Lecture 2: Exchange Economics with and without Commitment

21 Ideas

1. Begin the study of unsecured or reputational borrowing, without any commitment to repay or any form of collateral or asset backing. Suppose loans are backed by borrowers reputation which vanishes forever if borrower defaults.

2. Assume perfect information but limited enforcement
   Contracts are enforced by the threat of exclusion from asset markets:

   - Kehoe-Levine: perpetual two-sided exclusion (no borrowing or lending after default) with stochastic monitoring: default is detected with probability ϕ, and goes unpunished with probability $1 - \phi$
   - Bulow-Rogoff: perpetual one-sided exclusion (can lend out after default but cannot borrow ever)

3. Complete markets leave no social role for default in equilibrium (default deterred for all borrowers in all possible histories of events by endogenous debt limits on each borrower)

4. Binding debt limits reduce capital mobility and inhibit equalization of MRS's among consumers and MRK's among firms.
   $\Rightarrow$ misallocation of resources (Hsieh and Klenow 2009; Azariadis and Kaas 2009)

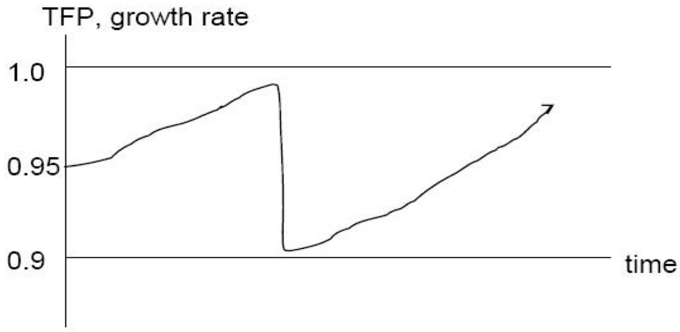
5. Idiosyncratic shocks are one order of magnitude larger than common or aggregate shocks (Davis and Haltiwanger). Here we de-emphasize aggregate disturbances, and focus instead on idiosyncratic shocks and also on the intermediate case of sectoral shocks.

22 Goals

Study misallocation in exchange economies and in production economies. What can monetary or fiscal policy contribute to improve capital mobility and lessen misallocation?

Explore reasons why, in economies with complete markets and limited enforcement of unsecured loans, *the first welfare theorem fails and second welfare theorem holds.* Some equilibria are socially optimal, others are not.

Demonstrate that in economies with sectoral or idiosyncratic shocks, aggregate TFP (total factor productivity) and growth rate may fluctuate, as shown below, even though the economy's production possibilities frontier (PPF) is stable. More generally, sectoral shocks can amplify or propagate common or aggregate shocks.



23 Setting

We start from a pure exchange economy with constant total income in which assets are claims on future income and exist in zero net supply. We focus on the distribution of consumption over households first and later on the distribution of production over firms, all of which are assumed to suffer from idiosyncratic or sectoral shocks. Throughout these notes we suppose often but not always that aggregate shocks do not exist. In particular:

Assumption 2.1: Total income and consumption possibilities are constant in each exchange economy.

Assumption 2.2: The aggregate production possibilities frontier is fixed in each production economy.

We analyze economies with idiosyncratic shocks which are independent draws from a common probability distribution; and also economies with sectoral shocks, that is environments in which income and productivity fluctuations are common to a significant fraction of households and firms, but not to all of them. All households have the same expected income or productivity in the long run. States of nature are represented by $s \in \{L, H\}$ for idiosyncratic shocks or $s \in \{1, 2\}$ for sectoral shocks, that is, by a binary Markov process with transition probabilities

$$\pi(s, s') = prob(s_{t+1} = s'|s_t = s)$$

where $\pi_{ss} \equiv prob\{s_{t+1} = s|s_t = s\}$, all s. State histories are represented by $s^t = (s_0, \cdots, s_t)$. For every household, the long-run probability of state $s \in \{L, H\}$ is

$$\pi_H = \frac{1 - \pi_{LL}}{2 - \pi_{LL} - \pi_{HH}}, \pi_L = 1 - \pi_H \tag{1}$$

Because of the law of large numbers, the probabilities (π_H, π_L) also describe the *fractions* of households in states H and L. The *asymptotic* values (π_H, π_L) are steady states for the probabilities (π_τ^H, π_τ^L) of being in state H or L after τ periods, conditional on the current state. From the Markov transition matrix, we obtain

$$\pi_\tau^H = \pi_{HH}\pi_{t-1}^H + (1 - \pi_{LL})(1 - \pi_{t-1}^H)$$

which leads to the difference equation

$$\pi_{\tau}^{H} = \rho_c \pi_{\tau-1}^{H} + 1 - \pi_{LL} \tag{2a}$$

Here the correlation coef?cient measures the persistence of Markov process.

$$\rho_c \equiv \pi_{HH} + \pi_{LL} - 1 \in (-1, 1) \tag{2b}$$

It is easy to check the equation (1) describes the unique stable steady state of the difference equation (2a).

Here are some additional assumptions:

Assumption 2.3: Let $\omega(\alpha, s')$ be the endowment of agent $\omega \in [0,1]$ in history s'. Then

$$\omega(\alpha, s') = \begin{cases} \omega_L = 1 - \alpha & \text{if } s_t = L \\ \omega_H = 1 + \frac{1-\pi_H}{\pi_H}\alpha & \text{if } s_t = H \end{cases}$$

This specification of income means that the long-term expected income for each agent α equals 1. Expected income in the long run is:

$$\bar{\omega}(\alpha) = \pi_H(1 + \frac{1-\pi_H}{\pi_H}\alpha) + (1-\pi_H)(1-\alpha) \quad \forall \alpha$$

Assumption 2.4: Households share a common utility function. Household α in history s^t has payoff

$$\upsilon(\alpha, s^t) = E\{\sum_{j=t}^{\alpha} \beta^{j-t} u[c(\alpha, s^j)] | s^t\}$$

where $c(\alpha, s^t)$ is the consumption by α in history s^t.

Assumption 2.5: An economy with *idiosyncratic shocks* is populated by a continuum of households indexed $\alpha \in [0,1]$ and distributed on the unit interval according to the p.d.f. G. Each household's history is an independent draw from a common Markov process. Endowments are described in Assumption 3. The initial distribution of households over states is the same as the stationary distribution in equation (1).

Remark: Because this economy has a unit population with average income of 1 unit, aggregate income also equals 1.

Assumption 2.6: An economy with *sectoral shocks* contains two types of individuals, indexed by $i = 1, 2$ with unit mass each. Sectoral shocks are indexed $s_t \in \{1, 2\}$ and follow a common, binary, *symmetric* Markov process such that

$$Pr\{s_{t+1} = s_t | s_t\} = \pi, \quad \forall\, s_t \in \{1, 2\}$$

Endowments for type $i = 1, 2$ in history s^t are

$$\omega(i, s^t) = \omega(i, s_t) = \begin{cases} 1 + \alpha & \text{if } s_t = i \\ 1 - \alpha & \text{if } s_t \neq i \end{cases}$$

Households have high income if the "state" agrees with their "type". We assume a symmetric Markov process to ensure that steady state equilibria exist. For that to happen we need agent 1 in state 1 to solve the same consumer problem as agent 2 in state 2.

24 Complete Markets with Perfect Enforcement: A Planning Problem

A central planner chooses consumption allocations $c(\alpha, s') \in \{c(\alpha, H), c(\alpha, L)\}$, *which depend on current events only*, to maximize a social welfare function subject to an aggregate resource constraint, i.e.,

$$\int_0^1 [\pi_H c(\alpha, H) + (1 - \pi_H) c(\alpha, L)] G(d\alpha) \leq 1$$

The social welfare function (SWF) weighs households by their population fractions and by their type, that is,

$$W = \int_0^1 [\pi_H V(\alpha, H) + (1 - \pi_H) V(\alpha, L)] \lambda(\alpha) G(d\alpha)$$

Here $\lambda(\alpha)$ is the social weight of household α and $V(\alpha, s)$ is the expected value of discounted utility (or *continuation payoff*) for household α in current state $s = H, L$. Note that a recursive definition of the payoff satisfies:

$$V(\alpha, H) = u[c(\alpha, H)] + \beta \pi_{HH} V(\alpha, H) + \beta(1 - \pi_{HH}) V(\alpha, L) \tag{3a}$$

$$V(\alpha, L) = u[c(\alpha, L)] + \beta \pi_{LL} V(\alpha, L) + \beta(1 - \pi_{LL}) V(\alpha, H) \tag{3b}$$

To solve equations (3a) and (3b) for $\{V(\alpha, H), V(\alpha, L)\}$, we define ρ_c from equation (2b). (Then $\rho_c = 0$ means that idiosyncratic shocks are i.i.d, when $\rho_c = 1$ shocks are perfectly positively correlated; when $\rho_c = -1$, shocks are perfectly negatively correlated). Solving equations (3a) and (3b), we obtain

$$(1 - \beta)(1 - \beta\rho_c) V(\alpha, H) = (1 - \beta\pi_{LL}) u[c(\alpha, H)] + \beta(1 - \pi_{HH}) u[c(\alpha, L)] \tag{3c}$$

$$(1 - \beta)(1 - \beta\rho_c) V(\alpha, L) = (1 - \beta\pi_{HH}) u[c(\alpha, L)] + \beta(1 - \pi_{LL}) u[c(\alpha, H)] \tag{3d}$$

To solve the planner's problem defined above, we look for a saddlepoint of the Lagrangean

$$L = W + \mu\{1 - \int_0^1 [\pi_H c(\alpha, H) + (1 - \pi_H) c(\alpha, L) c(\alpha, L)] \lambda(\alpha) G(d\alpha)\}$$

Set $\frac{\partial L}{\partial c(\alpha, s)} = 0$ for $s \in \{L, H\}$ and obtain

$$\lambda(\alpha) u'[c(\alpha, H)] = \frac{\mu}{1 - \beta_\rho} = \lambda(\alpha) u'[c(\alpha, L)], \qquad \forall\, \alpha \in [0, 1]$$

$$\implies c(\alpha, s) = c^*(\alpha), \quad \forall\, s \in \{L, H\}$$

Therefore consumption may depend on agent type and an aggregate income but not on *idiosyncratic or sectoral state histories.*

Equal-treatment optimum:

$$\text{If} \quad \lambda(\alpha) = 1, \ \forall\ \alpha, \text{ then} \quad c(\alpha, s) = 1, \ \forall\ (\alpha, s), \text{ and} \quad V(\alpha, H) = V(\alpha, L) = \frac{u(1)}{1-\beta}$$

Question: If households were allowed to deviate permanently from the equal-treatment optimum allocations to autarky, would any household $\alpha \in [0, 1]$ ever do so?

Before we answer this question, we duplicate the planner's allocation through competitive markets.

25 Complete Markets with Perfect Enforcement: Competitive equilibrium

We decentralize the planner's optimal allocation by allowing households to trade Arrow securities contingent on future idiosyncratic or future sectoral states $s' = L, H$. Households buy securities contingent on $s' = L$, and sell securities contingent on $s' = H$. The former deliver income when individual endowment is low; the latter pledge to give up income when individual endowment is high. Let $p(s; s')$ be the price at $s_t = s$ of security paying 1 unit of consumption at $s_{t+1} = s'$ and paying zero if $s_{t+1} \neq s'$. Then the standard de?nition of competitive equilibrium is:

Definition 2.1: An equilibrium is a list of security prices $p(s, s')$, security holdings $Q(\alpha, s')$ and consumption allocations $c(\alpha, s)$ for each $\alpha \in [0, 1]$ and $(s, s') \in \{L, H\} \times \{L, H\}$ such that:

1. Household α maximizes the continuation payoff at time $t = 0$,

   $$V(\alpha, s_0) = E\{\sum_{t=0}^{\infty} \beta^t u[c(\alpha, s_t)] | s_0\}$$

   under the budget constraints

   $$c(\alpha, s) + \sum_{s'} p(s, s') Q(\alpha, s') = \omega(\alpha, s) + Q(\alpha, s) \tag{4}$$

   The household takes as given the initial state s_0, prices $p(s, s')$ and initial security holdings $Q(\alpha, s_0)$.

2. Markets clear, that is

   $$\int_0^1 \{\pi_H[c(\alpha, H) - \omega(\alpha, H)] + (1 - \pi_H)[c(\alpha, L) - \omega(\alpha, L)]\} G(d\alpha) = 0 \tag{5a}$$

   in the goods market; and

   $$\pi_H \int_0^1 Q(\alpha, H) G(d\alpha) + (1 - \pi_H) \int_0^1 Q(\alpha, L) G(d\alpha) = 0 \tag{5b}$$

   in the claim market.

The market clearing condition in the goods market states that the excess demands of high income people and low income people sums to zero when weighted by their population proportions. The corresponding securities market condition means that the supply of claims contingent on future states of high income is exactly balanced by the demand for claims contingent on future states of low income.

As expected, an equilibrium with perfect enforcement delivers an optimal allocation of resources for any initial distribution of individual securities that satisfies equation (5b). One can easily show the following result:

Theorem 2.1: There is a unique equilibrium with complete consumption smoothing. In particular,

1. $c(\alpha, s) = c^*(\alpha)$
2. Security prices satisfy $p(s, s') = \beta\pi(s, s')$
3. Sectoral or idiosyncratic holdings $(Q(\alpha, H), Q(\alpha, L))$ satisfy the budget constraints (4) at the prices listed in part (2)
4. There exists an initial distribution of assets that replicates itself and supports a symmetric equilibrium with equal consumption for all agents, i.e., such that $c^*(\alpha) = 1$, $\forall$ $\alpha \in [0, 1]$. Specifically,

$$Q^*(\alpha, H) = -\frac{1}{(1-\beta)(1-\beta\rho_c)}[(1-\beta\pi_{LL})\omega(\alpha, H) + \beta(1-\pi_{HH})\omega(\alpha, L) + \beta\rho_c - 1]$$
$$Q^*(\alpha, L) = -\frac{1}{(1-\beta)(1-\beta\rho_c)}[(1-\beta\pi_{HH})\omega(\alpha, L) + \beta(1-\pi_{LL})\omega(\alpha, H) + \beta\rho_c - 1]$$

where $\rho_c \equiv \pi_{HH} + \pi_{LL} - 1 \in [-1, 1]$

Example 2.1: (Deterministic symmetric economy): Suppose $\pi_{HH} = \pi_{LL} = 0$, i.e., $\rho_c = -1$. Then state histories for the two agents or sectors are

$$s^t = \begin{cases} H, L, H, L, \ldots \\ L, H, L, H, \ldots \end{cases}$$

and the asymptotic distribution of states is $(\pi_H, \pi_L) = (1/2, 1/2)$. Assume that endowments are

$$\omega(\alpha, L) \equiv y_L(\alpha) = 1 - \alpha$$
$$\omega(\alpha, H) \equiv y_H(\alpha) = 1 + \alpha$$

Then theorem 2.1.4 says that we can choose an initial distribution of asset holdings

$$Q(\alpha, H) = -\frac{1 + \alpha + \beta(1-\alpha) - 1 - \beta}{1 - \beta^2} = -\frac{\alpha}{1+\beta} = -Q(\alpha, L)$$

which generates for each agent a constant consumption stream $c_t^1 = c_t^2 = 1$, $\forall$ t.

This distribution aggregates to zero and replicates itself in equilibrium. The present value of consumption equals $Q(\alpha, s)$ plus the PV of endowment income. For example, in current state L, the PV is Q plus the value of the endowment stream $(1-\alpha, 1+\alpha, 1-\alpha, 1+\alpha, \ldots)$ discounted at the equilibrium yield $R = 1/\beta$. This turns out to be

$$PV(\text{income}) + Q = \frac{1-\alpha+\beta(1-\alpha)}{1-\beta^2} + \frac{\alpha}{1+\beta} = \frac{1}{1-\beta}$$

The same is true in current state H. Therefore, $c(\alpha, L) = c(\alpha, H) = 1$. To understand this example, note that the initial holding $Q(\alpha, L)$ is equivalent to adding income $\delta = (1-\beta)\alpha$ to the endowment of low income agents and subtracting the same amount from the endowment of high income agents. In fact, Q_L is the present value of the endowment stream $(\delta, 0, \delta, 0, \ldots)$. Figure 3 shows how the vector $(c_H, c_L) = (1, 1)$ is the common claim of all households $\alpha \in [0, 1]$ maximizing utility when $\beta R = 1$ and endowment is $1 + \alpha - \delta$ when $s = H$, and $1 - \alpha + \delta$ when $s = L$.

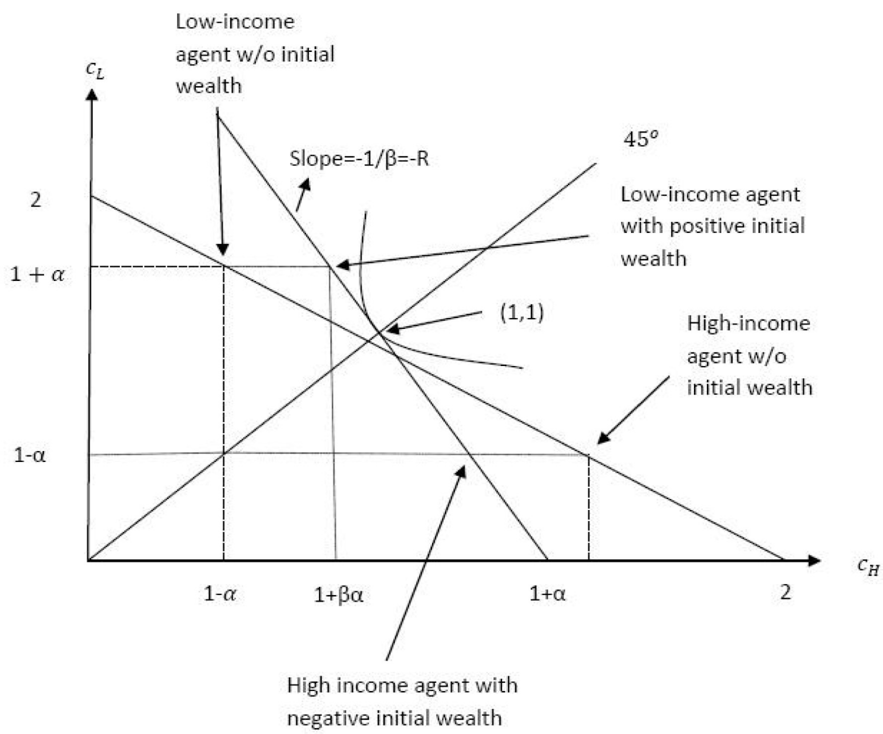


Figure 2.1: Symmetric equilibrium with perfect enforcement

Now we return to the question asked previously

Question: What if we allow high state households to deviate from the allocation $c(\alpha, s) = 1$, $\forall(\alpha, s)$, for one period and pay some penalty? Would any household exercise this option?

The answer clearly depends on the economic consequences of deviating from $c(\alpha, s) = 1$. Suppose we specify the following penalty: Let $\phi \in [0, 1]$ be the exogenous probability of detecting default. Then a detected defaulter goes into perpetual autarky. An undetected defaulter obtains costless debt relief. Under this arrangement, the payoff from solvency is:

$$V(\alpha, s) = \frac{u(1)}{1-\beta}, \quad \forall\, (\alpha, s)$$

Payoff from default for a high-state household is:

$$V_D(\alpha, H) = \phi V_A(\alpha, H) + (1-\phi)\frac{u[1 + (1-\beta)\alpha/(1+\beta)]}{1-\beta} \tag{6a}$$

where by analogy with equation (3c), we replace consumption with income and obtain

$$(1-\beta\rho_c)(1-\beta)V_A(\alpha, H) = (1-\beta\pi_{LL})u[\omega(\alpha, H)] + \beta(1-\pi_{HH})u[\omega(\alpha, L)] \tag{6b}$$

Here V_A is the value of autarky for a high income household and $(1-\beta)\alpha/(1+\beta)$ is the permanent consumption increment for undetected defaulters. The present value of that increment is $\alpha/(1-\beta)$ and equals the value of the forgiven debt $Q_H = -\alpha/(1+\beta)$.

Remember that the payoff from autarky in the high state satisfies equation (6b). In the deterministic example, $\rho_c = -1$ and the autarky payoff becomes

$$(1-\beta)V_A(\alpha, H) = \frac{u(1+\alpha) + \beta u(1-\alpha)}{1+\beta} \tag{5c}$$

From (6a) and (6b) it is easy to see that any household $\alpha \in [0, 1]$ with deterministic income will not deviate from the first-best outcome if and only if

$$(1+\beta)u(1) \geq \phi[u(1+\alpha) + \beta u(1-\alpha)] + (1-\phi)(1+\beta)u(1 + \frac{1-\beta}{1+\beta}\alpha) \tag{7}$$

A particularly interesting special case of (7) is that of perfect monitoring $\phi = 1$, when (7) reduces to

$$(1+\beta)u(1) \geq u(1+\alpha) + \beta u(1-\alpha) \tag{*}$$

As shown in the following figure, condition (*) is satisfied, and the first-best allocation determines autarky, if and only if the endowment point $\Omega = (1+\alpha, 1-\alpha)$ is between E' and F; (*) fails iff Ω is in the interval (EE').

Inspecting Figure 5 reveals that (*) is likely to fail for $\phi = 1$ if β is low or α is low or u($\cdot$) is nearly linear.

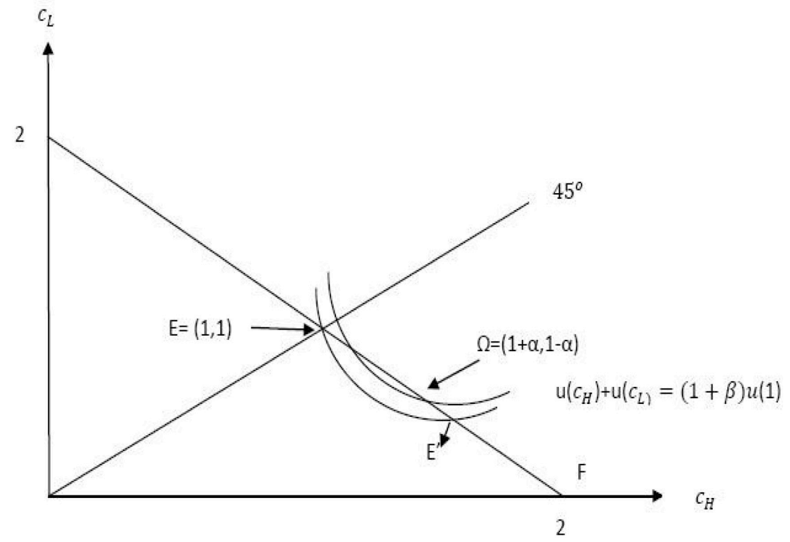


Figure 2.2: Symmetric Equilibrium with Limited Enforcement

In addition, if (*) holds for $\phi = 1$, it will still fail for smaller values of ϕ unless ϕ is "large enough", because it is violated at $\phi = 0$.

In a more general stochastic setting of random monitoring and random income which obeys a symmetric two state Markov process with $\pi_{HH} = \pi_{LL} = \pi$ and correlation coefficient $\rho_c = 2\pi - 1$, equation (7) generalizes easily by combining (6a) with (6b) to:

$$(1+\beta-2\beta\pi)u(1) \geq \phi[(1-\beta\pi)u(1+\alpha)+\beta(1-\pi)u(1-\alpha)]+(1+\beta-2\beta\pi)(1-\phi)u(1+\frac{1-\beta}{1+\beta}\alpha) \tag{8}$$

When individual income shocks are very persistent and π is close to 1, this relation will fail for all values of (α, β, ϕ). Problem 3.2 asks you to prove this result and explain it.

Problem 2.1: An economy consists of two groups of agents $i = 1, 2$ with mass 1 each and constant income stream $y_t^i = 1$ for all agents i and all times t. Initial assets are zero for all i. Utility functions $u_i = \sum_{t=0}^{\infty} \beta_t^i \log(c_t^i)$ with $0 < \beta_1 < \beta_2 < 1$.

1. Describe the equilibrium outcome under perfect enforcement. What are the asymptotic values of consumption and of the rate of interest?
2. Is the perfect enforcement outcome achievable under limited enforcement? If so, for what parameter values?

[Assume default is punished with perpetual autarky with probability 1]

Problem 2.2: Show that high-income households will want to default at the symmetric equilibrium outcome of an Arrow-Debreu economy if their household incomes follow a highly autocorrelated two-state symmetric Markov process.

Problem 2.3: Study *asymmetric* equilibrium outcomes in Arrow-Debreu economies and show that agents with high enough initial debt will always want to deviate. What is the critical stock of initial debt that will lead a household into default?

Problem 2.4:

1. Extend theorem 2.1 to a deterministic exchange economy with a continuum of agents with mass 1. Agent types are indexed on the income variability parameter $\alpha \in [0, 1]$ with a known distribution.

2. Describe the symmetric Arrow-Debreu allocation and find a condition that will prevent agent α from defaulting.

3. Show that the no-default condition you discovered in part (b) cannot hold for a high income household with low enough value of the income variability parameter α.