Quantitative Macroeconomics: On Calibration and Decomposition Analysis[1]

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Why calibrating?

- Economists attempt to understand observed patterns and events as well as to draw inferences
- Econometric estimation is to condition on the data to search for the economic world most likely to have generated them
- Calibration is to regard the appropriate data or measurements as to be determined in part by the features of the theory
- Econometric estimation and calibration are thus not competing but complementing methodologies (Hansen-Heckman JEP 1996; Cooley OxfREP1997)

Why calibrating?

- Calibration is particularly useful in the absence of suitable data for structural econometric analysis,
  - Lack of measurement
  - Lack of data with sizable sample and large variations
- By counterfactual analysis, calibration enables model-based policy experiments beyond simple numerical exercises

What is calibration?

- Calibration analysis consists of the following steps:
  1. Establish stylized facts empirically and questions to address
  2. Establish a theory consistent with stylized facts and appropriate for addressing the questions
  3. Calibrate the theoretical model to fit the data and adjust the theory if necessary

What is calibration?

- Particularly, the 3rd step is to calibrate structural parameters based on empirical observations, which are usually
  - Time series first and second moments and autocorrelation
  - Cross-sectional first and second moments and spatial autocorrelation
  - In forms of ratios or growth rates or in logs, rather than simple levels (unit free)

Ground rules: Cooley (1997)

1. Do not justify parameter choices by referring to prior studies
2. Honor the theory
3. Respect the measurements
4. Calibration and estimation are complements not substitutes: estimations based on microeconomic observations on individual behavior are particularly useful to incorporate
5. Do not proliferate free parameters: adding more for better fit beyond addressing the questions potentially biases the results
6. Match the measurements to the model: create proper measures based on theory
7. Match the model to the measurements: design algorithms for proper parametrization

Preliminaries

- The calibration process that match the measurements to the model and match the model to the measurements
- Count endogenous variables ($\#D$) and structural parameters ($\#M$) in the **calibrated** model
- Collect observables related to the theoretical model, differentiated by exogenous ($\#X$) and endogenous ($\#N$)
- Collect commonly used parameter values ($\#P$) **with caution** (Ground Rule 1)
- Count:
  - the remaining structural parameters ($\#S = M - P$)
  - model equations linked to the aforementioned observables in data ($\#E$)
  - the remaining endogenous variables ($\#V = D - N$)

Calibration: Identification

- Conduct calibration: it depends crucially on
  - the remaining structural parameters (S)
  - model equations linked to the aforementioned observables in data (E)
  - the remaining endogenous variables (V)
- Their relative magnitudes lead to three very different cases

Calibration: Identification

- Case 1: $V < E \leq S + V$
  (a) $E = S + V$ (exact identification): compute all of the S remaining structural parameters
  (b) $V < E \leq S + V$ (under identification): choose commonly used values $\#C \leq S$ parameters without violating Ground Rule 1
    i. if $C = S$: compute $E - V$ parameters from the model equations (the use of outside parameter values makes the calibration exactly identified
    ii if $C < S$: use minimizing distance (minimizing the sum of squared errors – possibly with unequal weight – to pin down the S remaining structural parameters

Calibration: Identification

- Case 2: $S + V < E < S + V + N$: this is a typical over identification case in calibration
  - "give up" $\#R = E - S - V$ observed endogenous variables that are likely with measurement errors or not as crucial for the purpose of the study (Ground Rule 3: respect the measurements)
  - use $S + V$ model equations to calibrate S parameters and to compute V remaining endogenous variables (Ground Rule 2: honor the theory)
  - test the model predictions for overly identified R endogenous variables (Ground Rule 3: respect the measurements) – if not too much off, the calibrated model is viewed appropriate to be used for quantitative analysis

Calibration: Identification

- Case 3: $E > S + V + N$: this is an over-identification case that **cannot** be used for calibration analysis, as it violates Ground Rule 2: honor the theory)
- One must change the fundamental model structure by
  - changing some functional forms to permit more structural parameters and hence to raise $\#S$
  - changing the model to have more endogenous decisions that are linked to observable outcomes and hence to increase $\#N$

The Framework

- The optimization problem:

$$\max \int_0^\infty \frac{c^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}} e^{-\rho t} dt$$
$$s.t. \quad \dot{k} = (1-\tau) A(\tau) k - \delta k - c, A(\tau) = \alpha \tau^\gamma$$

where $\{c, k, A, \tau\}$ denote consumption, capital, TFP and income tax rate; $\rho > 0$, $\sigma, \delta \in (0,1)$

- The dynamical system is governed by:
  1. Keynes-Ramsey (KR): $\frac{\dot{c}}{c} = \sigma [(1-\tau) A(\tau) - \delta - \rho]$
  2. Capital Evolution (CE): $\frac{\dot{k}}{k} = (1-\tau) A(\tau) - \delta - \frac{c}{k}$

Theoretical Result

- Endogenous growth rate (from KR):
  $\theta = \sigma \left[(1 - \tau) A(\tau) - \delta - \rho \right]$
- Created measures by theory (Ground Rule 6):
  - Consumption-capital ratio (from CE):
    $$\frac{c}{k} = \left[(1 - \tau) A(\tau) - \delta \right] - \theta$$
  - Capital-output ratio:
    $$\frac{k}{y} = \frac{1}{A(\tau)}$$
- Set functional form to suit for proper parametrization (Ground Rule 7): $A(\tau) = \alpha \tau^{\gamma}$, $\alpha > 0$, $\gamma \in (0, 1)$

Preparing for Calibration to the US

- Observed endogenous variables: ($\#N = 3$)
  - Growth rate: $\theta = 1.8\%$
  - Capital-output ratio: $k/y = 3$
  - Consumption-output ratio: $c/y = 0.6$ (so, $c/k = (c/y)/(k/y) = 0.6/3 = 0.2$)
- Observed exogenous variables: $\tau = 0.2$ ($\#X = 1$)
- Commonly used parameters: ($\#P = 2$)
  - Time preference rate: $\rho = 0.05$
  - Depreciation rate: $\delta = 0.05$
- Other structural parameters: σ, α, γ ($\#S = 3$)
- Model equations: KR, CE, k/y ($\#E = 3$)
- Remaining endogenous variables: none ($\#V = 0$)

Conducting Calibration

- $S + V (= 3 + 0) = E (= 3) < S + V + N (= 3 + 0 + 3)$
  - So, in principle, we can calibrate $S = 3$ parameters
- Problem: $1 - 1 \neq 0$
  - $k/y = 1/A(\tau) => A(\tau) = 1/3 = \alpha \cdot \tau^{\gamma}$, which cannot be used to pin down both α and γ
  - $\theta = \sigma \left[(1 - \tau) A(\tau) - \delta - \rho \right] => \sigma = 0.108$, which is too low – typically in the range of $(1/4, 2/3)$
  - $\frac{c}{k} = \left[(1 - \tau) A(\tau) - \delta \right] - \theta$ over-identifies, which provides a test on $c/k = 0.1987$ (pretty good as it is very close to the data, 0.2)

Completing Calibration

- Possible solutions:
  - (A) Set $\alpha = 1$ (normalization)
    $=> \gamma = \ln(1/3)/\ln(0.2) = 0.683$
  - (B) Set $\gamma = 1$ (linear $A(\tau)$) $=> \alpha = (1/3)/0.2 = 1.667$
- But check the associated growth-maximizing tax rates:
  $d\theta/d\tau = 0 => \tau^*/\,(1-\tau^*) = \gamma$
  - (A) $\tau^* = 0.41$
  - (B) $\tau^* = 0.50$
  - such high growth-maximizing tax rates are unlikely for the US, inconsistent with empirical evidence documented in the literature pioneered by Aschauer, Barro and Sahasukul

Completing Calibration

- Alternative:
  - (C) assume government selects growth maximizing $\tau^* = 0.2 => \gamma = \tau^*/(1-\tau^*) = 0.25$
  - Then, $\alpha \cdot \tau^\gamma = 1/3 => \alpha = 0.498$
  - That is, $A(\tau) = 0.5 \cdot \tau^{1/4}$
  - This suggests the productive government spending elasticity of TFP to be 1/4, by and large consistent with the empirical literature

Completing Calibration

- Fine-tuning – on the intertemporal elasticity of substitution:
  - To ensure is σ proper, consider $\sigma = 2$
  - $k/y = 1/A(\tau) = 3$ and $\theta = 2\left[(1-0.2)/3 - \delta - \rho\right] = 0.018 => \delta + \rho = (1-0.2)/3 - 0.018/2 = 0.258$
  - That is, even by setting depreciation at rate $\delta = 10\%$, one still needs to consider high time discounting at $\rho = 0.15$ to get intertemporal elasticity of substitution right
  - This is related to a vast literature pioneered by Hall (JPE 1988) (cf. Campbell-Mankiw, NBER Macro Annual 1989; Lawrance, JPE 1991; Atkeson-Ogaki, JME 1996; Barsky-Juster-Kimball-Shapiro, QJE 1997; Ogaki-Atkeson, REStat 1997; Ogaki-Reinhart, JPE 1998; Guvenen, JME 2006)
  - Possible fixes: nonhomothetic or recursive utility, risky asset, endogenous labor, borrowing constraint – be **cautious** (Ground Rules 4 and 5)

The Framework: A Simplified Model of Garriga-Hedlund-Tang-Wang (2021)

- Discrete, stochastic dynamic programming
- Infinitely lived, rational agents, heterogeneous in
  1. mobility (disutility) costs: type ϵ drawn from $\Psi(\epsilon)$
  2. labor income shocks drawn:
     - 2.1 permanent shocks: s_t with transition $\pi(s_{t+1} \mid s_t)$ and initial drawn from stationary distribution $\Pi(s_t)$
     - 2.2 transitory shocks: e_t drawn from $G(e_t)$
  3. ex post residency: rural vs. urban renters and urban owners residing in farm houses h_f, rental apartments h_a and owner-occupied houses h, respectively

Production Technology

- Goods production (using labor only):
  1. agricultural: $Y_{ft} = Z_{ft} N_{ft}$
  2. manufacture: $Y_{mt} = Z_{mt} N_{mt}$
- Rental apartment and housing production (using structure, labor, land):
  1. rental unit: $Y_{at} = Z_{at} F^{a} \left(L_{at}, \Upsilon \left(S_{at}, N_{at} \right) \right)$
  2. house: $Y_{ht} = Z_{ht} F^{h} \left(L_{ht}, \Upsilon \left(S_{ht}, N_{ht} \right) \right)$
  3. owner-occupied house production is more land-intensive

Rural

- The optimization problem:

$$V_t^{rural}(\epsilon) = \max_{x_f, x_m} u(x_{ft}, x_{mt}, h_f)$$
$$+\beta \max\left\{V_{t+1}^{rural}(\epsilon), EV_{t+1}^{rent}(y_{t+1}, s_{t+1}) - \xi_{t+1}\epsilon\right\}$$
$$s.t. \quad p_{ft}x_{ft} + x_{mt} = p_{ft}Z_{ft}$$
$$y_{t+1} = e_{t+1}s_{t+1}w_{t+1} + \mathcal{T}_{t+1}$$

where $\{x_f, x_m, Z_f, w, \mathcal{T}\}$ denote agricultural/manufactured good consumption, urban manufacturing TFP, wage income, and government transfer to insure an income floor for survival; $\xi > 0$, $\beta \in (0, 1)$

Urban Renter

- The optimization problem:

$$V_t^{rent}(y_t, s_t) = \max_{x_m, x_f, b_{t+1}} u(x_{ft}, x_{mt}, h_a)$$
$$+\beta \max\left\{EV_{t+1}^{rent}(y_{t+1}, s_{t+1}), EV_{t+1}^{buy}(y_{t+1}, s_{t+1})\right\}$$
$$s.t. \quad p_{ft}x_{ft} + x_{mt} + p_{at}h_{at} + b_{t+1} = y_t$$
$$y_{t+1} = e_{t+1}s_{t+1}w_{t+1} + (1 + i_{t+1})\, b_{t+1} + \mathcal{T}_{t+1}$$

where b_{t+1} denotes beginning-of-period asset with real interest rate i_{t+1}

Urban Buyer

- The optimization problem:

$$V_t^{buy}(y_t, s_t) = \max_{x_m, x_f, b_{t+1}, d_{t+1}, h_{t+1}} u(x_{ft}, x_{mt}, \varsigma h_{t+1})$$

$$+\beta \max\left\{EV_{t+1}^{rent}\left(y_{t+1}^{rent}, s_{t+1}\right), EV_{t+1}^{own}\left(y_{t+1}^{own}, h_{t+1}, d_{t+1}, h_{t+1}\right)\right\}$$

$$\begin{aligned} s.t. \quad & p_{ft}x_{ft} + x_{mt} + (1 + \tau_b + \delta_h) p_{ht} h_t + b_{t+1} = y_t + d_{t+1} \\ & y_{t+1}^{rent} = e_{t+1}s_{t+1}w_{t+1} + (1 + i_{t+1}) b_{t+1} \\ & \qquad + (1 - \tau_s) p_{h,t+1} h_{t+1} - (1 + r_{t+1}) d_{t+1} + \mathcal{T}_{t+1} \\ & y_{t+1}^{own} = e_{t+1}s_{t+1}w_{t+1} + (1 + i_{t+1}) b_{t+1} \\ & d_{t+1} \leq (1 - \theta_t) p_{ht} h_{t+1} \end{aligned}$$

where $\{d_{t+1}, \tau_b, \tau_s, \delta_h\}$ denote housing mortgage, transaction cost (buying/selling) and depreciation; $\varsigma > 0$ captures home-ownership premium

Urban Buyer

- The optimization problem:

$$
\begin{aligned}
V_t^{own}\left(y_t, h, d_t, s_t\right) &= \max_{x_m, x_f, b_{t+1}} u\left(x_{ft}, x_{mt}, \varsigma h\right) \\
+\beta \max &\left\{EV_{t+1}^{rent}\left(y_{t+1}^{rent}, s_{t+1}\right), EV_{t+1}^{own}\left(y_{t+1}^{own}, h, d_{t+1}, h_{t+1}\right)\right\} \\
s.t. \quad & p_{ft}x_{ft} + x_{mt} + \delta_h p_{ht} h_t + \left(\gamma + r_t\right) d_t + b_{t+1} = y_t \\
& y_{t+1}^{rent} = e_{t+1} s_{t+1} w_{t+1} + \left(1 + i_{t+1}\right) b_{t+1} \\
& \quad + \left(1 - \tau_s\right) p_{h,t+1} h_{t+1} - \left(1 + r_{t+1}\right) d_{t+1} + \mathcal{T}_{t+1} \\
& y_{t+1}^{own} = e_{t+1} s_{t+1} w_{t+1} + \left(1 + i_{t+1}\right) b_{t+1} \\
& d_{t+1} = \left(1 - \gamma\right) d_t
\end{aligned}
$$

where $\gamma > 0$ measures mortgage amortization rate

Close the Model

- Government:
  1. optimization: $NR_t = \max p_{Lt} L_t - \frac{\vartheta_t}{2} L_t^2$, $\vartheta_t > 0$, $L_t = \int_{rent} L_{at} di + \int_{own} L_{ht} di$
  2. budget balance: $NR_t = \int \mathcal{T}_t di + I_t$, I_t measures all other government spending (thrown into the ocean)
- Migration equilibrium: rural-urban migration occurs when $\epsilon \leq \epsilon^*_{t+1} \equiv \frac{1}{\xi_{t+1}} \left[EV^{rent}_{t+1} \left(y^{rent}_{t+1}, s_{t+1} \right) - V^{rural}_{t+1} \left(\epsilon \right) \right]$
- Housing evolution: $H_t = \left(1 - \delta_h\right) H_{t-1} + Y_{ht}$
- Housing market clearing: $\int h d\Phi^{own}_t = \left(1 - \delta_h\right) H_{t-1} + Y_{ht}$
- Labor market clearing: $N_{ft} = 1 - \Psi\left(\epsilon^*_t\right) = N_{mt} + N_{at} + N_{ht}$

Functional Form Specification

- $\Upsilon\left(S_{at}, N_{at}\right) = S_a^{\alpha_S} N_a^{1-\alpha_S}$; $F^a = L_a^{\alpha_{La}} \Upsilon\left(S_a, N_a\right)^{1-\alpha_{La}}$, $F^h = L_a^{\alpha_{Lh}} \Upsilon\left(S_a, N_a\right)^{1-\alpha_{Lh}}$, $\alpha_{La} < \alpha_{Lh}$
- $u\left(x_f, x_m, x_h\right) = U\left(C(x_f, x_m), x_h\right)$:
  - $C = \left[\phi_f\left(x_f - \underline{x}_f\right)^{\frac{\nu_f - 1}{\nu_f}} + \left(1 - \phi_f\right)\left(x_m\right)^{\frac{\nu_f - 1}{\nu_f}}\right]^{\frac{\nu_f}{\nu_f - 1}}$
  - $U = \left\{\phi_c C^{\frac{\nu_c - 1}{\nu_c}} + (1 - \phi_c) x_h^{\frac{\nu_c - 1}{\nu_c}}\right\}^{\frac{\nu_c}{\nu_c - 1}}$
- $\Psi\left(\epsilon\right) = 1 - \left(\frac{\epsilon}{\epsilon_{\min}}\right)^{-1/\kappa}$; $\ln\left(\xi_t\right) = \ln\left(q_t\right) + \ln\left(\tilde{\xi}_t\right)$, with q_t measuring average urban housing quality (the hedonic factor)
- $\ln\left(s_t\right) = \rho \ln\left(s_{t-1}\right) + \varepsilon_t$, $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ and ρ a 3-state Markovian (Rouwenhorst 1995); $\ln\left(e_t\right) \sim N\left(0, \sigma_e^2\right)$
- $\mathcal{T}_t\left(e_t s_t\right) = \max\left\{0, p_{ft}\underline{x}_f + p_{at} h_{at} + \left(0.5 \cdot \underline{es} - e_t s_t\right) w_t\right\}$

Conducting Calibration

- Calibrating to Chinese data over 2001-2014
- Fit transformed initial steady state to 2001 data
- Set transformed final steady state (denoted ∞) to be 50 years after the end of sample period (2014)
  - use logistic extrapolation
  - use smooth pasting pre-/post-end of sample
- # parameters far exceed observed moments, thus requiring a proper care

Calibration Strategy

- normalize some initial scaling parameters to 1; set $\nu_c = 2$
- use time series data to pin down trends in TFPs
- use micro estimation to pin down income process (Fan-Song-Wang, 2010), mobility cost distribution (Liao-Wang-Wang-Yip, 2021), land and structure shares (Deng-Tang-Wang-Wu, 2022; Favilukis-Ludvigson-Van Nieuwerburgh, 2017), housing transactions parameters (Guren-McKay-Nakamura-Steinsson, 2020, Garriga-Hedlund 2020)
- jointly calibrate $\left\{\beta, \varsigma, \nu_f, \phi_c, \phi_f, \epsilon_{\min}, \tilde{\xi}_\infty\right\}$ to fit 7 observed moments (2001/2014 rural pop & agricultural spending shares, ownership rate in 2000, financial assets-GDP ratio in 2007, average housing expenditure share) by minimizing distance

Completing Calibration

- Joint calibration results: $\beta = 0.842, \varsigma = 1.30, \nu_f = 2.11, \phi_c = 0.047, \phi_f = 0.287, \epsilon_{\min} = 7.26, \tilde{\xi}_\infty = 0.736$
- Fit (model vs. data):

Description	Model	Data	Source
2001 Rural Population	62.3%	62.3%	CSY[a] 2016
2014 Rural Population*	45.2%	45.2%	CSY[a] 2016
2001 Agricultural Spend Share	14.1%	14.1%	CSY[a] 2016
2014 Agricultural Spend Share*	9.2%	9.2%	CSY[a] 2016
Homeownership Rate	82.0%	82.6%	Census[b] 2000
Financial Assets to GDP	1.48	1.5	UHS[c] 2007
Housing Spend Share (Owners)	24.9%	24.5%	CFPS[d] 2014, 2016

The Use of Calibrated Models

- Once the model is fully calibrated, one may conduct various quantitative comparative statics and counterfactual policy experiments
- One may also conduct general counterfactual analysis to investigate how a particular driver may affect the model economy
- By comparing the benchmark with counterfactual, one can isolate the contribution of a particular driver
- By doing so for all underlying drivers, one may then obtain decomposition outcomes

Counterfactual Analysis

- We begin by differentiating the model variations of y from the data variations, with the difference called errors as it measures the model prediction errors (denoted ERR):

$$ERR = \text{data variation} - \text{model variation} \equiv var^{data} - var$$

Counterfactual Analysis

- Let there be k factors that account for the variations of a model variable y over time. Then we can write the model variations of y as:

$$var = \sum_{j=1}^{k} var_j + 2 \sum_{j'>j} \sum_{j=1}^{k} cov_{jj'}$$

- By counterfactual anlaysis, we can derive counterfactual variations as var^{-i}, where superscript $-i$ indicates the counterfactual by eliminating factor i:

$$var^{-i} = \sum_{j\neq i}^{k} var_j + 2 \sum_{j'>j, j'\neq i} \sum_{j\neq i} cov_{jj'}$$

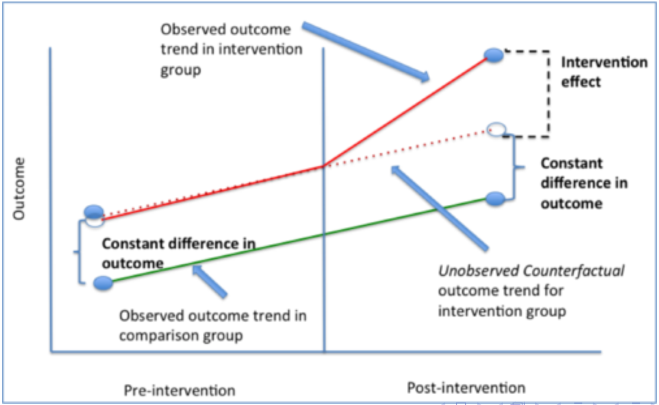
Diff-Diff in Calibration

- Taking difference, we have a "counterfactual diff-diff" (cDID) measure with respect to factor i:

$$\begin{aligned}
\triangle^{i} &= var - var^{-i} \\
&= \sum_{j=1}^{k} var_j + 2\sum_{j'>j}\sum_{j=1}^{k} cov_{jj'} - \sum_{j\neq i} var_j - 2\sum_{j'>j,j'\neq i}\sum_{j\neq i} cov_{jj'} \\
&= var_i + 2\sum_{j'>i,j'\neq i} cov_{ij'}
\end{aligned}$$

Diff-Diff Graph

- Think of the impact of factor i as a pre- and post-intervention comparison (Columbia Public Health Methods):



Total Diff-Diff

- Summing up over all factors, we obtain total diff-diff (TD):

$$
\begin{aligned}
TD &= \sum_{i=1}^{k} \triangle^{i} = \sum_{i=1}^{k} \left(var_i + 2 \sum_{j'>i, j' \neq i} cov_{ij'} \right) \\
&= var + 2 \sum_{i=1}^{k} \sum_{j'>i, j' \neq i} cov_{ij} \\
&= var + TCOV
\end{aligned}
$$

- Thus, the total covariance term $TCOV$ is positive (negative) iff total diff-diff exceeds (falls below) the model variation.

The Nexus Between Theory and Data

- In any paper, one may use model as well as data to figure out the sign of each relevant element of the variance-covariance matrix:

$$\begin{bmatrix} sign(var_1) & sign(cov_{12}) & sign(cov_{13}) & \cdots & sign(cov_{1k}) \\ & sign(var_2) & sign(cov_{23}) & \cdots & sign(cov_{2k}) \\ & & sign(var_3) & \cdots & sign(cov_{3k}) \\ & & & \ddots & \vdots \\ & & & & sign(var_k) \end{bmatrix}$$

  where $sign(var_i) > 0$ and the lower-triangular cov terms are symmetric to the upper-triangular (which can thus be ignored).
- This will help out the intuition concerning the sign of $TCOV$.

Procedure of Decomposition

- Looking at the expression

$$TD = var + TCOV$$

- Noting that $var > 0$, one can see that, in order for total diff-diff TD to have a consistent sign as the model variation var (i.e., to have $TD > 0$), we must have:

1. $TCOV > 0$, or,
2. $-var < TCOV < 0$.

Procedure of Decomposition

- In either case, one can normalize an individual factor i's diff-diff (denoted $\tilde{\triangle}^i$) to compute its contribution (denoted CON^i).
- Specifically, we have:

$$\begin{aligned}\tilde{\triangle}^i &= \frac{var}{TD} \cdot \triangle^i \\ CON^i &= \frac{\tilde{\triangle}^i}{var} = \frac{\frac{var}{TD} \cdot \triangle^i}{var} = \frac{\triangle^i}{TD}\end{aligned}$$

where adding up implies: $\sum_{i=1}^{k} \frac{\triangle^i}{TD} = 1$, i.e., total contribution of all factors sums up to one.

Procedure of Decomposition

- In the unfortunate case when $TCOV < -var$, we need to acknowledge the large covariance term and conduct decomposition analysis accordingly (no need for normalization):

$$CON^i = \frac{\triangle^i}{var}$$

$$CON^{TCOV} = \frac{|TCOV|}{var}$$

where adding up again implies: $\frac{\sum_{i=1}^{k}\triangle^i + |TCOV|}{var} = 1$.

- Thus, the calibrated decomposition analysis is basically replaced by a "variance decomposition" exercise, which is not ideal but can resolve the problem caused by a negative $TCOV$ of large magnitude.

Procedure of Decomposition

- Sometimes one may use data variations by adding a "residual" (denoted *RES*) factor that summarizes both model prediction errors and the covariance term:

$$
\begin{aligned}
CON^i &= \frac{\triangle^i}{var^{data}} \\
CON^{RES} &= \frac{|TCOV| + ERR}{var}
\end{aligned}
$$

  where it is easy to check the adding-up condition: $\frac{\sum_{i=1}^{k} \triangle^i + |TCOV| + ERR}{var^{data}} = 1$.
- Whether to use one against another depends on how exhaustive the underlying drivers are included in the analysis – a residual should be added if the list of drivers is not comprehensive.

Wrapping Up

- Regardless of which formulation to be used, one shall be thorough in dealing with the "names" of the underlying factors.
- For example, over time, factor j may be rising most of the time, factor $j\prime$ falling most of the time, whereas factor j'' moves up and down.
- Then in conducting decomposition, one may call the factors as: expansion in j, reduction in j' and changes in j''.
- Many people leave just the name of each factor, but adding the terms of expansion and reduction in tables make the reader easier to understand the analysis.

Concluding Remarks

In summary, calibration is both science and art, requiring data and institutional background, theoretical fundamentals as well as knowledge to properly link theory to practice. It is complement rather than substitute to econometric work. Once a model is carefully calibrated to fit data, quantitative analysis can be conducted to

- quantify various comparative-static effects
- assess quantitatively various policy experiments
- promote better understanding of the relative contributions of the underlying drivers and hence the relative importance of the underlying channels incorporated in the model.

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Useful WebLinks: Fundamentals

1. Lawrence Christiano and Jesús Fernández-Villaverde (NBER Mini-Course: Ramsey, New Keynesian, DSGE):
https://faculty.wcas.northwestern.edu/lchrist/course/NBER_2011/syllabus.html
2. Ben Moll (CEPR Lecture on YouTube: continuous-time heterogeneous-agent models):
https://www.youtube.com/watch?v=KlmwBqDP2TU
3. Thomas Sargent (Sloan Foundation QuantEcon Lecture: new classical, dynamic programming with Python/Julia):
https://quantecon.org/lectures/

Other Useful WebLinks

1. Christopher Carroll (Econ-ARK/HARK toolkit for heterogeneous agent macro):
   http://www.econ2.jhu.edu/people/ccarroll/Courses/Topics/Syllabus-Oslo.pdf
2. Jesús Fernández-Villaverde (computational methods):
   https://www.sas.upenn.edu/~jesusfv/teaching.html
3. Alisdair McKay (Python, discrete-time, cycles):
   https://alisdairmckay.com/Notes/HetAgents/index.html

Other Useful WebLinks

4. Ben Moll (MATLAB, continuous-time, growth & mean-field games):
https://benjaminmoll.com/lectures/ (lectures); https://benjaminmoll.com/codes/ (codes)

5.Makoto Nakajima (Computational Methods for Macroeconomics):
https://makotonakajima.github.io/comp/

6. Julien Pascal (MATLAB, BMK algorithm for aggregate uncertainty in heterogeneous macro):
https://notes.quantecon.org/submission/5ea288cb833c72001a988e4d

7. Jesse Perla (Computational and Quantitative Macroeconomics):
https://www.jesseperla.com/post/computational-macro/