma = 0.90$	0.99813	Mexico	$\sigma = 0.36$	0.99552
Malaysia	$\sigma = 0.52$	0.99930	Philippines	$\sigma = 0.73$	0.99264

Note: The coefficients of determination regarding the fitted value of the right-hand side of Eq. (52) are provided for the selected σ

Appendix E. Calibrated θ_j and η_j

Table 7 Calibrated θ_j and η_j

		late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
(i) Advanced countries												
Germany	θ	4.423	4.767	5.292	5.633	5.851	6.348	9.892	7.972	11.169	17.884	7.268
	η	0.402	0.485	0.536	0.649	0.413	0.737	1.978	1.245	2.871	4.257	0
Japan	θ	7.322	8.717	12.722	10.232	10.529	8.344	11.534	8.063	7.210	7.263	6.999
	η	0.911	1.386	2.749	2.118	2.372	0	2.980	1.170	0.416	0	0
U.K.	θ	5.274	5.965	6.534	5.488	5.822	9.190	7.426	11.613	12.806	13.587	9.728
	η	0.656	0.790	0.980	0.059	0.052	1.797	0.147	2.781	3.393	3.829	0
U.S.	θ	7.199	8.097	6.523	6.985	7.222	7.751	8.680	11.542	11.865	12.648	14.222
	η	1.462	1.732	0	0.262	0.435	0.249	1.704	5.384	5.448	7.006	10.598
(ii) Fast-growing economies												
Hong Kong	θ			1.319	2.100	3.122	4.988	4.992	6.422	4.989	9.842	13.148
	η			0.044	0.116	0.221	0.538	0.436	0.745	0.441	1.606	2.925
S. Korea	θ		0.365	0.406	0.592	0.602	0.827	1.017	0.938	1.318	1.064	1.209
	η		0.012	0.014	0.022	0.022	0.037	0.048	0.051	0.086	0.063	0.059
Singapore	θ			1.753	1.830	2.742	1.944	3.241	3.746	3.098	5.124	6.886
	η			0.044	0.048	0.150	0.015	0.175	0.245	0.196	0.490	0.788
Taiwan	θ	2.386	2.471	2.967	2.881	3.170	4.341	3.804	4.464	4.336	6.050	4.999
	η	0.016	0	0.054	0	0.010	0.221	0	0.233	0	0.946	0
(iii) Emerging growing economies												
China	θ		0.301	0.453	0.467	0.386	0.897	0.539	0.929	1.097	1.305	1.403
	η		0.001	0.005	0.001	0	0.014	0	0.016	0.031	0.032	0.033
Greece	θ	2.938	3.354	3.638	3.577	3.525	3.013	6.695	3.177	3.474	3.575	2.812
	η	0	0	0	0	0.029	0	0.766	0	0	0	0

Table 7 continued

		late 50s	early 60s	late 60s	early 70s	late 70s	early 80s	late 80s	early 90s	late 90s	early 00s	late 00s
Malaysia	θ		2.813	3.651	3.354	4.749	3.627	6.058	6.395	5.661	8.614	6.931
	η		0.143	0.247	0	0.422	0.012	0.671	0.776	0.692	1.517	0.119
(iv) Development laggards												
Argentina	θ	5.735	11.026	10.678	8.748	18.526	6.900	6.768	7.808	13.518	11.003	12.961
	η	0	2.096	2.125	0	4.869	0	1.268	0.087	3.108	1.928	3.143
Mexico	θ	5.142	7.822	8.134	7.380	14.113	8.868	6.202	8.255	7.042	5.159	6.922
	η	0.629	1.101	1.271	0	2.863	1.724	0	1.563	1.068	0	1.298
Philippines	θ	2.406	2.477	3.322	2.824	2.794	2.260	2.563	2.209	3.890	3.566	2.909
	η	0	0	0.100	0	0	0.051	0.002	0	0.145	0.117	0

Note: The values of calibrated θ_j and η_j can be compared year by year within a country and cannot be compared across countries. This is because we use the data for the real GDP and capital stock at constant 2011 national prices assembled from PWT9.0

Appendix F. Development scores and underlying drivers of the middle-income trap

Table 8 Development scores

	Flying	Inconclusive	Trapped	Development score
(i) Advaned countries				
Germany	9	0	2	0.636
Japan	5	0	6	−0.091
U.K.	9	0	2	0.636
U.S.	9	1	1	0.727
Average	8.00 (72.7%)	0.25 (2.3%)	2.75 (25%)	0.477
(ii) Fast growing economics				
Hong Kong	6	0	3	0.333
S. Korea	8	0	2	0.600
Singapore	6	0	3	0.333
Taiwan	10	0	1	0.818
Average	7.50 (77%)	0.00 (0%)	2.25 (23%)	0.538
(iii) Emerging growing economies				
China	5	0	5	0.000
Greece	7	0	4	0.272
Malaysia	7	0	3	0.400
Average	6.33 (61.3%)	0.00(0%)	4.00 (38.7%)	0.225

Table 8 continued

	Flying	Inconclusive	Trapped	Development score
(iv) Development laggards				
Argentina	7	0	4	0.272
Mexico	6	1	5	0.091
Philippines	2	1	8	−0.545
Average	5.00 (44%)	0.67 (6.0%)	5.67 (50%)	− 0.059

Note: As indicated in Tables 2, 3, 4 and 5, 1 is assigned to each of the flying episodes, 0 to inconclusive, and −1 to trapped, and then the average development scores are computed for each country and each group

Table 9 Underlying drivers to middle-income trap

Drivers	Large technology downgrading (θ)	Large increase in barriers (η)	TFP slowdowns
(i) Advanced countries			
	Germany: early 90s		Germany: late 00s
	late 00s		
	Japan: early 70s	Japan: late 70s	Japan: early 70s
	early 80s/early 90s		early 90s/late 00s
	late 90s		
	U.K.: late 80s		U.K.: late 00s
	late 00s		
	U.S.: late 60s		U.S.: late 60s
(ii) Fast growing economies			
	Hong Kong: late 60s	Hong Kong: early 00s	Hong Kong: late 90s
	late 90s		
	S. Korea: early 90s		
	early 00s		
	Singapore: early 80s	Singapore: early 00s	Singapore: late 90s
	late 90s		
	Taiwan: late 00s		
(iii) Emerging growing economies			
	China: early 60s	China: late 60s	China: early 60s
	late 70s/late 80s	early 80s	
	Greece: late 70s	Greece: late 80s	Greece: late 70s
	early 80s/late 00s	late 00s	early 80s/late 00s
	Malaysia: early 80s	Malaysia: early 00s	Malaysia: late 00s
	late 00s		

Table 9 continued

Drivers	Large technology downgrading (θ)	Large increase in barriers (η)	TFP slowdowns
(iv) Development laggards			
	Argentina: early 70s	Argentina: late 60s	Argentina: late 60s
	early 00s	late 80s	early 70s/early 00s
	Mexico: late 80s	Mexico: late 60s	Mexico: late 00s
	early 00s	early 90s/late 00s	
	Philippines: early 60s	Philippines: late 50s	Philippines: early 50s
	late 70s/early 80s	early 70s/early 80s	early 70s/early 80s
	early 90s/late 00s	late 90s	late 90s

Note: We focus on (i) large technology downgrading with θ falling by more than 5%, (ii) large increase in barriers with η rising by more than 5%, and (iii) TFP slowdowns with negative TFP growth

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