Economics 4021, Spring 2013, Mid-Term 1 Solution

1 Question 1 (50%)

(a)(20%) Gini coefficient takes the difference between all pairs of incomes and simply totals the absolute difference in the economy. In this question, there are only two people. Hence, Gini coefficient is simply the income difference of those two, $G(x) = x - (1-x) = 2x - 1$ for $1/2 \leq x \leq 1$. Check: $G(1/2) = 0$ and $G(1) = 1$.

(b)(30%)

Country	Gini index	GDP per capita (PPP-adjusted, US$)	Relative GDP to US (US=100)	year of information
5 countries with high Gini coef.				
Namibia	70.7	3547.05	9.29	2003
South Africa	65.0	6766.79	15.93	2005
Lesotho	63.2	829.06	3.00	1995
Botswana	63.0	4818.92	18.94	1993
Sierra Leone	62.9	737.14	3.34	1989
5 countries with low Gini coef.				
Iceland	25.0	40447.93	95.21	2005
Norway	25.0	62714.23	134.25	2008
Denmark	24.0	34677.48	81.63	2005
Slovenia	24.0	22908.74	53.93	2005
Sweden	23.0	33958.52	79.94	2005

Sources: Gini rankings are from CIA World Factbook. PPP-adjusted GDP per capita and relative GDP are from Penn World Table. Years of GDP per capita correspond to the years of Gini index.

The table shows 5 countries with high Gini coefficient and 5 countries with low Gini coefficient. There seems to be a negative relationship between Gini and GDP per capita, in that countries with high(low) Gini index tend to have low(high) GDP per capita, if not the causation.

Grading point: For GDP per capita, you should specify the following information: in what year? current US \$? PPP-adjusted or non-adjusted? Otherwise, you never know what you are comparing with.

2 Question 2 (50%)

(a)(15%) PVLR = 80 + 120 = 200. A consumption plan that yields the highest utility will be $(c, c^f) = (100, 100)$. To this end, a household must borrow 20 in period 1 and pay this back in period 2. The utility from this consumption plan is $min\{c, c^f\} = 100$.

(b)(15%) Now the household can borrow only upto 10 units. Therefore, the desired consumption plan $(c, c^f) = (100, 100)$ is infeasible. The maximum utility under the constraint one will obtain will be $c = 90$ and $c^f = [90, 110]$. That is, no matter how much the household consumes in period 2, more than 90 units will add no utility. The household welfare is $min\{c, c^f\} = 90$, 10 units dropped from no borrowing case.

(c)(20%) Since the assumption on borrowing constraints changes the results, either one (or both) of the following cases will be correct. The key point of this question is Ricardian equivalence.

(A1) No borrowing constraint: to maintain the same tax revenue, it must be that $T_1 + T_2 = 50$ where T_t denotes tax collected in period t. If there is no borrowing constraint, there is no alternative tax system which (strictly) improves the household's welfare of 100.
(A2) Borrowing limit of 10 units: for instance, a tax system of $(T_1, T_2) = (0, 50)$ allows the household to consume $c = 100$ in period 1 and $c^f = 100$ in period 2 without any borrowings. This strictly improves the household's welfare to be $min\{c, c^f\} = 100$.

3 Question 3 (50%)

(a)(10%) Optimal labor demand will be determined at the point where the marginal product of labor equals wage, i.e. $MPN = w$. Since firm is a price taker, wage is exogenously given. The marginal product of labor is the output changes with respect to an additional unit increase in labor. Algebraically, differentiate the output Y with respect to labor demand N^d, which gives you $MPN = \frac{\partial Y}{\partial N^d} = \left(\frac{K}{N^d}\right)^{1/2}$.

(b)(10%) Let $K = 1$ and $N^s = w$. Since we know from the above question that optimal labor demand will be determined at $MPN = \left(\frac{1}{N^d}\right)^{1/2} = w$ and demand and supply must be equal in the equilibrium, $N^d = N^s$. Therefore, we have $\left(\frac{1}{w}\right)^{1/2} = w$ or $w = 1$. Hence, $N^s = N^d = 1$. Total wage income is $wN = 1$.

(c)(15%) When the government imposes a minimum wage $w = 2$, then $N^s = 2$ and $N^d = 1/4$. So there is excess supply of labor. Therefore, the total wage income is $wN = 2 \times 1/4 = 1/2$, since only 1/4 of household can find a job.

(d)(15%) To maximize the wage income, the government should impose minimum wage at the level that it does not bind. That is,
1) $w_{min} >$ equilibrium wage
Total wage income $= w_{min}N^d = w^{-1}$
Hence total wage income increases as minimum wage approaches to the equilibrium wage.
2) $w_{min} <$ equilibrium wage
Since minimum wage is not binding, the equilibrium wage is in effect. Hence, the total wage income does not change as minimum wage changes.