Search for the Best-Fitting: Evaluating Minimum Wage Policy in a Multi-Tier Search and Wage-Posting Framework with Cross-Market Substitutions

C. Y. Kelvin Yuen, HKUST(GZ) and HKUST

Ping Wang, Washington University in St. Louis and NBER

January 2024

Abstract: While minimum wage policy is widely adopted in the real world, can it effectively raise the average wage of lower paid jobs without generating large detrimental employment consequences? The empirical literature fails to establish robust findings. We develop a general-equilibrium search and wage-posting framework with heterogeneous workers and heterogeneous tasks matching in multi-tier labor markets: abstract, routine high-skilled, routine middle-skilled, manual middle-skilled and manual low-skilled. We incorporate rich cross-market spillover and compositional effects from individual responses to market thickness. As a result of minimum wage hikes, we show that (i) the unemployment rate at the minimum wage binding market is higher, while all other markets enjoy a lower unemployment rate; (ii) employment in the manual low-skilled jobs is lower, whereas employment in the routine high-skilled and manual middle-skilled markets is higher due to cross-market substitutions; and, (iii) employment in other markets has ambiguous responses due to conflicting effects on potential worker entry and unemployment. By calibrating the model to fit the U.S. data, we evaluate the impacts of the federal minimum wage hike (2007-2009) and the on-going minimum wage increase in Seattle (2017-2021). We find that the minimum wage effects on employment on the binding markets depend crucially on the magnitudes of spillover and compositional effects and that the employment effects may be weak in a nonbinding market. Moreover, our results suggest that, while a minimum wage hike may reduce aggregate employment and output, they only generate small effects on submarket average and overall average wages.

JEL Classification: D83, E24, E60, J64.

Keywords: Labor Search and Matching, Substitution of Heterogeneous Workers and Tasks, Market Spillover, Minimum Wage Policy.

Acknowledgment: We are grateful for valuable comments and suggestions from John Kennen, Derek Laing, Rody Manuelli, Chris Moser, and Yongs Shin. Needless to say, the usual disclaimer applies.

Correspondence: C. Y. Kelvin Yuen, Urban Governance and Design Thrust, The Hong Kong University of Science and Technology (Guangzhou) and Department of Economics, The Hong Kong University of Science and Technology; E-mail: kelvinyuen@ust.hk; Ping Wang, Department of Economics, Washington University in St. Louis, St. Louis; E-mail: pingwang@wustl.edu.

"*Upon the basis of the wages system the value of labouring power is settled like that of every other commodity; and as different kinds of labouring power have different values, or require different quantities of labour for their production, they must fetch different prices in the labour market. To clamour for equal or even equitable retribution on the basis of the wages system is the same as to clamour for freedom on the basis of the slavery system.*" (Karl Marx, *Wage-Labour and Capital Value, Price and Profit*, 1865; NY: International Publishers, 1969)

"*No person can maximize the American dream on the minimum wage.*" (Benjamin Todd Jealous, president of the National Association for the Advancement of Colored People, 2008-2013)

"*A survey of aea members in 1992 found that 79% of respondents agreed that a minimum wage increases unemployment among young and low-skilled workers.*" (*The Economist*, August 14, 2020)

1 Introduction

Since the pivotal contribution by Stigler (1946), there has been numerous studies on the labor market consequences of minimum wage policy. While minimum wage policy is designed to mitigate poverty and to diminish the power of "monopsony employers," the literature has identified conflicting empirical results about the minimum wage effect on employment, unemployment, and market wage distribution. Of greater concern is the consequences of employment, because it is essential for a successful minimum wage policy not to hinder labor hires by much. In particular, some studies do find significantly negative effects of minimum wage on employment (e.g., Aaronson and French 2007; Stewart 2004), though others fail to establish such effects in a robust manner (e.g., Card and Krueger 1995; Dube, Lester and Reich (2010); Strobl and Walsh 2011).[1] To some degree, the lack of a robust employment effect is not surprising because only a small fraction of workers is responsive to the policy changes (cf. MaCurdy 2015). But to understand why such effects are not identified consistently, one must go deeper to reveal the underlying mechanism through which a hike in the minimum wage influences labor matching and equilibrium wage outcomes, which is the main purpose of our paper.

Let us take a closer look at the empirical findings focusing on low-skilled workers in low-paid markets and if the minimum wage hike is large and binding. In this case, it is more likely to find an

[1] For earlier studies, the reader is referred to a comprehensive survey by Brown (1999).

increase in the minimum wage to raise unemployment (e.g., Fitoussi 1994). More recently, Aaronson and Phelan (2019) find that with minimum wage hikes, relative employment declines at cognitively routine occupations (which may be viewed as a market with routine task filled by skilled labor) but not manually routine or non-routine low-wage occupations. On the contrary, Cengiz et al. (2019) do not find any evidence for substitution away from routine-task intensive occupations, including both cognitively routine and manually routine occupations. This recent literature points to a new direction: the employment consequences of minimum wage policy should depend crucially on the nature of tasks and the skill level of workers. That is, while it is clear that *no person can maximize the American dream on the minimum wage*, the *harmful effect of minimum wage on employment* may not be as robust as nearly four-fifth AEA economists have anticipated.

Accordingly, in our paper, we incorporate explicitly multi-tier labor markets with workers of different skills. To be more specific, we consider three types of workers – high-skilled, middle-skilled, and low-skilled workers – and three types of tasks created by firms – abstract, routine, and manual. There are five segmented labor markets under consideration: abstract, routine high-skilled, routine middle-skilled, manual middle-skilled, and manual low-skilled. Each of the routine and the manual firms may create vacancies in different markets, whereas each of the better skilled may choose to enter one of the potential markets. Thus, both workers and firms search for best-fitting markets through which they interact and their responses subsequently affect the workings of various minimum wage policies.

Our modeling strategy is to be rooted on the job-search and wage-posting framework developed by Burdett and Mortensen (1998), as its tractable structure and empirical relevance are useful for analyzing *the value of laboring power*, job turnover and wage dispersion (cf. Mortensen 2003; Moscarini and Postel-Vinay, 2016). To suit the purpose of our study, we generalize the Burdett-Mortensen framework in several significant and important ways. First of all, we have considered multi-tier labor markets that are segmented with different market thickness in which workers of identical skills may be paid with different wages. Second, firms may post the same type of tasks in different markets featuring different productivities. Third, we allow job finding rates to be endogenously determined.[2] As to be elaborated below, these elements of generalization, particularly, endogenous job finding together with task substitution, either from the firm or from the worker side, will play a crucial role to determine how a minimum wage hike in a given market may generate a "spillover" effect on other markets. This general structure allows us to study labor market per-

[2]Moscarini and Postel-Vinay (2016) also allow for endogenous meeting rate in a similar wage posting game. However, their paper focuses on business cycle dynamics in the absence of minimum wage considerations.

formance in each of the segmented markets as well as in all aggregate measures, including potential labor pools, vacancies, unemployment rates, market tightness, mean wages, wage dispersion, and most importantly employment and output.

Theoretically, we find that even when a minimum wage only binds in the lowest market (manual low-skilled), it would still generate rich spillover effects on other markets through worker and task substitutions. For example, an increase in the minimum wage would lead to the manual task firms posting more vacancies for the middle-skilled workers. As a result, middle-skilled workers have more incentive to work in manual task jobs, entailing a smaller potential labor pool in the routine middle-skilled market, which would in turn discourage vacancy postings in the same market. Thus, this spillover effect features market spillover and creates a chain effect propagating through all the markets in the economy. In equilibrium, we find that while the unemployment rate at the minimum wage binding market is higher, all other markets enjoy a lower unemployment rate. This leads to unambiguously lower employment in the manual low-skilled jobs and higher employment in the manual middle-skilled market as a result of market spillover from the firm side. For other markets, such as those abstract and routine middle-skilled jobs, the effect on employment is ambiguous due to the conflicting effects on the potential labor pool and the unemployment rate.

We next conduct quantitative analysis by calibrating the model to fit the data from the U.S. labor market. Specifically, under a standard market discount rate and matching elasticity of workers, we jointly calibrate matching efficacy, task-specific productivity, and job separation rates to fit the targets of relative wages (of a higher market to the lowest manual low-skilled market) as well as submarket unemployment rates. We then evaluate the impacts of the federal minimum wage increase during the great recession (2007-2009) and the ongoing local minimum wage increase in Seattle (2017-2021) on the relevant (national or local) labor market. Such experiments are intriguing. In particular, prior to such minimum wage hikes, the minimum wage is binding only in the lowest market. After the Federal minimum wage hike, the minimum wage becomes binding in the two lower markets, whereas after implementing the local minimum wage policy in Seattle, it is binding in the three lower markets.

Our quantitative results suggest that with the 2007-2009 federal minimum wage hikes, the minimum wage becomes binding not only in the manual low-skilled market, but also in the manual middle-skilled market. We show that the market spillover effect is relatively small at the beginning until the minimum wage is large enough, after which the spillover effect starts affecting the labor market significantly. We also find that in general the minimum wage elasticities become larger in absolute value when the minimum wage becomes binding in the second lowest market. This is because the market spillover effect is amplified by the direct effect from the binding market. With the federal minimum wage increase, we find the employment in the manual low-skill market falling by 19.7%, but that in the manual middle-skilled market dropping only by 1%. The combined employment effect on the two binding markets with their different employment size weighed in is found to be negative. For the non-binding markets, through market spillovers, we find an employment rise in the routine middle-skilled jobs by 0.88% as well as an employment contraction in the routine high-skilled market by 16.2%, while employment in the abstract market increases by 5.66% despite its more remote chain effect through market spillover from the firm and the worker sides and because of its special requirement of high skilled labor. In addition, the minimum wage increase suppresses wage inequality in the binding markets, a drop of 35.37% in the manual low-skilled market and a fall of 3.4% in the manual middle-skilled market; its effects on the wage dispersion in non-binding markets are relatively small. Overall, the federal minimum wage hike raises the aggregate unemployment rate by 0.24 percentage point, while increasing the aggregate employment and aggregate output by 1.50%. The aggregate effects on the average wage and the wage inequality in the whole economy are found to be relatively small due to conflicting compositional effects in various markets. Finally, we find the density of workers at the minimum wage in the binding markets to be positively correlated with the firm's profit in these markets.

In the case of the relatively large local minimum wage increase in Seattle from \$11 to \$15, we find that the minimum wage now becomes effective also in the third market (routine middle-skilled). Unlike the federal case, the total impact on the economy is dominated by the market spillover effects throughout the local minimum wage hike. As a result, while the qualitative effects of the minimum wage hike are similar to the federal case, the quantitative effects turn out to be much larger. For instance, the drop in employment and output in the manual low-skilled workers is sizable at 24.38%, while the increase in the employment and output in the manual middle-skilled market is more dramatic at 44.05%. On the aggregate level, the local minimum wage increase leads to a much larger 1.73 percentage point increase in overall unemployment rate and 1.44% drop in aggregate employment and output.

The main takeaway is that a full evaluation of the minimum wage policy must account for (i) differential job tasks and differential worker skills, (ii) compositional changes in various markets where the minimum wage may or may not bind, and (iii) rich market spillovers from both firm and worker sides. Our paper provides a first step toward addressing this consideration by conducting a systematic evaluation of the minimum wage policy under a unified search and matching framework. Our quantitative analyses suggest such compositional changes and spillover effects serve to better understand the conflicting empirical observations, thus offering better assessment of the widely adopted minimum wage policy in the real world.

Related literature

The most relevant work to ours is one by Burdett and Mortensen (1998) whose search and matching framework is the root of our model structure (see also the work by van den Berg and Ridder, 1998, and Mortensen, 2003). In their paper, however, the job finding rate is assumed to be exogenous. Hence, when workers are homogenous, unemployment rate is constant with respect to minimum wage, which simply shifts the whole wage distribution (see our Proposition 3). They show that when workers have different unemployment benefits, there exists inefficient unemployment and minimum wage may actually increase employment in this case. In our model, the job finding rate is endogenously determined, and minimum wage may have equilibrium impact and spillover effects on unemployment rates, even though workers are homogenous within each market.

A closely related paper is by Kumar (2008). Although the focus of his paper is on how the interaction between the minimum wage and the capital tax policies may affect labor market outcomes, a baseline finding with minimum wage hikes but without corresponding capital tax changes can be directly compared with our quantitative results on aggregate measures (see Table 4 in his paper and Table 6 in our paper). In both papers, aggregate employment, aggregate output and aggregate wage dispersion are lower whereas aggregate unemployment is higher. While mean wage in Kumar's paper is higher, it is lower in our paper. The main drivers of this difference are two folds. On the one hand, we have 5 markets with heterogeneous tasks, which can naturally lead to different aggregate outcome. On the other hand, labor supply in Kumar (2008) shrinks in response to higher minimum wage; however, potential labor pools in our 4 non-binding markets may rise or fall, adjusting in an alternating way (see our Proposition 4), which may again result in different aggregate outcome.

In another related paper, Engbom and Moser (2018) study the effects of a large minimum wage increase (119%) in Brazil. In their model, firms with different productivities post wages in different markets segmented by workers' abilities. They find that a higher minimum wage leads a decline in inequality, and that there is a spillover effect all the way up to the 80th percentile. A key difference between their model and ours is that, since firms in their model solve employment problem separately for each market and workers are restricted to the market of their own type, a minimum wage increase has spillover effect only *within* a binding market, but not across the non-binding ones. By contrast, our model allows for substitutions between different markets; as a consequence, a minimum wage hike generates spillover effect not only *within* but also *across* binding and non-binding markets. Quantitatively, they find that the minimum wage increase reduces the variance of earnings by 11 log points and employment rate by about 2 percentage points. We estimate that the relatively large local minimum wage increase (25.5%, still far smaller than the hike in Brazil) would lead to up to 60% drop in the variance of the *log wage* in the lowest binding market, and a 2.8% decrease in employment and output.

There are a few more remotely related studies. For example, Fitoussi (1994) finds that an increase in the minimum wage would cause an increase in unemployment rate among the low-skilled workers, yet it would lead to an increase or a decrease in the real wage of the higher skilled labor depending on whether the high-skilled and low-skilled workers are substitutes or complements. In our model, we reconfirm his argument by establishing that middle-skilled wage is rising with minimum wage because middle and low-skilled workers are substitutes. In the two-country model of Davis (1998) in the presence of a binding minimum wage, technical progress leads to higher unemployment and possibly higher relative wage for the unskilled workers. In contrast, we obtain much richer results in our sensitivity analysis when there is capital biased technological change. Postel-Vinay and Robin (2002) construct an on-the-job search model with productivity differences to generate wage distribution endogenously and find a thin tail at the lower end that is consistent with institutionalized minimum wage. Brochu and Green (2013) incorporate learning about matching quality into the Burdett-Mortensen–Pissarides framework to explain the empirically identified negative effects of minimum wage on job finding and job separations rates. More recently, Berger, Herkenhoff and Mongey (2019) links minimum wage to markdown via labor market power, showing that a higher minimum wage may induce some firms to compress their markdowns and raise wages while expanding employment along workers' labor supply curves. Thus, in a market featuring a greater density of workers at the minimum wage, one would expect firms to earn lower profits whereas the market's average wage is expected to be higher. We support their finding based on a very different mechanism – a minimum wage hike in a binding market forces some low offer vacancies to post higher wage and the upward shift of the wage distribution raises the wage cost, thus lowering the profit accrued.

2 The Model

There are three types of workers: high-skilled (h), middle-skilled (m), and low-skilled (l) workers, each with a fixed measure $\bar{n}_i,\ i \in \{h, m, l\}$. On the firm's side, there are three types of tasks: abstract

(a), routine (r), and manual (n). Each task has a total number of vacancies $\bar{v}_j,\ j \in \{a, r, n\}$. In the data, a vast majority of abstract tasks are performed by high-skilled employees, whereas most of low-pay manual tasks are filled by low-skilled workers. Accordingly, in the model, we consider a natural benchmark with only high-skilled workers conducting abstract tasks and the low-skilled taking low-pay manual tasks. To allow for worker and task substitution, we consider that routine tasks can be done by both high-skilled and middle-skilled workers, and manual tasks can be filled by both middle-skilled and low-skilled workers.

Accordingly, there are five markets, labeled by $\tau \in \{a, rh, rm, nm, nl\}$, each having different jobs τ, ranked from high to low by task complexity. One may think of the abstract market ($\tau = a$) features jobs such as professionals, engineers and managers, the high-routine market ($\tau = rh$) contains high-level officers/production supervisors, the middle-routine market ($\tau = rm$) consists of all the standard routine tasks such as manufacture and sales, the middle-level manual market ($\tau = nm$) may be called manually routine, whereas the low-manual market ($\tau = nl$) covers all the standard low-skilled manual jobs such as traditional services. Different task-skill pairs carry *different* productivities. Hence, workers with different skill levels are *imperfect substitutes* from the perspective of the firm. Thus, our model from the perspective of the firm features heterogeneous productivities and imperfect substitution of workers, differing from the canonical framework (cf. van den Berg and Ridder, 1998; Mortensen, 2003).

In the baseline setting, we focus on the "extensive margin" of vacancy creation in forms of endogenous choice of posting in different segmented markets. That is, $\bar{v}_j$ is given but vacancies posted in market τ, denoted v_τ, is endogenous. In so doing, one can clearly see the market spillover effects of a minimum wage hike. Later, we will show that our results are robust to allowing for the "intensive margin" of vacancy creation with endogenous $\bar{v}_j$.

There is directed search for each job at the relevant market. Let n_τ and v_τ be the measure of workers (both employed and unemployed) and vacancies in the τ market. Under the aforementioned worker-task matching structure, we have $n_a + n_{rh} = \bar{n}_h$ and $v_{rh} + v_{rm} = \bar{v}_r$. Figure 1 summarizes the labor market structure of firms and workers.

This labor market model structure captures realistic features of the economy. Specifically, workers with different skill levels tend to work in different tasks, and hence the segregated market structure. Figure 2 shows the proportion of task for each skill level[3]. For example, about 90% of the high-skilled workers work in either abstract or routine task. Similarly, about 80% of middle-skilled workers work in either routine or manual task, while 80% of low-skilled workers are employed

[3] See Section 4 for the definitions of task and skill level.

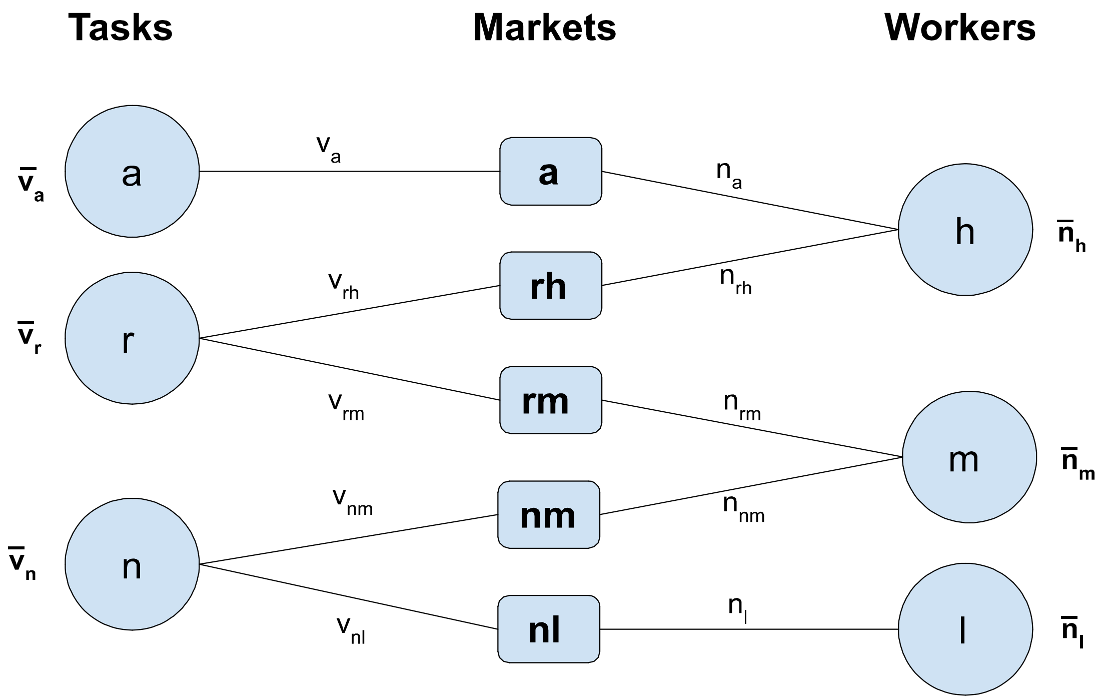


Figure 1: Labor market allocations

in manual task. Therefore, our market structure covers the vast majority of the workers in the US labor market without loss of generality.

Our modeling strategy is rooted on the job-search and wage-posting framework developed by Burdett and Mortensen (1998). This strategy is proper. Particularly, in **?** there is no mass point anywhere in the wage distribution – that is, there is no "bunching" around the minimum wage even after a hike. Figure 3 shows that this is indeed the case. After a series of minimum wage increases from 2007 to 2009 in the US, the wage distribution appears more truncated by the minimum wage without significant bunching around the minimum wage, which is consistent with the prediction by a standard **?** model.

2.1 Workers

Unemployed workers with previous experience in the τ market receives a flow utility b_τ. With flow probability $\phi(\theta_\tau)$ they receive a wage offer, where $\theta_\tau = \frac{v_\tau}{n_\tau}$ is the (endogenously determined) labor market tightness measure in the τ market where a higher theta means a tighter market for employers to recruit workers. Thus, similar to Moscarini and Postel-Vinay (2016), we allow job finding rates to be endogenously determined, rather than exogenously fixed as in the original wage-posting framework developed by Burdett and Mortensen (1998). Let $F_\tau(w)$ be the wage offer

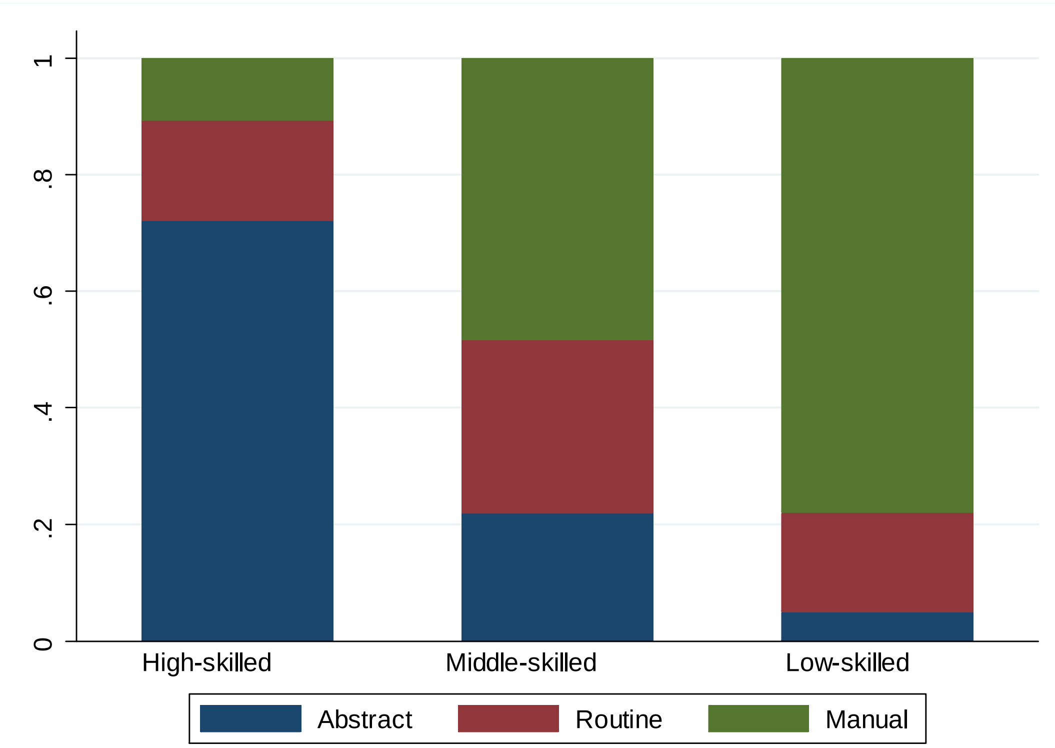


Figure 2: Proportion of tasks by skill level

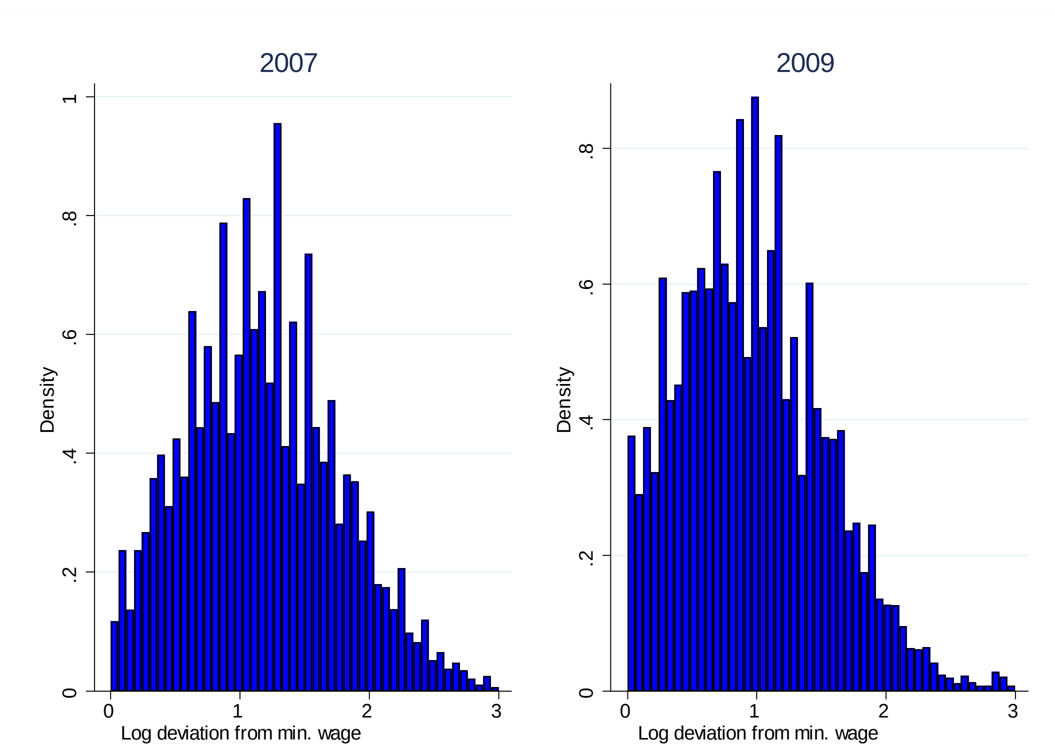


Figure 3: Wage distribution before and after a minimum wage hike

distribution, then the value of being unemployed with experience in the τ market but searching in the τ' market must satisfy the Bellman equation:

$$rU^{\tau}_{\tau'} = b_{\tau} + \phi\left(\theta_{\tau'}\right)\left[\int \max\left\{U^{\tau}_{\tau'}, E_{\tau'}\left(w'\right)\right\} dF_{\tau'}\left(w'\right) - U^{\tau}_{\tau'}\right] \tag{1}$$

where $E_{\tau'}(w')$ is the value of being employed at the wage w' in the τ' market and the term in the square bracket measures the change in value from unemployed to employed. The entry into each relevant market (say, a high-skilled may enter market a or rh) is balanced because more entry reduces the tightness and lowers the flow probability to receive an offer.

Clearly, there exists a reservation wage $R^{\tau}_{\tau'}$ below which unemployed workers would never accept the offer. In the special case when workers are always searching in the same market, it can be shown that

$$R^{\tau}_{\tau} = b_{\tau} \tag{2}$$

That is, the reservation wage is just the flow benefit of unemployment.

As mentioned before, high-skilled workers can choose to search between the a and rh market, and middle-skilled workers can choose between rm and nm market. On the contrary, low-skilled workers can only search in the nl market. Hence, the values of being unemployed for these types of workers are given by $U^{a}_{h} = \max\{U^{a}_{a}, U^{a}_{rh}\}$, $U^{rh}_{h} = \max\left\{U^{rh}_{a}, U^{rh}_{rh}\right\}$, $U^{rm}_{m} = \max\{U^{rm}_{rm}, U^{rm}_{nm}\}$, $U^{nm}_{m} = \max\{U^{nm}_{rm}, U^{nm}_{nm}\}$, and $U^{nl}_{l} = U^{nl}_{nl}$.

Employed workers receives a wage w and faces an exogenous job separation probability δ_{τ}. For simplicity, we assume that currently employed workers in market τ can only search in her own market (say, as a result of own market advantages) and face the same on-the-job-search probability $\phi(\theta_{\tau})$ as the unemployed searching in this market. Hence, the value of being employed follows:

$$rE_{\tau}(w) = w + \phi(\theta_{\tau}) \int \left[\max\left\{E_{\tau}\left(w'\right), E_{\tau}(w)\right\} - E_{\tau}(w)\right] dF_{\tau}\left(w'\right) + \delta_{\tau}\left[\max\{U^{\tau}_{\tau}, U^{\tau}_{\tau'}\} - E_{\tau}(w)\right] \tag{3}$$

which states that the flow value of being employed must be equal to the current wage, plus the expected flow value changing from current to another job and the expected flow value changing to unemployed. Two clarification remarks are in order. First, to an employed high paid worker, it is in general difficult to find a better job because it is more likely to have $\max\{E_{\tau}(w'), E_{\tau}(w)\} = E_{\tau}(w)$. Second, the probability to switch job also depend on the density of the wage distribution $f_{\tau}(w')$. As a result, taking $\phi(\theta_{\tau})$ the same as the unemployed is innocuous.

2.2 Firms

In this economy, each firm in a market τ posts a wage á la Burdett and Mortensen (1998), subject to the statutory minimum wage $w_{\min}$ considered in our paper. The posted wage must maximize the profit and the maximized profit π_τ must be equalized between all firms in the market. Specifically, each firm in market τ solves the following problem:

$$\pi_\tau = \max_{w \geq \{R_\tau, w_{\min}\}} (p_\tau - w)\, \ell_\tau(w) \tag{4}$$

where p_τ is the productivity of the job τ and $\ell_\tau(w)$ is the number of available workers per firm at the wage w. That is, firm's output (sales) is given by $y_\tau = p_\tau \ell_\tau$. It is noted that $\ell_\tau(w)$ is not the typical labor demand function in neoclassical theory of production, but the available workers for each wage that yields identical profit. Thus, it is immediate that for a higher wage posted, more workers can be attracted to the job post.

Clearly, the minimum wage has an impact in the firm's offer distribution only if $w_{\min} > R_\tau$. In this case, we call the minimum wage as *effective* in the market τ.

Similar to the occupational choice problem of the unemployed workers, firms of different tasks (except a) can choose to post vacancies in two different markets. Hence, the profits of routine and manual tasks are given by

$$\pi_r = \max\{\pi_{rh}, \pi_{rm}\}; \quad \pi_n = \max\{\pi_{nm}, \pi_{nl}\} \tag{5}$$

3 Equilibrium

We consider a *stationary wage posting equilibrium* where a non-degenerate distribution of firms is posting wages in each market, workers search in the same market, and, by equilibrium selection,

$$U_a^{rh} = U_{rh}^{rh}; \quad U_{rm}^{nm} = U_{nm}^{nm} \tag{6}$$

In other words, unemployed high-skilled workers who previously worked in the abstract task strictly prefer abstract task ($U_a^a > U_{rh}^a$), but those with previous experience in rh job are indifferent between a and rh jobs – a no-arbitrage condition for workers. Similarly, unemployed middle-skilled workers previously in the routine job strictly prefer routine task ($U_{rm}^{rm} > U_{nm}^{rm}$), but those with previous experience in nm job are indifferent between rm and nm jobs.

There are 6 market clearing conditions, one each for the three types of vacancy tasks and one each for the three types of workers:

$$v_a = \bar{v}_a; \quad v_{rh} + v_{rm} = \bar{v}_r; \quad v_{nm} + v_{nl} = \bar{v}_n$$
$$n_a + n_{rh} = \bar{n}_h; \quad n_{rm} + n_{nm} = \bar{n}_m; \quad n_{nl} = \bar{n}_l \tag{7}$$

It is clear that both $U^{\tau}_{\tau'}$ and π_τ depend on labor market tightness θ_τ. Hence, the market tightness θ_τ measures for all markets $\tau \in \{a, rh, rm, nm, nl\}$, the 5 types of vacancies posted in each market and the 5 types of workers in each market can be determined by the five market tightness definitions ($\theta_\tau = \frac{v_\tau}{n_\tau}$), the 6 market clearing conditions and the 4 indifference boundary conditions given below:

$$\pi_{rh}(\theta_{rh}) = \pi_{rm}(\theta_{rm}); \quad \pi_{nm}(\theta_{nm}) = \pi_{nl}(\theta_{nl})$$
$$U_a^{rh}(\theta_a) = U_{rh}^{rh}(\theta_{rh}); \quad U_{rm}^{nm}(\theta_{rm}) = U_{nm}^{nm}(\theta_{nm}) \tag{8}$$

These indifferent boundary conditions determine labor market tightness endogenously.

3.1 Comparative Static Analysis

In this subsection, we characterize the stationary wage posting equilibrium. Our chief goal is to analyze the effects of changes in the minimum wage. To do so, we shall begin by examining a key channel via the endogenously determined market tightness of each market τ.

Proposition 1 ***Lemma 2*** *(The Relationship between Market Tightness and Wage Distribution and Profits)*

(i) *The wage distribution rises (in the first-order stochastic dominance sense) with a tighter market to employers, i.e.,*

$$\frac{\partial F_\tau(w;\theta_\tau)}{\partial \theta_\tau} < 0 \tag{9}$$

(ii) *Suppose the separation rate is less than the job finding rate, i.e.* $\delta_\tau < \phi(\theta_\tau)$ *, then firm's profit declines when the market is tighter to employers, i.e.,*

$$\frac{\partial \pi_\tau}{\partial \theta_\tau} < 0 \tag{10}$$

Regarding part (i) of Lemma 1, it is clear that when market is tighter to firms, they must post higher wage and as a result the cdf of a particular wage w must be lower. To understand part (ii), note that a tighter market implies a higher job finding rate and hence higher wage. More complicated, there are three effects on profit. First, there is a *direct wage cost effect*: higher wage lowers the markup $p-w$, thus reducing profit directly. Second, there is a *production scale effect*: an increase in market tightness means that worker's job finding rate is higher. The higher wage would attract more workers as well, thus raising the profit. Finally, there is an *employment outflow effect*: more worker inflows must be accompanied by more outflows as a result of on-the-job search in the steady state, and such outflows offset part of the production scale effect and hence lower profit. Under the condition specified in part (ii), the first and third effects outweigh the second, resulting in a reduction in profit.

Using Lemma 1, one can characterize how exogeneous changes in the minimum wage in a particular market may affect that market's wage distribution and profit – the "own-market" effects:

Proposition 3 *(Minimum Wage, Wage Distribution and Profits) Suppose minimum wage is effective in market τ. Then, with a rise in the minimum wage in this market,*

(i) *the wage distribution rises (in the first-order stochastic dominance sense), i.e.,*

$$\frac{\partial F_{\tau}\left(w;\theta_{\tau},w_{\min}\right)}{\partial w_{\min}}<0 \tag{11}$$

(ii) *the profit accrued by firms in this market also decreases, i.e.,*

$$\frac{\partial \pi_{\tau}\left(\theta_{\tau};w_{\min}\right)}{\partial w_{\min}}<0 \tag{12}$$

Part (i) of Proposition 3 is easily understood: when a rise in a market's minimum wage is effective, it forces some low offer vacancies to post higher wage, thus reducing the cdf of a particular wage w. This immediately leads to the finding in part (ii): given the market tightness, a higher posted wage has a direct wage cost effect, which lowers the profit accrued by low offer vacancies and hence all firms in the market as a result of profit equalization.

The reader should be warned that the result in Proposition 3 is under a fixed measure of market tightness. To characterize the equilibrium effects of minimum wage, we must investigate how minimum wage affects the 4 indifference boundaries given in (8). Consider a rise in the minimum wage in the lowest market $\tau = nl$. With Proposition 3, we are ready to analyze the indifferent boundaries for each routine firm and each manual firm to post in two possible markets. Yet, we must also study the indifferent boundaries for each high skilled and each middle skilled to enter into two potential markets. To do so, we impose a mild regularity condition that the own-market effect of market tightness on its search workers' values is all positive:

Condition R *(Regularity Condition)*

$$\frac{\partial U_{a}^{rh}\left(\theta_{a}\right)}{\partial\theta_{a}}>0;\ \frac{\partial U_{rh}^{rh}\left(\theta_{rh}\right)}{\partial\theta_{rh}}>0;\ \frac{\partial U_{rm}^{nm}\left(\theta_{rm}\right)}{\partial\theta_{rm}}>0;\ \frac{U_{nm}^{nm}\left(\theta_{nm}\right)}{\partial\theta_{nm}}>0$$

In our calibration analysis, the validity of Condition R will be checked. In the following propositions, we perform several sets of comparative statics of changes in the minimum wage in the lowest market $\tau = nl$.

Proposition 4 *(Minimum Wage and Labor Market Allocation) Suppose that minimum wage is effective only in market nl, i.e.* $w_{\min} > R_{nl}$*, and that Condition R holds. Then as the minimum wage rises, the equilibrium possesses the following properties:*

(i) *(effect on the potential labor pool of each market) There is a market spillover effect on the worker side such that potential labor pools adjust in an alternating way as follows:*

$$\frac{\partial n_a}{\partial w_{\min}} < 0,\ \frac{\partial n_{rh}}{\partial w_{\min}} > 0,\ \frac{\partial n_{rm}}{\partial w_{\min}} < 0,\ \frac{\partial n_{nm}}{\partial w_{\min}} > 0 \tag{13}$$

(ii) *(effect on the vacancies in each market) There is a market spillover effect on the firm side such that vacancies adjust in an alternating way as follows:*

$$\frac{\partial v_{rh}}{\partial w_{\min}} > 0,\ \frac{\partial v_{rm}}{\partial w_{\min}} < 0,\ \frac{\partial v_{nm}}{\partial w_{\min}} > 0,\ \frac{\partial v_{nl}}{\partial w_{\min}} < 0 \tag{14}$$

Recall from Proposition 3 that higher minimum wage lowers the profit of the firms operating in the nl market. Hence, there is incentive for the manual task vacancies to shift to nm market to hire middle-skilled workers instead. This is a *market spillover effect* on the firm side. Therefore, we have v_{nl} drops but v_{nm} rises. Now since there are more vacancies in the nm market, more middle-skilled workers are attracted to that market, which is a typical *matching externality effect.* Hence, we have n_{nm} increases and n_{rm} falls, where the latter entails a market spillover effect on the worker side. Similarly, seeing less workers available in the rm market, routine task firms choose to post more vacancies in the rh market instead. Thus, we have v_{rm} reduces and v_{rh} increases. Finally, with more high-skilled workers being attracted to the rh market, n_{rh} rises and n_a falls. In short, Proposition 4 shows the presence of cross-market spillover effects that lead to alternating changes in the potential labor pools and vacancy postings through the chain of adjacent labor markets (from low to high), as a result of spillovers on both sides of the labor market in conjunction with matching externalities.

We turn next to examining the effect of lowest market minimum wage on market tightness ($\theta_\tau = \frac{v_\tau}{n_\tau}$):

Proposition 5 *(Minimum Wage and Labor Market Tightness) Suppose that minimum wage is effective only in market nl* ($w_{\min} > R_{nl}$) *and that Condition R holds. Then as the minimum wage* *rises, the lowest market nl is tighter, but all the other markets are thicker to employers, i.e.,*

$$\frac{\partial \theta_\tau}{\partial w_{\min}} > 0, \;\; \tau \in \{a, rh, rm, nm\} \tag{15}$$

$$\frac{\partial \theta_{nl}}{\partial w_{\min}} < 0$$

Using Proposition 4, we can see that, with higher minimum wage in the lowest market nl, its vacancies are lower whereas the labor pool in the highest market a is larger, implying a tighter market in nl but a thicker market in a. Concerning the next highest market rh, we learn from Proposition 4 that there are more vacancies posted which in turn attracts more high-skilled workers. Notice however that in this market it does not pay as much as in market a. Thus, to maintain indifference boundary $U_a^{rh}(\theta_a) = U_{rh}^{rh}(\theta_{rh})$, it must be that market rh is also thicker. Regarding market rm, Proposition 4 indicates that the vacancies in market rm are now lower and its labor pool is smaller. Since market rh is thicker and firm profit $\pi_{rh}(\theta_{rh})$ reduces (Proposition **??**(ii)), to maintain indifference boundary $\pi_{rh}(\theta_{rh}) = \pi_{rm}(\theta_{rm})$ requires that market rm is thicker too. Finally, we turn to market nm and note from Proposition 4 that its vacancies are higher and its labor pool becomes larger. Because market rm is thicker, to maintain indifference boundary $U_{rm}^{nm}(\theta_{rm}) = U_{nm}^{nm}(\theta_{nm})$, we must have a thicker market nm as well.

Since higher market tightness leads to higher job finding rates, unemployment must fall unambiguously. Thus, the following proposition is an immediate consequence of Proposition 5.

Proposition 6 *(Minimum Wage and Unemployment) Suppose that minimum wage is effective only in market nl ($w_{\min} > R_{nl}$) and that Condition R holds. Then as the minimum wage rises, the lowest market has a higher unemployment rate, but unemployment rates in all the other markets are lower, i.e.,*

$$\frac{\partial u_\tau}{\partial w_{\min}} < 0, \;\; \tau \in \{a, rh, rm, nm\} \tag{16}$$

$$\frac{\partial u_{nl}}{\partial w_{\min}} > 0 \tag{17}$$

Since higher market tightness leads to higher job finding, which in turn entails lower unemployment, the result follows immediately from the Proposition 5. The result is interesting: although a minimum wage hike has a direct consequence raising the unemployment rate in the lowest market, it helps reduce unemployment in all other markets. Of course, the overall unemployment effect of the entire economy remains a quantitative question.

Finally, we investigate the minimum wage effects on employment in each market, namely, $e_\tau = n_\tau (1 - u_\tau)$ for $\tau \in \{a, rh, rm, nm, nl\}$. This is the heart of our research, as elaborated in the introduction.

Proposition 7 *(Minimum Wage and Employment) Suppose that minimum wage is effective only in market nl* ($w_{\min} > R_{nl}$) *and that Condition R holds. Then as the minimum wage rises, the equilibrium possesses the following properties:*

(i) *Employment in the lowest market nl falls, i.e.,*

$$\frac{\partial e_{nl}}{\partial w_{\min}} < 0 \tag{18}$$

(ii) *Employment in the routine high-skilled and manual middle-skilled markets are higher, i.e.,*

$$\frac{\partial e_{rh}}{\partial w_{\min}} > 0, \; \frac{\partial e_{nm}}{\partial w_{\min}} > 0 \tag{19}$$

(iii) *Changes in employment in the abstract and routine middle-skilled markets are generally ambiguous.*

The results are direct consequences of Propositions 4(i) and 6. Whenever the minimum wage effects on the potential labor pool and the unemployment rate are opposite, its effect on employment is unambiguous. Specifically, in markets rh and nm, the effects of a minimum wage hike are to enlarge their labor pools and to reduce their unemployment rates. Thus, equilibrium employment rises in these markets. In the lowest market nl, the labor pool is unchanged while the unemployment rate is higher, thereby suppressing its equilibrium employment. In the remaining two markets, a and rm, the potential labor pools shrink but the unemployment rates are lower as well. Hence, the net effects turn out to be ambiguous. What is interesting is the unambiguous beneficial effects of a minimum wage hike on the employment measures in the high-routine market (rh) and the middle-manual market (nm). *Such a finding is a consequence of a combination of two novel channels – the market spillover effects on both sides of the labor market and the matching externality effects in all labor markets.* Since $Y_\tau = p_\tau e_\tau$, the effects of a minimum wage hike on total output in each market follows immediately.

3.2 Endogenous intensive-margin vacancy creation

So far, we have the intensive margin of vacancy creation assumeing that the total number of the vacancy for each task $\bar{v}_j$ remain unchanged, following a change in the minimum wage. One could argue, however, that an increase in minimum wage, which lowers the profit (Proposition **??**), would also lower the incentive for firms to post vacancies, and thus the number of vacancies would decrease. How would the effects of minimum wage change when the intensive margin of vacancy creation is endogenous?

To address this, we allow the total number of vacancies for each task, $\bar{v}_j,\ j \in \{a, r, n\}$, to be increasing in the respective profit:

$$\frac{\partial \bar{v}_j}{\partial \pi_j} > 0,\ j \in \{a, r, n\} \tag{20}$$

It turns out that Propositions 3 through 7 remain qualitatively true under such extension (See Appendix for a proof).

Intuitively, when the profit affects vacancy creation positively, profits and utility functions becomes less sensitive to changes in the minimum wage. For example, suppose there is a change in minimum wage such that the profit π_j increases. As a result, the number of vacancy $\bar{v}_j$ will increase as well, which raises the market tightness and hence a lower profit. Therefore, the effect on profits will be muted (with the same direction) when vacancy creation is endogenously determined. The same thing goes for the utility as a function of market tightness. This implies that changes in minimum wage will have the same qualitative impact on the labor market.

The resulting quantitative effects, however, may be larger or smaller. On the one hand, lower sensitivity of the profits and utility functions entails that adjustment in the labor market allocation would have to be larger to restore market equilibrium. On the other hand, the muted responses from the firm's profit also mean that less adjustment is needed.

3.3 Taking stock

Propositions 3-7 provide a general guideline toward understanding the effects of minimum wage hikes. We have shown that a minimum wage hike in the lowest-pay low-manual market raises the wage distribution in the first-order stochastic dominance sense and reduces firm profit accrued. Moreover, under a regularity condition that restricts the own-market effect of market tightness on its search workers' values to be all positive, such a hike (i) generates a cross-market spillover effect leading to alternating changes in the potential labor pools and vacancy postings through the chain of adjacent labor markets, (ii) tightens the lowest market but eases all other markets, (iii) raises the unemployment rate of the lowest market but reduces those of other markets, and (iv) decreases the employment in the lowest market but increase it in the routine high-skilled and manual middle-skilled markets.

Thus, even the regularity condition, the net employment effects of a minimum wage hike on the abstract market and the middle-routine markets and the overall employment effect of the entire economy are generally ambiguous, When the regularity condition does not hold, when minimum wage hikes occur in more than one market (say the lowest two markets), and when the potential labor pools and vacancy postings are both changing, the effects of minimum wage hikes become a quantitative question, to which we now turn.

4 Quantitative analysis

In this section, the model is calibrated to the US labor market. We then evaluate the long-run impacts of the federal minimum wage increase during the great recession and the ongoing local minimum wage increase in Seattle on the labor market. In addition to the market spillover effects analyzed in the previous section, we also consider skill upgrading (Dustmann et al., 2021) and vacancy adjustment over time in the quantitative exercise. As stressed in the introduction, the empirical literature does not generate robust findings regarding the impacts of minimum wage hikes. For illustrative purposes, however, we shall cite some of such findings from various work whenever our results can be directly compared.

4.1 Data

To compute wages and other labor market statistics for each task in the US economy, we use the micro-data from the CPS Annual Social and Economic Supplement (ASEC), which is conducted every March and asks detailed questions about each individual's labor income. Our sample contains workers aged 16 to 64 from 2003 to 2017. We exclude armed forces and those out of labor force. Workers' occupations are aggregated into ten occupation groups according to the definition in ASEC. Following the theory-based occupational classifications in Jaimovich and Siu (2018) and Foote and Ryan (2015), we then categorize these ten occupation groups into abstract (a), routine (r), and manual (m) tasks, as shown in Table 1 [4].

[4] Specifically, we identify Nonroutine Cognitive as Abstract tasks, Routine Cognitive as Routine tasks, and Routine Manual and Nonroutine Manual as Manual tasks.

Task	Occupation
Abstract	Professional & related, Management, business, & financial operations
Routine	Office & administrative, Sales & related
Manual	Production, Transportation, Installation, Construction & Service

Table 1: Occupations and tasks

We calculate the hourly wage by dividing the total annual wage and salary earnings by the product of the number of weeks worked and the usual hours workers per week. All wages are deflated to 2003 dollars using the Consumer Price Index from the Bureau of Labor Statistics. As is standard in the literature, we trim wages at the 0.1 and 99.9 percentiles.

Finally, we define skill level of a worker by her education level. Specifically, we take low-skilled workers as those with less than high school diploma, middle-skilled workers as those who completed high school but less than four-year college, and high-skilled workers as those with at least four-year college.

4.2 Calibration and specification

We assume a standard constant-returns-to-scale matching function $M(n,v) = An^{\alpha}v^{1-\alpha}$. Hence the job finding rate is given by

$$\phi(\theta_\tau) = \frac{M(n_\tau, v_\tau)}{n_\tau} = M(1, \theta_\tau) = A\theta_\tau^{1-\alpha} \tag{21}$$

One period in the model corresponds to one year. We set the real interest rate as $r = 5\%$, as commonly used in macro studies. The matching elasticity of workers in the matching function, α, is taken to be 0.5, which is also standard in the literature (e.g., see Petrongolo and Pissarides, 2001). We will nonetheless perform sensitivity analysis with regard to α.

The population of the economy is normalized to one. We consider skill upgrading (Dustmann et al., 2021) and vacancy adjustment over time. Hence, $\{\bar{n}_h, \bar{n}_m, \bar{n}_l\}$ is taken to the fraction of workers (both employed and unemployed) for each skill in each time period. For each task $k \in \{a, r, n\}$, we also measure the effective number of jobs $\bar{v}_k$ as the number of workers working in each task in each time period.[5] Again, sensitivity analysis will be performed by considering task polarization à la Autor and Dorn (2013).

[5] Specifically, we measure the labor force and the number of employees for each skill and task before and after the minimum wage hike, and then apply a linear transition in between.

The rest of the parameters $\{\delta_\tau, p_\tau, A\}$ are jointly calibrated to match the US labor market statistics. In particular, the separation rates are chosen to match the unemployment rate for each market, and the labor productivity of each job is calibrated to match the relative wages of each job in the data, after normalizing the labor productivity of job nl as one. Finally, the flow utility of unemployment is taken to be 40% of the productivity of each market, following Shimer (2005). In the baseline calibration, we set the level of minimum wage as the 2007 federal minimum wage ($5.15), which corresponds to 0.407 in terms of labor productivity unit in the model.

Table 2 shows the baseline parameters, and Table 3 shows the comparison of data vs. model with respect to the selected target moments. Lastly, Table 4 shows the baseline results on aggregate output, employment, and potential labor pool in each market.

4.3 Federal minimum wage increase (2007 - 2009)

Over a decade ago, there was a federal minimum wage hike, where $w_{\min}$ increased from $5.15 in 2007 to $7.25 in 2009 over several incremental rises in two years. For comparison purposes, we convert all values in 2003 dollars. As such, this minimum wage hike corresponds to an increase from $4.57 to $6.22.

In the theory established, we conduct comparative statics with respect to a small increase of minimum wage that prevails only in the lowest (nl) market. In our quantitative analysis, the size of minimum wage increase is large. Thus, it is possible that the minimum wage may become effective in the second lower (nm) or even the middle ranked (rm) market. As it can be seen below, in the case of the federal minimum wage increase, minimum wage at the end becomes effective in both the nl and the nm markets; for the local minimum wage increase in Seattle, minimum wage by 2021 will be effective in all three lower markets, nl, nm and rm.

For illustrative purposes, think of the federal case where minimum wage binds in both the nl and the nm markets. On the one hand, the minimum wage hike in the nl market generates a market spillover effect from the firm side affecting all labor market measures in the nm market. On the other, there is a direct minimum wage effect in the nm market because it is binding after the change. When the former market spillover effect and the latter direct effect affect a labor market measure in the nm market in the same direction, the net effect is amplified. When the former and the latter direct effects are conflicting with each other, the net effect is dampened and the sign may even be switched if the latter is stronger.

By plotting a labor market measure against the minimum wage, we can see the emerge of a "kink" that is corresponding to the binding of the minimum wage in a higher (than the lowest)

Parameters	Meaning	Values	Target/Source
b_a	Flow utility of unemployment for job a	1.12	40% of productivity
b_{rh}	Flow utility of unemployment for job rh	0.92	40% of productivity
b_{rm}	Flow utility of unemployment for job rm	0.68	40% of productivity
b_{nm}	Flow utility of unemployment for job nm	0.53	40% of productivity
b_{nl}	Flow utility of unemployment for job nl	0.40	40% of productivity
δ_a	Separation rate of job a	0.00084	Unemployment rate of job a
δ_{rh}	Separation rate of job rh	0.0034	Unemployment rate of job rh
δ_{rm}	Separation rate of job rm	0.0016	Unemployment rate of job rm
δ_{nm}	Separation rate of job nm	0.0065	Unemployment rate of job nm
δ_{nl}	Separation rate of job nl	0.0017	Unemployment rate of job nl
p_a	Labor productivity of job a	2.79	Relative wage of job a
p_{rh}	Labor productivity of job rh	2.29	Relative wage of job rh
p_{rm}	Labor productivity of job rm	1.70	Relative wage of job rm
p_{nm}	Labor productivity of job nm	1.33	Relative wage of job nm
p_{nl}	Labor productivity of job nl	1	Normalization
r	Interest rate	0.05	/
α	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
A	Matching efficacy	0.053	Job finding rate
$\bar{v}_a$	Firm size of task a	Varied by time	Number of employees in task a
$\bar{v}_r$	Firm size of task r	Varied by time	Number of employees in task r
$\bar{v}_n$	Firm size of task n	Varied by time	Number of employees in task n
$\bar{n}_h$	Number of h workers	Varied by time	Labor force with skill h
$\bar{n}_m$	Number of m workers	Varied by time	Labor force with skill m
$\bar{n}_l$	Number of l workers	Varied by time	Labor force with skill l
$w^0_{\min}$	Baseline minimum wage	0.41	2003; in terms of labor productivity

Table 2: Parameters (Baseline calibration)

Moment	Wage ratio $\frac{w_a}{w_{nl}}$	$\frac{w_{rh}}{w_{nl}}$	$\frac{w_{rm}}{w_{nl}}$	$\frac{w_{nm}}{w_{nl}}$	Unemployment rate u_a	u_{rh}	u_{rm}	u_{nm}	u_{nl}
Data	3.02	2.31	1.76	1.36	0.018	0.032	0.048	0.067	0.098
Model	2.93	2.38	1.75	1.35	0.018	0.033	0.045	0.069	0.097

Table 3: Selected moments, data vs. model

Market	Aggregate output (Y)	Employment (e)	Potential labor pool (n)
a	0.81	0.29	0.30
rh	0.02	0.01	0.01
rm	0.74	0.44	0.46
nm	0.15	0.11	0.12
nl	0.10	0.10	0.11

Table 4: Baseline results

market. For the sake of brevity, we present only such changes for potential labor pool (Figure 4) and unemployment (Figure 5) in each market and in aggregation, while relegating others to the Appendix. Panel (a) shows the overall impact while panel (b) shows the market spillover effect only, by holding the number of workers and vacancy constant. First, as it can be seen at the kink, the minimum wage becomes binding in the nm market when it reaches about \$5.95. Second, the market spillover effects, which are consistent with the theory developed in the previous section, is relatively small compared to the overall effect until the minimum wage reaches the kink, after which the market spillover effect starts affecting the labor market significantly. In particular, after reaching the kink, potential labor pools of the nm and the rh markets rise sharply while that of the rm market drops by a noticeable margin.

In order to understand the net effect of a minimum wage hike, we thus compute how responsive of each labor market measure changes before and after the kink (i.e., the minimum wage elasticities $\frac{w_{\min}\partial x_{ij}}{x_{ij}\partial w_{\min}}$, evaluated at \$4.57 and \$6.22, respectively). These elasticities are summarized in Table 5.

The results suggest that the minimum wage elasticities of potential labor pool in market $\tau \in \{a, rh, rm, nm\}$ become larger in absolute value when the minimum wage becomes binding in the nm market. This is consistent with Proposition 4: the market spillover effect from the firm side and the direct effect on the potential labor pool in the nm market are both positive and reinforcing,

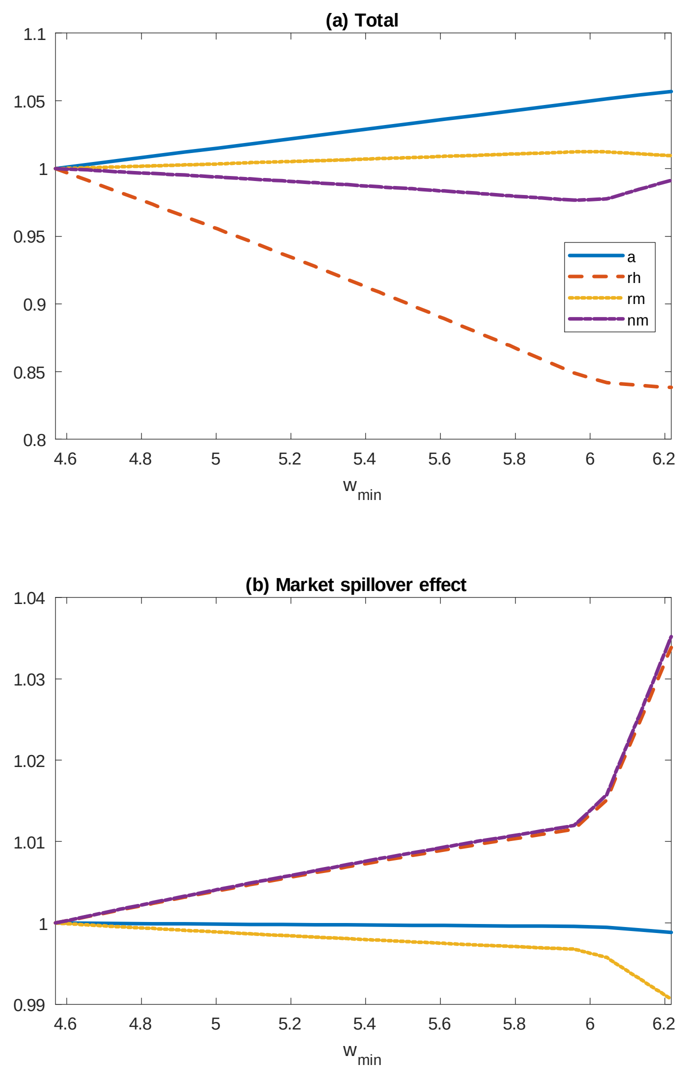


Figure 4: Potential labor pool (federal minimum wage increase), normalized

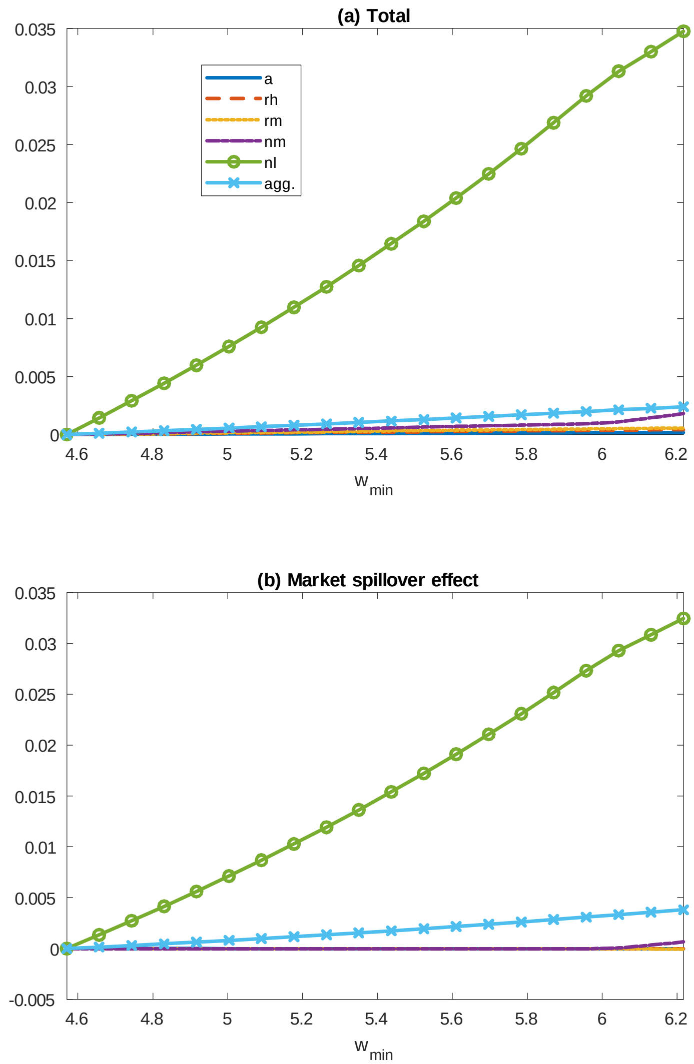


Figure 5: Unemployment rate (federal minimum wage increase)

	At w_{min}				At w_{min}		
Measure	4.57	6.22	Change	Measure	4.57	6.22	Change
Potential labor pool				Vacancy allocation			
n_a	-0.002	-0.023	-0.021	v_a	0	0	0
n_{rh}	0.047	0.838	0.791	v_{rh}	0.049	0.868	0.819
n_{rm}	-0.013	-0.190	-0.177	v_{rm}	-0.011	-0.159	-0.148
n_{nm}	0.048	0.726	0.677	v_{nm}	0.051	0.029	-0.022
n_{nl}	0	0	0	v_{nl}	-1.734	-2.217	-0.483
Unemployment rate				Employment or Output			
u_a	-0.001	-0.011	-0.010	e_a or Y_a	-0.002	-0.023	-0.021
u_{rh}	-0.001	-0.015	-0.014	e_{rh} or Y_{rh}	0.047	0.838	0.792
u_{rm}	-0.001	-0.015	-0.014	e_{rm} or Y_{rm}	-0.013	-0.189	-0.176
u_{nm}	-0.001	0.324	0.325	e_{nm} or Y_{nm}	0.049	0.701	0.652
u_{nl}	0.786	0.966	0.180	e_{nl} or Y_{nl}	-0.084	-0.146	-0.062

Table 5: Minimum wage elasticities (federal minimum wage increase)

leading to a larger increase in n_{nm} and hence larger changes in other potential labor pool via market spillover effects from both sides. Similar reinforcing effect is found in firm profit in which a minimum wage hike causes a larger drop after it becomes binding in the nm market. Turning now to unemployment, we can see that the negative market spillover effect from the firm side is small but there is a sizable direct and positive effect on u_{nm} (recall Proposition 6). This induces a sign switch from (small) negative to (large) positive. As a consequence, the minimum wage elasticity of market tightness θ_{nm} also turns from positive to negative.

With the above discussion in mind, we are ready to examine how much percentage changes in each labor market measure this federal minimum wage hike induces. The results are summarized in Table 6.

Let us begin with the effects of federal minimum wage hike on employment (or output), which is the heart of this study. Recall that $e_\tau = n_\tau(1 - u_\tau)$ and $Y_\tau = p_\tau e_\tau$. It should be noted that the total numbers of workers and jobs are adjusting after the minimum wage hike, which is not captured by the spillover effects. As a result of the reinforcing positive effects from market spillover and direct minimum wage changes, the potential labor pool in the a market features the largest 5.68% increase. Through market spillover from the worker side, it also induces a 16.15% drop in the

Measure	Percentage Change (%)				
Potential labor pool	n_a	n_{rh}	n_{rm}	n_{nm}	n_{nl}
	5.68	-16.15	0.94	-0.84	-16.49
Vacancy allocation	v_a	v_{rh}	v_{rm}	v_{nm}	v_{nl}
	3.61	-18.29	-1.70	-6.22	-58.34
Unemployment rate	u_a	u_{rh}	u_{rm}	u_{nm}	u_{nl}
	0.017	0.042	0.058	0.180	3.477
Average wage	$w_{mean,a}$	$w_{mean,rh}$	$w_{mean,rm}$	$w_{mean,nm}$	$w_{mean,nl}$
	-0.01	-0.02	-0.03	-0.02	-0.31
Employment or	e_a or Y_a	e_{rh} or Y_{rh}	e_{rm} or Y_{rm}	e_{nm} or Y_{nm}	e_{nl} or Y_{nl}
Aggregate output	5.66	-16.19	0.88	-1.04	-19.70
Income inequality	$\sigma^2(\log(w))_a$	$\sigma^2(\log(w))_{rh}$	$\sigma^2(\log(w))_{rm}$	$\sigma^2(\log(w))_{nm}$	$\sigma^2(\log(w))_{nl}$
	0.96	1.21	1.20	-3.40	-35.37
Aggregate measures	u_{agg}	w_{agg}	Y_{agg}	$Y_{productivity}$	$\sigma^2(\log(w))_{agg}$
	0.24	1.64	1.50	1.75	-4.62

Table 6: Percentage change of economic variables (federal minimum wage increase)

rh market potential labor pool and through market spillover from the firm side a 16.49% drop in the nl market potential labor pool. While the binding minimum wage in the nl market results in a sizable 3.48 percentage point increase in its unemployment rate, the conflicting effects from market spillover and direct minimum wage changes in the nm market lead to a modest rise of 0.18 percentage point in its unemployment. This is consistent with the empirical results by Aaronson and Phelan (2019), who find that while an increase in minimum wage leads to a drop in employment of cognitively routine occupations, there is no significant employment on manual occupations. Similarly, Lordan and Neumark (2018) estimates that a rise in minimum wage decreases significantly the employment share of automatable jobs, which are mostly routine in nature. Additionally, Cengiz et al. (2019) find little change in low-wage jobs. Our results are, however, contrasting with Card and Krueger (1995) and the follow-up empirical studies cited in the introduction.

Now, the increase in the nl market unemployment and the decrease in the potential labor pool directly translates into a 19.70% drop in its employment and output. The reduction in the labor force and vacancies in the nm market labor force together with a small rise in its unemployment lead to a large 14.52% increase in its employment and output. Through market spillovers, employment and output in the rm market increases by 1.04% and those in the rh market drops by 16.19%, largely driven by their potential labor pool and vacancies changes. With increasing labor force and vacancies, output and employment in the a market rises by 5.66% despite the presence of market spillover effects.

Turning to other labor market measures, we note that federal minimum wage hike induces a large 58.34% drop in vacancies in the nl market, together with a 6.22% vacancy decrease in the nm market., reflecting also the skill upgrading from the worker's side The sizes of vacancy creation in the a and rh markets are comparable to the respective labor force changes. These together lead to changes in market tightness. Changes in average wages in each market are essentially negligible due to offsetting changes within and across markets. Nonetheless, the minimum wage effects on wage inequalities measured by variances of log wages are nonnegligible. In particular, a higher minimum wage significantly lowers wage inequality in the nl market; it also suppresses wage inequality in the nm market when the minimum wage becomes binding. While wage inequality in the nl market falls by 35.37%, that in the nm market drops by 3.4%. Comparable changes in other markets purely through market spillover are positive, though being generally small. Autor et al. (2016) estimate that minimum wage reduces income inequality for both low and high income earners, suggesting the possiblity of a spillover effect on inequality which is consistent with our findings. Cengiz et al. (2019) also find spillover effects up to $3 above the minimum wage.

Market	D(density)	D(profit)
a	0.00000	0.0003
rh	0.00000	0.0005
rm	0.00000	0.0005
nm	0.0484	0.0001
nl	0.0769	0.0001

Table 7: Change in the density of workers at the minimum wage vs. change in the profit (federal minimum wage increase)

Overall, in the entire economy, the offsetting effects of federal minimum wage hike only raise the unemployment rate by 0.24 percentage point, but increases output by 1.5 percentage by the compositional effect. The negative effect on employment is also estimated by Aaronson and French (2007). Our result is also in line with Clemens (2015) and Clemens and Wither (2019), who estimate empirically the impact of the federal minimum wage increase during the Great Recession, resulting in a decline in the overall employment by about half a percentage point.

As a result of the direct minimum wage effect, the average wage of the economy increases by 1.64%. While overall wage inequality reduces by 4.62%, overall productivity increases by 1.75 percentage. The effect on inequality is consistent with Lee (1999) and Autor et al. (2016), who find that the falling federal minimum wage in the 1980s is responsible for the rising income inequality during the same time.

To this end, we would like to apply our model to a recent discussion on linking minimum wage to markdown via labor market power (see, e.g., Berger, Herkenhoff and Mongey 2019). Specifically, an increase in the minimum wage may induce some firms to compress their markdowns and raise wages while expanding employment along workers' labor supply curves. Thus, in a market featuring a greater density of workers at the minimum wage, one would expect firms to earn lower profits whereas the market's average wage is expected to be higher (See Figure 6). When the number of workers and jobs are held constant, the predictions on profits and average wages are consistent with our partial equilibrium effects shown in Propositions 3(ii) (profits) and 3(i) (wage distribution). However, in a dynamic environment when there is skill upgrading and subsequent change in vacancy creation, the overall change in the profits turns out to be positive, as shown in Table 7.

Remark: An important remark is in order. Specifically, while some of the responses to minimum wage hikes may seem to be large, such effects may be dampened if switches in jobs and vacancy

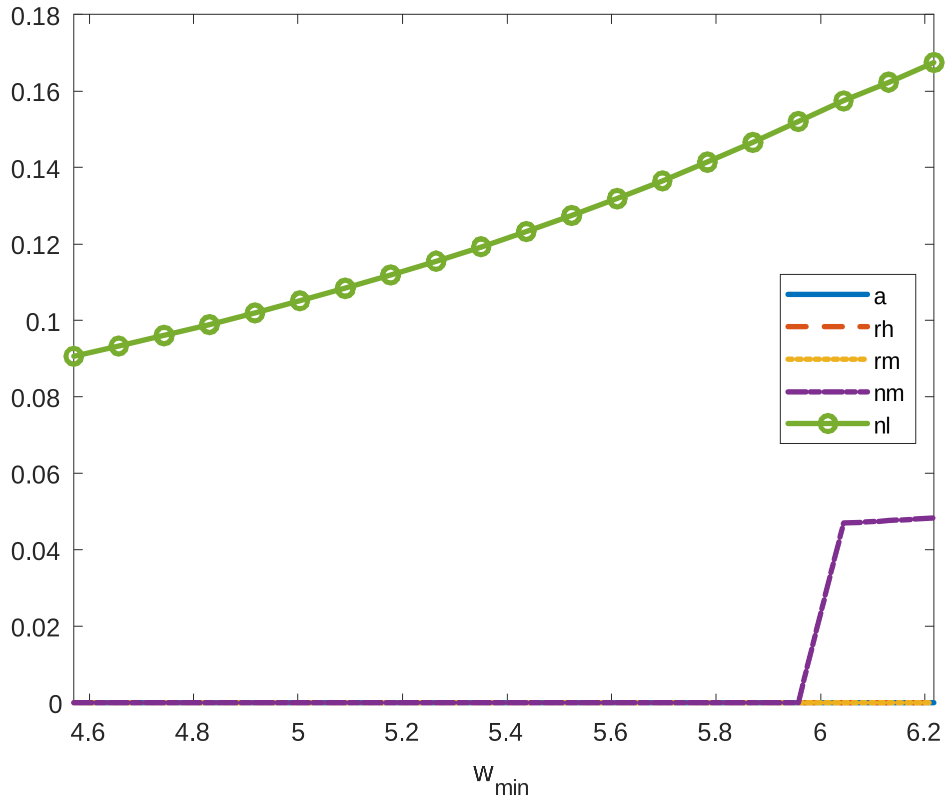


Figure 6: Density of workers at the minimum wage (federal minimum wage increase)

postings take time. That is, the minimum wage impacts estmated in our calibrated economy should be viewed as intermediate- to longer-run effects.

4.4 Local minimum wage increase (2017 - 2021)

While the federal minimum wage has been stagnant since 2009, a number of states and municipals have been implementing and raising their local minimum wage to a much higher level. For example, the state of California had a minimum wage of $11 in 2018 and is set to increase it to $15 in 2022. Similarly, Massachusetts also plans to raise its local minimum wage from $11 in 2018 to $15 in 2023.

Here we are considering the similar case of Seattle, where the minimum wage will be increasing from $11 in 2017 to $15 in 2021. To evaluate its consequences, we convert these figures into 2003 dollars and project an annual price increase of 2.07% from 2017 to 2021, which is the annual inflation rate from 2003 to 2017. This translates to the increase of minimum wage from $8.26 to $10.37 in 2003 dollars. We then project the annual growth rate of the baseline productivity from 2017 to 2021 to be roughly the same as the annual growth rate in real wage for the nl group from 2003 to 2017, which is about 1.00%.

As we have seen in the previous section, the minimum wage starts to bind in the nm market when it is above $5.95. In fact, we can see below that the minimum wage is effective also in the rm market when it reaches $8.59. In this case, the market spillover effect can spill over to both higher and lower markets, which, when coupled with the direct effects on all three binding markets, complicates the analysis for evaluating the overall effects. As we will see in a moment, however, in most of the cases the effects of minimum wage are amplified once it becomes binding in rm market.

Figure 7 and 8 show the effects of local minimum wage change on the potential labor pool and unemployment. We can see clearly that for the effects on a and rh markets, there is a kink at $8.59, which corresponds to the point when the minimum wage is also blinding in the rm market. Unlike the federal minimum wage, however, the market spillover effect is strong enough at the beginning so that the total effects on the labor market is dominated by the market spillover effects.

Since the minimum wage is not binding in those two highest markets, we conclude the amplification is purely due to the additional market spillover effect spilled over from the rm market. As with the federal case, the large increase in local minimum wage would cause labor flows from the a market to rh market, and from the rm market to the nm market. It is perhaps surprising that the direct binding effect in the rm market does not have much mitigating impact on the falling of n_{rm}. This shows that the indirect market spillover effects from the lower binding markets (i.e. nm and nl) are much stronger than the direct effect in this case. The unemployment rate, however, tells a

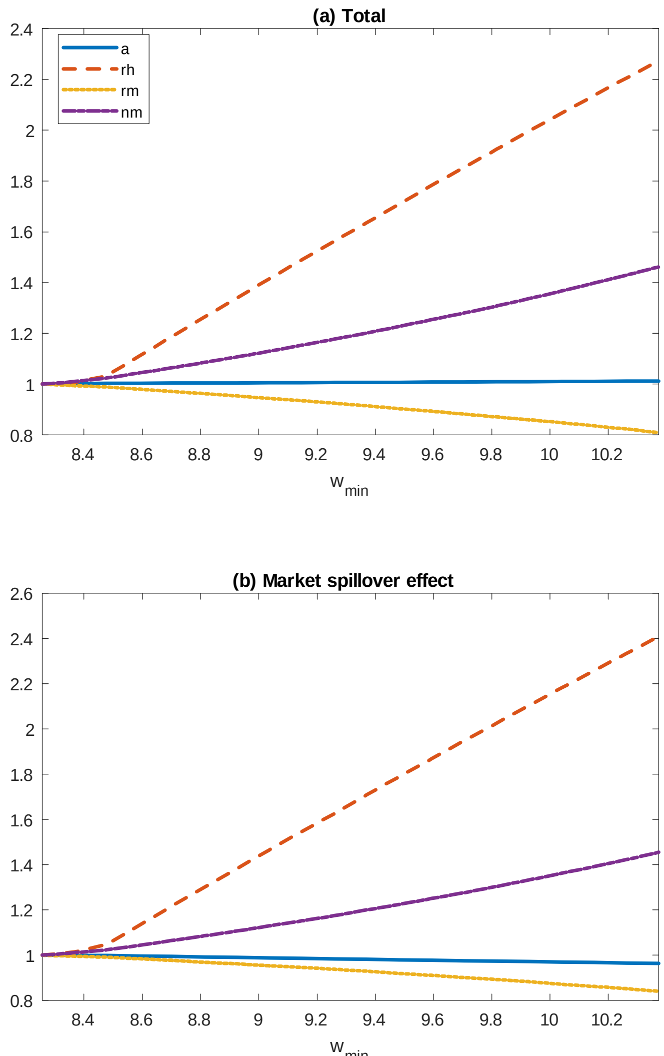


Figure 7: Potential labor pool (local minimum wage increase), normalized

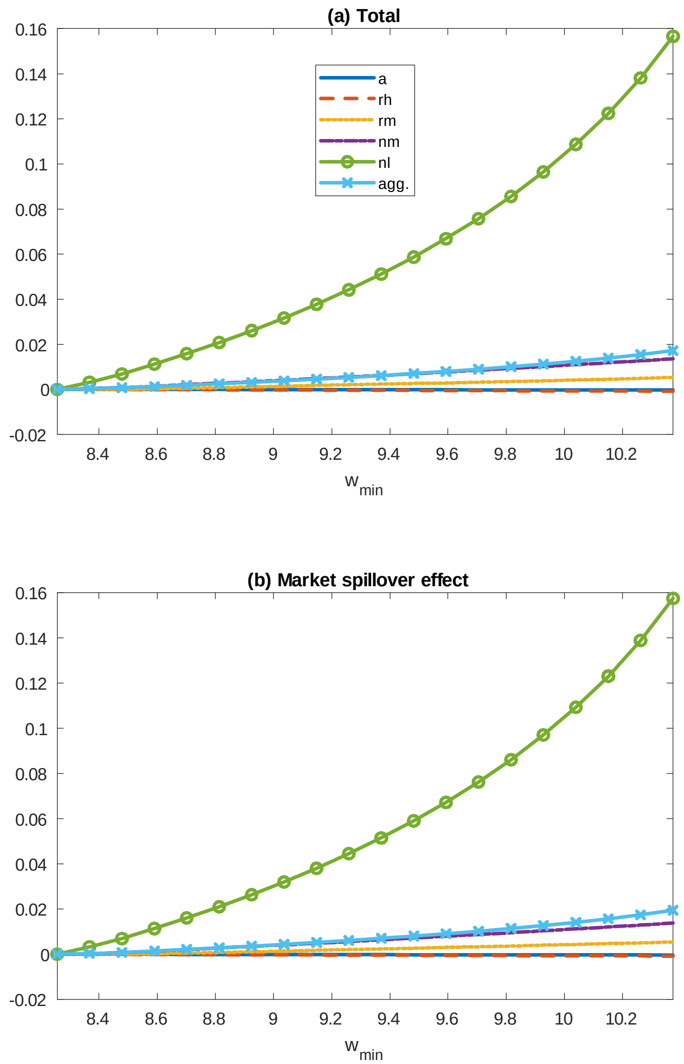


Figure 8: Unemployment rate (local minimum wage increase)

Measure	At w_{min} 8.26	10.37	Change	Measure	At w_{min} 8.26	10.37	Change
Potential labor pool				Vacancy allocation			
n_a	-0.035	-0.213	-0.178	v_a	0	0	0
n_{rh}	1.361	3.679	2.319	v_{rh}	1.406	3.961	2.554
n_{rm}	-0.359	-1.584	-1.225	v_{rm}	-0.312	-3.293	-2.981
n_{nm}	1.019	2.492	1.473	v_{nm}	0.026	0.020	-0.006
n_{nl}	0	0	0	v_{nl}	-4.251	-20.336	-16.085
Unemployment rate				Employment or Output			
u_a	-0.017	-0.105	-0.087	e_a or Y_a	-0.035	-0.211	-0.176
u_{rh}	-0.022	-0.136	-0.113	e_{rh} or Y_{rh}	1.361	3.684	2.322
u_{rm}	-0.022	0.814	0.837	e_{rm} or Y_{rm}	-0.358	-1.627	-1.269
u_{nm}	0.459	1.125	0.666	e_{nm} or Y_{nm}	0.982	2.382	1.400
u_{nl}	1.780	6.990	5.211	e_{nl} or Y_{nl}	-0.355	-3.332	-2.977

Table 8: Minimum wage elasticities (local minimum wage increase)

different story. We can see that while u_{nm} and u_{nl} are increasing due to the fact that the minimum wage is binding in these markets, u_{rm} initially falls mildly due to the spillover of the matching externality effects from the lower markets. Nevertheless, once the minimum wage becomes binding in the rm market, the own matching externality effect dominates and hence leads to an increase in the unemployment rate, similar to what we have seen in the federal case. Overall, the market spillover effect accounts for almost all the increases in the potential labor pools of the rh and the nm markets; it also accounts for a lion's share of the fall in the potential labor pool of the rm market.

Again we can also look at the minimum wage elasticity to understand the mechanism and to evaluate the quantitative significance of the impact of the local minimum wage change. Table 8 shows the minimum wage elasticity with respect to the local minimum wage before and after the kink (i.e., at \$8.26 and \$10.37 respectively).

First, we see that in general the minimum wage elasticity for the a and rh markets becomes much larger after the rm market becomes binding. For example, the elasticity for v_{rh} after kink becomes three times as high as that before the kink. The elasticity for the unemployment in the a market becomes six times as high. Similar pattern can be observed for other variables such as the aggregate output. This shows that the spillover effects from the lower markets become much stronger with

Measure	Percentage Change (%)				
Potential labor pool	n_a	n_{rh}	n_{rm}	n_{nm}	n_{nl}
	1.299	127.421	-18.946	46.212	-6.900
Vacancy allocation	v_a	v_{rh}	v_{rm}	v_{nm}	v_{nl}
	5.325	139.370	-36.111	1.445	-83.708
Unemployment rate	u_a	u_{rh}	u_{rm}	u_{nm}	u_{nl}
	-0.034	-0.081	0.542	1.371	15.654
Average wage	$w_{mean,a}$	$w_{mean,rh}$	$w_{mean,rm}$	$w_{mean,nm}$	$w_{mean,nl}$
	0.019	0.047	0.033	0.204	-0.877
Employment or	e_a or Y_a	e_{rh} or Y_{rh}	e_{rm} or Y_{rm}	e_{nm} or Y_{nm}	e_{nl} or Y_{nl}
Aggregate output	1.334	127.611	-19.406	44.045	-24.379
Income inequality	$\sigma^2(\log(w))_a$	$\sigma^2(\log(w))_{rh}$	$\sigma^2(\log(w))_{rm}$	$\sigma^2(\log(w))_{nm}$	$\sigma^2(\log(w))_{nl}$
	-1.860	-2.349	-21.228	-34.435	-59.829
Aggregate measures	u_{agg}	w_{agg}	Y_{agg}	$Y_{productivity}$	$\sigma^2(\log(w))_{agg}$
	1.725	-0.424	-1.441	0.381	1.263

Table 9: Percentage change of economic variables (local minimum wage increase)

the addition of rm market binding effects. This is mainly due to the strong substitution effects on the firm side for the routine tasks. Second, we also observe that the elasticity for n_{nm}, the potential labor pool in the nm market, becomes significantly larger. This is a consequence of the strong substitution from the worker's side. Interestingly, the direct binding effect of the minimum wage on the rm market does not induce middle-skilled workers to switch from nm to rm market. In fact, they *increasingly substitute away* from the rm market. Third, as discussed before, the existence of the direct minimum wage impact also induces a sign change of the elasticity of u_{rm} from negative to positive. Finally, the binding effects of the rm market induce a larger reduction of the income inequality. This is especially true for the nl market, where the wage dispersion becomes twice as large as that before the kink.

To evaluate the overall impact of the policy change, Table 9 shows the percentage change of the labor market variables under the local minimum wage change.

Compared to the federal minimum wage change, we can see that the percentage changes are much larger in this case, though the qualitative effects are essentially alike. A noticeable exception is the unemployment rate in the a and rh market, where the hike in the minimum wage now leads to a *decrease* in u_{rm} instead of an increase as in the federal minimum wage case. The reversal of the sign is due to the now much stronger spillover effects predicted by the theory, which contributes to a much larger positive impact on aggregate output in the rh market, where the production increases by more than 100%. As in the case of the federal minimum wage change, the effects on employment and aggregate output are dominated by the effects on the potential labor pool n and vacancies v. Hence, we observe that the percentage change in employment is similar to that in the potential labor pool, except for the nl market, where the change is amplified by the large reduction in the vacancy as well. Overall, the quantitative impact on average wage in each market remains relatively small.

On the aggregate level, the local minimum wage leads to a 1.73 percentage point increase in overall unemployment and 1.44% drop in aggregate production, both of which are quantitatively significant. This is consistent with Meer and West (2016), who find significant negative effect of local minimum wage on overall employment. The aggregate effects on aggregate wages and income inequality, however, remain relatively small. The local minimum wage change leads to a slight increase of 1.26% in the overall wage dispersion. Compared to the much larger reduction in each individual market, we can see that the substitution of labor among these markets partly mitigate the impact on inequality. The composition change of labor toward lower market also leads to a slight drop in aggregate wages.

As in the Federal case, we also identify a negative relationship between the density of workers at the minimum wage and firm profits across different markets (See Figure 9 and Table 10). Also similarly, a local minimum wage hike now increases total employment in the three lower markets where minimum wage is binding after the change, whereas average wage over these market turns out to be lower due again to a dominating general equilibrium effect.

4.5 Sensitivity Analysis

In this section, we perform robustness check. In particular, we check our findings when we alter each of the following preset parameters case-by-case: (i) set the worker matching elasticity α as 0.72 (Shimer 2005); (ii) set the worker matching elasticity α as 0.4 (Blanchard and Diamond 1990); (iii) consider *task polarization* (Autor and Dorn, 2013) such that the number of vacancies of abstract and manual tasks increases by 10% and the number of vacancies of routine task decreases by 10%. To be brief, we focus only on the key indicators, namely, unemployment, employment/output, mean wage, and wage dispersion, to contrast with the baseline calibration outcomes. Table 11 shows the

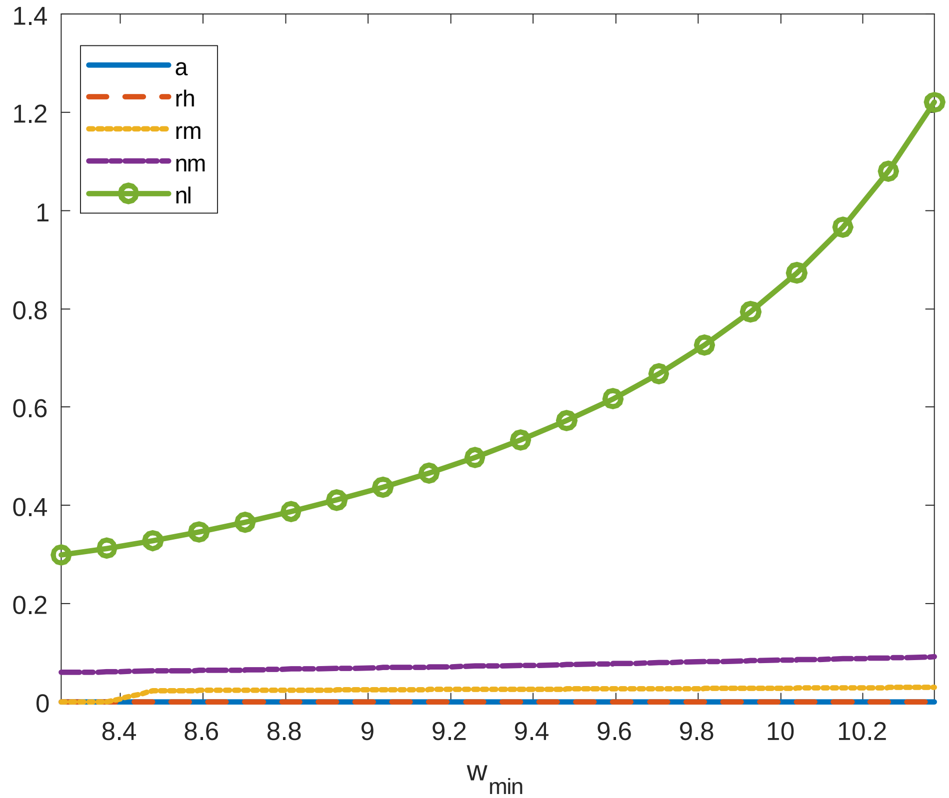


Figure 9: Density of workers at the minimum wage (local minimum wage increase)

Market	D(density)	D(profit)
a	0.0000	-0.0005
rh	0.0000	-0.0010
rm	0.030	-0.0010
nm	0.031	-0.0035
nl	0.922	-0.0035

Table 10: Change in the density of workers at the minimum wage vs. change in the profit (local minimum wage increase)

results.

When matching elasticity of workers α rises to 0.72, there is a sizable employment switch from the rh and nm markets to the rm-market, while mean wages do not change much. From (21), we have $\ln \phi_\tau = \ln A + (1-\alpha)\ln \theta_\tau$, so the job finding rate is increasing (resp. decreasing) in α whenever $\theta_\tau < 1$ (resp. $\theta_\tau > 1$). Under one-for-one matching, our baseline results of $\theta_{rm} < 1$, $\theta_{rh} > 1$ and $\theta_{nm} > 1$ imply that v_{rm}, n_{rh} and n_{nm} are on the short side of the respective market. Thus, for given θ_τ, higher α translates into a higher job finding rate ϕ_{rm} but lower job finding rates ϕ_{rh} and ϕ_{nm}. The direct effect of a higher job finding rate ϕ_{rm} is to encourage more potential workers to enter into the rm-market, whereas that of lower job finding rates ϕ_{rh} and ϕ_{nm} is to discourage their entry into the rh- and the nm-markets.

Given the workers' indifference boundary condition $U_{rm}^{nm}(\theta_{rm}) = U_{nm}^{nm}(\theta_{nm})$, the above argument implies U_{rm}^{nm} is higher but U_{nm}^{nm} is lower. To restore the equilibrium thereby requires θ_{rm} to drop and θ_{nm} to rise. Now look at the firms' indifference boundary condition $\pi_{rm}(\theta_{rm}) = \pi_{rh}(\theta_{rh})$. Higher ϕ_{rm} but lower ϕ_{rh} imply π_{rm} is lower but π_{rh} is higher. Recalling that π_τ is decreasing in θ_τ, so it requires θ_{rm} to decrease but θ_{rh} to increase in order to restore the equilibrium. Combining both and applying similar arguments to the θ_a and the θ_{nl} markets, we obtain a lower θ_{rm} but higher θ_{rh} and θ_{nm} in general equilibrium. Should these general equilibrium effects be outweighed by the direct effects mentioned above (which is the case in our quantitative analysis), we would expect to see the flow of employment from the rh and nm markets to the rm-market. This is yet another illustration of the important mechanism, namely, *market spillover effect* on both the worker (when comparing the rm- and the rh-markets) and the firm side (when comparing the rm- and the nm-markets).

On the contrary, when matching elasticity of workers drops to $\alpha = 0.4$, the opposite outcomes arise. Of particular interest, the aggregate income inequality drops, while the income inequality in each individual market barely changes. This is due only to changes in employment shares. Particularly, the employment share of the nm market rises sharply whereas that of the rm market drops substantively.

Finally, when there is task polarization as described above, there is a sizable switch from the rh to the a market and from the rm to nm market, Intuitively, when there is more abstract and manual tasks but less routine tasks, it becomes relatively easier to find jobs for abstract and manual tasks. This motivates high-skilled workers to switch from the rh to the a market, and middle skilled workers from the rm to the nm market.

Measure	Baseline	$\alpha = 0.4$	$\alpha = 0.72$	Task polarization
u_a	0.018	0.018	0.017	0.017
u_{rh}	0.033	0.034	0.031	0.032
u_{rm}	0.045	0.047	0.042	0.043
u_{nm}	0.069	0.072	0.062	0.065
u_{nl}	0.096	0.101	0.086	0.091
u_{agg}	0.046	0.048	0.040	0.043
Y_a	0.815	0.809	0.829	0.843
Y_{rh}	0.023	0.027	0.012	0.001
Y_{rm}	0.743	0.690	0.884	0.747
Y_{nm}	0.152	0.191	0.047	0.150
Y_{nl}	0.101	0.101	0.103	0.102
Y_{agg}	1.834	1.818	1.874	1.843
$w_{mean,a}$	2.768	2.767	2.770	2.769
$w_{mean,rh}$	2.247	2.246	2.251	2.249
$w_{mean,rm}$	1.656	1.654	1.659	1.657
$w_{mean,nm}$	1.281	1.279	1.286	1.284
$w_{mean,nl}$	0.946	0.944	0.952	0.949
w_{agg}	1.867	1.852	1.904	1.874
$\sigma^2(\log(w))_a$	0.003	0.003	0.003	0.003
$\sigma^2(\log(w))_{rh}$	0.006	0.006	0.006	0.006
$\sigma^2(\log(w))_{rm}$	0.008	0.008	0.007	0.008
$\sigma^2(\log(w))_{nm}$	0.012	0.012	0.011	0.011
$\sigma^2(\log(w))_{nl}$	0.015	0.016	0.014	0.014
$\sigma^2(\log(w))_{agg}$	0.127	0.131	0.116	0.127

Table 11: Sensitivity analysis

5 Concluding Remarks

In this paper, we have constructed a general equilibrium search and matching framework incorporating multi-tier labor markets with workers of different skills. With different combinations of tasks and workers, we have characterized equilibrium outcomes in five segmented labor markets, abstract, routine high-skilled, routine middle-skilled, manual middle-skilled and manual low-skilled, where firms may post the same type of tasks in different markets featuring different productivities (routine high-skilled versus routine middle-skilled, or manual middle-skilled versus manual low-skilled) and workers of the same skill may choose enter into different markets (abstract versus routine high-skilled, or routine middle-skilled versus manual middle-skilled). Thus, this general structure has allowed for compositional changes in various markets where the minimum wage may or may not bind and rich spillovers through market spillover from both the firm and the worker sides. With these in hand, we have established that when minimum wage rises, (i) while the unemployment rate at the minimum wage binding market is higher, all other markets enjoy a lower unemployment rate, (ii) while employment in the manual low-skilled jobs is lower, employment in the routine high-skilled and manual middle-skilled markets is higher due to market spillovers, and (iii) employment in other markets has ambiguous responses due to conflicting effects on the potential labor pool and the unemployment rate.

We have also conducted quantitative analysis in a calibrated economy fitting the U.S. labor market, which is subsequently used to evaluate the impacts of the federal minimum wage increase during the great recession (2007-2009) and the ongoing local minimum wage increase in Seattle (2017-2021). We have discovered that the minimum wage becomes binding not only in the manual low-skilled as well as the manual middle-skilled markets with the 2007-2009 federal minimum wage hikes, while in all three lower markets the minimum wage becomes binding with the local minimum wage hike. We have found that the minimum wage elasticities are larger in absolute value when the minimum wage binds beyond the lowest market, as a result of the amplifying market spillover effects from the firm and the worker sides, whereas the minimum wage effects on employment on the binding markets depend crucially on not only the magnitudes of these market spillover effects but also the compositional effects as a result of endogenous changes in the size of each relevant market. We have also identified that the employment effects may be weak in a nonbinding market that is positioned away from along the chain from the binding market, especially when the required skill is less substitutable. Because there are more markets with binding minimum wage, the local minimum wage policy has been found to yield larger effects than the federal minimum wage hike.

We find, however, that the effect of both federal and local minimum wage changes on the aggregate output is negative. We have also quantified the minimum wage effects on submarket average and overall average wages as well as wage inequality, generally pointing to small aggregate effects due to conflicting compositional effects in various markets. We have further obtained a positive correlation between the density of workers at the minimum wage in the binding markets and the firm's profit in these markets, consistent with the literature on labor power.

Our paper has thus provided a systematic evaluation of the two minimum wage policies within a unified search and matching framework, highlighting that the compositional changes in binding and nonbinding markets and market spillover effects from both firm and worker sides together may serve to better understand the conflicting empirical observations and to ensure better assessment of the such minimum wage hikes.

Along these lines, there are two avenues of extensions that are particularly interesting. On the one hand, one may consider "worker flexibility" in the sense that the hours at work may be adjusted in response to a minimum wage hike. That is, by reducing the hours at work, the potentially detrimental effect on employment could be mitigated. On the other hand, one may allow for "firm flexibility" in the sense that the capital intensity of some tasks may be adjusted upward to substitute away from low-paying jobs. In this case, a minimum wage hike may lead to a reinforcing negative effect on employment. Thus, by incorporating these two dimensions of flexibilities, whether minimum wage hikes may hinder employment by more or by less would be yet another quantitative question for future studies.

Appendix

A major portion of the appendix is not intended for publication.

A Solving the model

As is standard, the steady state unemployment rate in the market τ is given by

$$u_\tau = \frac{\delta_\tau}{\delta_t + \phi(\theta_\tau)}$$

Let $G_\tau(w)$ be the cumulative distribution of employed workers earning a wage less than or equal to w in the τ market. In the steady state, the inflow and the outflow of the workers would be the same, i.e.

$$\phi(\theta_\tau) F_\tau(w) u_\tau = (\delta_\tau + \phi(\theta_\tau)(1 - F_\tau(w)))(1 - u_\tau) G_\tau(w)$$

and hence

$$G_\tau(w) = \frac{F_\tau(w)\delta_\tau}{\delta_\tau + \phi(\theta_\tau)(1 - F_\tau(w))}$$

Therefore, the density of workers per firm earning wage w is given by

$$l_\tau(w) = \frac{\delta_\tau \phi(\theta_\tau)}{[\delta_\tau + \phi(\theta_\tau)(1 - F_\tau(w))]^2}$$

Putting into the profit function and let $w = \underline{w}_\tau$, the lower limit of the wage distribution, we have

$$\pi_\tau = (p_\tau - \underline{w}_\tau) \frac{\delta_\tau \phi(\theta_\tau)}{(\delta_\tau + \phi(\theta_\tau))^2} \tag{A.1}$$

Finally, by setting the profit be the same for all wages in the support of $F_\tau(w)$, we have the wage distribution

$$\begin{aligned} (p_\tau - \underline{w}_\tau) \frac{\delta_\tau \phi(\theta_\tau)}{(\delta_\tau + \phi(\theta_\tau))^2} &= (p_\tau - w) \frac{\delta_\tau \phi(\theta_\tau)}{[\delta_\tau + \phi(\theta_\tau)(1 - F_\tau(w))]^2} \\ \Rightarrow F_\tau(w) &= \left(1 + \frac{\delta_\tau}{\phi(\theta_\tau)}\right) \left[1 - \left(\frac{p_\tau - w}{p_\tau - \underline{w}_\tau}\right)^{\frac{1}{2}}\right] \end{aligned} \tag{A.2}$$

where

$$\underline{w}_\tau = \max\{R_\tau, w_{\min}\}$$

The upper limit $\bar{w}_\tau$ of the wage distribution can be obtain by setting $F_\tau(\bar{w}_\tau) = 1:$

$$\bar{w}_\tau = \underline{w}_\tau + (p_\tau - \underline{w}_\tau) \left[1 - \left(\frac{\delta_\tau}{\phi(\theta_\tau) + \delta_\tau}\right)^2\right]$$

B Proofs

B.1 Proof of Lemma 1

By differentiating π_τ with respect to θ_τ, we have

$$\begin{aligned}\frac{\partial \pi_\tau(\theta_\tau)}{\partial \theta_\tau} &= (p_\tau - \underline{w}_\tau)\frac{(\delta_\tau + \phi(\theta_\tau))^2 \delta_\tau \phi'(\theta_\tau) - 2\delta_\tau \phi(\theta_\tau)(\delta_\tau + \phi(\theta_\tau))\phi'(\theta_\tau)}{(\delta_\tau + \phi(\theta_\tau))^4} \\ &= \frac{\delta_\tau \phi'(\theta_\tau)(p_\tau - \underline{w}_\tau)}{(\delta_\tau + \phi(\theta_\tau))^3}(\delta_\tau - \phi(\theta_\tau))\end{aligned}$$

Hence, if $\delta_\tau < \phi(\theta_\tau)$, then $\frac{\partial \pi_\tau}{\partial \theta_\tau} < 0$.

By differentiating $F_\tau(w)$ with respect to θ_τ, we have

$$\frac{\partial F_\tau(w;\theta_\tau)}{\partial \theta_\tau} = -\left(\frac{\delta_\tau \phi'(\theta_\tau)}{(\phi(\theta_\tau))^2}\right)\left[1 - \left(\frac{p_\tau - w}{p_\tau - \underline{w}_\tau}\right)^{\frac{1}{2}}\right] < 0$$

for all w is the support of $F_\tau(w)$.

B.2 Proof of Proposition 3

If the minimum wage is effective in the τ market, then

$$\underline{w}_\tau = w_{\min}$$

It is clear from (A.1) and (A.2) that the profit and wage distribution function are decreasing in $\underline{w}_\tau$, given the same θ_τ, hence we have

$$\begin{aligned}\frac{\partial \pi_\tau(\theta_\tau; w_{\min})}{\partial w_{\min}} &< 0 \\ \frac{\partial F_\tau(w;\theta_\tau, w_{\min})}{\partial w_{\min}} &< 0\end{aligned}$$

for all w is the support of $F_\tau(w)$.

B.3 Proof of Proposition 4

By substituting marketing clearing conditions (7), the system of indifference conditions (8) can be written as

$$\begin{aligned}\pi_{rh}\left(\frac{v_{rh}}{n_{rh}}\right) &= \pi_{rm}\left(\frac{\bar{v}_r - v_{rh}}{n_{rm}}\right) \\ \pi_{nm}\left(\frac{v_{nm}}{\bar{n}_m - n_{rm}}\right) &= \pi_{nl}\left(\frac{\bar{v}_n - v_{nm}}{\bar{n}_l}; w_{\min}\right) \\ U_a\left(\frac{\bar{v}_a}{\bar{n}_h - n_{rh}}\right) &= U_{rh}\left(\frac{v_{rh}}{n_{rh}}\right) \\ U_{rm}\left(\frac{\bar{v}_r - v_{rh}}{n_{rm}}\right) &= U_{nm}\left(\frac{v_{nm}}{\bar{n}_m - n_{rm}}\right)\end{aligned}$$

Implicitly differentiating the system with respect to $w_{\min}$,

$$\left(\frac{\partial \pi_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial v_{rh}} - \frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial v_{rh}}\right)\frac{\partial v_{rh}}{\partial w_{\min}} + \frac{\partial \pi_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial n_{rh}}\frac{\partial n_{rh}}{\partial w_{\min}} = \frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial n_{rm}}\frac{\partial n_{rm}}{\partial w_{\min}} + \frac{\partial \pi_{rm}}{\partial w_{\min}}$$

$$\left(\frac{\partial \pi_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial v_{nm}} - \frac{\partial \pi_{nl}}{\partial \theta_{nl}}\frac{\partial \theta_{nl}}{\partial v_{nm}}\right)\frac{\partial v_{nm}}{\partial w_{\min}} + \frac{\partial \pi_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial n_{rm}}\frac{\partial n_{rm}}{\partial w_{\min}} + \frac{\partial \pi_{nm}}{\partial w_{\min}} = \frac{\partial \pi_{nl}}{\partial w_{\min}}$$

$$\left(\frac{\partial U_{a}}{\partial \theta_{a}}\frac{\partial \theta_{a}}{\partial n_{rh}} - \frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial n_{rh}}\right)\frac{\partial n_{rh}}{\partial w_{\min}} = \frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial v_{rh}}\frac{\partial v_{rh}}{\partial w_{\min}}$$

$$\frac{\partial U_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial v_{rh}}\frac{\partial v_{rh}}{\partial w_{\min}} + \left(\frac{\partial U_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial n_{rm}} - \frac{\partial U_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial n_{rm}}\right)\frac{\partial n_{rm}}{\partial w_{\min}} + \frac{\partial U_{rm}}{\partial w_{\min}} = \frac{\partial U_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial v_{nm}}\frac{\partial v_{nm}}{\partial w_{\min}} + \frac{\partial U_{nm}}{\partial w_{\min}}$$

Simplifying and using **Lemma 1 and** Proposition 3, we have

$$\frac{\partial n_{rm}}{\partial w_{\min}} = k_2\frac{\partial v_{rh}}{\partial w_{\min}} = k_2 k_4\frac{\partial \pi_{nl}}{\partial w_{\min}} < 0$$

$$\frac{\partial v_{rh}}{\partial w_{\min}} = k_4\frac{\partial \pi_{nl}}{\partial w_{\min}} > 0$$

$$\frac{\partial n_{rh}}{\partial w_{\min}} = k_1\frac{\partial v_{rh}}{\partial w_{\min}} = k_1 k_4\frac{\partial \pi_{nl}}{\partial w_{\min}} > 0$$

$$\frac{\partial v_{nm}}{\partial w_{\min}} = k_3\frac{\partial v_{rh}}{\partial w_{\min}} = k_3 k_4\frac{\partial \pi_{nl}}{\partial w_{\min}} > 0$$

where

$$k_0 = \frac{\frac{\partial U_a}{\partial \theta_a}\frac{\theta_a}{n_a}}{\frac{\partial U_a}{\partial \theta_a}\frac{\theta_a}{n_a} + \frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{\theta_{rh}}{n_{rh}}} > 0$$

$$k_1 = \frac{\frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{1}{n_{rh}}}{\frac{\partial U_a}{\partial \theta_a}\frac{\theta_a}{n_a} + \frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{\theta_{rh}}{n_{rh}}} > 0$$

$$k_2 = \frac{\frac{k_0}{n_{rh}}\frac{\partial \pi_{rh}}{\partial \theta_{rh}} - \frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial v_{rh}}}{\frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial n_{rm}}} < 0$$

$$k_3 = \frac{\frac{\frac{\partial \pi_{rh}}{\partial \theta_{rh}}}{\frac{\partial \pi_{rm}}{\partial \theta_{rm}}}\frac{k_0}{n_{rh}}\frac{\partial U_{rm}}{\partial \theta_{rm}} - \frac{\partial U_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial n_{rm}}k_2}{\frac{\partial U_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial v_{nm}}} > 0$$

$$k_4 = \frac{1}{\frac{k_0}{n_{rh}}\frac{\partial \pi_{rh}}{\partial \theta_{rh}}\frac{\frac{\partial \pi_{nm}}{\partial \theta_{nm}}}{\frac{\partial \pi_{rm}}{\partial \theta_{rm}}}\frac{\frac{\partial U_{rm}}{\partial \theta_{rm}}}{\frac{\partial U_{nm}}{\partial \theta_{nm}}} - \frac{\partial \pi_{nl}}{\partial \theta_{nl}}\frac{\partial \theta_{nl}}{\partial v_{nm}}k_3} < 0$$

B.4 Proof of Proposition 5

By (a), we have

$$\frac{\partial \theta_a}{\partial w_{\min}} = \bar{v}_a\frac{\partial}{\partial w_{\min}}\left(\frac{1}{n_a}\right) > 0$$

$$\frac{\partial \theta_{nl}}{\partial w_{\min}} = \frac{1}{\bar{n}_l}\frac{\partial v_{nl}}{\partial w_{\min}} < 0$$

Now given the system

$$\pi_{rh}(\theta_{rh}) = \pi_{rm}(\theta_{rm})$$

$$\pi_{nm}(\theta_{nm}) = \pi_{nl}(\theta_{nl}; w_{\min})$$

$$U_a(\theta_a) = U_{rh}(\theta_{rh})$$

$$U_{rm}(\theta_{rm}) = U_{nm}(\theta_{nm})$$

By implicitly differentiating, we have

$$\frac{\partial \theta_{rh}}{\partial w_{\min}} = \frac{\frac{\partial U_a}{\partial \theta_a}}{\frac{\partial U_{rh}}{\partial \theta_{rh}}} \frac{\partial \theta_a}{\partial w_{\min}} > 0$$

$$\frac{\partial \theta_{rm}}{\partial w_{\min}} = \frac{\frac{\partial \pi_{rh}}{\partial \theta_{rh}}}{\frac{\partial \pi_{rm}}{\partial \theta_{rm}}} \frac{\partial \theta_{rh}}{\partial w_{\min}} > 0$$

$$\frac{\partial \theta_{nm}}{\partial w_{\min}} = \frac{\frac{\partial U_{rm}}{\partial \theta_{rm}}}{\frac{\partial U_{nm}}{\partial \theta_{nm}}} \frac{\partial \theta_{rm}}{\partial w_{\min}} > 0$$

B.5 Proof of Proposition 6

The steady state unemployment rate is given by

$$u_\tau = \frac{\delta_\tau}{\delta_t + \phi(\theta_\tau)}$$

which is decreasing in θ_τ. Hence, the results follow Proposition 5.

B.6 Endogenous vacancy creation

In this case, only the proof of Proposition 4 has to be modified. To see that the sensitivity of the profits is muted, differentiating π_{rm} with respect to $w_{\min}$ yields

$$\frac{\partial \pi_{rm}}{\partial w_{\min}} = d_{r1}\left(\frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial v_{rh}}\frac{\partial v_{rh}}{\partial w_{\min}} + \frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial n_{rm}}\frac{\partial n_{rm}}{\partial w_{\min}}\right)$$

where

$$d_{r1} = \frac{1}{1 - \frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial \bar{v}_r}\frac{\partial \bar{v}_r}{\partial \pi_{rm}}} < 1$$

Hence, π_{rm} becomes less sensitive compared to the case when $\bar{v}_r$ is constant.

Now the system after implicit differentiation becomes

$$\left(\frac{\partial \pi_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial v_{rh}} - d_{r1}\frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial v_{rh}}\right)\frac{\partial v_{rh}}{\partial w_{\min}} + \frac{\partial \pi_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial n_{rh}}\frac{\partial n_{rh}}{\partial w_{\min}} = d_{r1}\frac{\partial \pi_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial n_{rm}}\frac{\partial n_{rm}}{\partial w_{\min}}$$

$$\left(\frac{\partial \pi_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial v_{nm}} - d_n\frac{\partial \pi_{nl}}{\partial \theta_{nl}}\frac{\partial \theta_{nl}}{\partial v_{nm}}\right)\frac{\partial v_{nm}}{\partial w_{\min}} + \frac{\partial \pi_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial n_{rm}}\frac{\partial n_{rm}}{\partial w_{\min}} = d_n\frac{\partial \pi_{nl}}{\partial w_{\min}}$$

$$\left(d_a\frac{\partial U_a}{\partial \theta_a}\frac{\partial \theta_a}{\partial n_{rh}} - \frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial n_{rh}}\right)\frac{\partial n_{rh}}{\partial w_{\min}} = \frac{\partial U_{rh}}{\partial \theta_{rh}}\frac{\partial \theta_{rh}}{\partial v_{rh}}\frac{\partial v_{rh}}{\partial w_{\min}}$$

$$d_{r2}\frac{\partial U_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial v_{rh}}\frac{\partial v_{rh}}{\partial w_{\min}} + \left(d_{r2}\frac{\partial U_{rm}}{\partial \theta_{rm}}\frac{\partial \theta_{rm}}{\partial n_{rm}} - \frac{\partial U_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial n_{rm}}\right)\frac{\partial n_{rm}}{\partial w_{\min}} = \frac{\partial U_{nm}}{\partial \theta_{nm}}\frac{\partial \theta_{nm}}{\partial v_{nm}}\frac{\partial v_{nm}}{\partial w_{\min}}$$

where

$$d_{r2} = 1 + \frac{\partial \pi_{rm}}{\partial \theta_{rm}} \frac{\partial \theta_{rm}}{\partial \bar{v}_r} \frac{\partial \bar{v}_r}{\partial \pi_{rm}} < 1$$
$$d_a = 1 + \frac{\partial \pi_a}{\partial \theta_a} \frac{\partial \theta_a}{\partial \bar{v}_a} \frac{\partial \bar{v}_a}{\partial \pi_a} < 1$$
$$d_n = \frac{1}{1 - \frac{\partial \pi_{nl}}{\partial \theta_{nl}} \frac{\partial \theta_{nl}}{\partial \bar{v}_n} \frac{\partial \bar{v}_n}{\partial \pi_{nl}}} < 1$$

Simplifying and using Lemma 1 and Proposition 3, we have

$$\frac{\partial n_{rm}}{\partial w_{\min}} = \tilde{k}_2 \frac{\partial v_{rh}}{\partial w_{\min}} = \tilde{k}_2 \tilde{k}_4 \frac{\partial \pi_{nl}}{\partial w_{\min}} < 0$$
$$\frac{\partial v_{rh}}{\partial w_{\min}} = \tilde{k}_4 \frac{\partial \pi_{nl}}{\partial w_{\min}} > 0$$
$$\frac{\partial n_{rh}}{\partial w_{\min}} = \tilde{k}_1 \frac{\partial v_{rh}}{\partial w_{\min}} = \tilde{k}_1 \tilde{k}_4 \frac{\partial \pi_{nl}}{\partial w_{\min}} > 0$$
$$\frac{\partial v_{nm}}{\partial w_{\min}} = \tilde{k}_3 \frac{\partial v_{rh}}{\partial w_{\min}} = \tilde{k}_3 \tilde{k}_4 \frac{\partial \pi_{nl}}{\partial w_{\min}} > 0$$

where

$$\tilde{k}_0 = \frac{d_a \frac{\partial U_a}{\partial \theta_a} \frac{\theta_a}{n_a}}{d_a \frac{\partial U_a}{\partial \theta_a} \frac{\theta_a}{n_a} + \frac{\partial U_{rh}}{\partial \theta_{rh}} \frac{\theta_{rh}}{n_{rh}}} > 0$$
$$\tilde{k}_1 = \frac{\frac{\partial U_{rh}}{\partial \theta_{rh}} \frac{\partial \theta_{rh}}{\partial v_{rh}}}{d_a \frac{\partial U_a}{\partial \theta_a} \frac{\theta_a}{n_a} + \frac{\partial U_{rh}}{\partial \theta_{rh}} \frac{\theta_{rh}}{n_{rh}}} > 0$$
$$\tilde{k}_2 = \frac{\frac{\partial \pi_{rh}}{\partial \theta_{rh}} \frac{k_0}{n_{rh}} + d_{r1} \frac{\partial \pi_{rm}}{\partial \theta_{rm}} \frac{1}{n_{rm}}}{-d_{r1} \frac{\partial \pi_{rm}}{\partial \theta_{rm}} \frac{\theta_{rm}}{n_{rm}}} < 0$$
$$\tilde{k}_3 = \frac{d_{r2} \left(\frac{\frac{\partial \pi_{rh}}{\partial \theta_{rh}}}{\frac{\partial \pi_{rm}}{\partial \theta_{rm}}} \frac{k_0}{d_{r1} n_{rh}} \right) \frac{\partial U_{rm}}{\partial \theta_{rm}} - \frac{\partial U_{nm}}{\partial \theta_{nm}} \frac{\theta_{nm}}{n_{nm}} k_2}{\frac{\partial U_{nm}}{\partial \theta_{nm}} \frac{\partial \theta_{nm}}{\partial v_{nm}}} > 0$$
$$\tilde{k}_4 = \frac{d_n}{\frac{\partial \pi_{nm}}{\partial \theta_{nm}} \left(\frac{d_{r2} k_0}{d_{r1} n_{rh}} \right) \frac{\frac{\partial U_{rm}}{\partial \theta_{rm}}}{\frac{\partial U_{nm}}{\partial \theta_{nm}}} \frac{\frac{\partial \pi_{rh}}{\partial \theta_{rh}}}{\frac{\partial \pi_{rm}}{\partial \theta_{rm}}} - d_n \frac{\partial \pi_{nl}}{\partial \theta_{nl}} \frac{\partial \theta_{nl}}{\partial v_{nm}} k_3} < 0$$

This verifies the validity of the propositions.

C Additional results

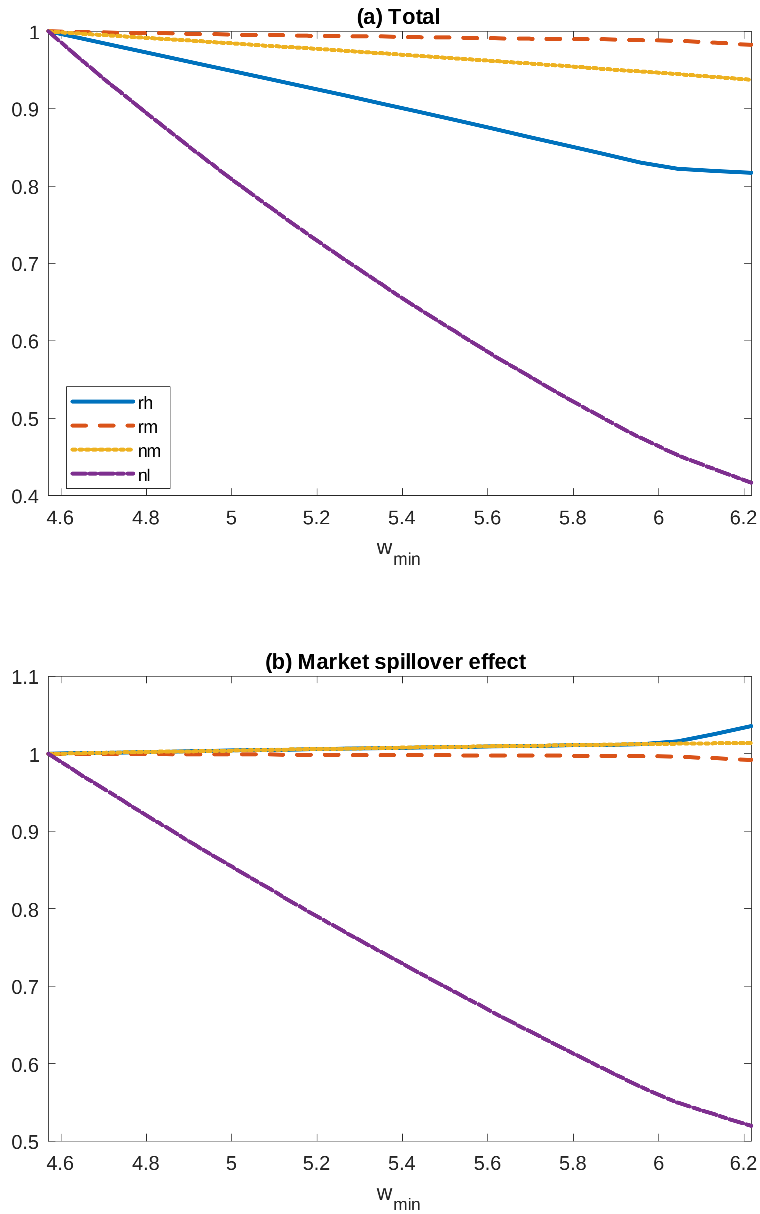


Figure C.1: Vacancy positing allocations (federal minimum wage increase)

\

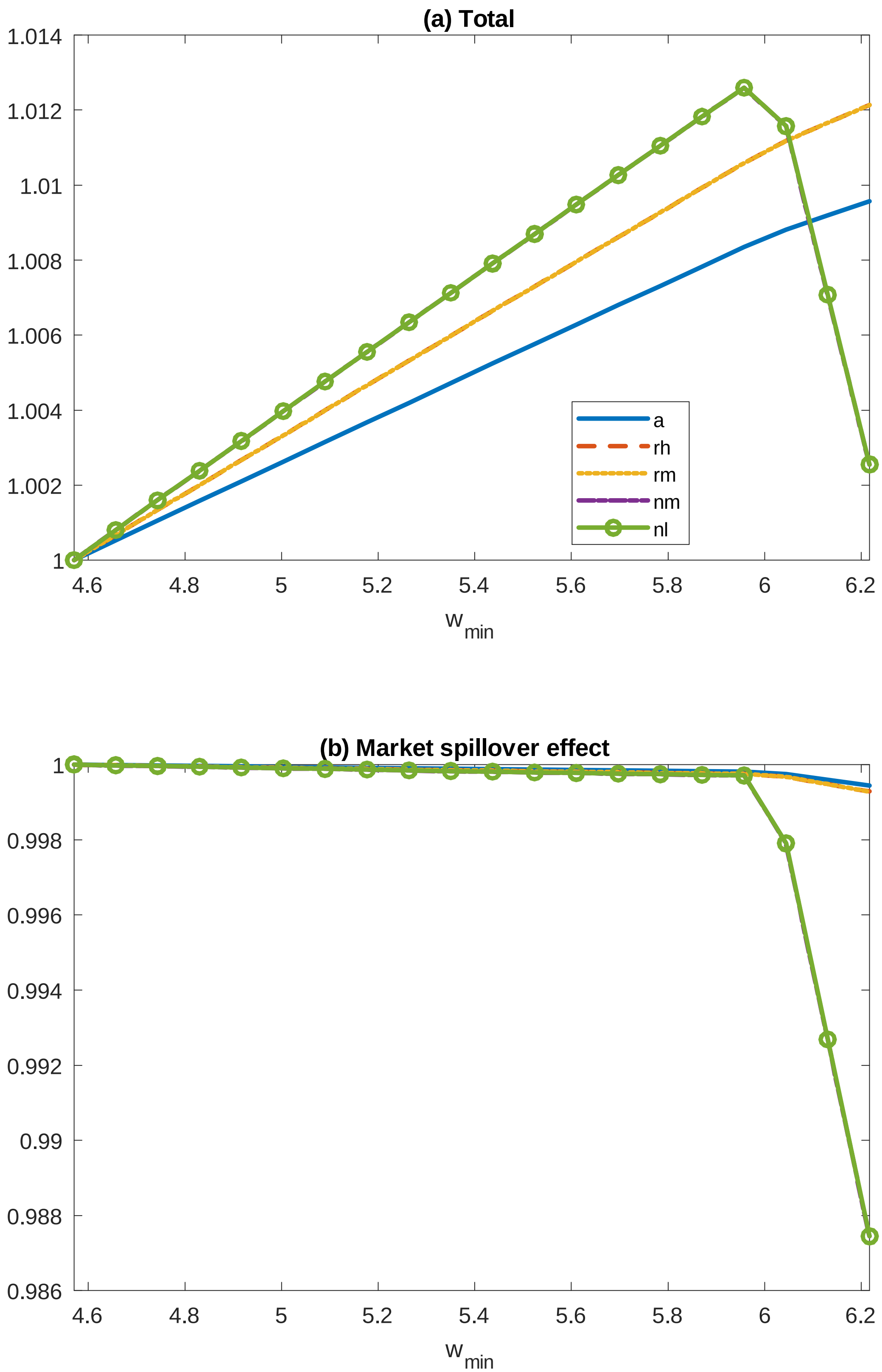
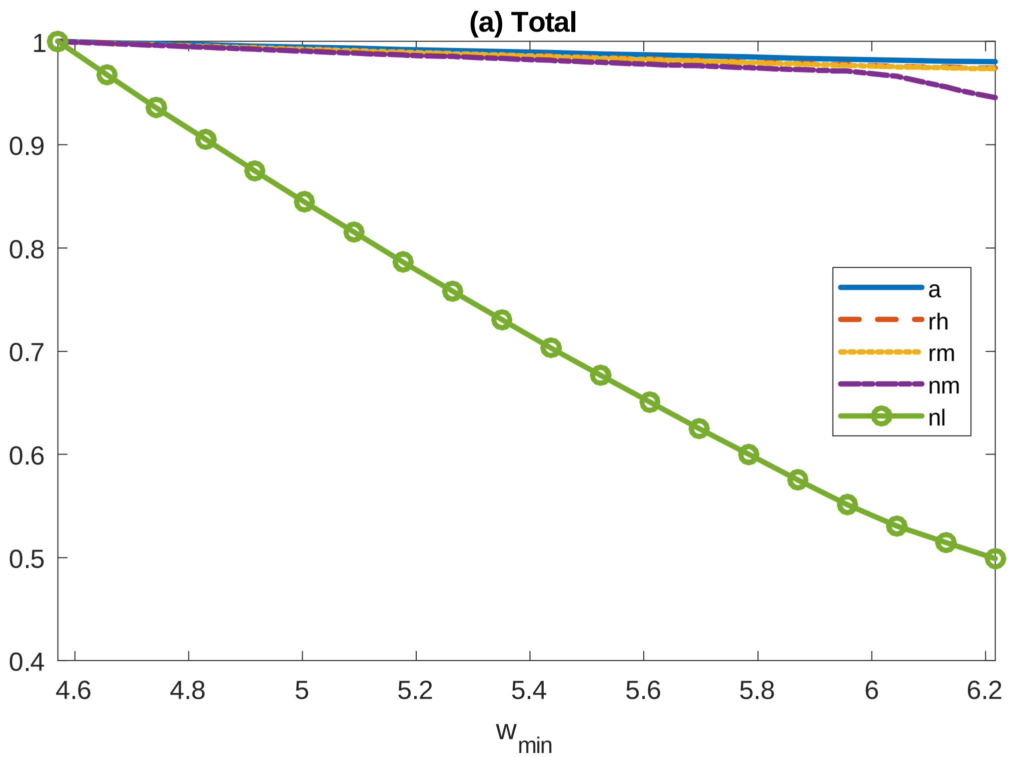


Figure C.2: Firm's profit (federal minimum wage increase)



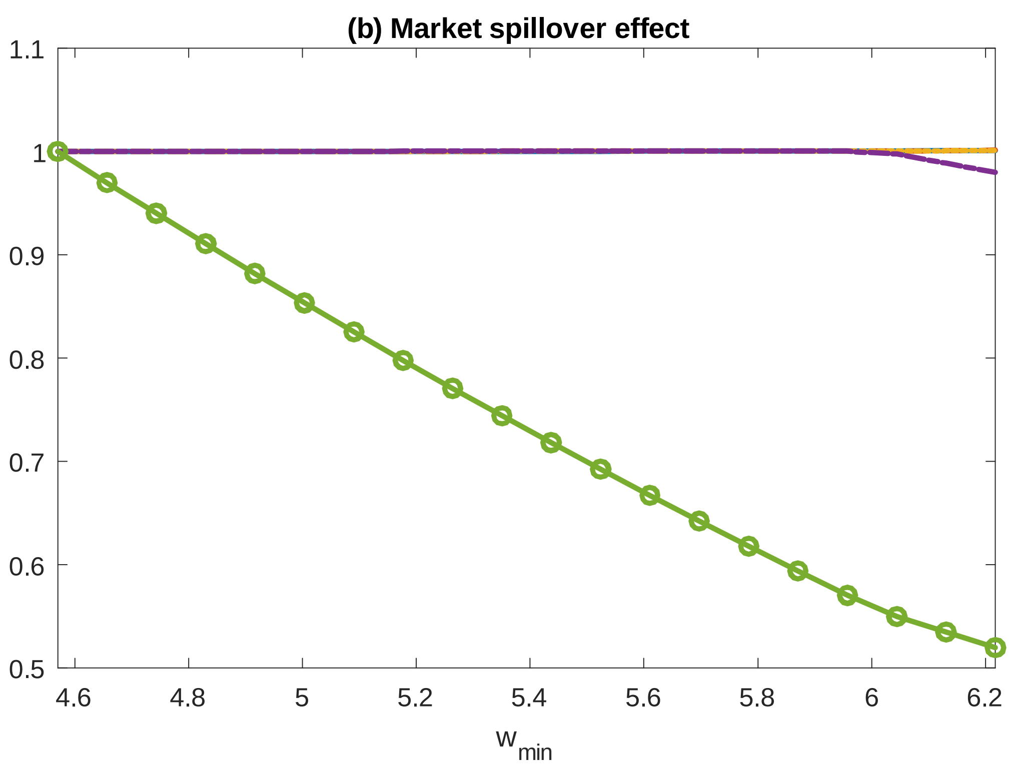


Figure C.3: Labor market tightness (federal minimum wage increase)

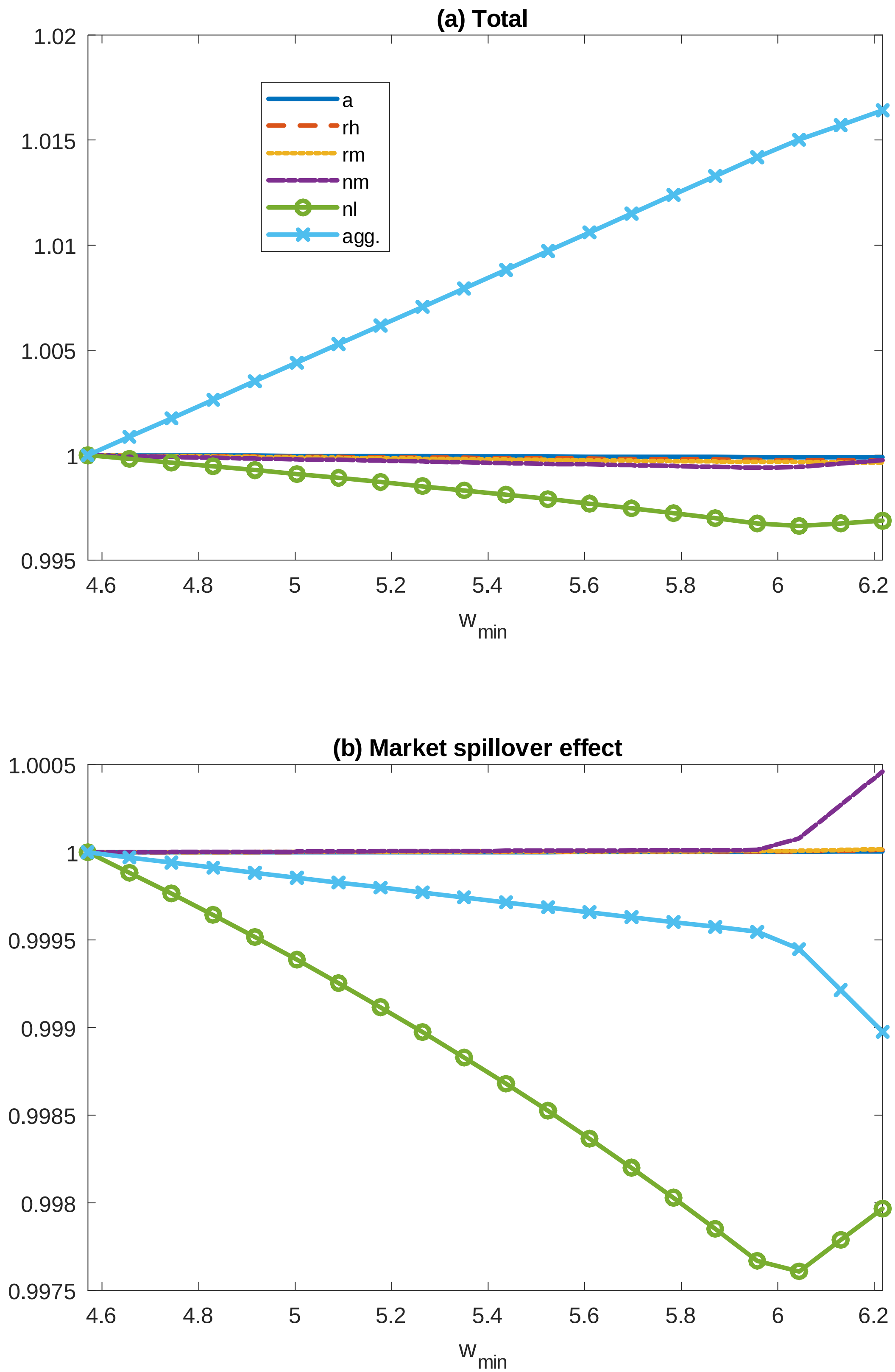


Figure C.4: Average wage (federal minimum wage increase)

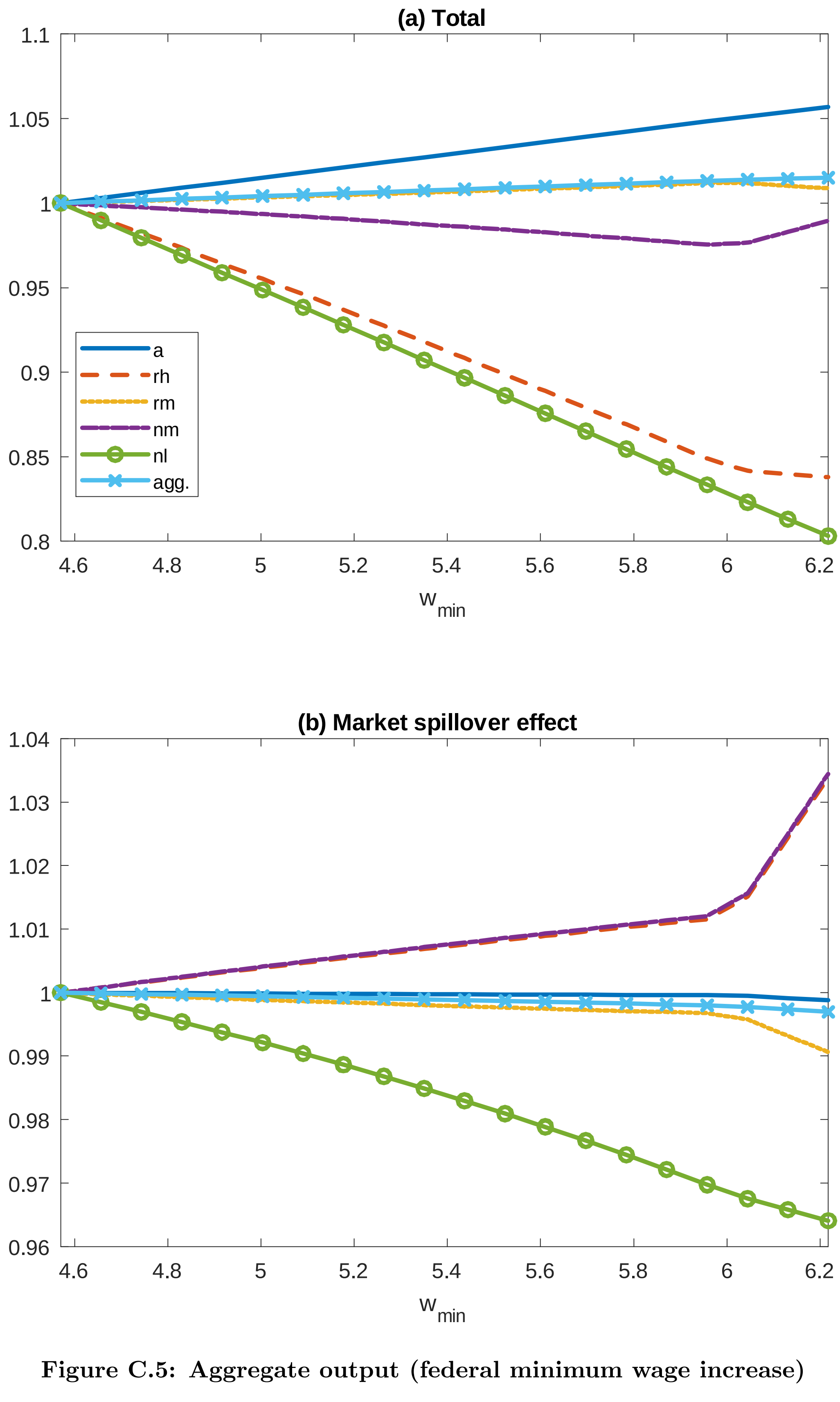


Figure C.5: Aggregate output (federal minimum wage increase)

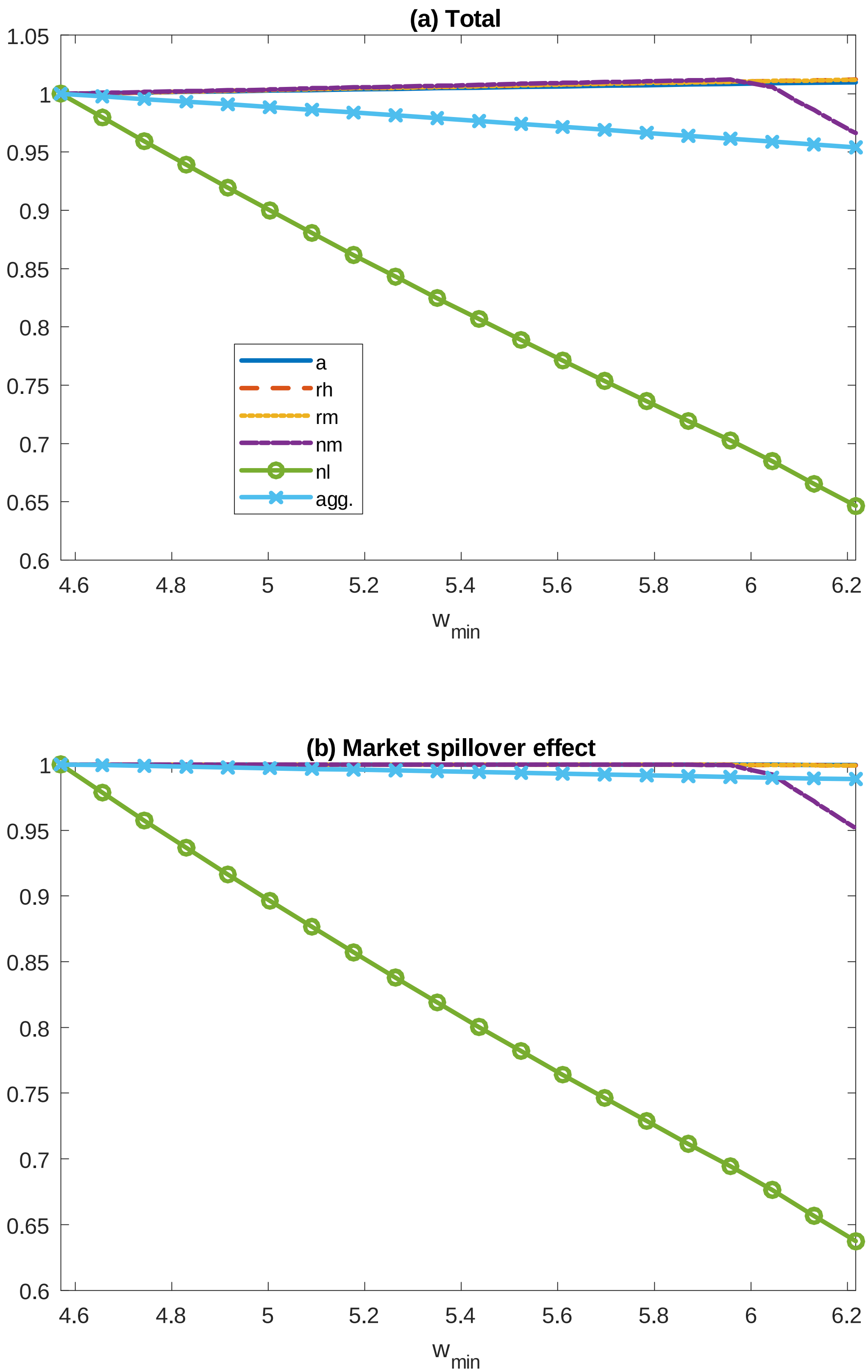


Figure C.6: Wage inequality (federal minimum wage increase)

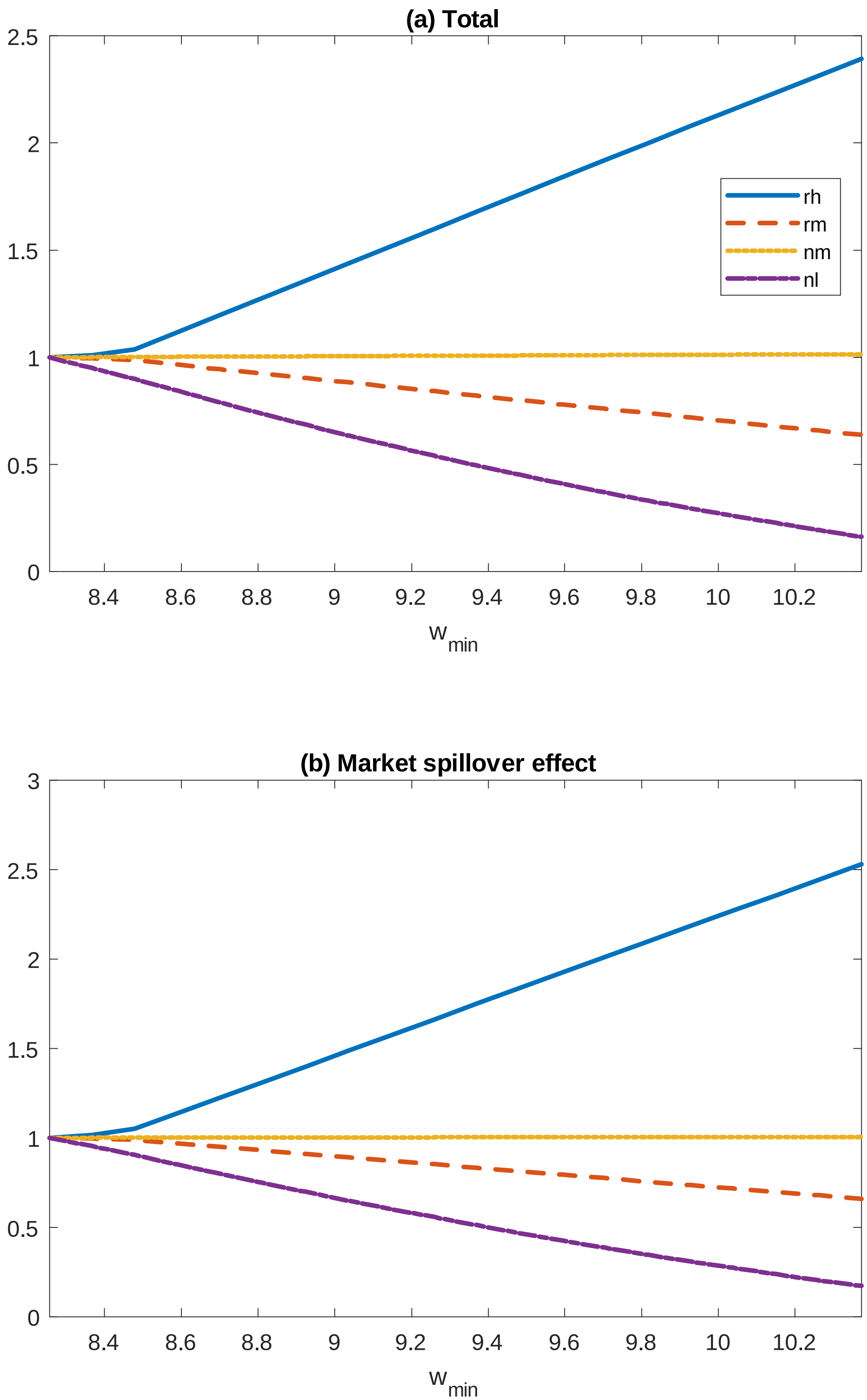


Figure C.7: Vacancy positing allocations (local minimum wage increase)

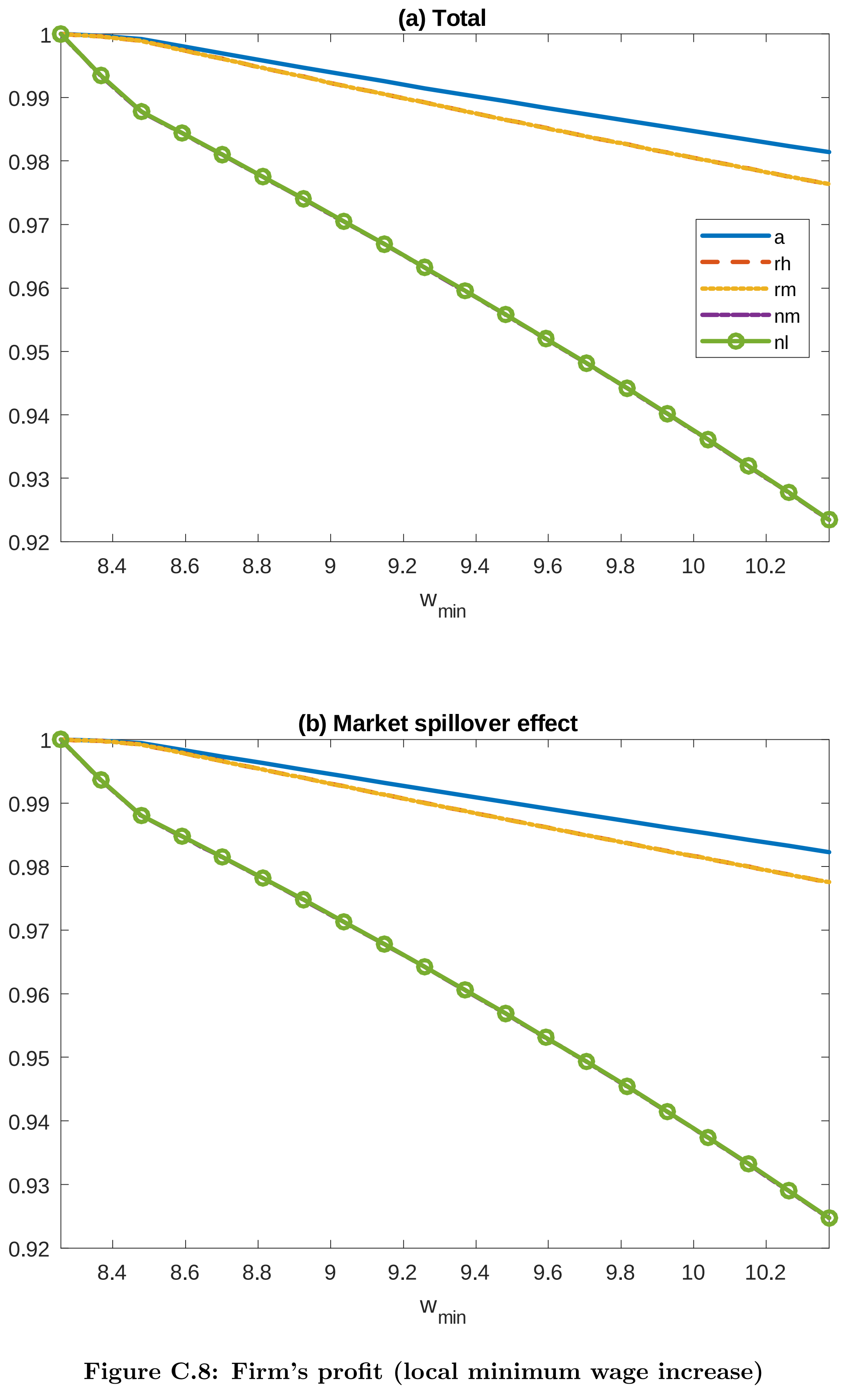


Figure C.8: Firm's profit (local minimum wage increase)

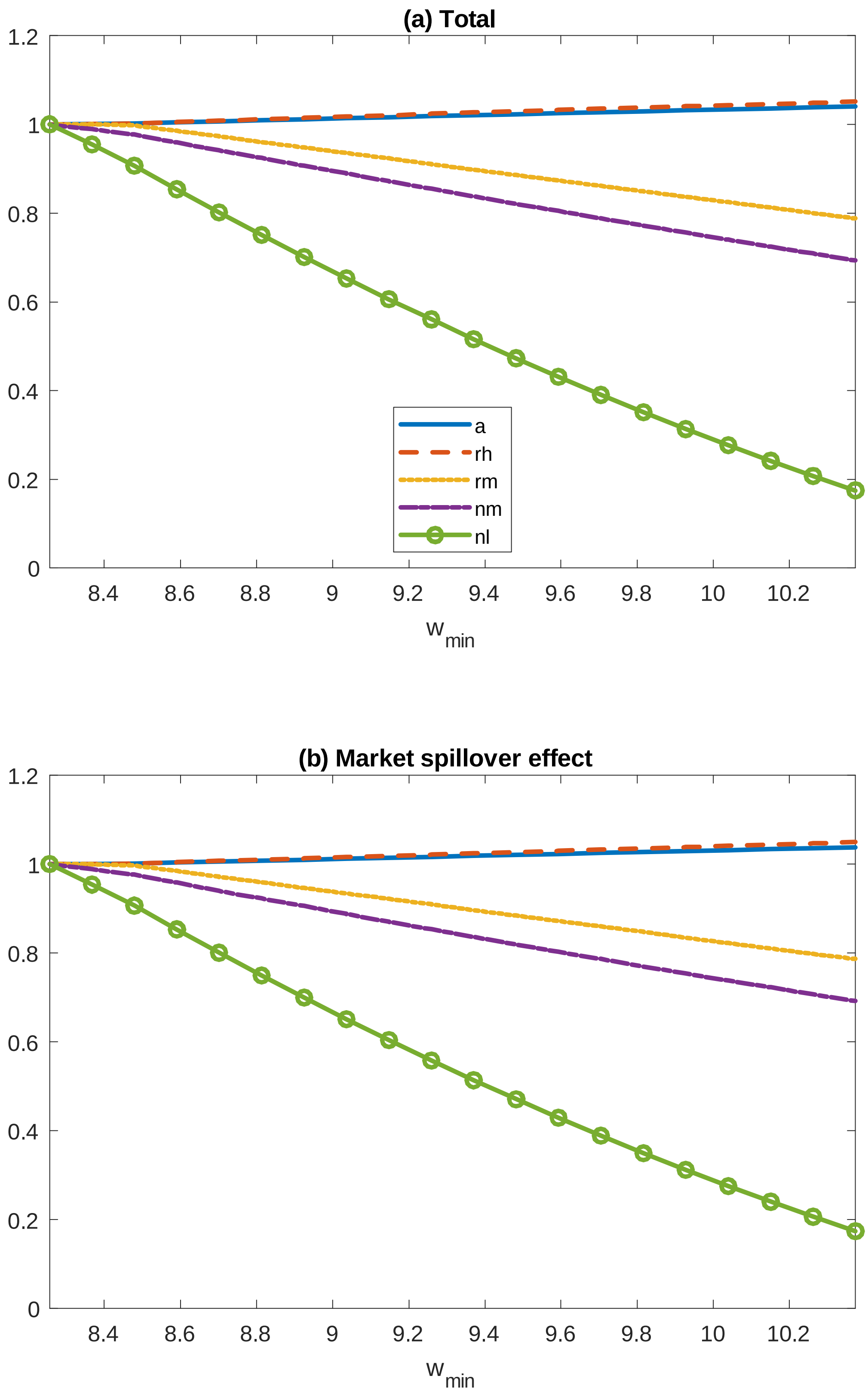


Figure C.9: Labor market tightness (local minimum wage increase)

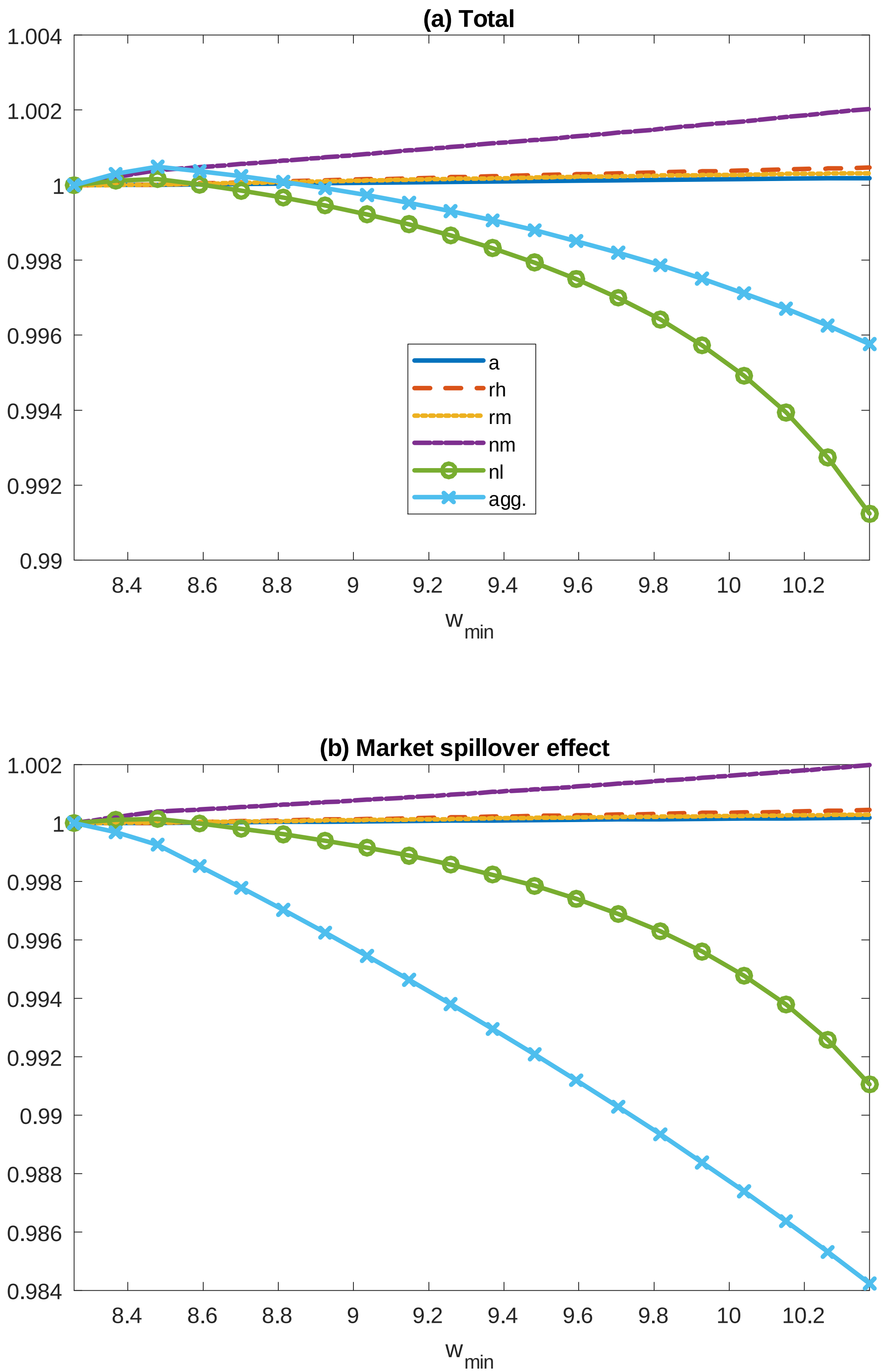
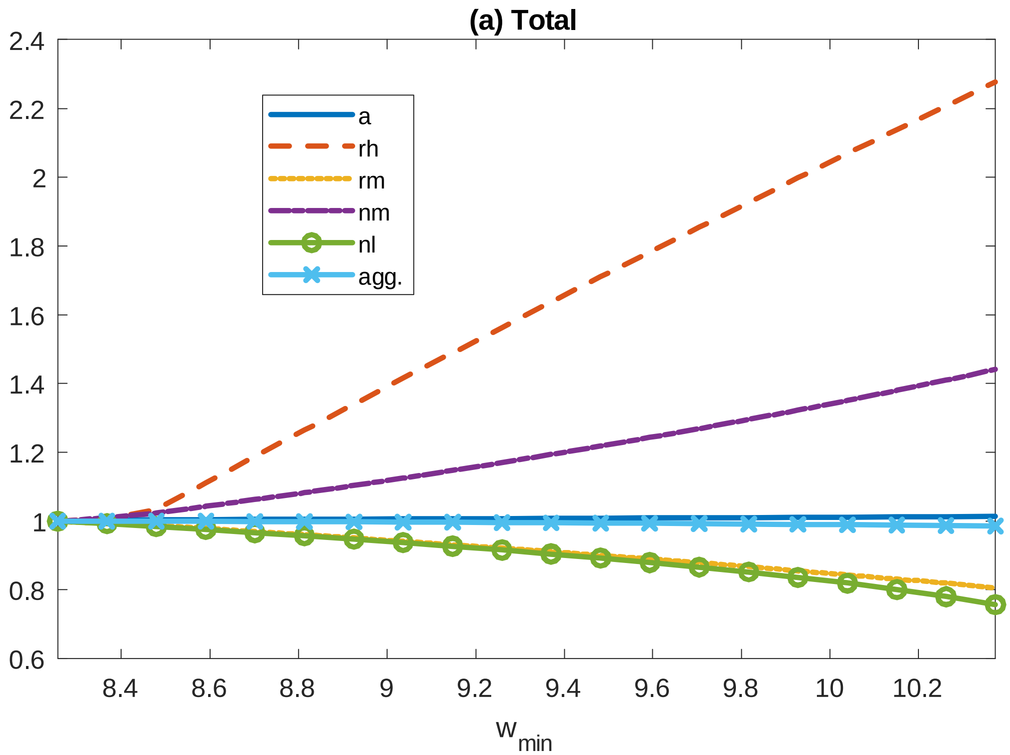


Figure C.10: Average wage (local minimum wage increase)



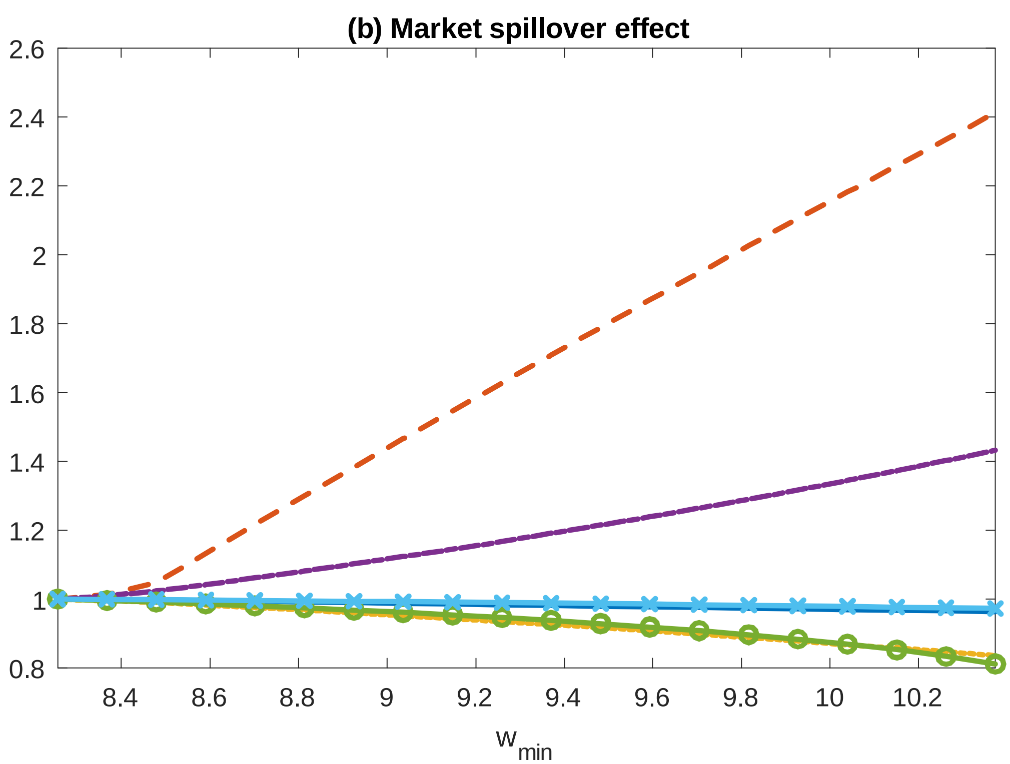


Figure C.11: Aggregate output (local minimum wage increase)

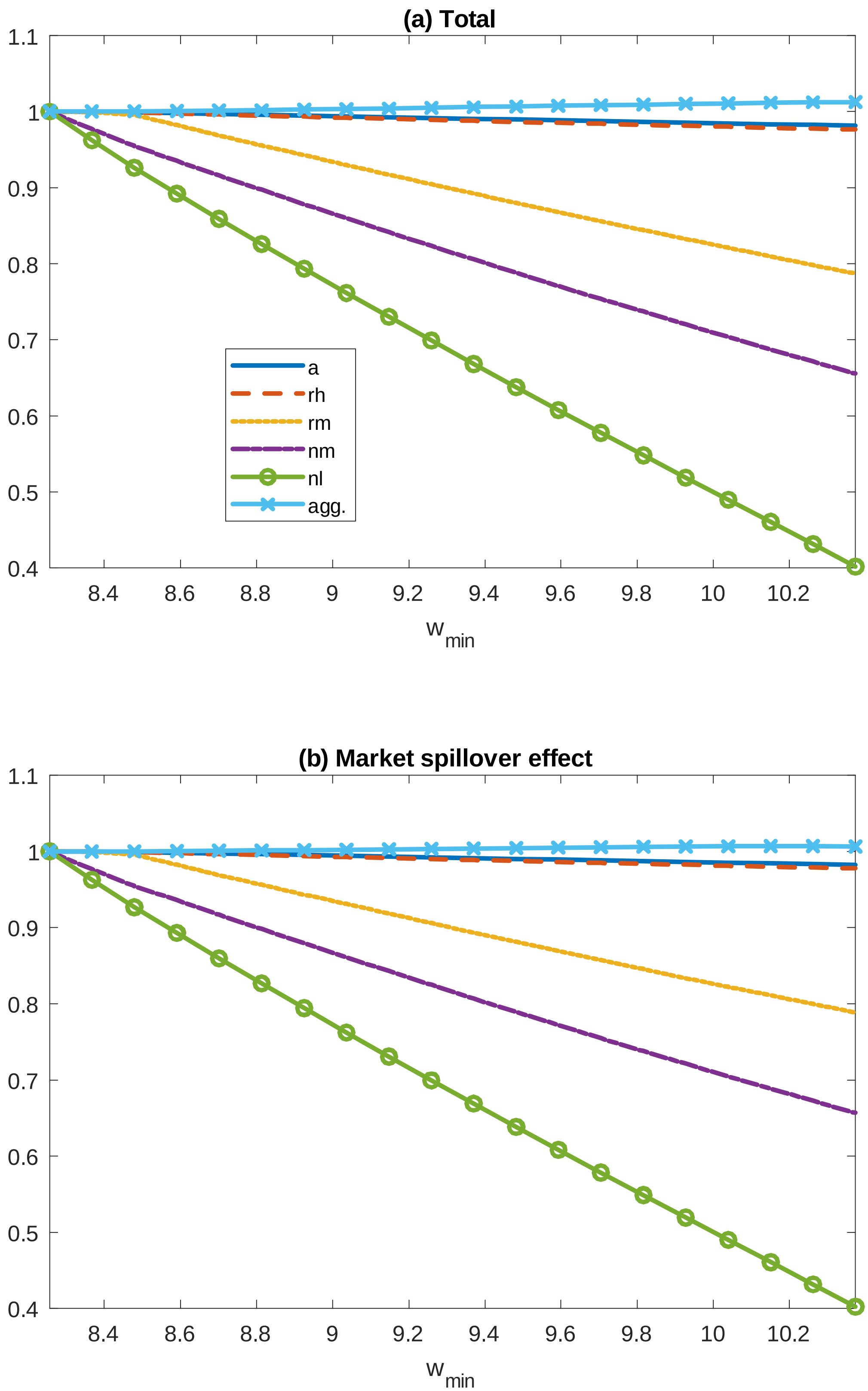


Figure C.12: Wage inequality (local minimum wage increase)