Demographic Transition and Growth

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A. Introduction

- **Long-term trend in population (Western Europe, Maddison 1982/1995):**
  - **500-1500: rising pop, no per capita output growth**
  - **1500-1870: rising pop, rising per capita output**
  - **after 1870: declining pop, rising per capita output**

- **Fertility decline since mid-1800:**
  - **reducing infant mortality enables lower fertility given the same desired quantity of children**
  - **rising income makes education more affordable and encourages the trade-off of quantity for quality in childbearing**
  - **rising opportunity cost increases childbearing cost**

- **Roles of demographic transition played in economic development:**
  - **high fertility is associated with low development (Malthusian trap)**
  - **high within-country fertility differentials are associated with high income inequalities (Kremer-Chen 2000)**

- **Literature:**
  - **Kuznets (1958), Becker (1960), Easterlin (1968): quality-quantity trade-off in childbearing under static, partial equilibrium**
  - **Lee (1987): quality-quantity trade-off in a dynamic, non-optimizing setup**
  - **Razin & Ben-Zion (1975), Becker-Barro (1989), *Wang-Yip-Scotese (1994)*, Palivos (1995): quality-quantity trade-off in exogenous growth setups**
  - ***Becker-Murphy-Tamura (1990)*: quality-quantity trade-off in an endogenous growth setting**
  - **Galor-Weil (2000): long-run demographic transition**
  - **de la Croix and M Doepke (2003): larger differentials in fertility induce widened inequalities**
  - ***Greenwood-Seshadri-Vandenbroucke* (2005): baby boom/baby burst**
  - **Moav (2005): better returns to child-labor can be a barrier to growth**
  - **Soares (2005): reduction in infant mortality leads to lower fertility rate and higher human capital accumulation, thereby raising long-run growth**
  - **Doepke-Zilibotti (2008): changing children quality via intergenerational behavioral transmission**
  - **Bara-Leukhina (2009): demographic transition and industrial revolution**
  - **Jayachandran & Lleras-Muney (2009): maternal mortality**
  - ***Jones (2022)*: unintended consequences of a declining population**
  - **Jiang-Lien-Wang-Y. Wang (2023): timing of childbearing**

B. Fertility Choice in Dynamic General Equilibrium: Wang-Yip-Scotese (1994)

- **Key: endogenous consumption/leisure/childbearing choice**

1. Optimization:

max $\int_0^\infty [U(c(t)) + V(\chi(t), \mu(t))] e^{-\rho t}\, dt$

s.t. $\chi(t) + \ell(t) + s(\mu(t)) = 1$

$c(t) + \dot{k}(t) = f(k(t), \ell(t)) - \mu(t)k(t)$

where childbearing/childrearing requires time and resources cost

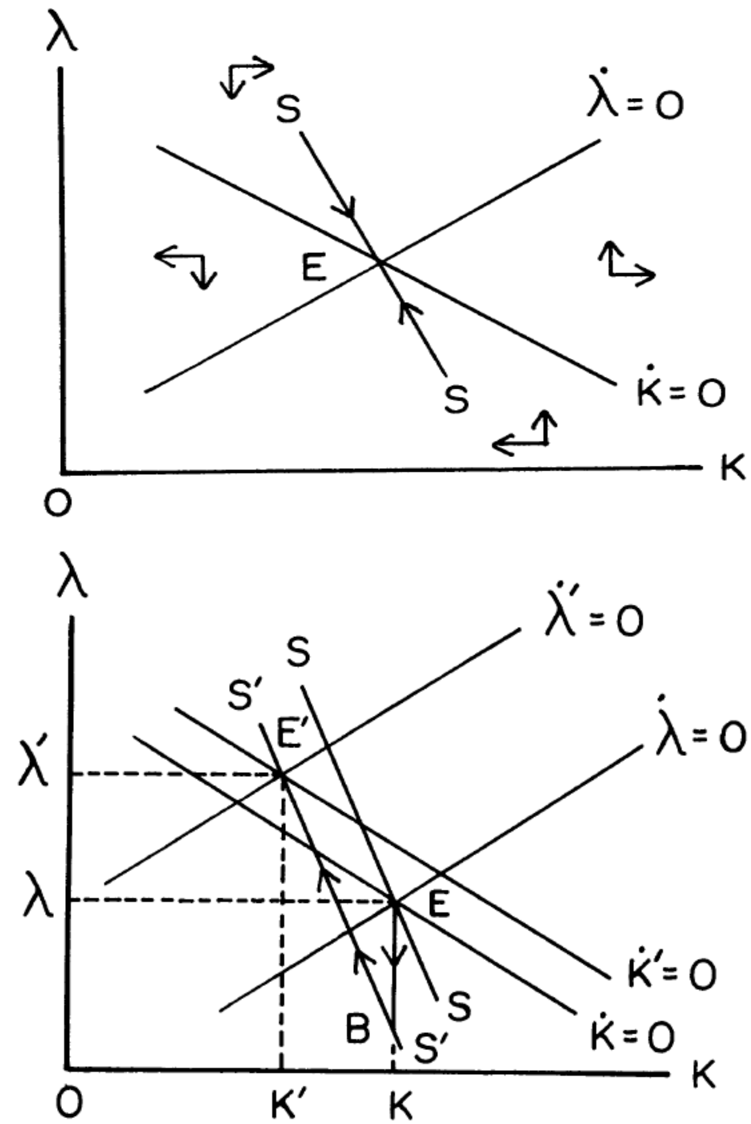
2. Equilibrium

- **Use the time constraint to eliminate χ and the FOCs (w.r.t. c, ℓ, μ) to derive:**

$c = c(k, \lambda); \quad \ell = \ell(k, \lambda); \quad \mu = \mu(k, \lambda)$

where $c_k = 0,\ c_\lambda < 0,\ \ell_k > 0,\ \ell_\lambda > 0,\ \mu_k < 0$ **and** $\mu_\lambda < 0$

- The above functionals can be plugged into the BC and the Euler eq. to derive the 2x2 dynamical system in (k,λ)
  - steady state: E
  - stable saddle: SS
- Comparative dynamics: effect of a preference shift toward the quantity of children (μ):
  - on impact, λ drops and k remains unchanged => μ ↑ and y ↓
  - in transition, λ rises and k reduces => μ may ↑ or ↓(↑ if k effect dominates); y may ↓ or ↑ (↓ if k-effect dominates)
- Empirical findings:
  - In response to a preference shift toward fertility, μ ↑ and y ↓
  - Fertility shocks explain about 80% of movements in fertility and 25% of movements in output



C. Quantity-Quality Trade-off in Becker-Murphy-Tamura (1990)

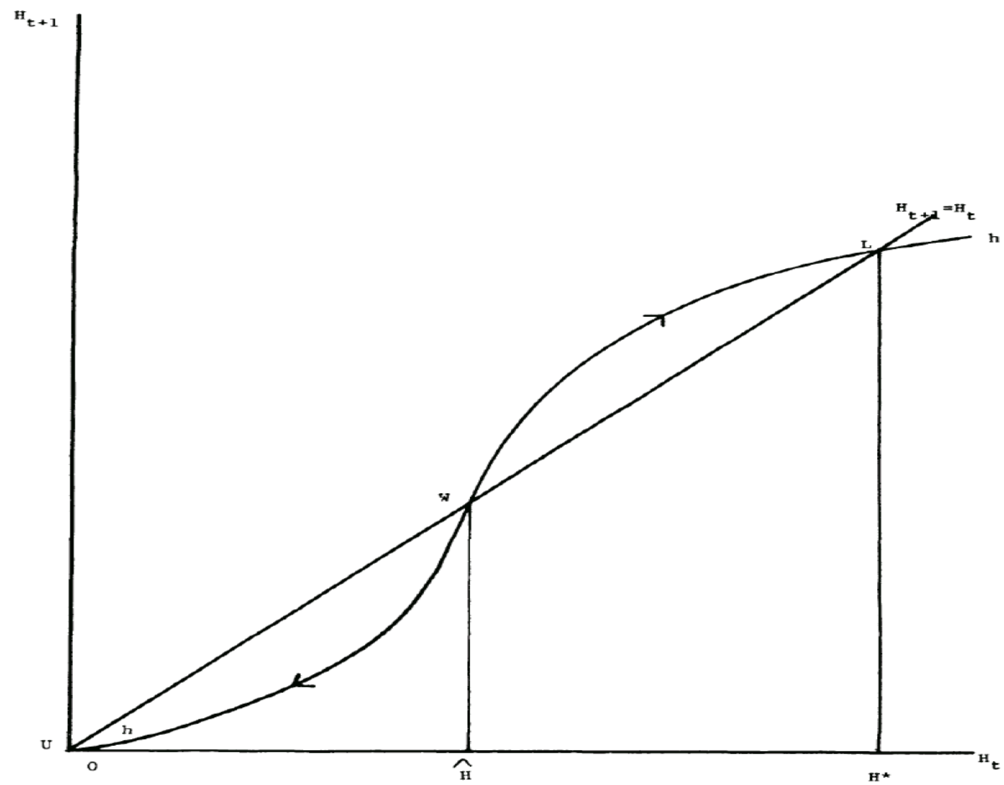
- **Becker-Barro (1989) dynasty preference + Ben-Paroth (1967) human capital accumulation**

1. The Model

- **Dynasty preference:** $V_t = u(c_t) + a(n_t)n_t V_{t+1}$**, with the degree of altruism per child, *a*(n), decreasing in the number of children (n)**
- **Human capital accumulation:** $H_{t+1} = Ah_t(bH^0 + H_t)^\beta$**, which depends on**
  - **child endowment (H^0)**
  - **parental human capital (H_t)**
  - **parental time devoted to childrearing (h)**
- **Budget constraint:** $c_t + fn_t = Dl_t(dH^0 + H_t)$
  - **total spending = consumption + childrearing expenses**
  - **output is linear in effective labor (D·L)**
- **Time constraint:** $T = l_t + n_t(v + h_t)$
  - **v = the exogenous time devoted to childrearing**
  - **h = the endogenous time input into childrearing**

2. Equilibrium (with b=d=β=1, $a(n) = \alpha n^{-\epsilon}$, $u(c) = \frac{c^{\sigma}}{\sigma}$):

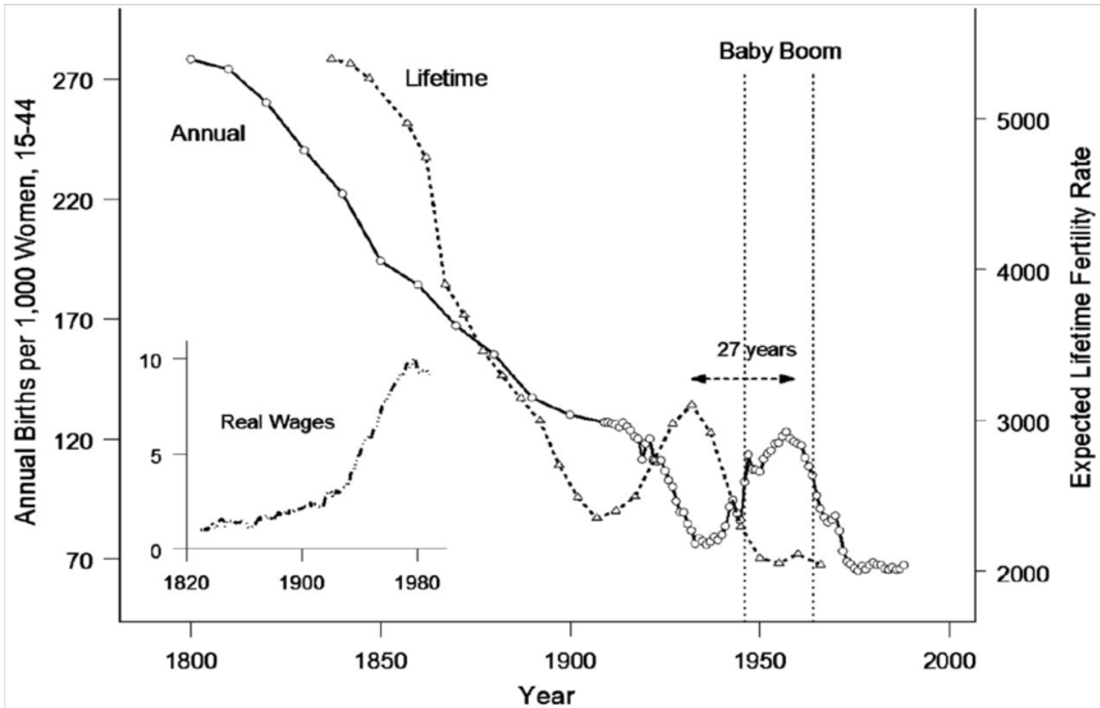
- **(c)** $\alpha^{-1} n_t^{\epsilon} \left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma} \geq R_{ht} = A(l_{t+1} + h_{t+1} n_{t+1})$
- **(n)** $(1 - \epsilon)\alpha n_t^{-\epsilon} V_{t+1} = u'(c_t)[(v + h_t)(H^0 + H_t) + f]$
- **Two BGPs:**
  - **high v => H = h = 0**
  - **low v => H > 0, h > 0**
- **Main Findings:**
  - **multiple equilibria**
  - **negative relatioship between population growth and output growth**
  - **higher exogenous childbearing time cost (v) can lead to a low growth trap**



- **Problems:**
  - **unrealistic characterization of the high/low equilibrium: low trap is more likely due to the following factors**
    - **subsistence consumption**
    - **infant mortality**
    - **high child-labor demand**
    - **poor early childhood development**
  - **inability to characterize the longer demographic transition between 1500 and 1870, particularly the observation that population growth and output growth are positively related, which is likely due to:**
    - **subsistence consumption**
    - **strong income effect of fertility choice at low level of economic development**

D. Baby Boom and Baby Bust: Greenwood-Seshadri-Vandenbroucke (2005)

- **Basic Idea:**
  - **better job opportunity or higher market wage for women**
    - => **higher opportunity costs for childbearing**
    - => **secular decline in fertility and increase in children education**
    - => **sustained increase in human capital & sustained growth**
  - **technical progress in producing household durables**
    - => **lower costs of rearing children**
    - => **post WWII baby boom**



1. The Model

- **OG with I+J period lived agents, with I periods of childhood and J periods of adulthood, fecund only in the 1st period of adulthood**
- **Age 1 adult preference:**

$$\sum_{j=0}^{J-1} \beta^j U(c_{t+j}^{j+1}) + \frac{1-\beta^J}{1-\beta} Q(n_t^1) = \sum_{j=0}^{J-1} \beta^j \phi \ln(c_{t+j}^{j+1} + \mathfrak{c}) + \frac{1-\beta^J}{1-\beta}(1-\phi)\frac{(n_t^1)^{1-\nu}-1}{1-\nu}$$

  - **$\mathfrak{c} > 0$ => consumption good is not necessary**
  - **φ higher => value children less**
- **Home production of children: $n_t^1 = H(l_t^1; x_t) = x_t (l_t^1)^{1-\gamma}$, depending on household technology *x***
- **Cost function of childbearing:**

$$C(n_t^1; x_t, w_t, g_t) = \min_{l_t^1}\{w_t l_t^1 + w_t g_t : n_t^1 = H(l_t^1, x_t)\} = w_t \left(\frac{n_t^1}{x_t}\right)^{1/(1-\gamma)} + w_t g_t$$

  - **g = the market price for purchasing household technology**
  - **homogeneous of degree one in w**

- **Budget constraint:** $\sum_{j=0}^{J-1} q_{t+j}c_{t+j}^{j+1} = \sum_{j=0}^{J-1} q_{t+j}w_{t+j} - q_t C(n_t^1; x_t, w_t, g_t)$

  where $q_{t+j} = q_{t+j-1}/r_{t+j}$ with $q_1 = 1$
- **Market goods production:** $y_t = F(k_t, e_t; z_t) = z_t k_t^{\alpha} e_t^{1-\alpha}$
- **Capital accumulation:** $k_{t+1} = \delta k_t + i_t$

2. Optimization

- **Household optimization:**
  - **intertemporal consumption tradeoff:** $U_1(c_{t+j}^{j+1}) = \beta r_{t+j+1} U_1(c_{t+j+1}^{j+2})$
  - **fertility decision:** $\frac{1-\beta^J}{1-\beta} Q_1(n_t^1) = U_1(c_t^1) C_1(n_t^1; x_t, w_t, g_t)$
- **Firm optimization:** $F_1(k_t, e_t; z_t) = r_t - \delta$ **and** $F_2(k_t, e_t; z_t) = w_t$

3. Equilibrium

- **Population:** $p_{t+1}^{j+1} = p_t^j$, for $j = 1, \dots, J-1$ **and** $p_{t+i}^1 = p_{t+i-I}^1 n_{t+i-I}^1$, for $i = 1, \dots, I$
- **MBC (flow):** $p_t^1 c_t^1 + p_t^2 c_t^2 + \cdots + p_t^J c_t^J + i_t = y_t$
- **Loanable fund market clearing (stock):**

- $\forall$ **t ≥ 0:** $p_t^1 s_{t+1}^2 + p_t^2 s_{t+1}^3 + \cdots + p_t^{J-1} s_{t+1}^J = k_{t+1}$
- **initial:** $p_1^2 s_1^2 + \cdots + p_1^J s_1^J = k_1$

- **Main results:**
  - **better household technology x => higher fertility n**
  - **lower household technology price g => higher fertility**
  - **higher market good technology z => lower fertility**

4. Calibration

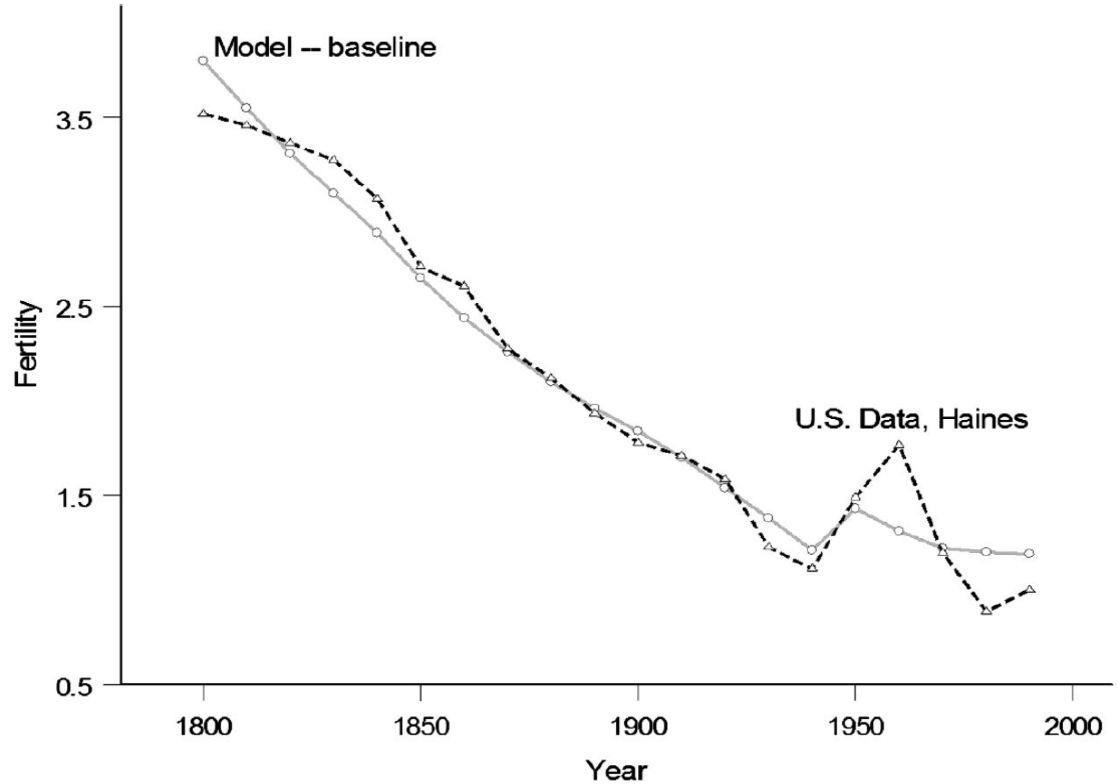
- **Baseline parametrization:**

Tastes	$\beta = 0.98^{10}$, $\phi = 0.83$, $\mathfrak{c} = 0.49$, $\mathrm{N} = 0.00$
Market technology	$\alpha = 0.30$, $\delta = (1 - 0.05)^{10}$
Home technology	$\gamma = 0.20$, $\{g_t\}_{t \in \mathcal{T}} = 0$, $x_{1800} = 2.53$, $x_{1950} = 2.83$, $x_{1960} = 2.84$
Generational structure	$I = 2$, $J = 4$

- **Model fitness**
  - **it generally fits the data well**
  - **by adjusting estimation weights, it fits the baby boom/bust better**

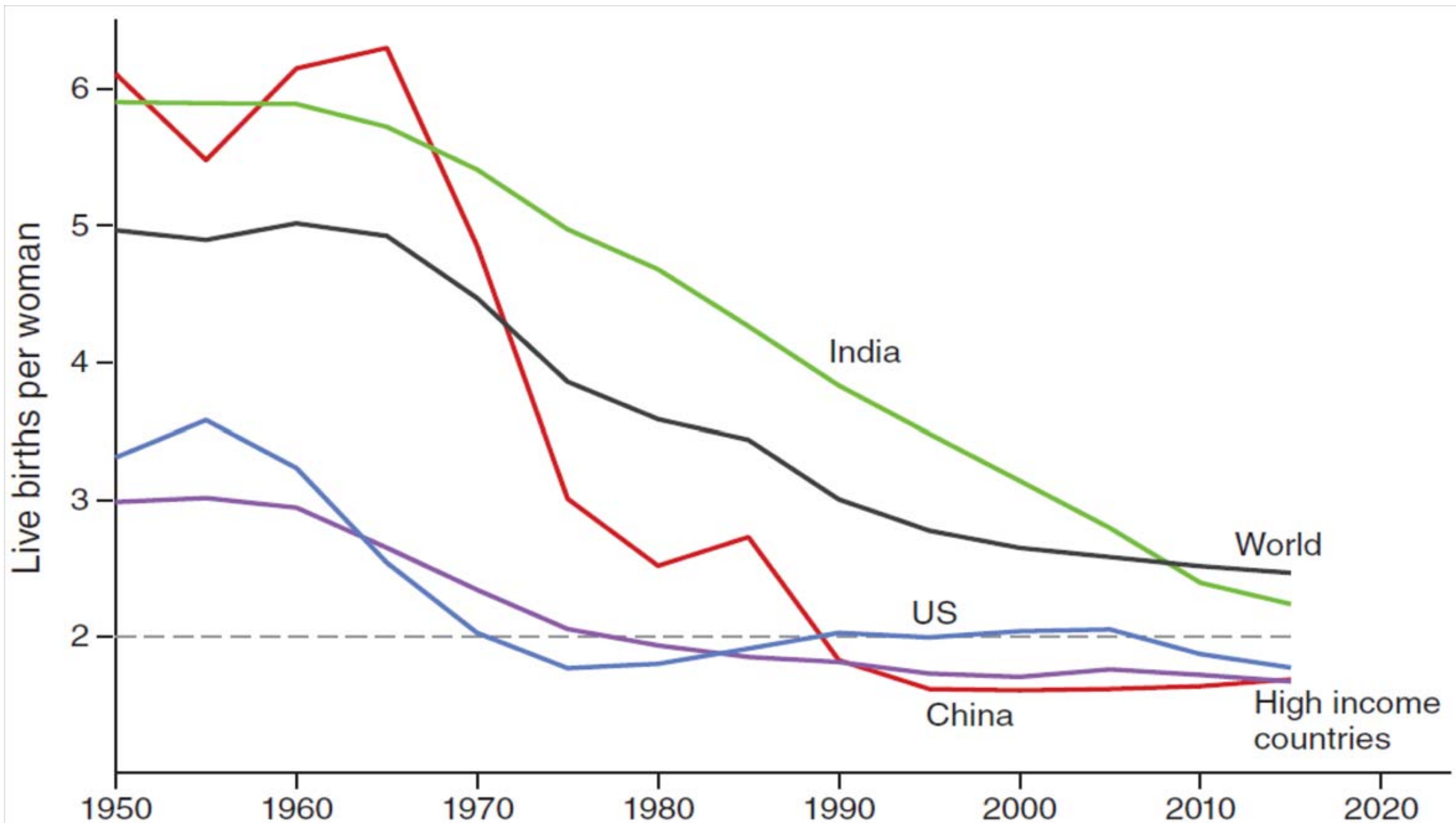
5. Problems

- **While the market wage channel (via market goods technology z) is well accepted, the household technology channel faces serious challenge:**
  - **the timing could be off by 20-50 years (http://1920newtechnologyhanyoung.weebly.com/appliances.html)**
  - **counterfactual test using Amish (Pennsylvania) shows little household technology-led fertility increase**



E. Population and Growth: Jones (2022)

- **In high income countries, fertility is below replacement. Could exponential growth of living standards continue?**
- **Total fertility rate (TFR):**



1. Fully Endogenous Growth Model

- **Basic setting with endogenous growth of knowledge rising in population:**

$$Y_t = A_t^{\sigma} N_t,$$
$$\frac{\dot{A}_t}{A_t} = \alpha N_t,$$
$$N_t = N_0 e^{-\eta t}, \ \eta > 0.$$

- **Equilibrium TFP:** $\frac{\dot{A}_t}{A_t} = \alpha N_0 e^{-\eta t}$ **=>** $\log A_t = \log A_0 + \frac{g_{A0}}{\eta}(1 - e^{-\eta t})$
- **Main finding: Empty Planet with A and y=Y/N converging to** $A^* = A_0 \exp\left(\frac{g_{A0}}{\eta}\right)$ **and** $y^* = y_0 \exp\left(\frac{g_{y0}}{\eta}\right)$ **asymptotically**

2. Semi-Endogenous Growth Model

- **Basic setting:**

$$Y_t = A_t^{\sigma} N_t,$$
$$\frac{\dot{A}_t}{A_t} = \alpha N_t^{\lambda} A_t^{-\beta},$$
$$N_t = N_0 e^{-\eta t}, \ \eta > 0.$$

(β>0=>new ideas becomes harder to find)

- **Equilibrium TFP:** $\frac{\dot{A}_t}{A_t} = \alpha N_0^{\lambda} e^{-\lambda\eta t} A_t^{-\beta}$ **=>** $A_t = A_0\left[1 + \frac{\beta g_{A0}}{\lambda\eta}\left(1 - e^{-\lambda\eta t}\right)\right]^{1/\beta}$

- **Main finding:** $A^* = A_0\left(1 + \frac{\beta g_{A0}}{\lambda\eta}\right)^{1/\beta}$ **and** $y^* = y_0\left(1 + \frac{g_{y0}}{\gamma\eta}\right)^{\gamma/\lambda}$ **with transitional growth falls to zero at a faster pace than $e^{-\lambda\eta t}$:**

$$\frac{\dot{y}_t}{y_t} = g_{y0} \times \left(\frac{y_t}{y_0}\right)^{-\frac{\lambda}{\gamma}} e^{-\lambda\eta t} = \frac{g_{y0} e^{-\lambda\eta t}}{1 + \frac{g_{y0}}{\gamma\eta}\left(1 - e^{-\lambda\eta t}\right)}$$

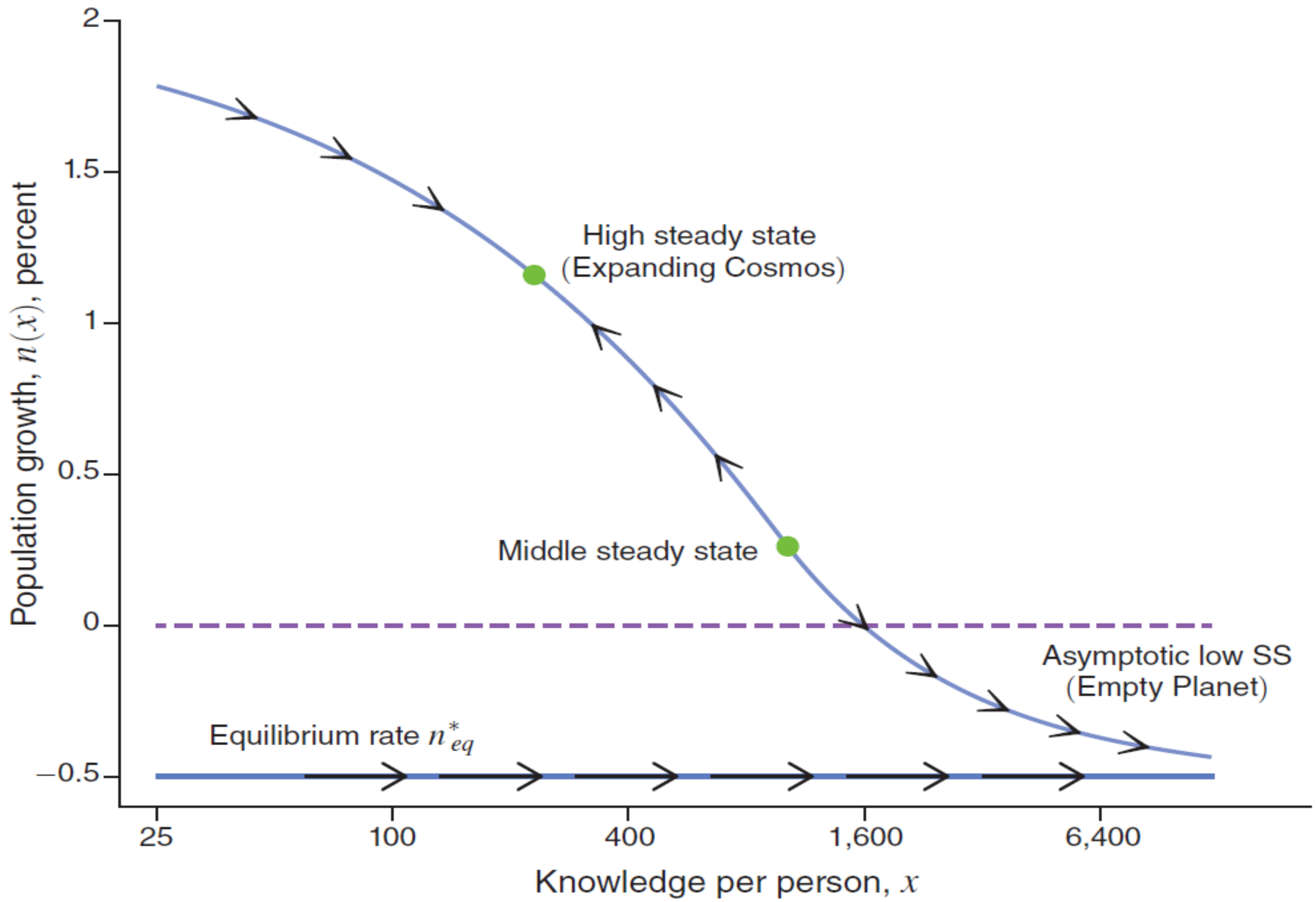
3. Endogenous Fertility

- **Unit labor requirement for the fertility rate = 1/$\bar{b}$ =>** $b(\ell_t) = \bar{b}\ell_t$
- **Constant death at δ => population growth rate:** $\frac{\dot{N}_t}{N_t} = n_t = b(\ell_t) - \delta$
- **Dynasty utility:** $U_0 = \int_0^{\infty} e^{-\rho t} u(c_t, N_t) dt,$ **with** $u(c_t, N_t) = \frac{(N_t^{\varepsilon} c_t)^{1-\theta}}{1-\theta}$ **,** $c_t = Y_t/N_t$ **and** $\rho \equiv \bar{\rho} + \delta$
- **Semi-endogenous growth:** $Y_t = A_t^{\sigma}(1 - \ell_t)N_t$ **and** $\frac{\dot{A}_t}{A_t} = N_t^{\lambda} A_t^{-\beta}$

- **Optimum:**
  - **(FOC) childbearing investment:** $$\underbrace{v_t N_t b'(\ell_t)}_{\text{MU of time in fertility}} = \underbrace{\frac{u_c(c_t, N_t)c_t}{1-\ell_t}}_{\text{MU of time in making goods}}.$$
  - **social value of new ideas:** $z_t \equiv \frac{\mu_t \dot{A}_t}{u_{ct} c_t} = \frac{\sigma g_{At}}{\rho - g_{\mu t} - (1-\beta) g_{At}}$
  - **childbearing investment:** $\ell_t = 1 - \frac{1}{\bar{b}\tilde{V}_t}$
  - **population growth:** $n_t = \bar{b} - \delta - \frac{1}{\tilde{V}_t}$
  - **Steady-state** z, n, and $\tilde{V}$ **are pinned down by:**
    - **(E1)** $z^* = \frac{\sigma g_A^*}{\rho - \varepsilon(1-\theta)n^* + [\beta - \sigma(1-\theta)] g_A^*}$
    - **(E2)** $n_{sp}^* = \bar{b} - \delta - \frac{1}{\tilde{V}_{sp}^*}$
    - **(E3)** $\tilde{V}_{sp}^* = \frac{\varepsilon + \lambda z^*}{\rho - \varepsilon(1-\theta)n^* - (1-\theta) g_c^*}$ **(shadow value of pop)**

- **Competitive equilibrium: max U_0 s.t.** $\dot{N}_t = (b(\ell_t) - \delta)N_t$, $c_t = w_t(1 - \ell_t)$
  - **childbearing investment:** $\ell_t = 1 - \frac{1}{\bar{b}\tilde{V}_t}, \quad where \quad \tilde{V}_t \equiv \frac{v_t N_t}{u_{ct} c_t}$
  - **population growth:** $n_t = \bar{b} - \delta - \frac{1}{\tilde{V}_t}$
  - **steady state:**
    - $$n_{eq} = \begin{cases} \frac{1}{\theta}\left(\bar{b} - \delta - \frac{\rho}{\varepsilon}\right), & if\ \bar{b} - \delta - \frac{\rho}{\varepsilon} < 0 \\ \frac{\bar{b} - \delta - \rho/\varepsilon}{1 - (1-\theta)\left(\frac{\varepsilon + \gamma}{\varepsilon}\right)}, & otherwise. \end{cases}$$
    - $$\tilde{V}_{eq} = \frac{\varepsilon}{\rho - \varepsilon(1-\theta)n_{eq} - (1-\theta)g_c^{eq}}$$
  - **thus, steady-state population growth could be negative if childbearing time cost ($1/\bar{b}$) is high or if parents are not much altruistic ($\varepsilon \to 0$)**
- **Main findings:**
  - **with positive population growth, CE is suboptimal**
  - **with negative population growth, CE is PO (Empty Planet)**
    - **guess $n^* < 0$, then $g_A = 0$ in the steady state**
    - **(E1) => $z^* = 0$**
    - **(E3) => identical shadow value of pop => Q.E.D.**

- **Multiple steady states and transitional dynamics (numerical):**



  - **equilibrium population approaches the empty planet planner solution**