Demographic Transition

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A. Introduction

- **Long-term trend in population (Western Europe, Maddison 1982/1995):**
  - **500-1500: rising pop, no per capita output growth**
  - **1500-1870: rising pop, rising per capita output**
  - **after 1870: declining pop, rising per capita output**
- **Fertility decline since mid-1800:**
  - **reducing infant mortality enables lower fertility given the same desired quantity of children**
  - **rising income makes education more affordable and encourages the trade-off of quantity for quality in childbearing**
  - **rising opportunity cost increases childbearing cost**
- **Roles of demographic transition played in economic development:**
  - **high fertility is associated with low development (Malthusian trap)**
  - **high within-country fertility differentials are associated with high income inequalities (Kremer-Chen 2000)**
  - **demographic transition and structural transformation interact with each other leading to widened world income dispersion (Cheung 2017; Jiang-Lien-Wang-Y. Wang 2019)**
  - **accompanied by the shift from quantity to quality, birth timing is delayed (Heckman-Walker 1990; Lien-Wang 2016; Jiang-Lien-Wang-Wang 2019)**

- **Literature:**
  - **Kuznets (1958), Becker (1960), Easterlin (1968): quality-quantity trade-off**
  - **Lee (1987): quality-quantity trade-off in a dynamic, non-optimizing setup**
  - **Razin & Ben-Zion (1975), Becker-Barro (1989), Wang-Yip-Scotese (1994), Palivos (1995): quality-quantity trade-off in exogenous growth**
  - **Becker-Murphy-Tamura (1990), *Galor-Weil* (2000), Jones (2022): quality-quantity trade-off in endogenous growth**
  - ***de la Croix-Doepke (2003)*: larger fertility differentials widen inequalities**
  - **Greenwood-Seshadri-Vandenbroucke (2005) and *Doepke-Hazan-Maoz (2015)*: baby boom/baby burst driven by household durable goods technology and female labor force participation, respectively**
  - **Moav (2005): better returns to child-labor can be a barrier to growth**
  - **Soares (2005): reduction in infant mortality leads to lower fertility rate and higher human capital accumulation, thereby raising long-run growth**
  - ***Doepke-Zilibotti (2008)*: intergenerational behavior transmission & quality**
  - **Bara-Leukhina (2009): demographic transition and industrial revolution**
  - **Jayachandran & Lleras-Muney (2009): maternal mortality**
  - ***Jiang-Lien-Wang-Wang (2023)*: endogenous birth timing, labor market development and demographic transition**

B. An Quick Overview of Demographic Transition: Galor-Weil (2000)

- **Key: to capture the three phases of demographic transition in Western Europe**
  - **Malthusian trap: pop grows, output stagnates**
  - **Post-Malthusian era: both pop and output grow**
  - **Modern growth regime: pop growth falls, output growth accelerates**

1. The Model

- **Agents: 2-period lived, with mass = L_t for cohort t**
- **Production:** $Y_t = H_t^{\alpha}(A_t X)^{1-\alpha}$ **=>** $y_t = h_t^{\alpha} x_t^{(1-\alpha)}$
  - **h = H/L = efficiency units of labor per worker**
  - **x = AX/L = effective land per worker**
  - **effective wage = w = $(x/h)^{1-\alpha}$**
  - **$g_{t+1} = (A_{t+1} - A_t)/A_t$ is endogenous**
- **Preference:** $u^t = (c_t)^{(1-\gamma)}(w_{t+1} n_t h_{t+1})^{\gamma}$, **where $w_{t+1}n_th_{t+1}$ = children gross consumption**
- **Budget Constraint (BC):** $w_t h_t n_t(\tau^q + \tau^e e_{t+1}) + c_t \leq w_t h_t$
  - **parental income is allocated to own consumption and childrearing**

- ○ $\tau^q + \tau^e e_{t+1}$ **= time devoted to childrearing for education (e) and non-education purposes**
- **Human Capital Accumulation (HC):** $h_{t+1} = h(e_{t+1}, g_{t+1})$
  - ○ **(A1) h is increasing and strictly concave in e**
  - ○ **(A2) h is decreasing in g (rapid technical progress grants HC obsolete)**
  - ○ **(A3) $dh_e/dg > 0$ (rapid technical progress raises MPE)**

2. Optimization (h_t is given to cohort t):

- **Substituting BC and HC into the utility function to eliminate c_t and h_{t+1} yields an optimization problem over (n_t, e_{t+1}) subject to nonnegativity constraints and subsistence consumption constraint (CC):** $w_t h_t[1 - n_t(\tau^q + \tau^e e_{t+1})] \geq \tilde{c}$
- **When CC is not binding:**
  - ○ **FOC(n_t):** $n_t[\tau^q + \tau^e e_{t+1}] = \gamma$
  - ○ **FOC(e_{t+1}):** $\gamma h_e(e_{t+1}, g_{t+1}) n_t[\tau^q + \tau^e e_{t+1}] \leq (1-\gamma)\tau^e h(e_{t+1}, g_{t+1})$
    (i.e., $MB_e \leq MC_e$; = 0 for $e_{t+1} > 0$)
- **When CC is binding:** $n_t[\tau^q + \tau^e e_{t+1}] = 1 - \tilde{c}/(w_t h_t)$

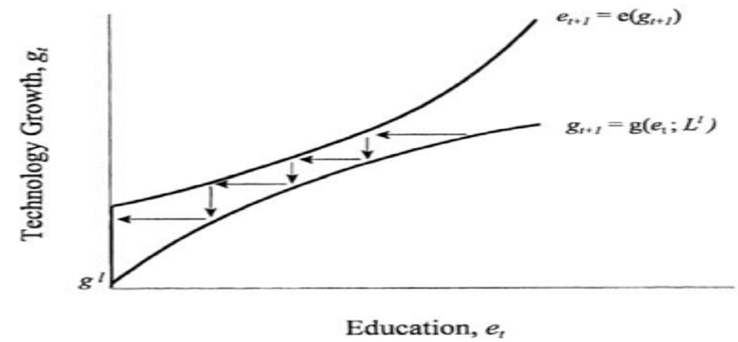
3. Equilibrium

- **Education decision (EE):**
  - ∃ a take-off critical value $\hat{g}$ s.t.

    $e_{t+1} = 0$ for $g_{t+1} \leq \hat{g}$

    $e_{t+1} = e(g_{t+1}) > 0$ for $g_{t+1} > \hat{g}$ (with $e_g > 0$)
  - (A4) $MB_e(0,0) \leq MC_e(0,0)$; $e_{gg} < 0$
- **Fertility decision:** $n_t = n(g_{t+1})$, with
  - $n_g \leq 0$ for $g_{t+1} > \hat{g}$
  - $n_g > 0$ for $g_{t+1} < \hat{g}$
- **Evolution of population:** $L_{t+1} = n(g_{t+1})L_t$
- **Technical progress (GG):** $g_{t+1} = g(e_t, L_t)$
  - (A5) $g_e > 0$, $g_{ee} < 0$, $g_L > 0$, $g_{LL} < 0$
- **Resource expansion:**

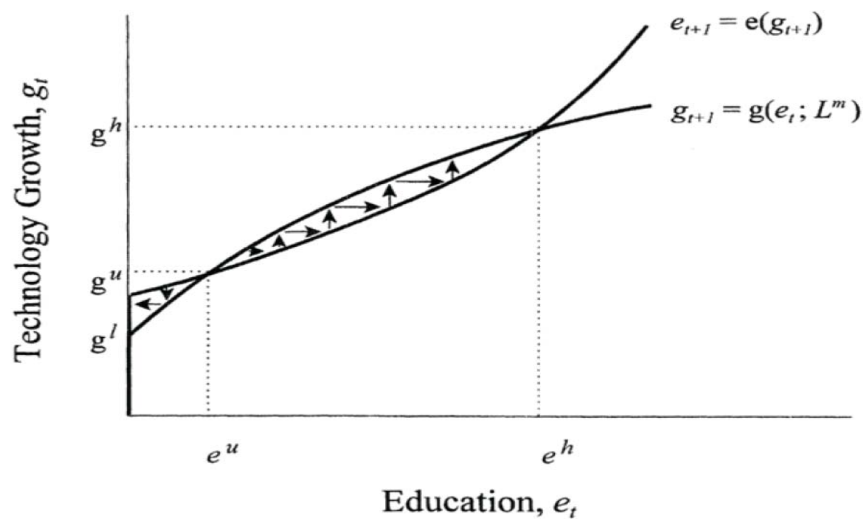
  $x_{t+1} = [(1+g_{t+1})/n(g_{t+1})]x_t$
- **Dynamical system:** 2 x 2 system, EE and GG, in (e, g) for given L

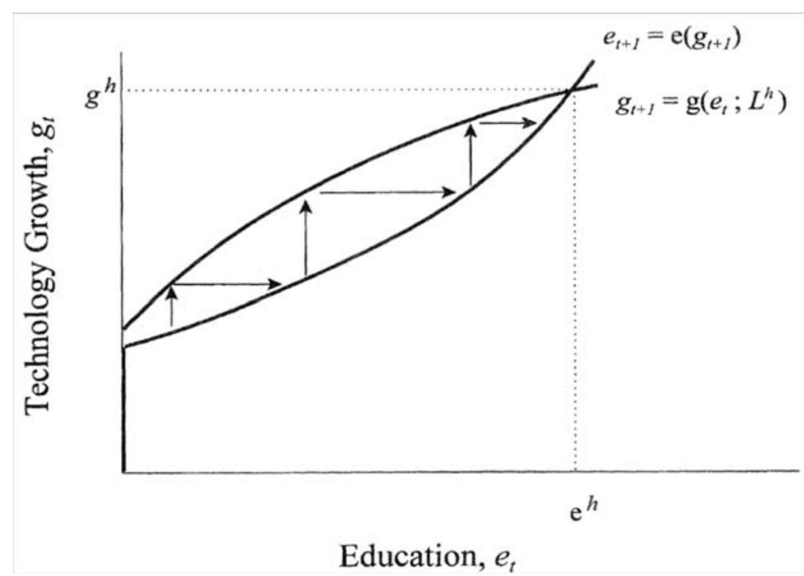


4. Main Findings

- **When EE is entirely above GG: Malthusian trap with $g = g^\ell$ and $e = 0$**

- When EE and GG intersect twice: post-Malthusian for $g > g^u \Rightarrow n_g > 0$
- When EE and GG intersects only once: modern growth regime featuring $e > 0$ and sustained growth
- Transition from Malthusian trap:
  - upward shift in GG: due to exogenous technical progress
  - downward shift in EE: due to reduction in τ^q
- Key drivers:
  - nonhomotheticity in τ^q and $\widetilde{c}$
  - growth-dependent population evolution
  - (A2) HC obsolescent
  - (A5) scale effect in technical progress





C. Fertility, Growth and Income Inequality: de la Croix-Doepke (2003)

- **Main issues:**
  - **What is the relationship between fertility and income disparity?**
  - **What is the effect of differential fertility on growth and inequality?**

1. The Model

- **2-period OG with agents working only when young**
- **Preference: $U^t = \ln(c_t) + \beta \ln(d_{t+1}) + \gamma \ln(n_t h_{t+1})$**
  - **value consumption when young (c) and when old (d)**
  - **value quality-augmented number of children (γ measures altruism)**
- **Human capital accumulation: $h_{t+1} = B_t(\theta + e_t)^\eta (h_t)^\tau (\bar{h}_t)^\kappa$, depending on**
  - **parental human capital (h_t) and average human capital (teacher quality)**
  - **schooling time (*e*)**
  - **scaling factor $B_t = B(1+\rho)^{(1-\tau-\kappa)t}$, with $\tau \in [0, 1\text{-}\kappa]$ measuring persistence of intergenerational human capital transmission and $\theta > 0$ indicating *e* unnecessary (i.e., $e = 0$ is a possible solution)**

- **Budget constraints:**
  - **When young:** $c_t + s_t + e_t n_t w_t \bar{h}_t = w_t h_t(1 - \phi n_t)$**, which equates consumption/saving/child-education expenses with income net of parents foregone earning from devoting time to childrearing**
  - **When old:** $d_{t+1} = R_{t+1} s_t$**, which equates old-age consumption with earning from saving when young**
- **Production:** $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$
- **Evolutionary dynamics:**
  - **population:** $P_{t+1} = P_t \int_0^{\infty} n_t \, \mathrm{d}F_t(h_t)$**, with F(h) = cdf of human capital**
  - **human capital distribution:**

$$F_{t+1}(h) = \int_0^{\infty} n_t I(h_{t+1} \leq h) \, \mathrm{d}F_t(h_t) \Big/ \int_0^{\infty} n_t \, \mathrm{d}F_t(h_t)$$

    - **I is an indicator function**
    - **average human capital =** $\bar{h}_t = \int_0^{\infty} h_t \, \mathrm{d}F_t(h_t)$

- **Market-clearing conditions**
  - **capital:** $K_{t+1} = P_t \int_0^\infty s_t \, \mathrm{d}F_t(h_t)$
  - **labor:** $L_t = P_t \left[\int_0^\infty h_t(1-\phi n_t) \, \mathrm{d}F_t(h_t) - \int_0^\infty e_t n_t \bar{h}_t \, \mathrm{d}F_t(h_t) \right]$

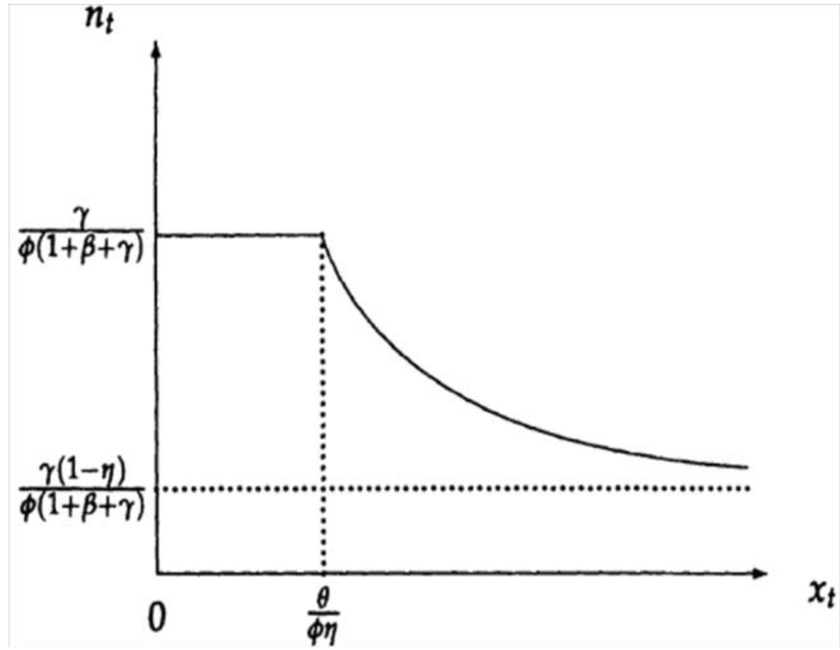
    (total labor supply) (teachers)

2. Optimization

- **Relative human capital:** $x_t \equiv \dfrac{h_t}{\bar{h}_t}$
- **Schooling decision: ∃ a critical value of *x* such that**
  - **those with** $x_t > \dfrac{\theta}{\phi\eta}$ **undertake schooling, with** $e_t = \dfrac{\eta\phi x_t - \theta}{1-\eta}$ **> 0 (↑ in *x*)**
  - **those below the critical value choose e = 0**

- **Saving:** $s_t = \dfrac{\beta}{1 + \beta + \gamma} w_t h_t$
- **Fertility decision:**

$$n_t = \frac{(1 - \eta)\gamma x_t}{(\phi x_t - \theta)(1 + \beta + \gamma)}$$ **(↓ in *x*)**



- **Differential fertility:**
  - **fertility rate drops as relative human capital rises**
  - **the higher η is, the larger the fertility differential will be**
    - **critical *x* falls**
    - **n_t curve becomes steeper**

3. Equilibrium

- **Output and population growth:** $g_t \equiv \dfrac{\bar{h}_{t+1}}{\bar{h}_t}, \quad N_t \equiv \dfrac{P_{t+1}}{P_t}$

- **For $\eta\phi > \theta$ and $\kappa = 1 - \tau$, BGP features:**

$$g = B\left(\frac{\eta(\phi - \theta)}{1 - \eta}\right)^{\eta} \quad \textbf{and} \quad N = \frac{(1 - \eta)\gamma}{(\phi - \theta)(1 + \beta + \gamma)}$$

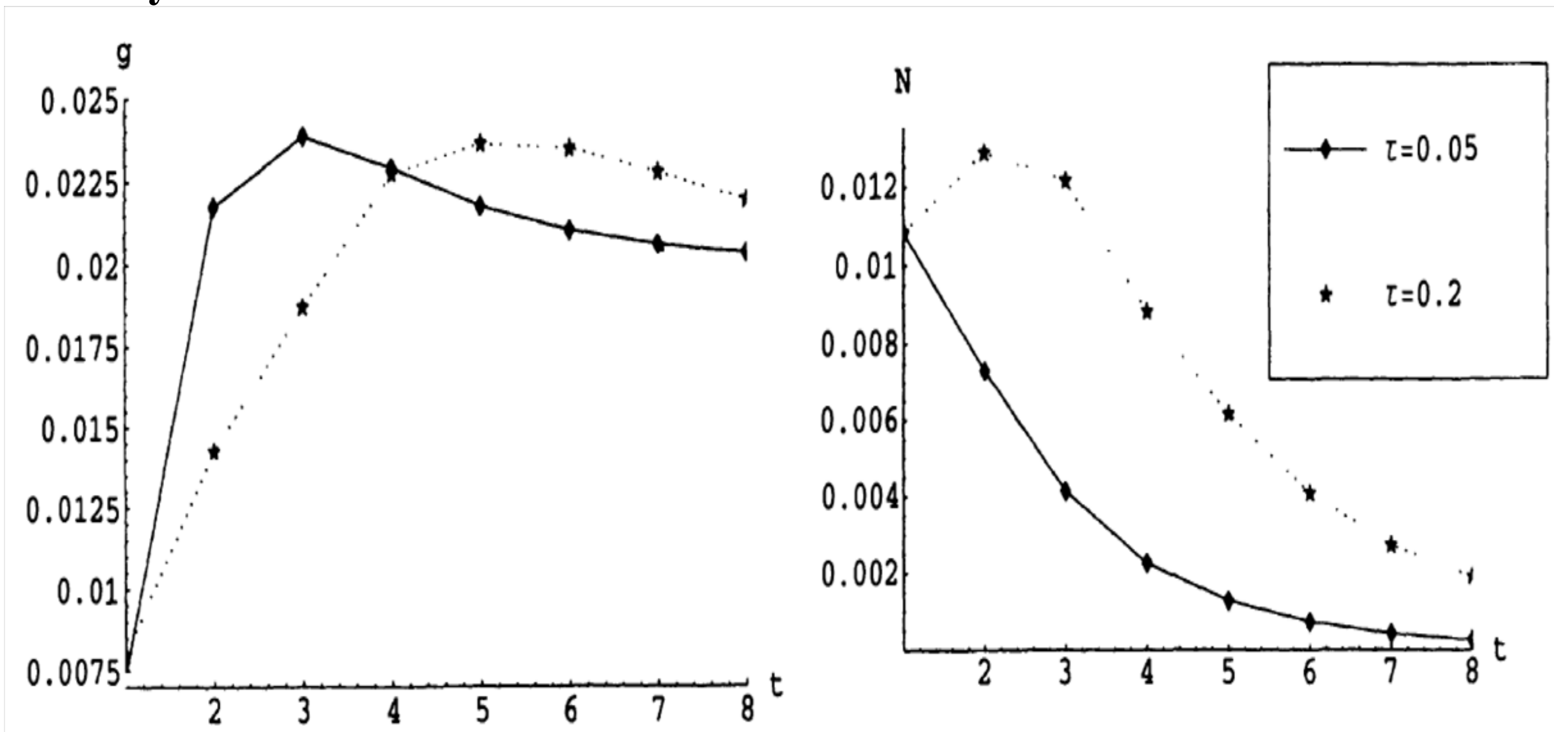
  - **higher η induces higher output growth and lower population growth**
  - **the less schooling-dependent human capital transmission is (high θ), the lower output growth and the higher population growth will be along BGP**

- **For $\kappa + \tau < 1$, $B = 1 + \rho$ (exogenous growth)**

- **By looking at the evolution of x, there are two fixed points (1, x*). $\exists\, \tau_c$ s.t. for $\tau < \tau_c = [(1-\eta)\varphi-\theta]/(\varphi-\theta)$, x = 1 is stable and x = x* is unstable; for $\tau \in [\tau_c, \eta)$, x = 1 is unstable and x = x* is stable (i.e., the dynamics of x feature transcritical bifurcation with parameter τ_c for $\tau < \eta$)**

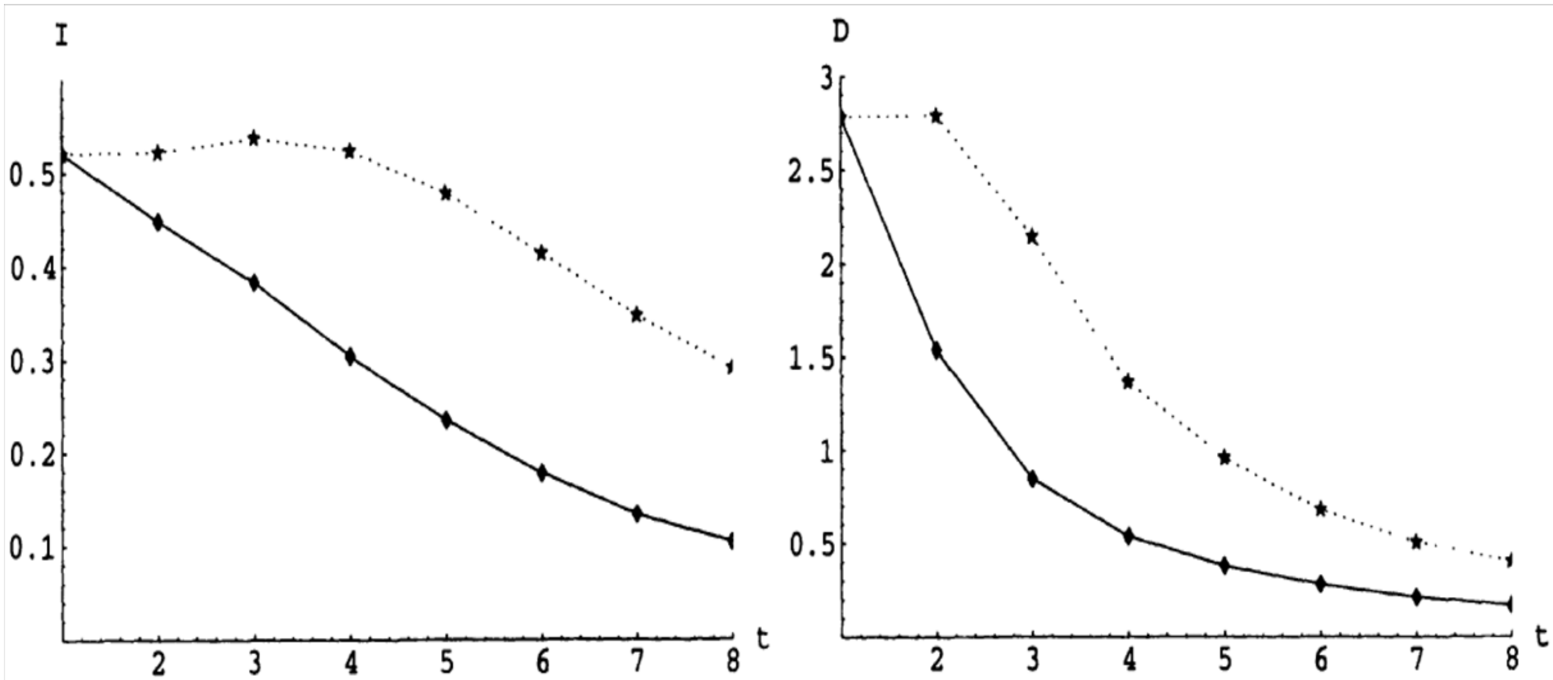
4. Calibrated Findings

- **Fertility and Growth**



  - **Need large persistence in intergenerational human capital transmission to support negative relationship between output and fertility growth**

- **Fertility Differential (D) and Income Inequality (I)**



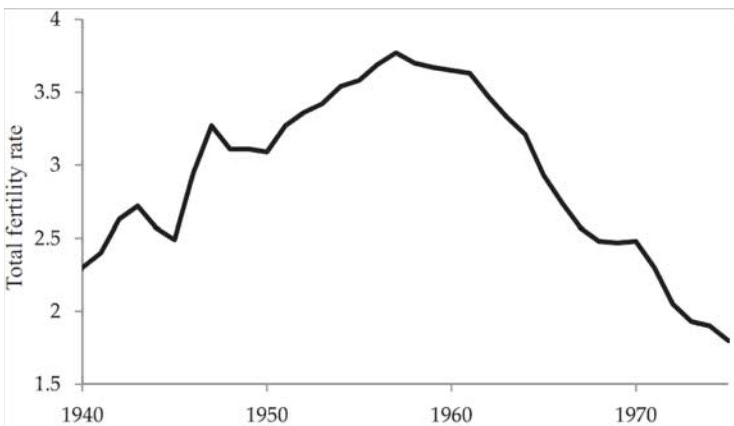
  - **As an economy develops, fertility differential and inequality can both rise and then decline (τ=.2)**

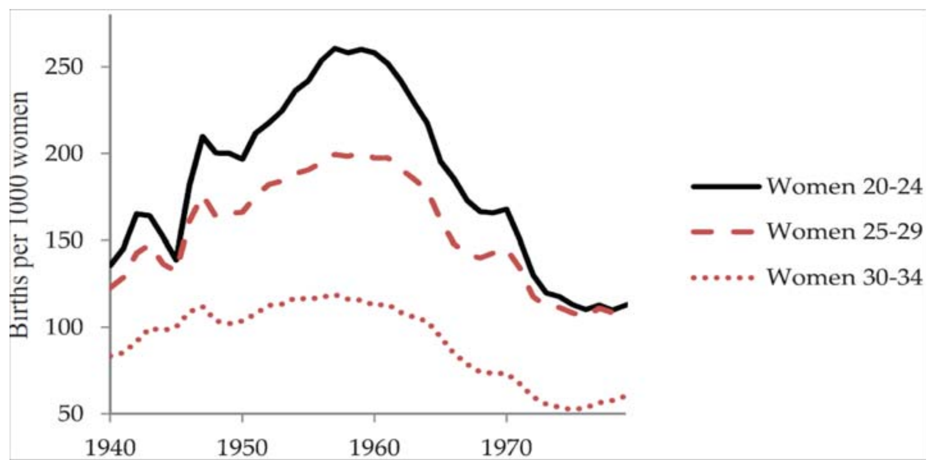
D. Baby Boom: Doepke-Hazan-Maoz (2015)

- **Greenwood-Seshadri-Vandenbroucke (2005) argue that technical progress in producing household durables has lowered costs of rearing children, thus inducing the post WWII baby boom in the U.S.**
- **In Doepke-Hazan-Maoz (2015), the main driver of baby boom is the rise in female labor force participation during WWII**

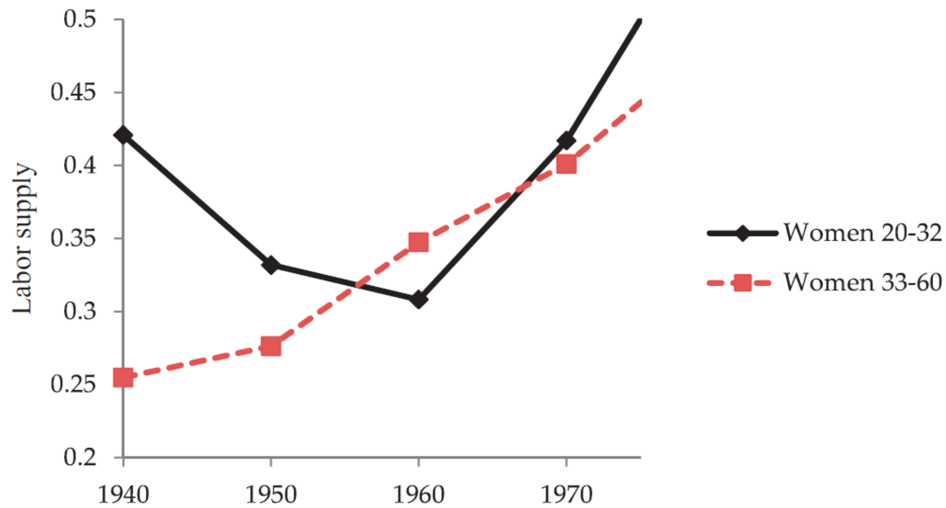
1. Data and Basic Idea

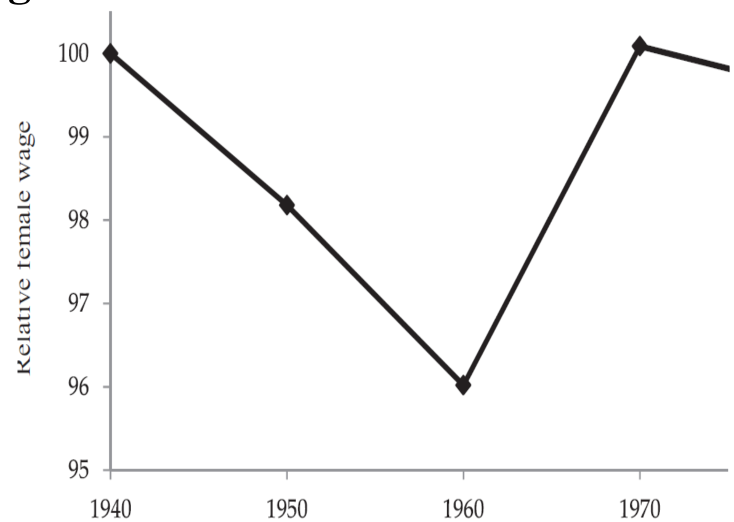
- **Total fertility rate (TFR)**





- **female relative labor supply and relative wage to men**





  - **WWII induced a large positive demand shock for female labor when men were fighting the war in Europe and Asia (Acemoglu-Autor-Lyle 2004)**
  - **this shock was persistent (culture, work experience, irreversible decision)**
  - **this shock had an asymmetric effect on different cohort of women**
    - **older experienced (33-60) women gained**
    - **younger (20-32) women lost during the war**
    - **these younger women after the war faced tougher competition from old women who remained in the labor force and from men returned from the war, so choosing optimally to bear more children instead**

2. The Model

- **Focus on married couples, making decisions on fertility and female labor force participation (discrete choices)**
- **They live for T+1 periods, from 0 to T**
  - **men always work continuously until period R and retire afterward**
  - **working women also retire after period R**
- **Preferences:** $U_t = E_t \left\{ \sum_{j=0}^{T} \beta^j \left[\log(c_{t,j}) + \sigma_x \log(x_{t,j} + x_{t,j}^W) \right] + \sigma_n \log(n_t) \right\}$
  - **x = female leisure;** $x_{t,j}^W = 0$ **if no war,** $= \bar{x}^W > 0$ **if entering the labor force during the war (this nonhomotheticity parameter is chosen to match data)**
  - σ_x **= heterogenous preferences for leisure, governing labor supply decisions**
  - **n = number of children**
- **Before-tax labor income:** $I_{t,j} = w_{t+j}^m e_{t,j}^m + w_{t+j}^f e_{t,j}^f l_{t,j}$
  - **e = market experience (labor supply in efficiency units)**
  - ***l* = female labor supply**
- **Budget constraint facing in period t+j by a couple turning adult in t :**

  $c_{t,j} + a_{t,j+1} = (1 + r_{t+j}) a_{t,j} + I_{t,j} - T_{t+j}(I_{t,j}, r_{t+j} a_{t,j})$
  - **consumption and net saving are equal to after-tax income**

- **Labor market experience evolution (for j < R):**

$$e^m_{t,j+1} = (1+\eta_{m,j})e^m_{t,j},$$

$$e^f_{t,j+1} = (1+\eta_{f,j}l_{t,j} + \nu(1-l_{t,j}))e^f_{t,j}$$

  - **η = gender and age-dependent return to experience**
  - **ν = return to age for whom not working**
- **Female leisure:**
  - **without children:** $x_{t,j} = h - l_{t,j} - z_{t,j}$
    - $z_{t,j} \in \{0, \bar{z}\}$ **= time adjustment cost for rejoining the labor force**
  - **with n^y young (non-adult) children:** $x_{t,j} = h - \phi(n^y_{t,j})^\psi - \kappa_j b_{t,j} - l_{t,j} - z_{t,j}$
    - **φ > 0, ψ > 0: childrearing cost parameters**
    - **b_j = 0, 1 (decision for having a baby in j)**
    - **κ = childbearing cost**
- **Aggregate production:** $Y_t = A_t^{1-\alpha} K_t^\alpha \left(\theta(L_t^f)^\rho + (1-\theta)(L_t^m)^\rho\right)^{\frac{1-\alpha}{\rho}}$
  - **men and women are not perfect substitutes**
  - **aggregate capital K depreciates at rate δ**
  - **productivity growth at rate γ:** $A_{t+1} = (1+\gamma)A_t$

- **The tax function:** $T_t(I_l, I_k) = \tau_{l,t} \max\{I_l - \xi_t, 0\} + \tau_{k,t} I_k + \tau_{LS,t}$**, including labor, capital and lump-sum taxes, where labor taxation is subject to exemption ξ**
- **Government budget constraint:**

$$G_t + w_t^m L_t^D + (1 + r_t) B_t = B_{t+1} + \sum_{s=1}^{T} P_{t-s} \int_0^\infty T_t(I_{t-s,s}, r_t a_{t-s,s}) \, dF(\sigma_x)$$

  - **P = cohort size**
  - **B = government debt**
  - **G = government spending (including military goods)**
  - **L^D = soldiers drafted into the military (in efficiency units of male only)**

3. Equilibrium

- **Goods (or loanable fund) market clearing:** $K_t + B_t = \sum_{s=1}^{T} P_{t-s} \int_0^\infty a_{t-s,s} \, dF(\sigma_x)$
  - **assets (savings) depending crucially on couple's leisure preference (σ_x)**

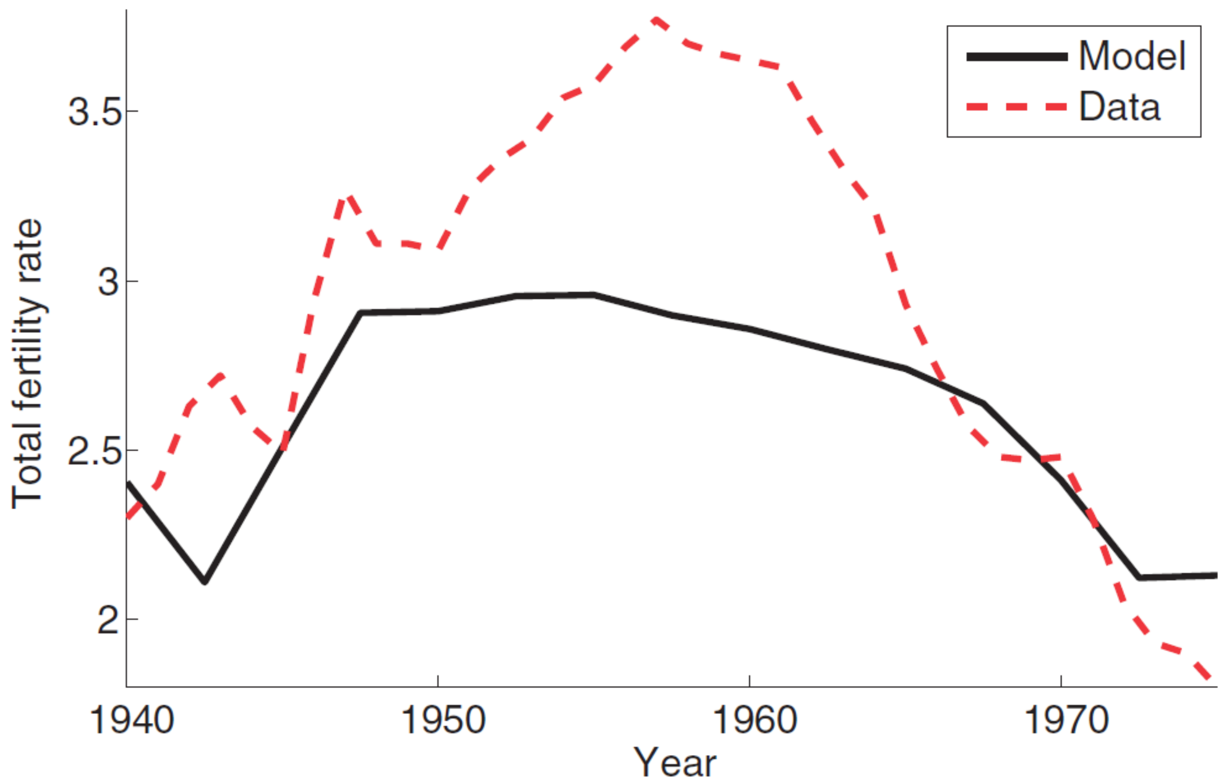
- **Male labor market clearing:** $L_t^m + L_t^D = \sum_{s=0}^{R} P_{t-s} \int_0^\infty e_{t-s,s}^m \, dF(\sigma_x)$
- **Female labor market clearing:** $L_t^f = \sum_{s=0}^{R} P_{t-s} \int_0^\infty e_{t-s,s}^f \, l_{t-s,s} \, dF(\sigma_x)$
- **Population evolution:** $P_{t+I} = \frac{1}{2} \sum_{s=0}^{M} P_{t-s} \int_0^\infty f_s b_{t-s,s} \, dF(\sigma_x)$
  - **fertility measured by individuals but cohort size by couples**
  - **f_s $b_{t\text{-}s,s}$ = expected number of births of cohort t-s**

4. Quantitative analysis

- **Life cycles:**
  - **set model period = 2.5 years (birth spacing)**
  - **childhood length: I = 8 (age 20, so adulthood starts at age 22.5)**
  - **maternal limit: M = 6 (age 22.5+15=37.5)**
  - **retirement: R = 15 (age 60)**
  - **length of life: T = 19 (age 70)**

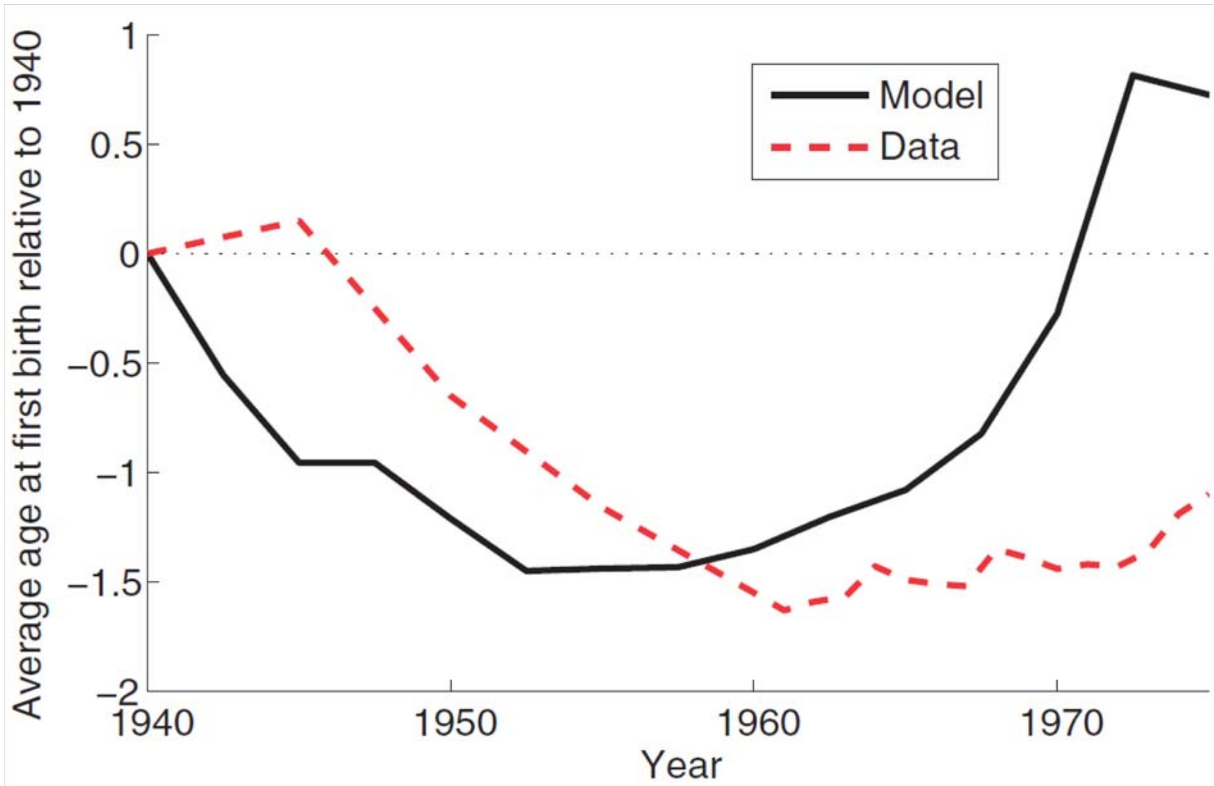
- **Key parameters:**
  - **share of female labor: $\theta = 0.35$**
  - **elasticity of substitution between male and female: 2.9 ($\rho = 0.65$)**
  - **return to experience:** $\eta_{f,j} = \eta_{m,j} = \exp(0.125 - 0.00053(12.5j - 6.25)) - 1$
  - **return to age: $\nu = 0.003$ (essentially nil)**
  - **childrearing cost function: $\varphi = 0.417$ and $\psi = 0.33$, based on**
    - **average private childrearing cost inclusive of forgone female earning = 40% of per capital income (Haveman-Wolfe 1995)**
    - **incremental childrearing cost**
  - **probability of conceiving a baby conditional on trying:**
    - **$f_j = 1$ for j up to 4 (impaired fecundity = 0 up to age 32.5)**
    - **$f_5 = 0.75$, $f_6 = 0.5$ (based on impaired fecundity in Menken et al. 1986)**
  - **leisure preference: σ_x uniformly distributed over [1.135, 1.589], calibrated to match**
    - **female labor force participation (and hence the level of fertility)**
    - **fertility response to the war shock**

- **Result on fertility:**



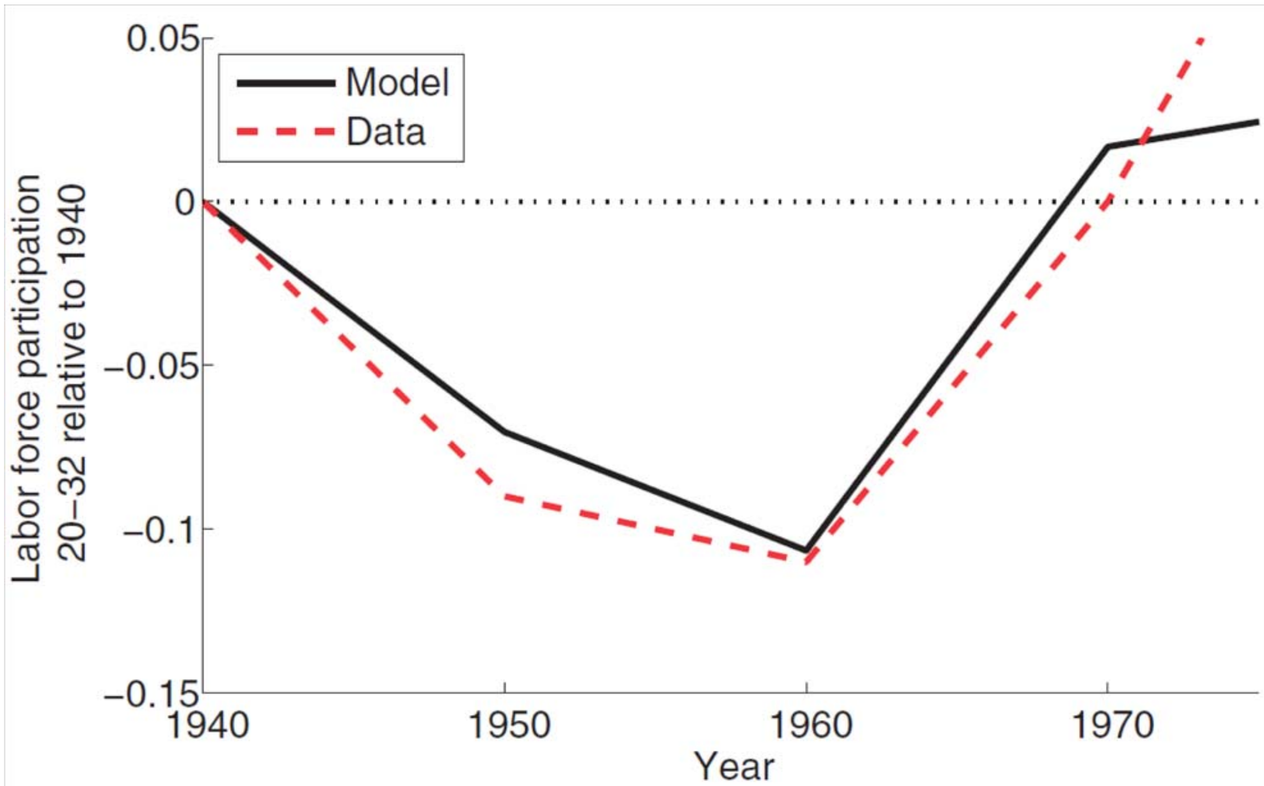
  - **counterfactual analysis => return to experience plays most important role**
  - **overall good fit to pattern, but underpredicting the baby boom with incorrect timing**

- **Result on timing of first birth**



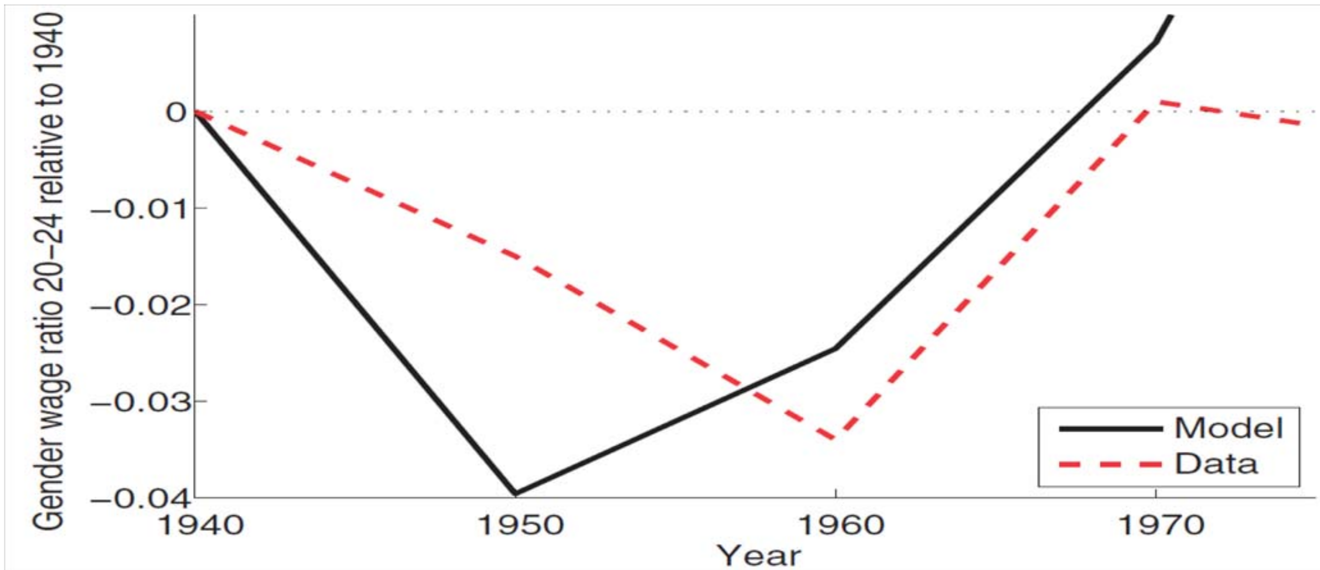
  - **counterfactual sharp rise in delayed birth after the baby boom**

- **Result on female labor force participation**



  - **very good fit in younger women's labor force participation**

- **Result on female-male wage gap**



  - **good fit to pattern, but timing is off (model predicts drop too quickly)**

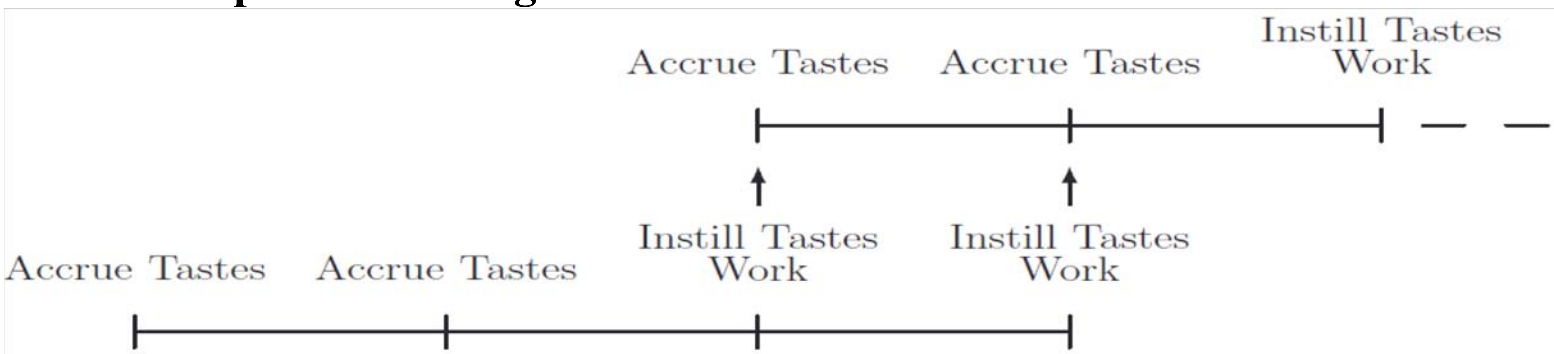
- **Open issues:**
  - **endogenous birth timing and spacing (Jiang-Lien-Wang-Wang 2019)**
  - **the role of social norms and consequent changes in labor markets**
  - **the role of part time jobs**
  - **household bargaining**

E. Changing Children Quality via Intergenerational Behavioral Transmission: Doepke and Zilibotti (2008)

- **Basic idea:**
  - **While the fertility literature emphasizes on quantity and quality tradeoff, the quality is measured exclusively by human capital following Becker-Murphy-Tamura (1990). However, parents can influence children significantly by shaping their children preferences in response to economic incentive**
  - **Particularly, after industrial revolution, middle-class parents shape children preference toward patience and good work ethic, causing landowning aristocracy replaced by middle-class industrial capitalism**
  - **Intergenerational transmission in child behavior may serve to explain more dispersed fertility decisions and the resulting changes in the speed of demographic transition and income growth**

1. The Model

- **OG with 4-period lived agents:**



- **Middle-class young adult lifetime utility:**

V=$(1-B)(\log(c_1)+A(1-n_1)-l_{A,1}-l_{B,1})+B(\log(c_2)+A(1-n_2)-l_{A,2}-l_{B,2})+z\,V_{\text{child}}(A'(l_A,A),B'(l_B,B))$

  - **endogenous preference formation in taste for leisure (A) and patience (B), $A'(l_A,A)$ and $B'(l_B,B)$, where $z \in (0,1)$**
  - **children preferences depend on parents preferences (A,B) and sustained effort ($l_{A,1}=l_{A,2}=l_A$, $l_{B,1}=l_{B,2}=l_B$) for shaping children preferences**
  - $A'(l_A,A)=\psi\bar{A}+(1-\psi)A+g(l_A)$ **and** $B'(l_B,B)=\psi\bar{B}+(1-\psi)B+f(l_B)$**, with**
    - $\bar{A}>0$ and $\bar{B}>0$ **= innate tastes and $\psi \in (0,1)$ = depreciation**
    - **g and f both increasing/concave in effort, satisfying: g(0)=f(0)=0**
    - $A_{\max}\equiv\bar{A}+g(\bar{l}_A)/\psi$ **and** $B_{\max}\equiv\bar{B}+f(\bar{l}_B)/\psi\le 1$

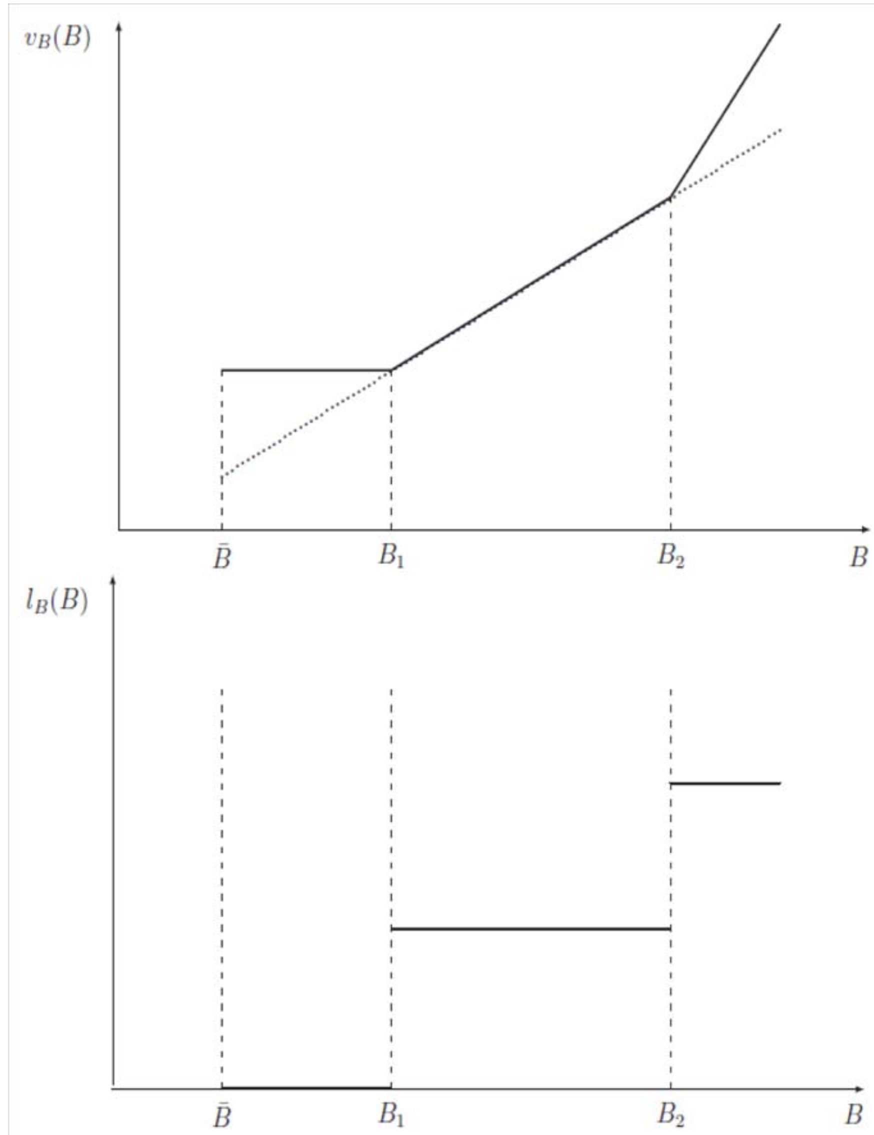
- **Labor productivity/wage earning:**
  - **occupations are indexed by $i \in \{1, 2, ..., J\}$, ordered by steepness of earning profile:** $w_{1,j} < w_{1,i}$ and $w_{2,j} > w_{2,i}$ **for all $j > i$**
  - **experience matters:** $w_{2,i} \geq w_{1,i} > 0$
- **Budget constraints:** $c_1 = w_{1,i} n_1$ and $c_2 = w_{2,i} n_2$ **(no borrowing-lending)**

2. Optimization and Equilibrium

- **Bellman:**

$$V(A,B) = \max_{i \in I, l_A, l_B, n_1, n_2} \{(1-B)(\log(w_{1,i} n_1) + A(1-n_1)) + B(\log(w_{2,i} n_2) + A(1-n_2)) - l_A - l_B + z V(A', B')\}$$

separable with $V(A,B) = v_A(A) + v_B(B)$



- $v_B(B)$, $l_B = l_B(B)$ and $w_{2,i}/w_{1,i}$ are all nondecreasing in B
- Marginal returns to effort l_B: R = $z \log(w_2/w_1)$, independent of B
- The law of motion of patience preferences $B'(l_B, B) = \psi \bar{B} + (1-\psi)B + f(l_B)$ converges to steady state (S-S), at which
  - parents and children choose the same patience investment
  - effort and patience stock are both increasing in R (earning profile)
- $v_A(A)$ and $l_A = l_A(A)$ are nondecreasing in A, and labor supply is $n = \min\{A^{-1}, 1\}$
- The law of motion of leisure preferences $A'(l_A, A) = \psi \bar{A} + (1-\psi)A + g(l_A)$ also converges to S-S, at which parents and children have the same work ethic

3. Landowning Rent-Seekers Behavior

- Land rent r, with a minimum rent $\underline{r}$
- Consumption: $c_1 = \underline{r}x + (r - \underline{r})xn_1$ and $c_2 = \underline{r}x + (r - \underline{r})xn_2$
- Patience investment: $l_B = 0$
- Leisure preferences: higher than those of middle-class agents
- Labor supply: $n = \max\left\{\min\left\{A^{-1} - \frac{\underline{r}}{r - \underline{r}}, 1\right\}, 0\right\}$ (less work due to land rent)
- Eventually, landowning aristocracy are replaced by middle-class capitalism

4. Historical Evidence

- **Occupational choice by Cambridge graduates:**

	1752–1799	1800–1849	1850–1899
Church	60	62	38
Land-owning	14	14	7
Teaching	9	9	12
Law	6	9	14
Administration	3	1	6
Medicine	1	2	7
Banking	0	0	2
Business	0	0	5
Other	7	3	9

  - **land-owning dropped by half**
  - **professional occupations rose by more than double**

F. Timing of Childbearing: Jiang-Lien-Wang-Wang (2023)

- **As significant as the shift from quantity to quality in fertility decisions, a rise in the median age at first birth has been commonly observed in the more developed world**
- **Such a positive trend is not only quantitatively large, but robust across regions (America, Asia, and Europe) and ethnic groups (given some noticeable disparities): Happel-Hill-Low (1984) and Cigno-Ermisch (1989)**
    - **By 1990, almost half (49%) of Swedish women in the 25-29 age group were still childless**
    - **The comparable figures for the U.S., Germany and the Netherlands were 42%, 57% and 61%, respectively**
- **Literature:**
    - **Conesa (2000), Iyigun (2000) and Caucutt, Guner and Knowles (2001): discrete-time models**
    - **Mullin and Wang (2002): continuous-time framework**
    - **Lien-Wang (2016): human capital and preference factors (empirical study of Taiwan)**
    - **Jiang-Lien-Wang-Wang (2019): productivity loss and job security factors (calibrating US in a continuous-time lifecycle model)**

1. The Model

- **A woman, married at M, lives for T = M+F years, with birth at M+B and childrearing until M+B+D**
- **3 key drivers:**
  - **preferences (altruism U_0 and disutility from childrearing ψ)**
  - **job security or support from husband income *θwh***
  - **productivity loss due to childbearing δ**
- **Lifetime utility aside from passive childhood valuation:**

$$V = \int_M^{M+F} \left[\frac{c^{1-\sigma}}{1-\sigma} + U_0 I(t \in [M+B, M+F]) - \psi I(t \in [M+B, M+B+D]) \right] e^{-\rho(t-M)} dt$$

- **Nonhuman wealth accumulation:**

$$\dot{a} = ra + [1 - \delta I(t \in [M+B, M+B+D])](1-\eta)wh + \theta wh - c$$

- **Human capital accumulation:** $\dot{h} = \Phi[1 - \delta I(t \in [M+B, M+B+D])]\eta h^{\gamma} H^{1-\gamma}$ **, with positive spillovers from *H***
- **Aggregate production: *y* = *AL*, with aggregate effective labor given by,**

$$L = \int_M^{M+F} \int_{i \,\in\, \text{cohort } \tau} \{[1 - \delta I(t \in [M+B, M+B+D])](1-\eta)h\} \, di \, d\tau$$

2. Optimization and Equilibrium

- **Strategy: solve a hypothetical BGP under a given B and then birth timing B**
- **Hypothetical BGP ($v = h/H$):**

$$\dot{c}/c = \frac{r-\rho}{\sigma} = g$$

$$\dot{p}/p = \dot{\lambda}_h/\lambda_h - \dot{\lambda}_a/\lambda_a = r - \Phi v^{\gamma-1}\left\{[1-\delta I(t\in[M+B, M+B+D])]\,(1-\eta+\gamma\eta)+\theta\right\} = 0$$

$$\frac{c}{a} = \frac{\rho+(\sigma-1)\,r}{\sigma} + [1-\delta I(t\in[M+B, M+B+D])](1-\eta)A\frac{h}{a} + \theta A\frac{h}{a}$$

$$\Phi[1-\delta I(t\in[M+B, M+B+D])]\eta v^{-(1-\gamma)} = \frac{r-\rho}{\sigma}$$

- $\eta = \dfrac{1}{\sigma r+(r-\rho)\,(1-\gamma)}\left[\dfrac{\theta\,(r-\rho)}{1-\delta I(t\in[M+B, M+B+D])}+r-\rho\right]$, **increasing in job security from husband income and in productivity loss – more better assortative matching encourages h investment whereas productivity loss discourages work and hence encourages h investment**
- $v = \left[\dfrac{\Phi\theta\sigma\eta}{\sigma\eta r-(r-\rho)\,(1-\eta+\gamma\eta)}\right]^{\frac{1}{1-\gamma}}$**, depending negatively on η – with more h investment, aggregate human capital rises more than individual human capital as a result of positive spillovers**

- **Under the hypothetical BGP, we obtain**
  - **the consumption path:**

$$\begin{aligned} c(t) &= \left\{ \frac{[\rho + (\sigma - 1)\, r]}{\sigma} a_M + \left[\frac{(1+\theta)\,[\sigma r - \gamma\,(r-\rho)]}{\sigma r + (r-\rho)\,(1-\gamma)} \right] \right\} e^{\left(\frac{r-\rho}{\sigma}\right)(t-M)} \\ &= c(M) e^{\left(\frac{r-\rho}{\sigma}\right)(t-M)} \end{aligned}$$

  - **the lifetime utility:** $V(B) = C_1(B) + C_2(B) + \frac{1}{\rho}\Omega(B)$ **, where**

$$\begin{aligned} C_1(B) &= \frac{c(M)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho + (\sigma-1)\, r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]F} - \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \right] \\ C_2(B) &= \frac{\tilde{c}\,(M+B)^{1-\sigma}}{1-\sigma} \frac{\sigma}{\rho + (\sigma-1)\, r} \left[1 - e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]D} \right] e^{-\left[\frac{\rho+(\sigma-1)r}{\sigma}\right]B} \\ \Omega(B) &= U_0 \left(e^{-\rho B} - e^{-\rho F} \right) - \psi \left(1 - e^{-\rho D} \right) e^{-\rho B} \end{aligned}$$

  - **birth timing is pinned down at: $V'(B) = 0$**
    - **enjoyment decreasing in B (first term of Ω)**
    - **utility loss decreasing in B (second term of Ω)**
    - **net consumption gain (first two terms of V) decreasing in B**

3. Comparative statics

Effects of an increase in	birth timing (B)
1. human capital or labor productivity (Ah_M)	$+$
2. husband's income or job security (θ)	$-$
3. productivity loss due to childbearing (δ)	$+$
4. preference for quality-adjusted children (U_0)	$-$
5. utility loss during childrearing years (ψ)	$+$
6. duration of childrearing (D)	$+$

4. Calibration and Counterfactual Analysis

- **Calibrating to US, focusing on 1940-45 and 1950-55 cohorts, before and after the baby boom**

- **Facts:**

	1940-1945 cohort		1950-1955 cohort		Difference	
	Mean	Median	Mean	Median	Mean	Median
Age at first marriage	22.135	22	22.472	22	0.337	0
Age at first birth	24.584	24	25.989	26	1.405	2.000
First spacing	2.449	2	3.517	3	1.068	1.000
Fertility	2.401	2	2.162	2	-0.240	0
Years of Schooling	13.962	14	14.243	14	0.282	0
Number of Observations	8918		6927			

- **Preset parameters/values: $\rho = 0.05$, $\sigma = 2.5$, $\delta = 15\%$ (Waldfogel 1998), $D = 4.111$ (Phipps et al. 2001), $\theta = 1.649$ (gender gap), $h = \exp(f(s))$ with f specified as a pairwise linear function in year of schooling s (Hall-Jones 1999)**
- **Calibrated parameters to fit average human capital growth, intertemporal no arbitrage, birth timing and birth timing differential: $\{\Phi,\gamma,U_0,\psi\} = \{0.0466, 0.8454, 0.00075, 0.00227\}$**

- **Fitness:**

		Data		Model	
Variable	Description	Average	Diff	Average	Diff
B	first spacing	2.9945	0.7313	2.9945	0.7313
$M+B$	age at first birth	25.2166	1.0905	25.2166	1.0905
η	human capital allocation	0.3770	0.0910	0.3770	0.0027
ν	relative human capital	1.0000	0.2415	1.2153	1.3509
l	time allocated to work	0.6230	-0.0910	0.6134	-0.0098

		Data		Model	
Variable	Description	High	Low	High	Low
B	first spacing	3.2490	2.5178	3.2491	2.5178
$M+B$	age at first birth	25.5962	24.5057	25.5962	24.5057
η	human capital allocation	0.4087	0.3176	0.3778	0.3751
ν	relative human capital	1.0840	0.8425	1.6855	0.3346
l	time allocated to work	0.5913	0.6824	0.6100	0.6198

- **Counterfactual analysis results:**
  - **fertility-related productivity loss and job security play a more important role than the conventional human capital channel in explaining the childbearing timing differentials between skill groups**
  - **women are more sensitive to changes in fertility preference as opposed to leisure loss**
  - **Compared with high-skilled women, low-skilled women are more vulnerable to changes in labor productivity, human capital, husband's income, fertility preference for children and leisure loss in raising children – as a result, low-skilled women push up or defer their timing of childbirth more relative to high-skilled women.**

G. Other and Open Issues

- **Soares (2005): Reduction in infant mortality lowers fertility for a given target**
  - **=> decreases childbearing cost & raises resource for education**
  - **=> enhances human capital accumulation & promote growth**
- **Moav (2005): Increase in high child-labor demand**
  - **=> raises opportunity costs for child education**
  - **=> reduces human capital accumulation & retard growth**
- **Doepke-Zilibotti (2008): extend Moav – child labor ban & fertility decline**
- **Open issues:**
  - **poor early childhood development has a long-lasting big effect: need multi-stage human capital accumulation**
  - **income effect of fertility choice is stronger at low level of economic development: need non-homothetic preferences**
  - **maternal mortality (cf. Jayachandran & Lleras-Muney 2009) and other health-related factors can affect fertility decisions (Y. Wang 2012)**
  - **social norms and joint household preferences: best hope to get timing right**
  - **switch from polygyny to monogamy (as a result of rising female inequality a la Gould-Moav-Simhon 2008) may have important fertility implications (Boldrin-Wang, in progress).**