Computational Imaging Group

Washington University in St. Louis
School of Engineering & Applied Science

Efficient and accurate inversion of multiple scattering with deep learning

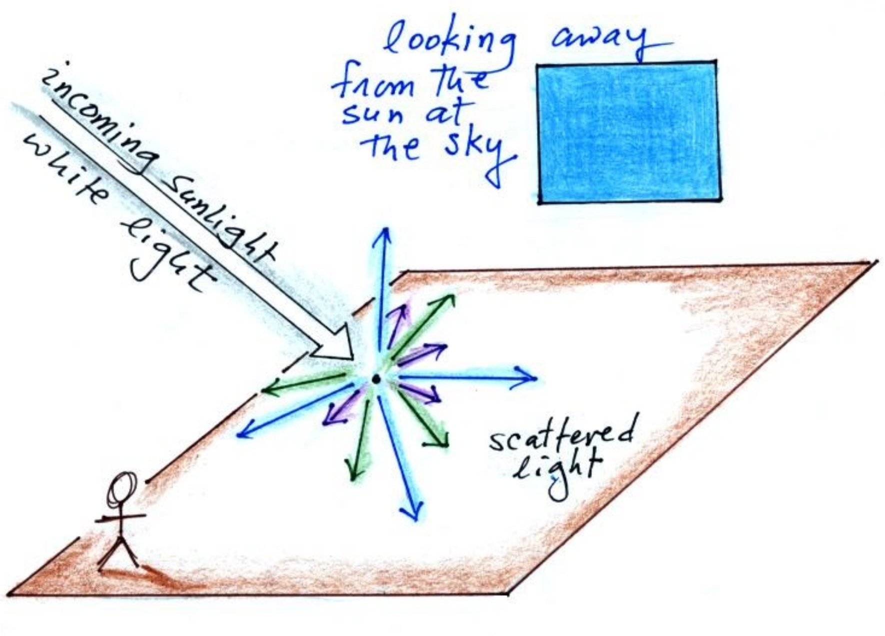
Yu Sun

iTWIST, CIRM, Marseille, France. November, 2018

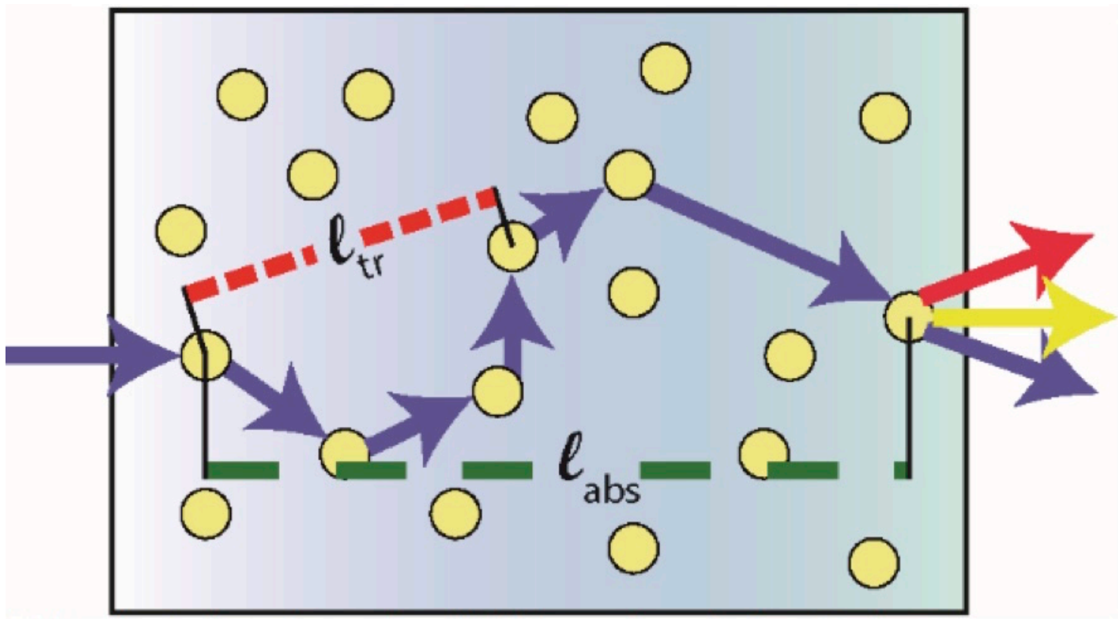
Thanks for the NSF support under rant No. 1813910.

What is multiple scattering ?

- **Interaction between wave and object**
- Related to refractive index



Single/**linear** scattering

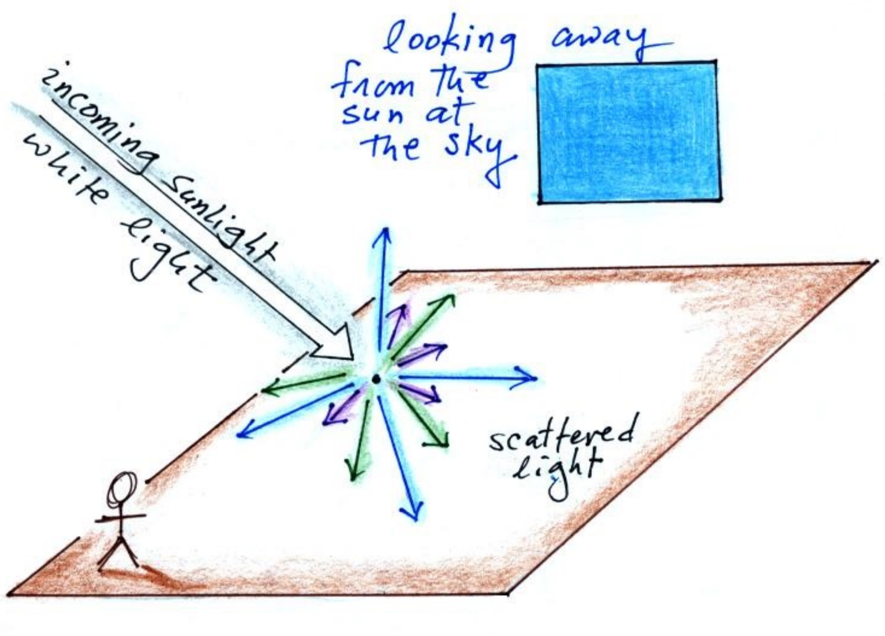


Multiple/**nonlinear** scattering

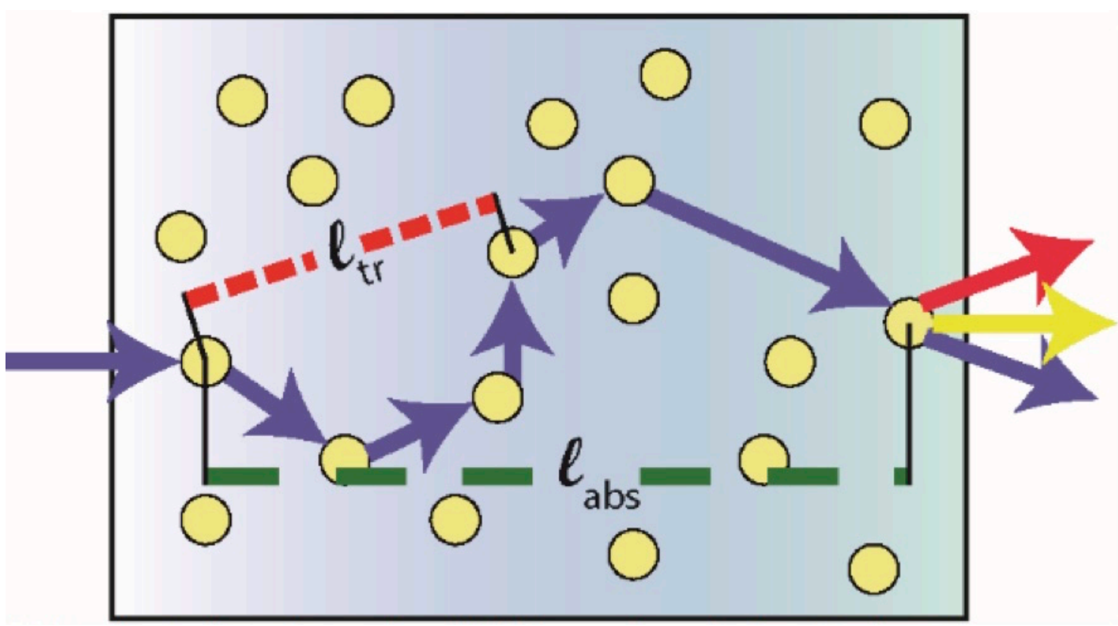
What is multiple scattering ?

- Interaction between wave and object
- **Related to refractive index**

$f \uparrow$ ~ **Scattering strength** $\uparrow$

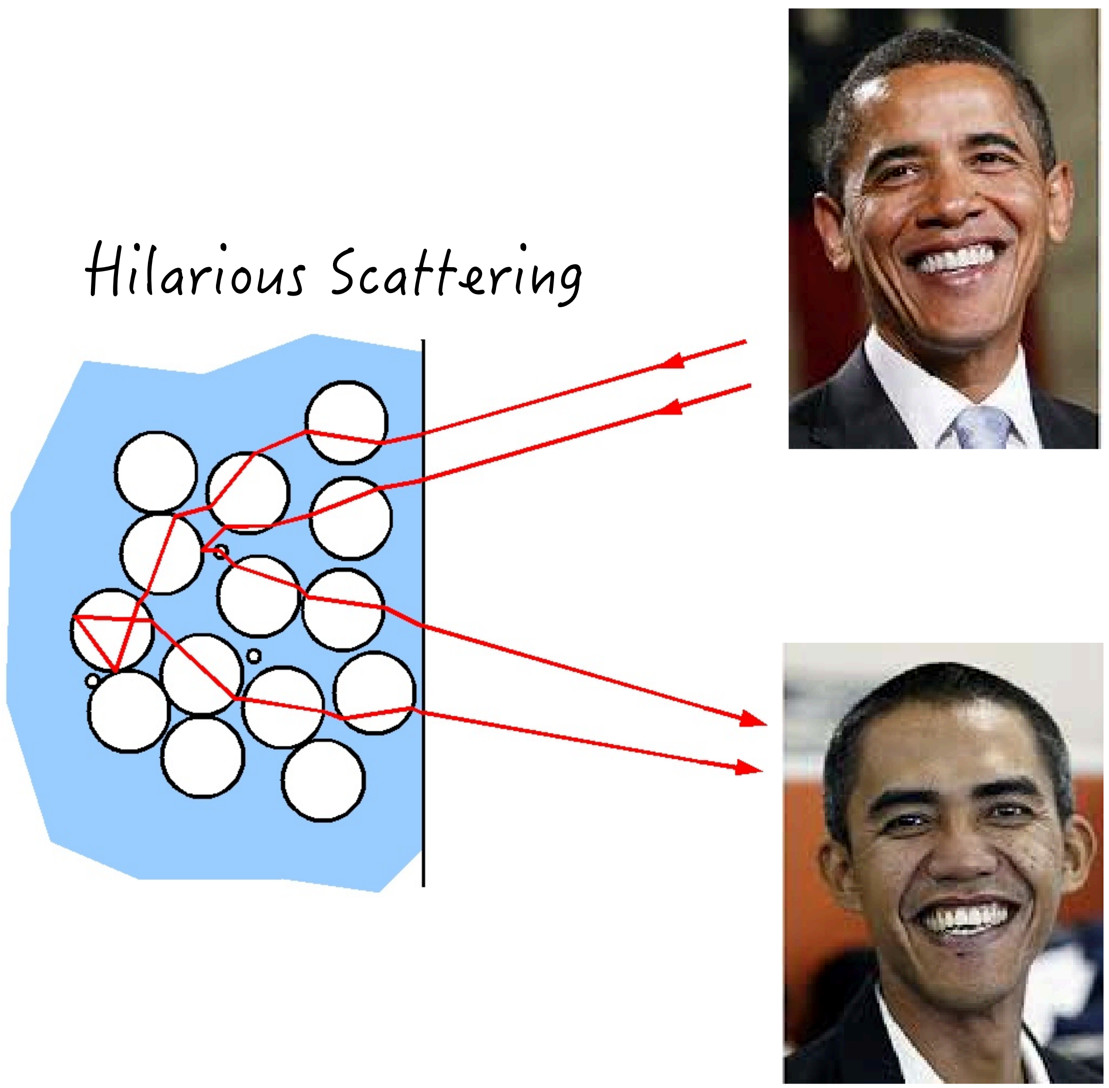


Single/**linear** scattering



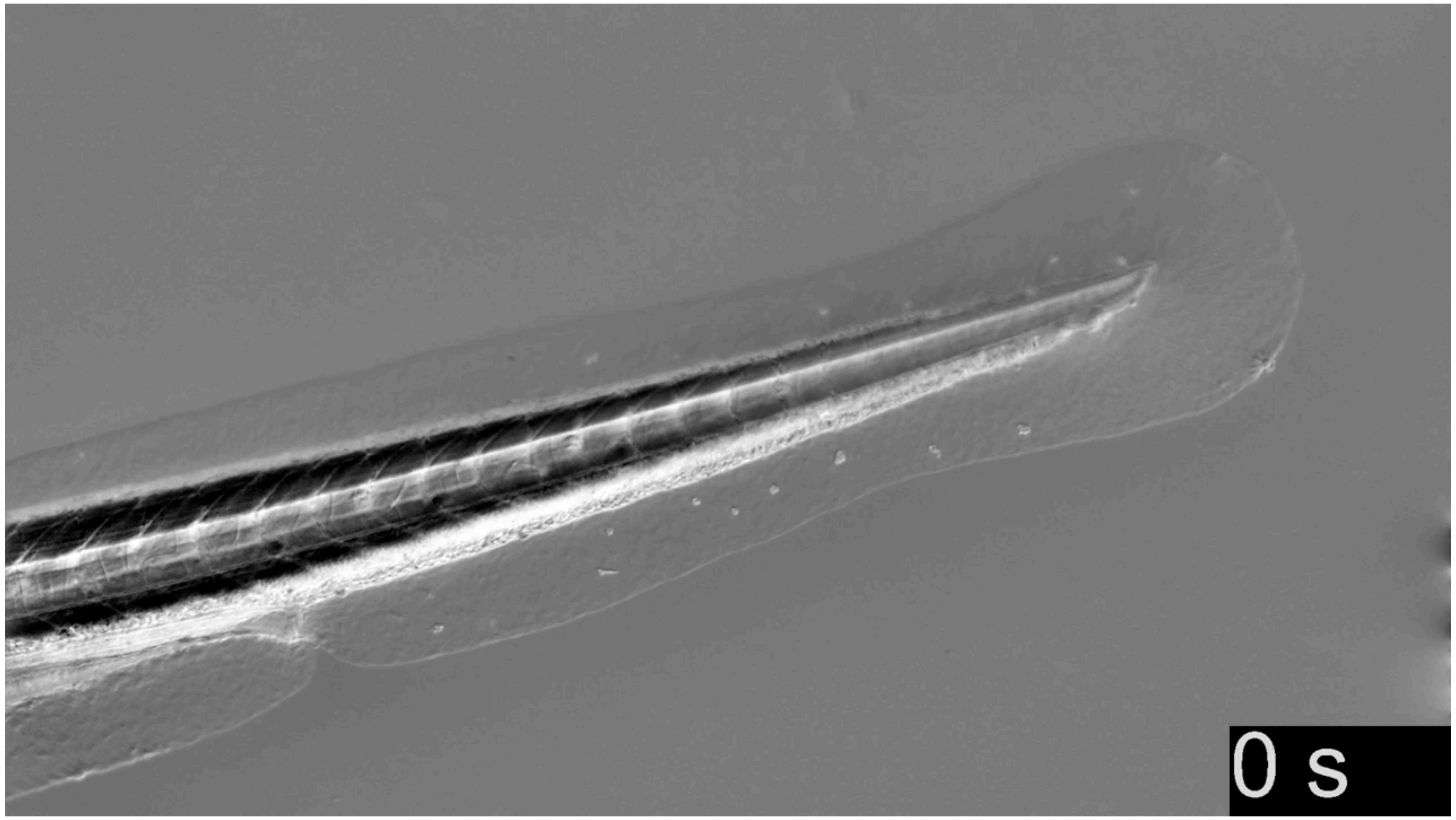
Multiple/**nonlinear** scattering

Scattering exists in wave imaging



Reconstruct image by inverting scattering

- Microwave imaging
- Ultrasound refraction tomography
- **Optical diffraction tomography [T. Kim et al., 2014]**
- …



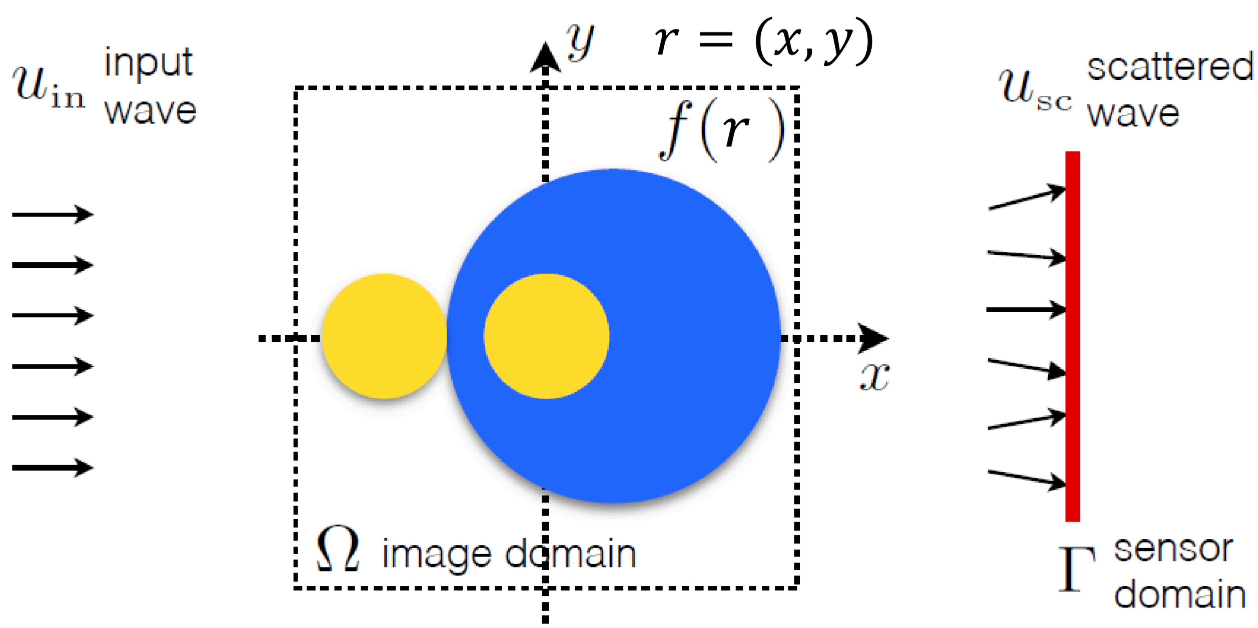
Zebra fish dynamics

T. Kim et al. *Nature Photonics* **8**, 256–263 (2014)

Inverting scattering as inverse problem

- **Lippmann-Schwinger equation [Lippmann & Schwinger, 1950]**
- Optimization formulation
- ISTA/FISTA/ADMM
- Effective in practice

$$u(\mathbf{r}) = u_{\text{in}}(\mathbf{r}) + k^2 \int_{\Omega} g(\mathbf{r}-\mathbf{r}')u(\mathbf{r}')f(\mathbf{r}')d\mathbf{r}', \quad \forall \mathbf{r} \in \mathbb{R}^d.$$



Lippmann, B. A.; Schwinger, J. (1950). *Phys. Rev. Lett.* **79**: 469.

Inverting scattering as inverse problem

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data-fidelity

regularizer

$$\widehat{\mathbf{f}} = \arg\min_{\mathbf{f}\in\mathbb{R}^N} \left\{ \frac{1}{2}\|\mathbf{y} - \mathbf{H}(\mathbf{f})\|_{\ell_2}^2 + \mathcal{R}(\mathbf{f}) \right\},$$

where $$\boxed{\mathbf{H}(\mathbf{f}) = \mathbf{S}\mathrm{diag}(\mathbf{u}(\mathbf{f}))\mathbf{f}}$$

$\mathbf{u}(\mathbf{f}) = \mathbf{u}_{\mathrm{in}}$ **linear ~ single scattering**

$\mathbf{u}(\mathbf{f}) \neq \mathbf{u}_{\mathrm{in}}$ **nonlinear ~ multiple scattering**

Inverting scattering as inverse problem

- Lippmann-Schwinger equation
- Optimization formulation
- **ISTA/FISTA/ADMM**
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where $\boxed{\mathbf{H}(\mathbf{f}) = \mathbf{S}\mathrm{diag}(\mathbf{u}(\mathbf{f}))\mathbf{f}}$

$\mathbf{u}(\mathbf{f}) = \mathbf{u}_{\mathrm{in}}$ **linear ~ single scattering**

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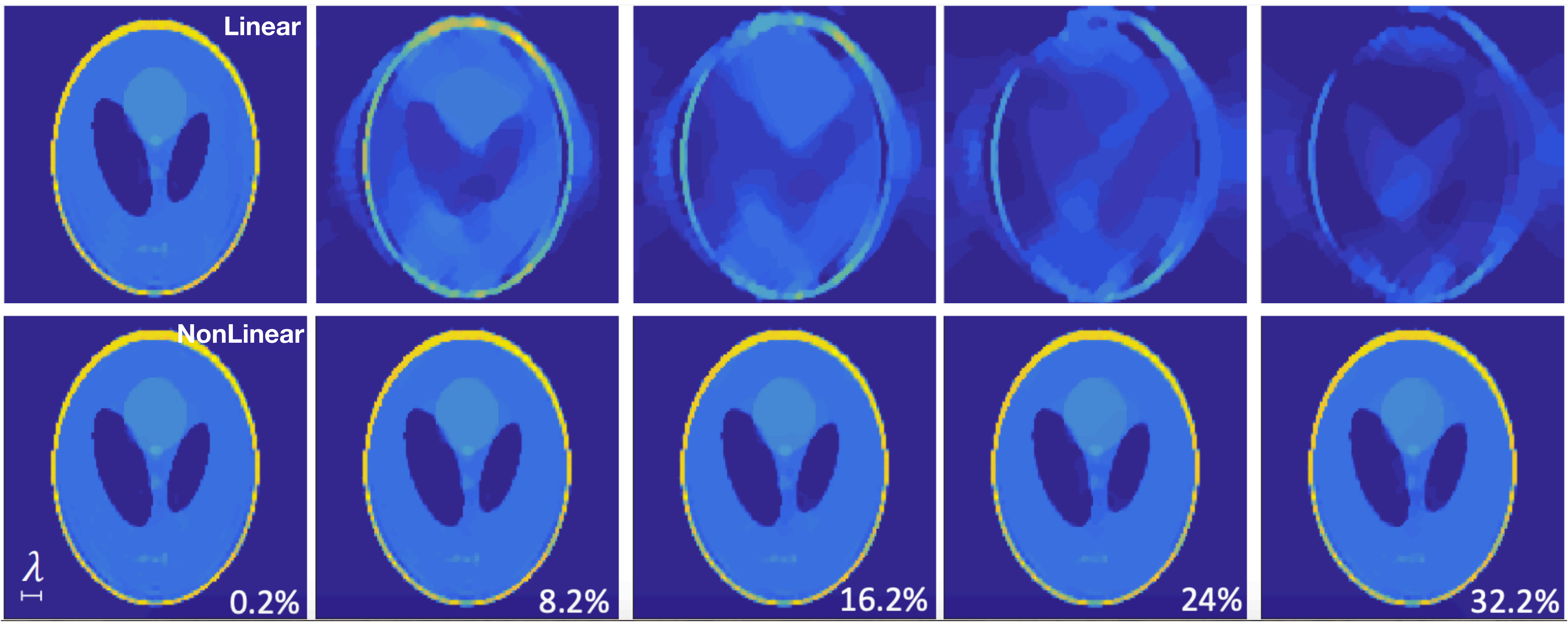
Results obtained by linear/nonlinear FISTA-TV

- **Effective in practice**



Nonlinear

Scattering Levels

Ma, et al., Proc. IEEE Int. Conf. Acoustics, Speech and Signal Process. (ICASSP 2018) (Calgary, Canada, March 15-20), pp. 6473-6477.

But, optimization-based methods have drawbacks

- **Slow reconstruction**
  - ~ hours for 1 example
- **Artifacts / inaccurate**
  - strong noise

$$u(\mathbf{r}) = u_{\text{in}}(\mathbf{r}) + k^2 \int_{\Omega} g(\mathbf{r}-\mathbf{r}')u(\mathbf{r}')f(\mathbf{r}')d\mathbf{r}', \quad \forall \mathbf{r} \in \mathbb{R}^d.$$

$$\boxed{\begin{aligned} \mathbf{u}^* &= \underset{\mathbf{u}}{\operatorname{argmin}} \|\mathbf{A}\mathbf{u} - \mathbf{u}_{\text{in}}\|_2^2 \\ \text{where} \quad \mathbf{A} &:= \mathbf{I} - \mathbf{G}\operatorname{diag}(\mathbf{f}) \end{aligned}}$$

H. Liu, et al., *IEEE Transactions on Computational Imaging*, vol. 4, no. 1, pp. 73-86, March 2018.

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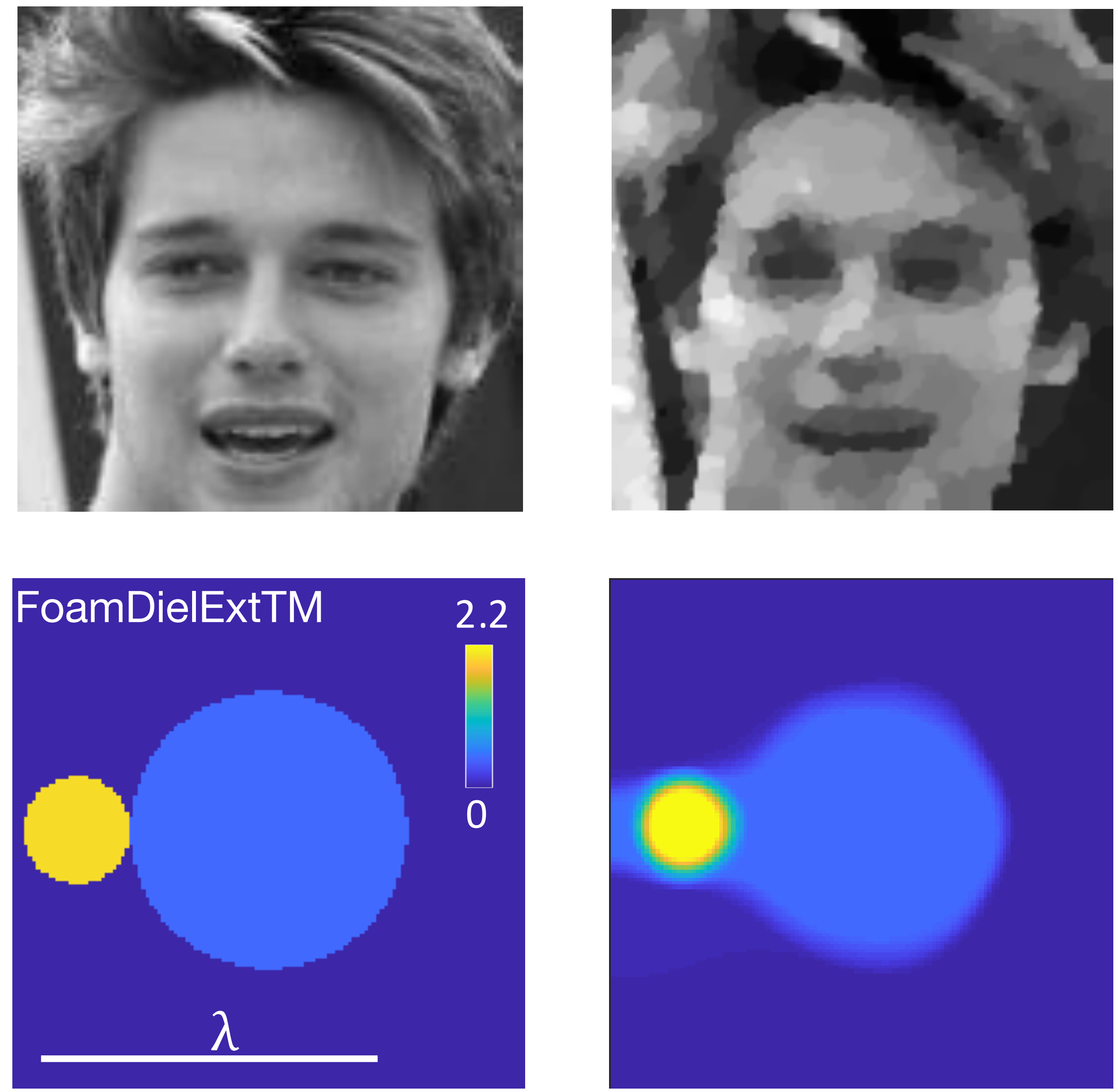
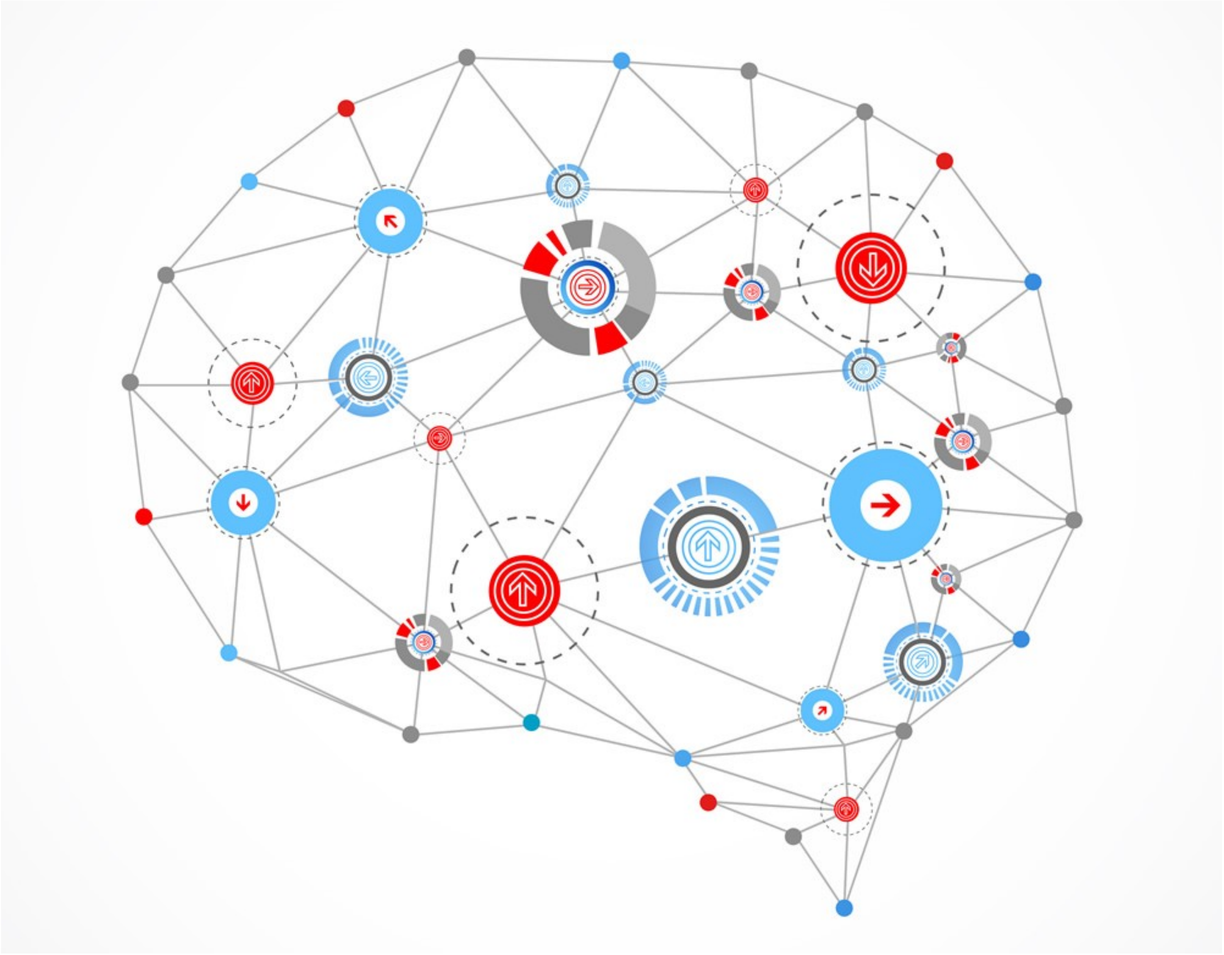


FISTA-TV, Input SNR = 20 dB, Strong scattering

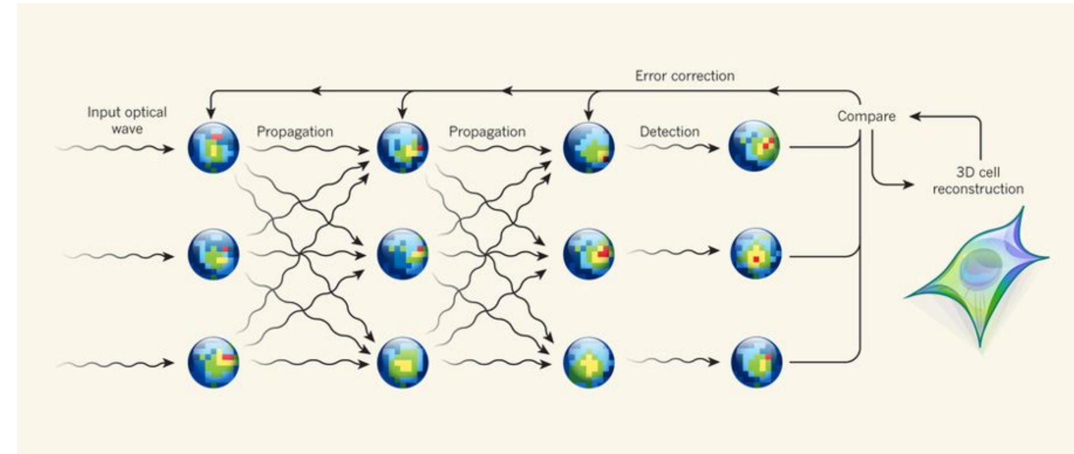
Can we do faster and more accurate simultaneously ?



Deep Learning !

Why we consider deep learning ?

- **Multiple scattering = repeated convolutions**
  - Inverting scattering = repeated deconvolution



Kamilov, et al., *Optica* vol. 2, no. 6, pp. 517–522, June 2015 (Nature News)

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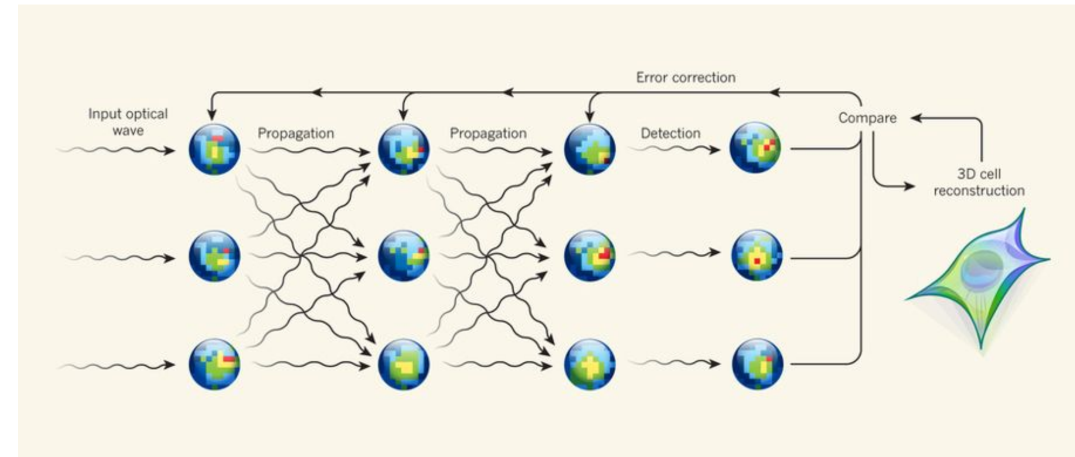
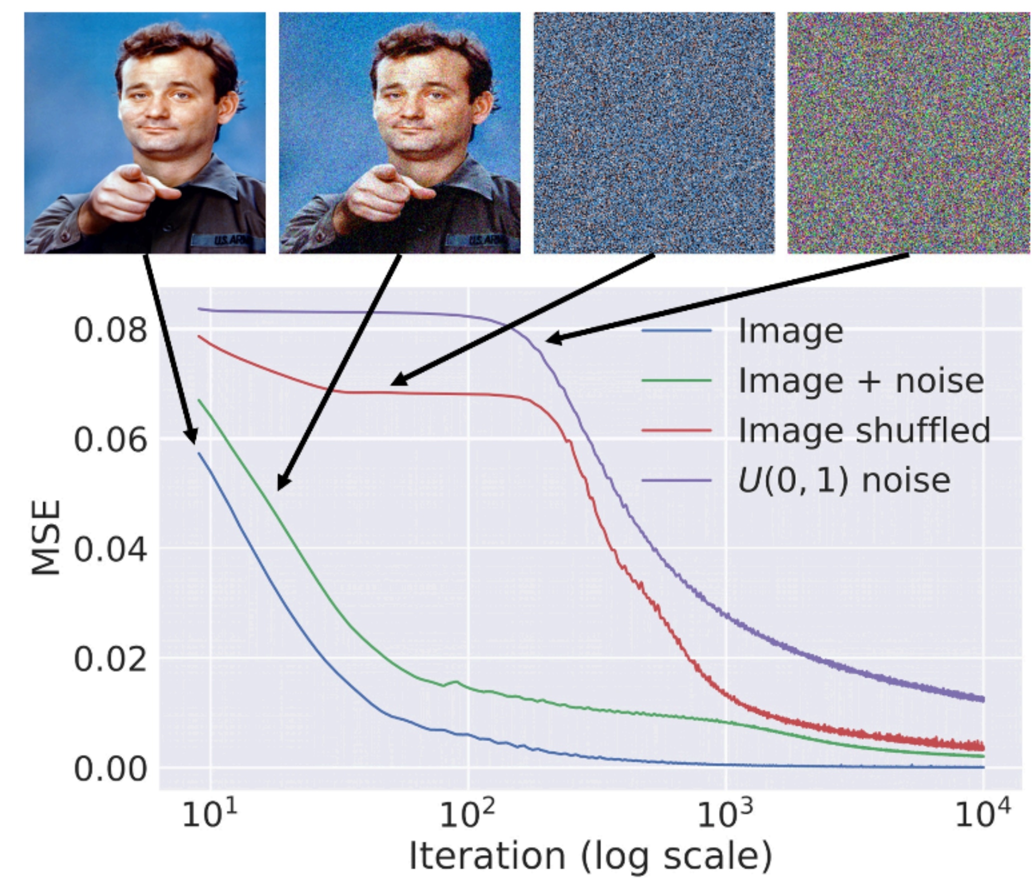
  - Denoising / impainting / deblurring





Ulyanov, et al., *CVPR* 2018

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- **Naturally fit to image**
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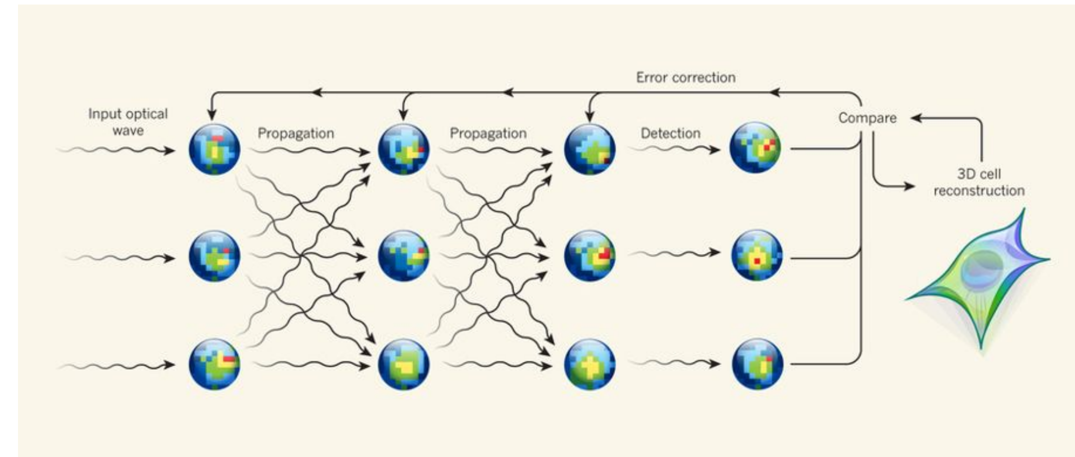
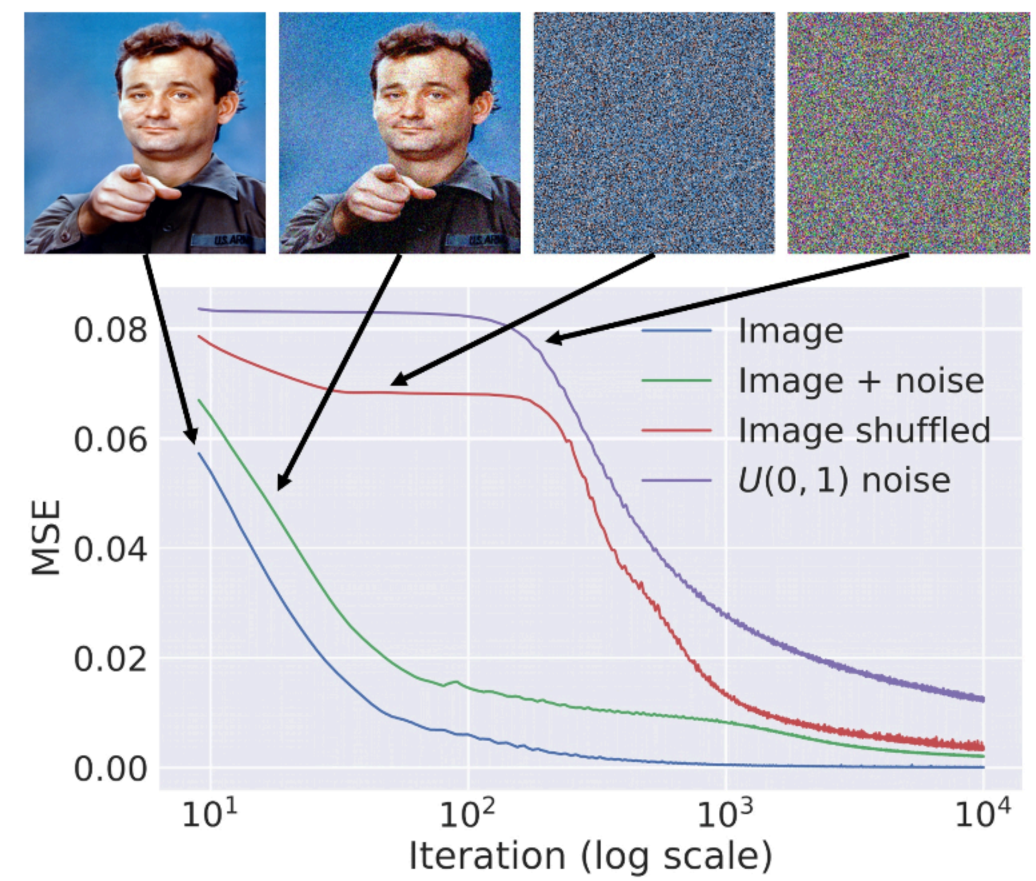
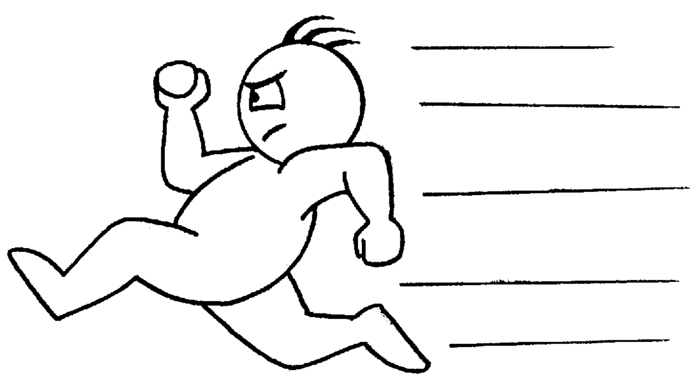
- **Fast prediction**
  - Single forward propagation

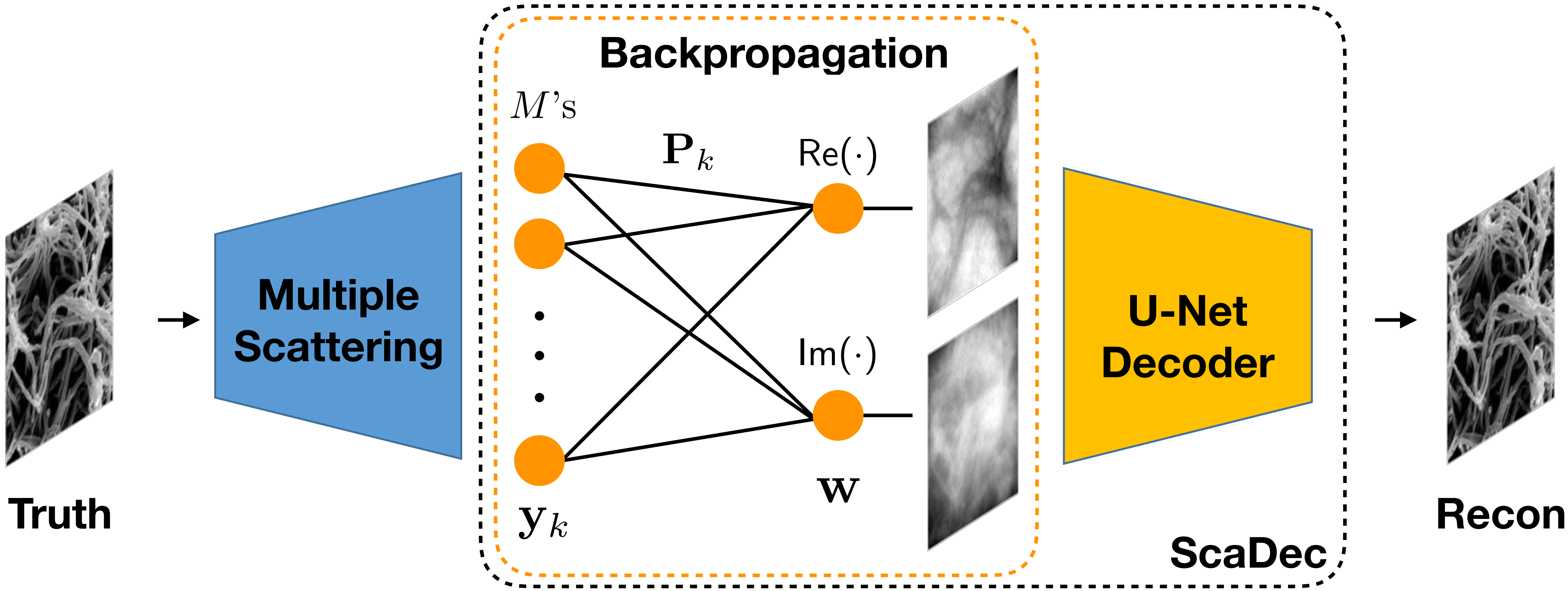






Ulyanov, et al., *CVPR* 2018

The framework of Scattering decoder (ScaDec)

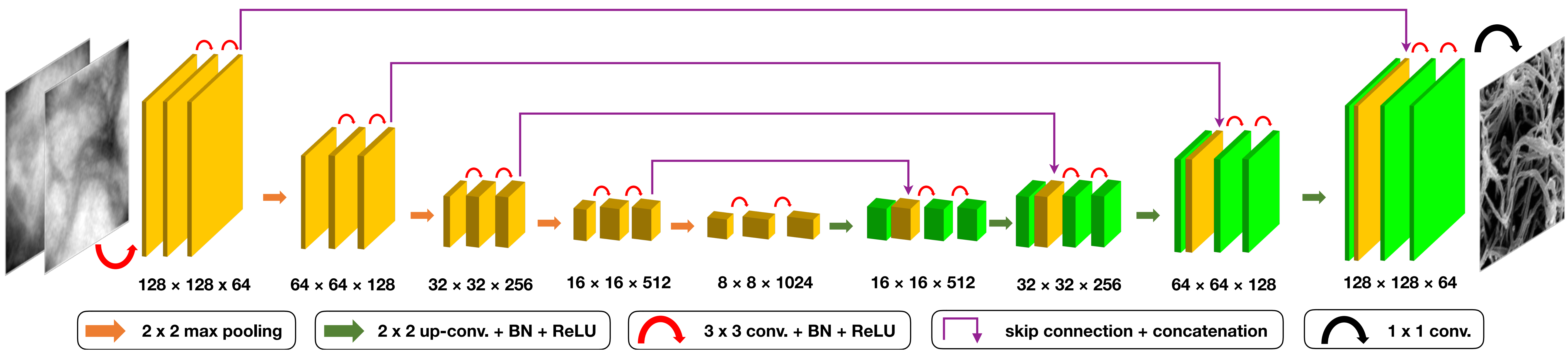


$$\mathbf{z}_k = \mathbf{P}_k \mathbf{y}_k \quad \text{with} \quad \mathbf{P}_k \triangleq \mathrm{diag}(\mathbf{u}_{in}^*)\mathbf{H}^{\mathsf{H}}$$

$$\mathbf{w} = \sum_{k=1}^{K} \mathbf{z}_k = \sum_{k=1}^{K} \mathbf{P}_k \mathbf{y}_k$$

Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

We design the decoder based on U-Net



- **U-Net [Ronneberger et al., 2015]**
  - Cited by 3314

Ronneberger, et al., *Medical Image Computing and Computer-Assisted Intervention (MICCAI)* 9351, 234–241 (2015)

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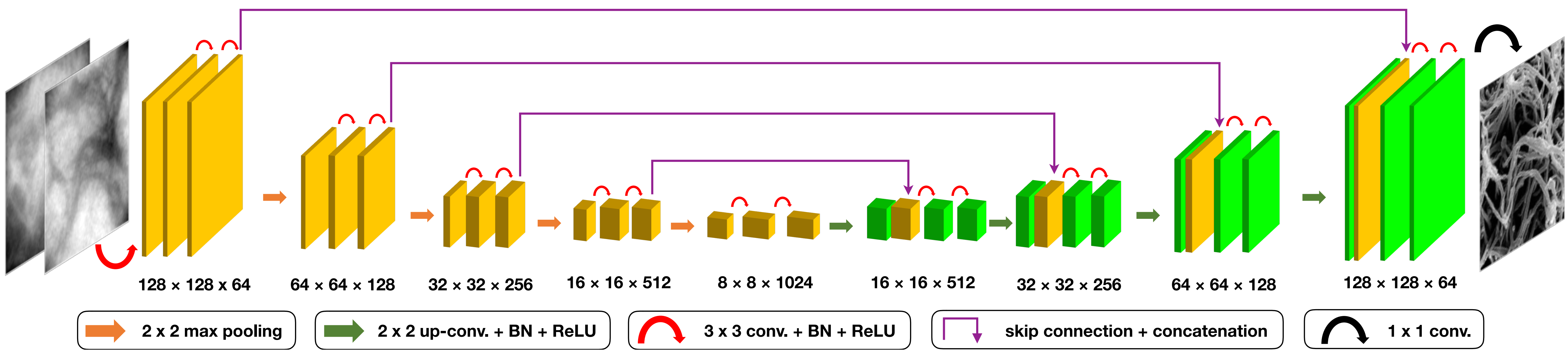
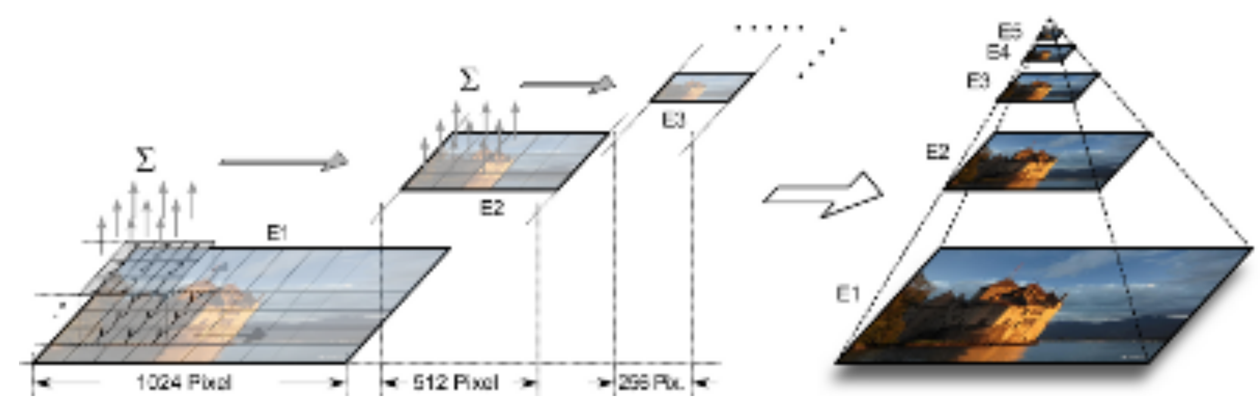


- **U-Net [Ronneberger et al., 2015]**
  - Cited by 3314
- **Multi-resolution decomposition**
  - Increasing effective receptive field



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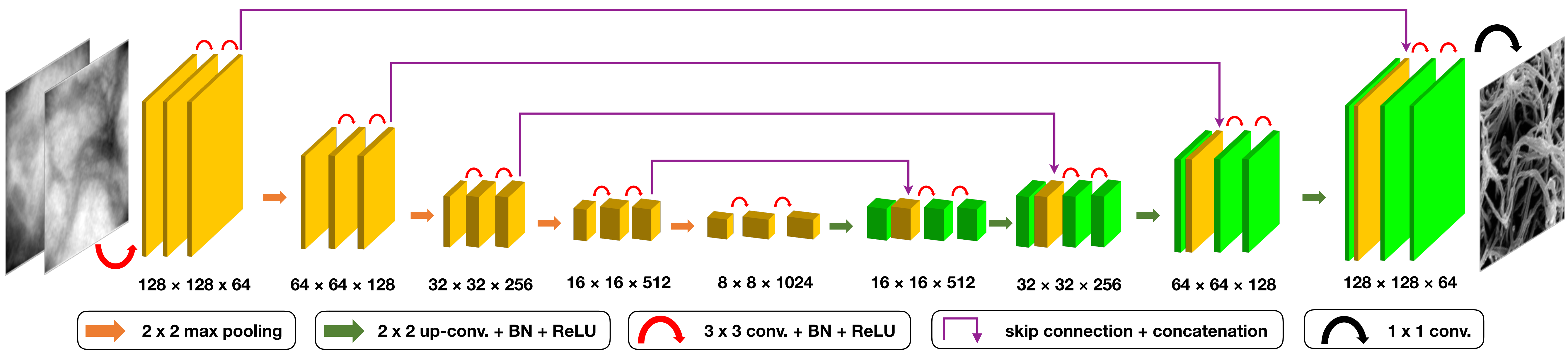


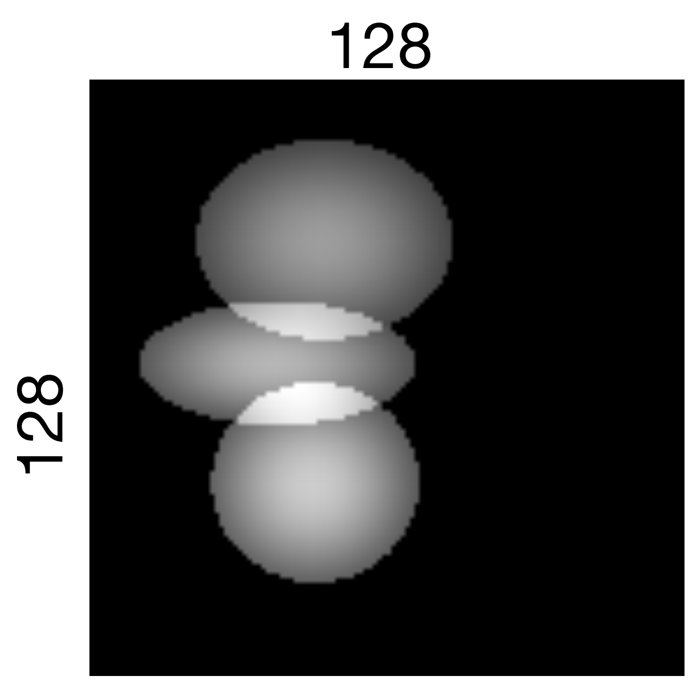
- **U-Net [Ronneberger et al., 2015]**
  - Cited by 3314
- **Multi-resolution decomposition**
  - Increasing effective receptive field
- **Local-global composition**
  - Local details + global features

Ronneberger, et al., *Medical Image Computing and Computer-Assisted Intervention (MICCAI)* 9351, 234–241 (2015)

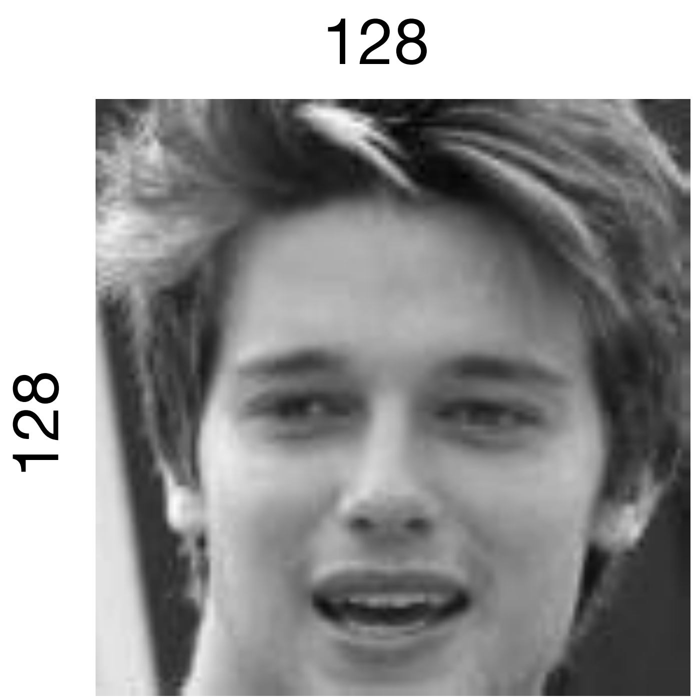
We firstly validate ScaDec on simulated datasets

Simulated data

- Piece-wise smooth & Human faces (CelebA)
- Use high fidelity model to simulate measurements
- Add noise with Input SNR = 20 dB
- Training: 1500 / Test: 24 (randomly selected)



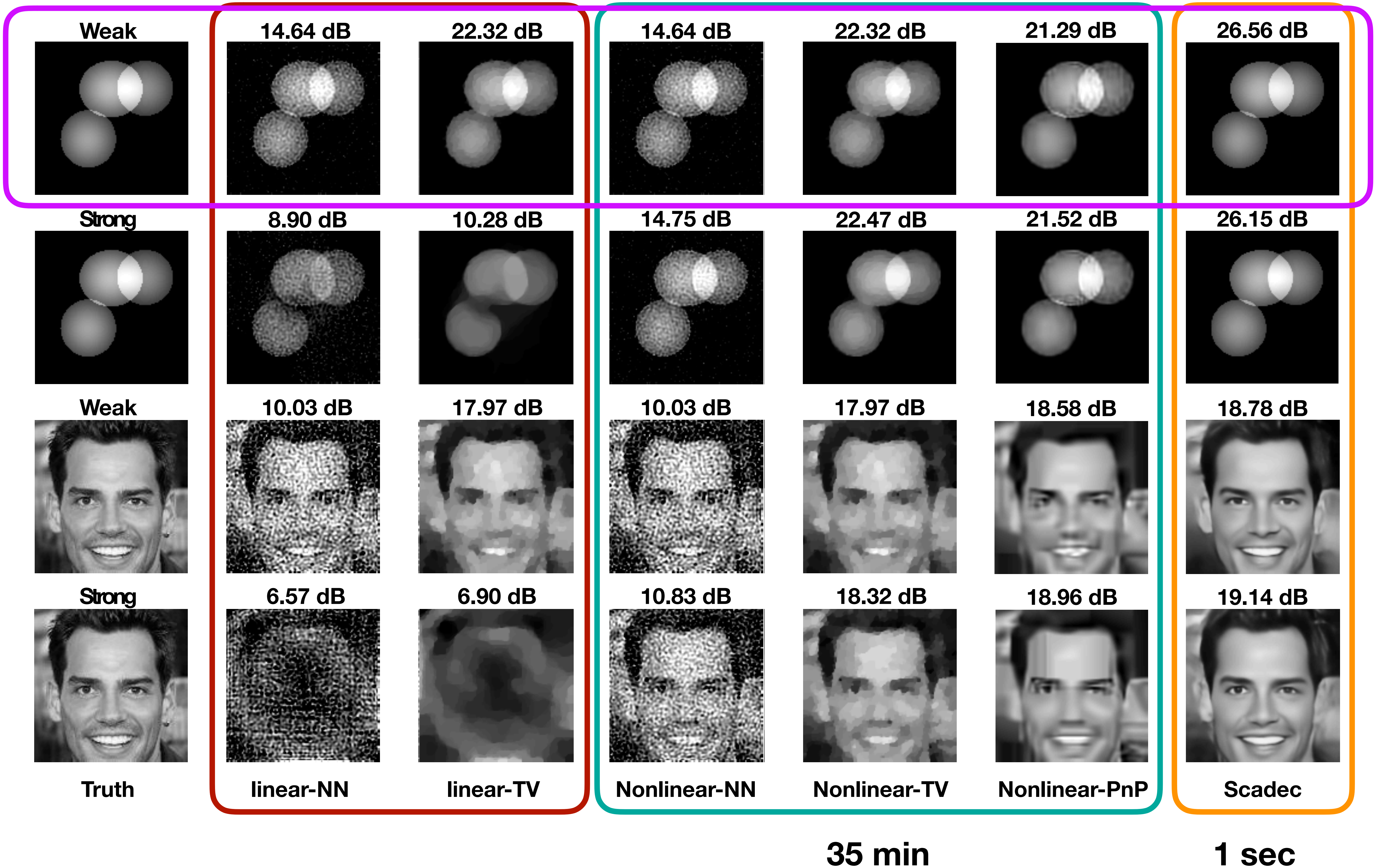
Piece-wise smooth



faces

Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

Visual comparison on Piece-wise smooth and Faces



Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

ScaDec is also competitive in terms of average SNR

Table 1. SNR (dB) comparison of six methods on two datasets

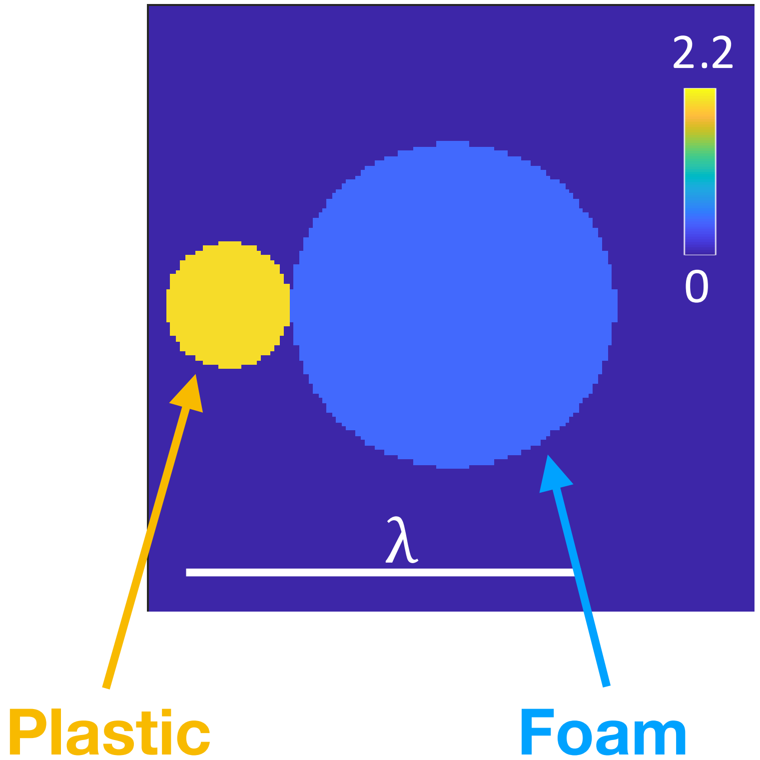
Method	**Average SNR over the dataset**			
	Piecewise-smooth		**Human faces**	
	Weak	Strong	Weak	Strong
FB-NN	16.49	12.79	10.39	6.61
LS-NN	16.49	16.74	10.39	10.85
FB-TV	23.04	15.53	19.79	7.08
LS-TV	23.04	22.57	19.79	20.12
LS-BM3D	21.54	21.72	20.48	20.99
ScaDec	26.14	26.19	20.26	20.21

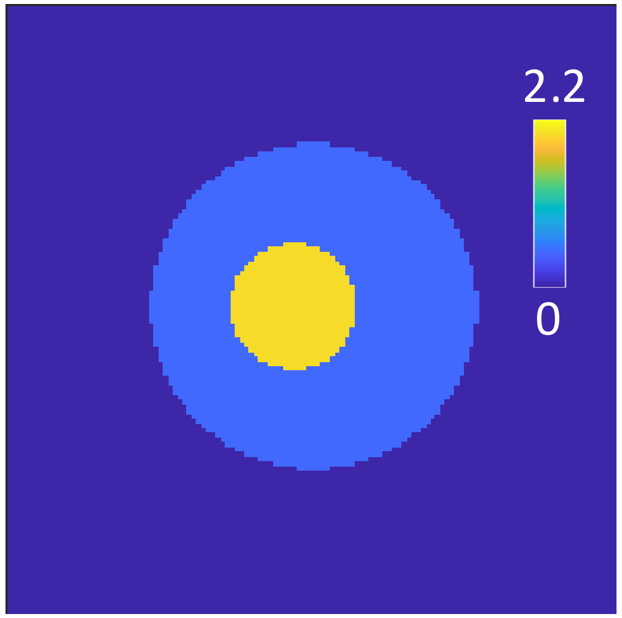
Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

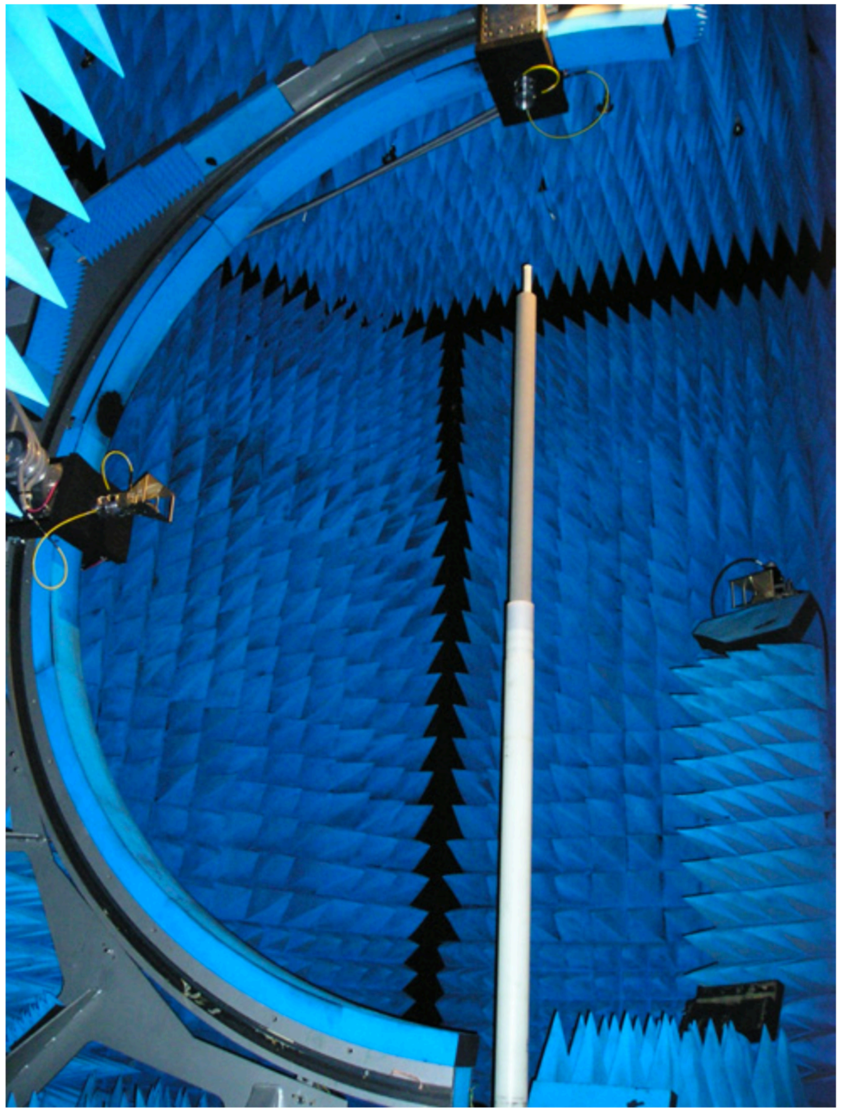
Validation on experimental microwave dataset

Experimental data

- **Plastic & foam (J.M. Geffrin, et al. 2005)**
- **Microwave / Very Noisy**
- Use 6500 synthesized data to train
- Strong prior







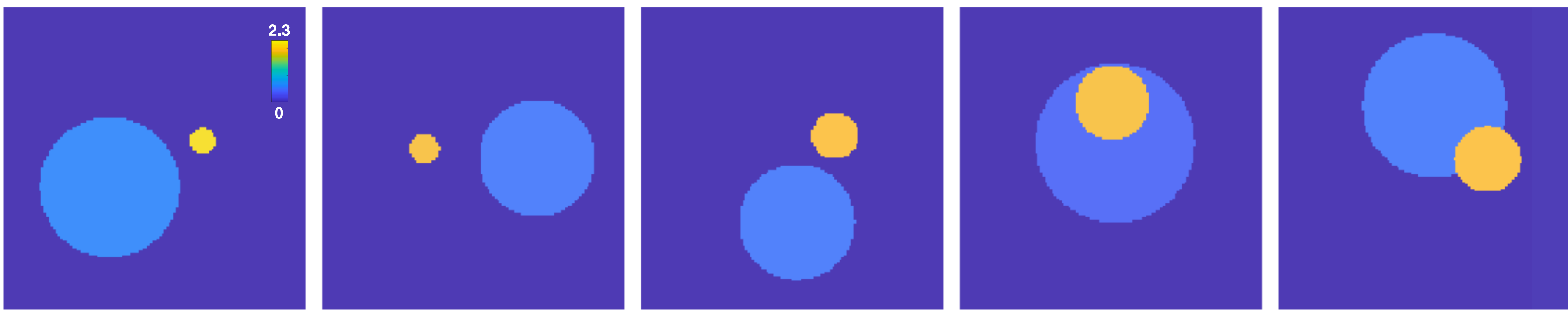
(a) Experimental setup

Jean-Michel Geffrin et al 2005 *Inverse Problems* 21 S117

Examples of synthesized data for training ScaDec

Experimental data

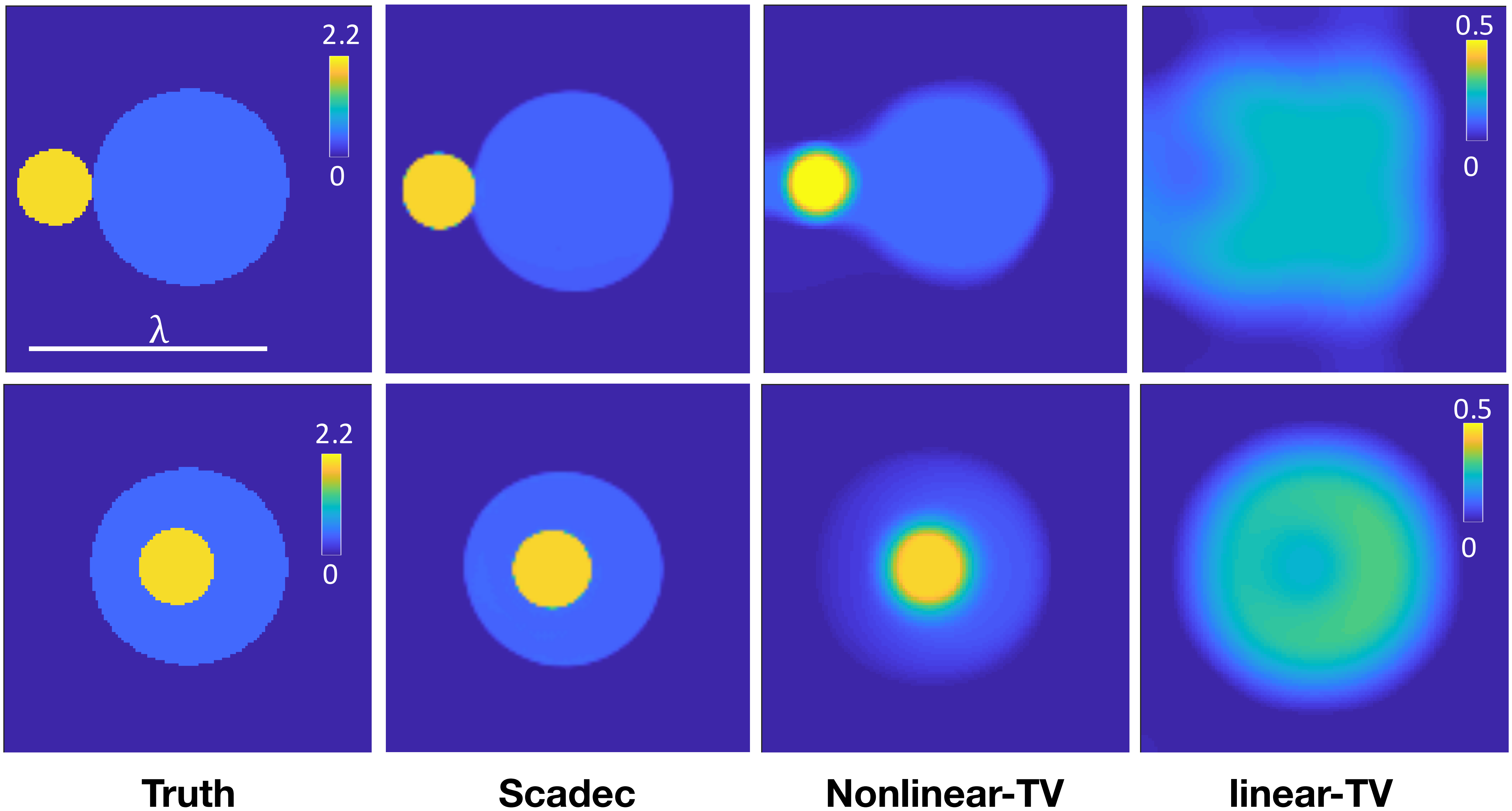
- Microwave data (J.M. Geffrin, et al. 2005)
- Very Noise
- **Use 6500 synthesized data to train**
- **Strong prior**



Synthesized examples

Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

ScaDec successfully reconstructs the image from real data



Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

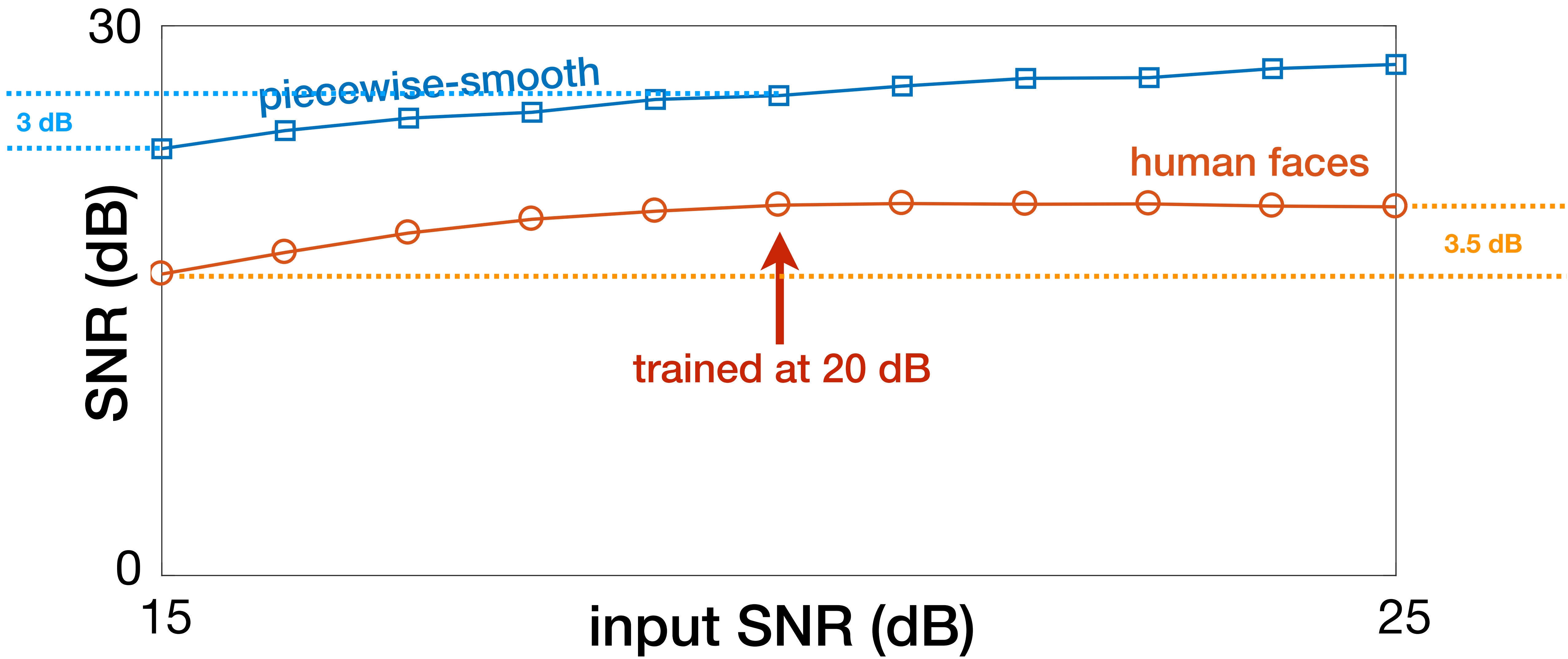
We test the stability of ScaDec with respect to noise and refractive index

Not enough, is the model robust ?

- Learning-based
- **Input noise**
- **Refractive index**

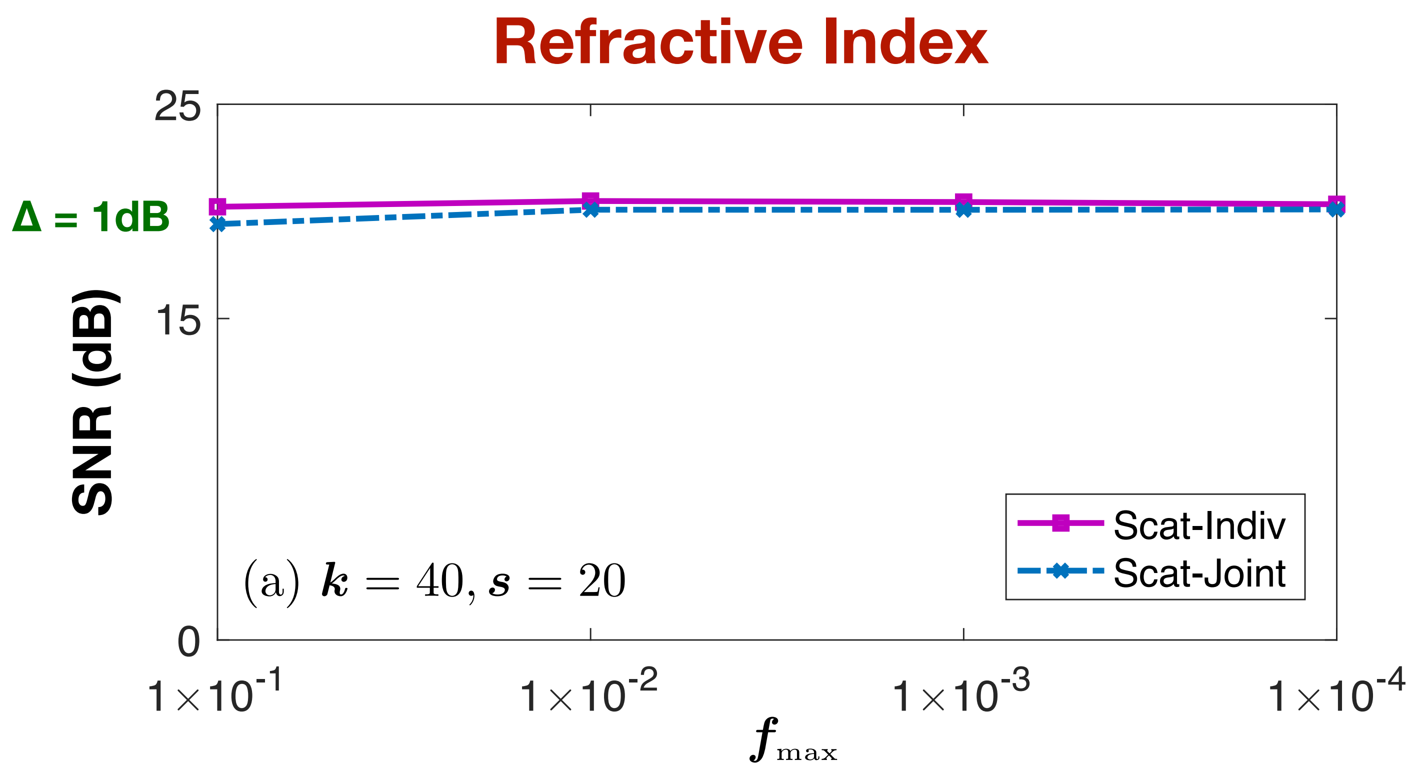
Yu Sun, et al., *Opt. Express* 26, 14678-14688 (2018)

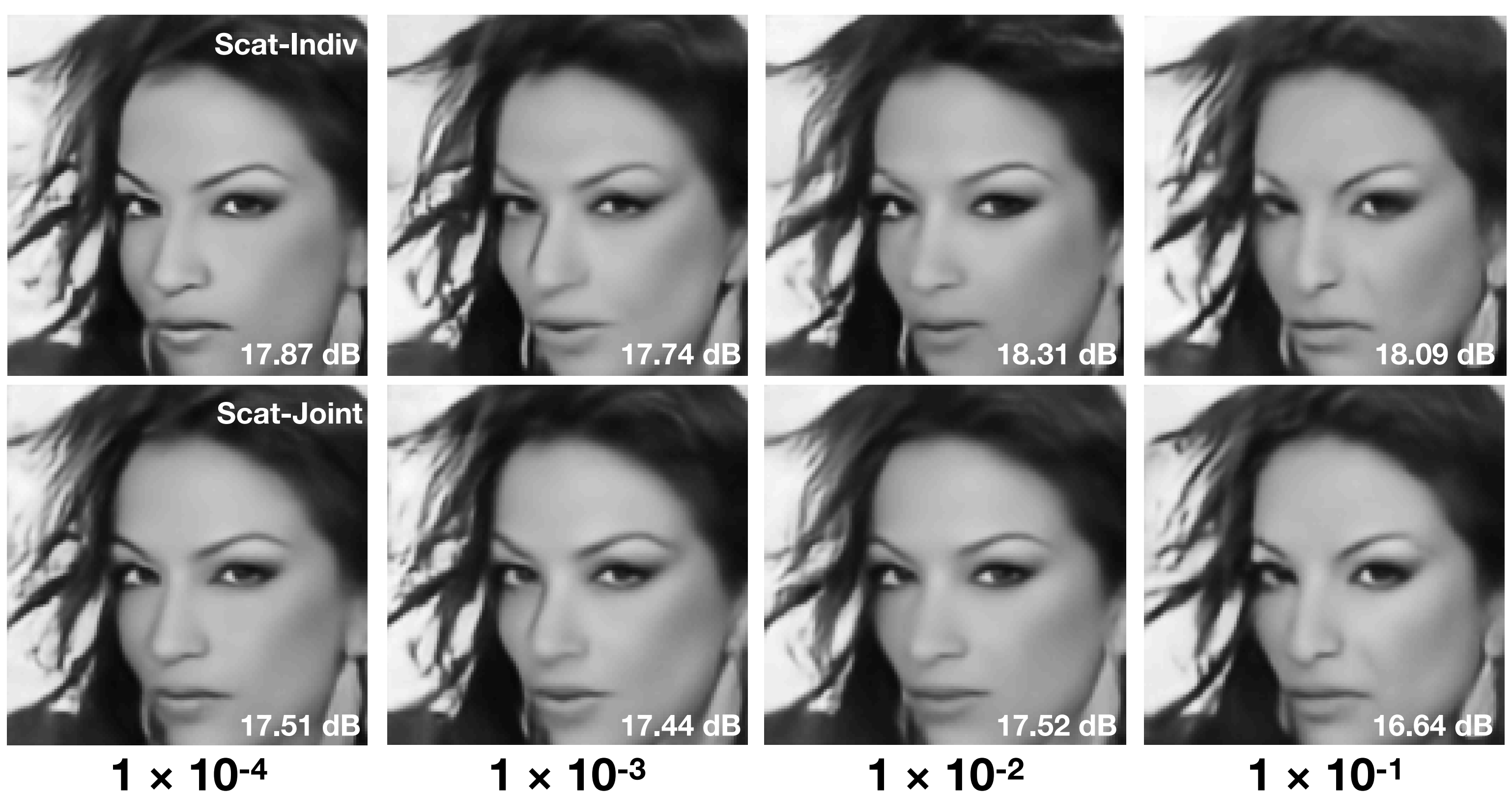
ScaDec degrades as noise level increases



Y. Sun and U. S. Kamilov, *iTWIST* 2018

ScaDec generalizes well by training jointly





Y. Sun and U. S. Kamilov, *iTWIST* 2018

Conclusion

- **Inverting Scattering is important & difficult**
- **ScaDec is shown to achieve fast and accurate reconstruction**
- **ScaDec is stable with respect to noise and scattering strength**
- **Code available here: https://github.com/sunyumark/ScaDec-deep-learning-diffractive-tomography**

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