Economics 4021, Spring 2013, 2nd Mid-Term Exam

A Suggested Solution

March 31, 2013

Question 1 (50%)

Company X announces a dividend policy that pays \$1 per share in odd-numbered years and \$2 per share in even numbered years. The relevant rate of return for companies like X is 5% per year.

(a) Suppose all investors are rational people who know and believe the announced dividend policy. What is the share price in odd years? In even years?

Answer: The share price of stock is discounted valuations of future dividends. A share bought today starts collecting dividends next period. Then,

$$\begin{aligned} P_{even} &= \frac{1}{1.05} + \frac{2}{(1.05)^2} + \frac{1}{(1.05)^3} + \frac{2}{(1.05)^4} + \dots \\ &= \frac{1}{1-(1.05)^{-2}}\left(\frac{1}{1.05} + \frac{2}{(1.05)^2}\right) \\ &\approx 29.76 \\ P_{odd} &= \frac{2}{1.05} + \frac{1}{(1.05)^2} + \frac{2}{(1.05)^3} + \frac{1}{(1.05)^4} + \dots \\ &= \frac{1}{1-(1.05)^{-2}}\left(\frac{2}{1.05} + \frac{1}{(1.05)^2}\right) \\ &\approx 30.24 \end{aligned}$$

(b) Next assume that all investors are myopic people who think (wrongly, as it turns out) that all dividend changes are permanent. Find the share prices in odd and even years.

Answer: In odd periods, the myopic investor observes that the dividend is \$1 and believes that the dividend will be \$1 every future period. In even periods, he sees \$2 dividend and believes this will go on. Thus,

$$P_{odd}^{myopic} = \frac{1}{1.05} + \frac{1}{(1.05)^2} + \cdots = \frac{\frac{1}{1.05}}{1-(1.05)^{-1}} = 20$$
$$P_{even}^{myopic} = \frac{2}{1.05} + \frac{2}{(1.05)^2} + \cdots = \frac{\frac{2}{1.05}}{1-(1.05)^{-1}} = 40$$

(c) Now suppose that some investors are myopic and others are rational. Which investor type would be selling company X stock in odd years? In even years?

Answer: In odd years, $P_{odd} > P_{odd}^{myopic}$. Thus, the myopic sell company X stock to the rational. In even years, $P_{even} < P_{even}^{myopic}$, and hence the rational sell company X stock to the myopic.

Question 2 (50%)

Study growth patterns in a Solow-type economy with zero population growth, 100% depreciation, 20% saving rate and technology $Y = 10K^{\alpha}N^{1-\alpha}$ with $\alpha = 1/2$.

(a) Suppose subsistence income is zero. Find the steady-state values of capital per worker and income per worker.

Answer: In a steady state, the following equation holds.

$$sy^* = (\delta + n)k^* \ where \ y = 10\sqrt{k}, \ \delta = 1, \ n = 0$$

$$0.2 \cdot 10\sqrt{k^*} = k^*$$

$$\therefore k^* = 4, \ y^* = 20$$

(b) Now assume that subsistence income is 2.5 units and that people save 20% of their income in excess of subsistence. Calculate the steady state values of capital per worker and income per worker.

Answer: Given the subsistence income level, the following equation holds in steady states.

$$s(y^* - 2.5) = (\delta + n)k^*, \ y \geq 2.5$$

After some algebra, we can get the following steady states.

$$0.2\left(10\sqrt{k} - 2.5\right) = k$$

$$2\sqrt{k} - 0.5 = k$$

$$2\sqrt{k} = k + 0.5$$

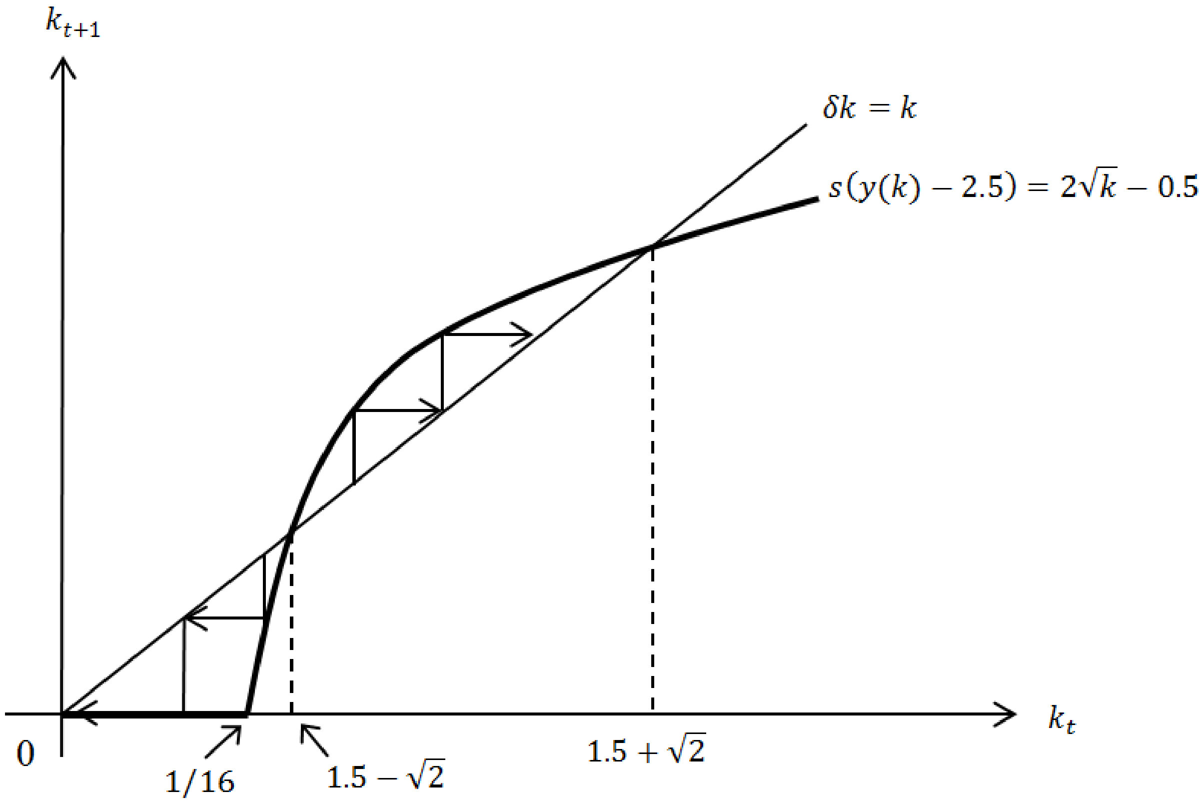
$$4k = k^2 + k + 0.25$$

$$k^2 - 3k + 0.25 = 0$$

$$k = \frac{3 \pm \sqrt{9-1}}{2} = 1.5 \pm \sqrt{2}$$

$$\therefore (k^*, y^*) \in \left\{(1.5 - \sqrt{2},\ 10 - 5\sqrt{2}),\ (1.5 + \sqrt{2},\ 10 + 5\sqrt{2})\right\}$$

To sum, there are two steady states with a trivial state such that $(k^*, y^*) = (0, 0)$.



(c) Describe the growth path of an economy that starts with initial income per worker $y_0 = 10$ units. What if $y_0 = 5$ units?

Answer: In both cases, income per worker grows until it reaches $10 + 5\sqrt{2}$. This is because both of the initial incomes per worker are greater than $10 - 5\sqrt{2}$. In other words, both of the initial capitals per worker are greater than $1.5 - \sqrt{2}$. We can see this from the above graph. If $y_0 < 10 - 5\sqrt{2} \approx 3$, then the economy has not enough initial income to save, it converges into $(k^*, y^*) = (0, 0)$.