OPTIMAL HEDGING OF STOCK PORTFOLIOS AGAINST FOREIGN EXCHANGE RISK: THEORY AND APPLICATIONS

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This paper applies a formula for the optimal hedge in a mean-variance framework to an investment in the Nikkei 225. It is shown that through hedging U.S. investors can construct a portfolio long in the Nikkei whose dollar excess return has the same volatility as the yen excess return of the Nikkei. There seems to be little gain from improving estimates of the exposure of the Nikkei to the dollar-yen exchange rate; in contrast, the performance of portfolios can be enhanced substantially by obtaining better forecasts of exchange rate changes.

I. INTRODUCTION

The extent to which internationally diversified portfolios should be hedged against exchange rate risk has recently attracted a lot of attention among practitioners and academics.[1] For hedging to be meaningful, the foreign currency exposure of the domestic return of a portfolio of common stocks has to be predictable and nontrivial. If hedging is feasible, investors have to compare the loss in expected return from hedging with the corresponding decrease in the volatility of the portfolio. In a mean-variance framework, the gain or loss in expected return from hedging has the same weight in the investor's objective function as does any change in expected return resulting from changes in portfolio weights. There is no difference between deciding how much to hedge and deciding how much of any asset to add to a portfolio. Consequently, in this framework, the hedging decision is straightforward once the exposures and the cost of hedging have been computed.

If the currency exposure of stocks in their own currency is trivial and hedging

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has no cost, the optimal hedge ratio is −1. The hedge ratio is defined as the dollar value of the position in foreign currency per dollar invested long in foreign stock. Hence, a hedge ratio of −1 implies that for each dollar invested abroad, a dollar's worth of foreign currency is borrowed or sold forward. The evidence on the own currency exposure of foreign stocks is mixed for studies that use weekly or monthly data. For instance, some studies find that yen returns on the Nikkei 225 are exposed to the dollar/yen exchange rate, whereas others argue that this exposure is trivial.[2]

The cost of hedging a portfolio against unanticipated exchange rate changes is simply the cost of taking currency forward positions. Abstracting from transaction costs, the expected gain from a short forward yen position is the difference between the forward price and the expected value of the yen at maturity of the forward contract. For the first ten years following the introduction of floating exchange rates, the consensus among economists was that the expected return to a short forward position is either equal to zero or cannot be predicted to be different from zero with any confidence. This consensus no longer exists, but it seems to be largely responsible for the common wisdom that the optimal hedge ratio should be computed without regard for the cost of hedging. The major dissenting view is that of Black [5], who uses a theoretical asset pricing model to show that, in equilibrium, hedging is costly and hence the optimal hedge ratio must be smaller than one in absolute value.

In this paper, we use daily data on stock returns to evaluate the exposure of the Nikkei 225 to the dollar/yen exchange rate. We show that, although this exposure can be predicted using time-series models, it seems difficult to achieve more variance reduction through forecasted hedge ratios than by simply assuming that the optimal hedge ratio is −1. Hence, we report no convincing evidence to suggest that a hedge ratio of −1 is not the minimum-variance hedge ratio. Using a hedge ratio of −1, it is possible to make an investment in the Nikkei insensitive to unanticipated changes in the dollar/yen exchange rate. Consequently, an American investor can have the same return volatility on an investment in the Nikkei as a Japanese investor can have. If hedging is costly, eliminating all currency exposure in a portfolio is not optimal. Based on historical dollar/yen hedging costs, we show that taking into account hedging costs leads to an optimal hedge ratio that differs substantially from −1.

The paper proceeds as follows: In the first section, we review the main determinants of the optimal hedge in a mean-variance framework. In following section, we estimate the dollar exposure of the Nikkei 225 using a variety of methods. We then formulate in another section the optimal hedge for a mean-variance investor taking into account the hedging cost and show, based on historical returns, the extent to which the optimal hedge ratio differs from −1. Concluding remarks are presented in the last section.

The Theoretical Determinants of the Hedge Ratio.

In this section, we show that the optimal hedge satisfies a simple formula in a mean-variance framework. This formula makes it possible to interpret and reconcile the various arguments related to hedging the currency exposure of portfolios of stocks. We consider a world of N countries. In country j, there are M_j risky assets. In each country, there is also a nominally riskless asset. R_j is the instantaneous return of the nominally riskless asset in country j. Let I_{ji} be the price in the currency of country j of the i-th asset in that country and e_j the price of the currency of country j in units of the currency of country 1. All prices, including the exchange rates, and all R_j are assumed to be continuous functions of S state variables, each following an Ito process. Rates of change of prices are assumed to be conditionally lognormal. Hence, over the next instant, the distribution of returns is characterized by the vector of mean returns and by the variance-covariance matrix of returns.

With our notation, the instantaneous return of the nominally risk-free asset of country j in terms of the currency of country 1 is

$$R_j + \frac{de_j}{e_j} \tag{1}$$

Hence, if one takes a long position in the i-th risky asset of country j, a short position in the nominally risk-free asset of that country, and a long position in the nominally risk-free asset of country 1, the return on that portfolio[3] is

$$\left(\frac{dI_{ji}}{I_{ji}} + \frac{de_j}{e_j} + \frac{de_j dI_{ji}}{e_j I_{ji}}\right) - \left(R_j dt + \frac{de_j}{e_j} - R_1 dt\right) = \frac{dI_{ji}}{I_{ji}} + \frac{de_j dI_j}{e_j I_j} - R_j dt + R_1 dt$$
$$= \frac{dI_{ji}}{I_{ji}} + \mathrm{Cov}\left(\frac{de_j}{e_j}, \frac{dI_{ji}}{I_{ji}}\right) dt - R_j dt + R_1 dt \tag{2}$$

The portfolio return is equal to the currency 1 return of an investment in asset i financed abroad plus an investment in the risk-free asset of country 1. The investment in asset i financed abroad has no cost in currency 1. The portfolio return is not a function of unanticipated changes in the exchange rate since the covariance between the rate of change of the exchange rate and the return of asset i is known **ex ante.**[4] In this sense, the portfolio corresponds to an investment in the i-th asset of country j instantaneously hedged against currency risk. In constructing this portfolio, the hedge ratio is -1 because each long position in asset i is accompanied by a sale of a corresponding unit of currency j of the nominally risk-free asset of country j. A hedge ratio of -1 is not the minimum-variance hedge ratio if local currency returns, i.e., dI_{ji}/I_{ji}, are correlated with the exchange rate since, by taking an additional hedge position to offset this covariance, it is possible to obtain a portfolio with a lower variance. Let dB^1_{ji}/B^1_{ji} be the return given in equation (2). We call the portfolio that yields that return a zero net foreign investment portfolio since it implies that all foreign investment is financed abroad. Note however that the same return could be achieved through an investment in the foreign asset i and a short position in an instantaneous forward contract in currency j. This is because, if covered interest rate parity holds, $R_j - R_1$ is equal to minus the forward premium and $R_j + \frac{de_j}{e_j} - R_1$ is equal to the return of a long position in currency j sold forward for currency 1.

If dB^h_{ij}/B^h_{ij} is the currency h return of a zero net investment portfolio long in asset i of country j, dB^h_{ij}/B^h_{ij} and dB^1_{ij}/B^1_{ij} have a correlation coefficient of one. To see that this is true, consider the covariance between the currency 1 return on a zero net foreign investment portfolio long in asset i of country j and the currency h return of a zero net foreign investment in asset i of country j. From the construction of a zero net investment portfolio long in asset i of country j, the only stochastic variable in the return of such a portfolio is the currency j return of asset i irrespective of the numeraire used to construct the portfolio. Hence, the instantaneous variance and covariances of zero net investment portfolios with other zero net investment portfolios do not depend on the numeraire used to construct them.[5]

It immediately follows from this discussion that two investors in different countries who invest in identical portfolios of nominally risky assets using zero net foreign investment portfolios have portfolios with identical instantaneous return volatilities. The expected excess returns on these portfolios differ only to the extent that the own-currency returns of nominally risky assets are correlated with the rate of change of the exchange rate between the two countries. Hence, if own-currency returns of nominally risky assets are not correlated with the rate of change of exchange rates, mean-variance efficient frontiers of zero net foreign investment portfolios are the same irrespective of the numeraire chosen to evaluate portfolios. If, further, there is no reward in terms of expected return to currency speculation, then there is no reason to hold foreign nominally riskless assets beyond the positions required to establish zero net foreign investment portfolios. Holding only nominally risky assets plus the domestic risk-free asset is optimal. Hence, if it is optimal not to take a position in foreign nominally riskless assets in excess of the one required to finance investment in foreign nominally risky assets all investors hold the same portfolio of nominally risky assets irrespective of their country of residence. Consequently, mean-variance investors located in different countries will have different portfolios of risky assets only to the extent that they find it optimal to take positions in foreign nominally risk-free assets beyond the positions required to finance investments in foreign nominally risky assets. One can show more generally that portfolios of risky assets differ only in their proportional holdings of nominally riskless assets, see Stulz [15].

We now consider the implications of our discussion for currency hedging in equilibrium. So far, we assumed that investors choose portfolios in the mean-variance space of nominal returns. One would expect investors to care about real rather than nominal returns; but if inflation is not volatile, the distinction between real and nominal returns is unimportant for portfolio choice. Suppose that inflation has zero volatility in country 1, that consumption baskets are similar across countries and the law of one price holds. In this case, country 1 returns are real returns for all investors in the world. Hence, we can choose the currency of country 1 as the numeraire currency for all investors. If investors do not differ in risk-tolerance across countries, they all select the same portfolio using the numeraire currency. Clearly, this means that in equilibrium they must hold the world market portfolio. If nominally riskless assets do not belong to that portfolio, no investor holds such assets. Hence, in equilibrium, investors cannot borrow to hedge and no hedging takes place.

Alternative assumptions about consumption baskets lead to different results about equilibrium hedging. To see this, suppose that instead of having all investors consume the same basket of goods, all investors of a given country consume the same basket of goods but investors consume different baskets of goods across countries.[6] In this case, if inflation is deterministic in each country, a country's nominally risk-free asset is riskless in real terms for investors of that country. Hence, if investors in a country are sufficiently risk averse, they would like to invest some of their wealth in the nominally risk-free asset of their country. To the extent that investors in one country want to be long in their country's nominally risk-free asset, it becomes possible for other investors to be short in that asset and hence to hedge currency risk. In this case, hedging takes place in equilibrium.

By making assumptions about what investors consume across countries and about the stochastic process followed by the price of their consumption basket in their local currency, one can derive a hedge ratio that holds in equilibrium. Depending on the assumptions, this hedge ratio may or may not be the same across investors of different countries. Typically, however, if risk-tolerances, consumption baskets and inflation processes vary across countries, there will be no universal hedge ratio. If there is no universal hedge ratio, one can still derive a hedge formula for investors in a particular country that does not depend on expected returns. However, the usefulness of such a formula is limited since it requires precise knowledge of the consumption baskets, risk-tolerances and stochastic processes of inflation across countries. In this case, an investor specific hedge formula that relies only on expected returns and the variance-covariance matrix of returns is easier to implement since investors typically use expected returns and the variance-covariance matrix of returns as inputs in portfolio allocation anyway.

We now investigate the optimal hedge ratio for an investor who uses currency 1 as the numeraire.[7] Let w_{ji}^{1} be the fraction of wealth an investor from country 1 chooses to invest in the i-th asset of country j using a zero net foreign investment portfolio. We choose asset 0 to be the nominally risk-free asset. If the i-th asset is asset 0, we define w_{j0}^{1} as the fraction of wealth invested in a long position in that asset. Straightforward manipulations show that the investor holds the following portfolio[8]

$$\underline{w} = (\alpha - R_1)(\underline{\mu}V^{-1}\underline{\mu})\ V^{-1}\underline{\mu} \qquad (3)$$

where w is the $\left(\sum_{j=1}^{N} M_j + (N-1)\right)$ column vector of investment proportions in assets risky in currency 1. Assets risky in currency 1 are all nominally risky assets and the assets nominally riskless in currencies other than currency 1. α is the investor's target expected return. μ is a vector of expected excess returns in currency 1. For the i-th asset in currency j, the expected excess return is defined to be the expected excess return of a zero net foreign investment portfolio with a long investment in asset j over the risk-free return in currency 1. The expected excess return for an investment in the nominal risk-free asset in currency j is $R_j + \mu_{e_j} - R_1$, where μ_{e_j} is the expected rate of change of the price of currency j in terms of currency 1. **V** is the variance-covariance matrix of returns of investments in zero net foreign investment portfolios of foreign risky assets and of investments in foreign nominal risk-free assets.

Consider now the exposure of the optimal portfolio with respect to currency j. We use the usual definition of exposure, namely the conditional covariance of the portfolio's return with the rate of change of e_j divided by the conditional variance of the rate of change of e_j. Thus, the exposure has the interpretation of a conditional regression coefficient in a regression of the portfolio's return on the rate of change of the exchange rate. This exposure can be computed as follows. Let $\underline{V}_{e_j}$ be a vector corresponding to the covariances of the rate of change of e_j with the returns on the zero net investment portfolios and the foreign nominally riskless assets. Hence, $\underline{V}_{e_j}$ is a column of matrix **V**. The exposure of the optimal portfolio with respect to currency j is

$$\begin{aligned} \underline{V}'_{e_j} w \sigma_{e_j}^{-2} &= \sigma_{e_j}^{-2} \underline{V}'_{e_j} (\alpha - R_1)(\mu \mathbf{V}^{-1} \mu) \mathbf{V}^{-1} \mu \\ &= \sigma_{e_j}^{-2} (\alpha - R_1)(\mu \mathbf{V}^{-1} \mu)(R_j + \mu_{e_j} - R_1) \end{aligned} \quad (4)$$

where $\sigma^2_{e_j}$ is the instantaneous volatility of the rate of change of the price of currency j. Let w_j be the investor's portfolio excluding holdings of the domestic risk-free asset and of the nominally riskless asset of country j in excess of the amount of that asset borrowed to finance investment in country j. Equation (4) can be rewritten after noticing that the left-hand side is the covariance of the return of portfolio w_j with the rate of change of the price of currency j, which we define as $\sigma_{e_j w_j}$. Let w_{j0} be the holdings of the nominally risk-free asset of country j in excess of the amount required to finance investment in country j. We can rewrite equation (4) as

$$\begin{aligned} w_{j0} &= (\alpha - R_1)(\underline{\mu} \mathbf{V}^{-1} \underline{\mu}) \left[\frac{R_j + \mu_{e_j} - R_1}{\sigma^2_{e_j}} \right] - \frac{\sigma_{e_j w_j}}{\sigma^2_{e_j}} \\ &= (\alpha - R_1)(\underline{\mu} \mathbf{V}^{-1} \underline{\mu}) \left[\frac{R_j + \mu_{e_j} - R_1}{\sigma^2_{e_j}} \right] - \beta^*_{e_j} \end{aligned} \quad (5)$$

where $\beta^*_{e_j}$ is the beta of the portfolio w with respect to the price of currency j. Note that the portfolio we used to derive the formula is one that holds positions in zero net foreign investment portfolios. Hence, the fraction of investment in the risk-free asset of country j given in equation (5) corresponds to the investment in that asset in excess of borrowing an amount equal to the initial investment in nominally risky assets of that country. For instance, if the fraction of wealth invested in country j's risky assets is 0.10, the investor takes a position in the risk-free asset of that country equal to $w_{j0} - 0.10$ since he has to borrow 0.10 times his wealth in the nominally riskless asset of country j to finance his investment in that country when constructing a zero net investment portfolio.

The formula given in equation (5) states that the foreign currency exposure of the portfolio is set equal to zero unless a forward contract on currency j has an expected payoff different from zero. If the expected payoff of the forward contract is positive, the investor will choose to take a long position in the foreign currency; if it is negative, he will choose to take a short position. Hence, the optimal hedge ratio is greater than one in absolute value if a long position in a forward contract has a negative expected payoff and it is smaller than one if it has a positive expected payoff.

The optimal hedge formula has a simple interpretation. If the expected payoff of a forward contract is zero, bearing foreign exchange risk amounts to adding volatility to the portfolio without earning a reward in the form of a higher expected return. Hence, in this case, the optimal portfolio has no foreign exchange exposure. If there is a payoff to bearing foreign exchange risk, the optimal portfolio has a position in foreign exchange that depends on the reward from bearing such risk relative to the additional risk resulting from this position. If the expected payoff from a long forward position is positive, less hedging takes place since hedging is costly.

Equation (5) can be rewritten so that the exposure one has to compute is the exposure of a portfolio with known distribution and weights that sum to one, as opposed to the portfolio $\underline{w}_j$ of equation (5) whose weights are investor specific. To rewrite equation (5), one needs to construct a portfolio with weights proportional to $\underline{w}_j$ that sum to one and then find out how much an investor with target rate of return a would invest in that portfolio. Let μ_P denote the mean excess return of such a portfolio and assume that the foreign investments of the portfolio are all financed abroad. In this case, equation (5) becomes

$$w_{j0} = (\alpha - R_1)(\mu_P - \beta^*_{e_j}\mu_f)^{-1}\{\underline{\mu}\mathbf{V}^{-1}\underline{\mu}\mu_f\mu_P\sigma^{-2}_{e_j} - \beta^*_{e_j}\} \qquad (5')$$

where μ_f is equal to $R_j + \mu_{e_j} - R_1$. In equation (5′), the only quantity that requires investor-specific information is a, the target rate of return of the portfolio. All other variables in equation (5′) can be obtained from the distribution of asset returns. Equation (5′) yields the optimal currency j position for a mean-variance efficient portfolio with expected excess return $\alpha - R_1$ irrespective of the asset universe from which the efficient portfolio is chosen. In the remainder of the paper, we take portfolio P to be the Nikkei 225 and assume that the investors can invest in P, dollar and yen risk-free assets. An alternative interpretation of our results is that the Nikkei 225 is a component of portfolio P and that the other components of that portfolio are uncorrelated with the dollar/yen exchange rate.

The Exposure of the Nikkei 225.

To optimally hedge a portfolio of common stocks, one has to estimate its exposure. Generally, the relevant portfolio is the tangency portfolio without the investment in the foreign currency. Here, however, we restrict our attention to the Nikkei 225 to examine the exposure of that portfolio to the dollar/yen exchange rate. There is no loss of generality if the yen rate of return of the Nikkei

225 is uncorrelated with the rate of change of the yen price of other currencies besides the dollar.[9] Hence, we have to compute the conditional beta of the dollar return of the Nikkei 225 and then use that beta to hedge. We focus on the conditional beta of the dollar return of an investment in the Nikkei financed in dollars to facilitate comparison with other work. This conditional beta equals 1 + $\beta^*(t)$, where $\beta^*(t)$ is the exposure of the Nikkei 225 computed according to the formula defined in the previous section, namely the exposure of a portfolio invested in the Nikkei 225 financed by borrowing in yen.

In our analysis, we use a database of Nikkei 225 daily returns from the beginning of 1982 to the end of 1989. Saturday returns are added to Friday returns when applicable. The return on the index is computed as $\log(I_t/I_{t-1})$ where I_t is the index level on day t. We assume that hedging involves borrowing at the three-month euro-yen rate. Consequently, for a day, the interest rate cost is the three-month euro-yen rate divided by 360. Similarly, excess returns are defined as excess returns over the three-month euro-dollar rate. Hence, the excess return on an unhedged position in the Nikkei is

$$\begin{aligned} r_{1t} &= \log(I_t e_t) - \log(I_{t-1}e_{t-1}) - R_{\$,t-1} \\ &= (\log I_t - \log I_{t-1}) + (\log e_t - \log e_{t-1}) - R_{\$,t-1} \end{aligned} \quad (6)$$

where e_t is the dollar price of the yen at date t and $R_{\$,t-1}$ is the three-month euro-dollar rate at date $t-1$ divided by 360. To compute the optimal hedge at date t, we would like to know the ratio

$$\beta_t = \frac{\mathrm{Cov}_t\left(\log \frac{(I_t e_t)}{I_{t-1}e_{t-1}}, \log \frac{(e_t)}{e_{t-1}}\right)}{\mathrm{Var}_t\left(\log \frac{(e_t)}{e_{t-1}}\right)} \quad (7)$$

where the covariance and variance for the period from date t to $t+1$ are computed with all information available at date t. We first use as our conditional forecast of β_{t+1} the in-sample estimate for β_t, where a period corresponds to one month and returns are observed daily. To estimate the in-sample β_t, we would like to observe exchange rates at the same points in time when we observe the market index. Unfortunately, this cannot be done with available published data sources. Instead, we estimate the in-sample β_t using either the yen/dollar exchange rate observed in Tokyo in the middle of the trading day or the yen/dollar exchange rate observed in New York at three p.m. Neither source for exchange rates yields exchange rate changes that are perfectly synchronous with the stock returns.

The results are reported in Table 1. For each year in the sample, we provide the mean and standard deviation of the monthly exposures. It is immediately obvious that the standard deviation is large, suggesting that the estimated betas change much over time. However, the average betas reported in Table 1 are not significantly different from one. Whereas we do not report the monthly betas, it is noteworthy that some of them exceed two or are negative and differ significantly from one.

Table 1
EXPOSURE ESTIMATES

Year	1	2	3	4	5	6
1982	1.2984	1.1233	1.4242	1.4255	1.3001	1.1526
	(0.2809)	(0.2955)	(0.4021)	(0.4897)	(0.2784)	(0.2393)
1983	1.2851	1.1796	1.2692	0.9729	0.8686	0.7938
	(0.3261)	(0.2899)	(0.3526)	(0.3648)	(0.2187)	(0.2093)
1984	1.3835	1.1948	1.3236	1.4446	1.3931	1.1542
	(0.5343)	(0.4918)	(0.8430)	(0.8550)	(0.3719)	(0.3596)
1985	0.9879	0.7903	0.9037	0.7927	1.0494	0.8158
	(0.6244)	(0.5847)	(0.4117)	(0.4356)	(0.1926)	(0.3397)
1986	1.1021	1.0671	1.1103	1.0414	1.0599	0.9734
	(0.4714)	(0.4654)	(0.7177)	(0.6073)	(0.4002)	(0.3907)
1987	0.6778	0.6533	1.2692	0.9729	0.8686	0.7938
	(0.6425)	(0.8065)	(0.8127)	(0.7392)	(0.4534)	(0.4718)
1988	0.8403	0.7020	0.8053	0.8205	0.8497	0.7751
	(0.6244)	(0.5847)	(0.4323)	(0.4575)	(0.3631)	(0.3208)
1989	1.2180	0.9086	1.1661	1.1757	1.2185	0.9453
	(0.2791)	(0.2568)	(0.2039)	(0.3069)	(0.1685)	(0.2264)
All	1.0970	0.9494	1.1463	1.1003	1.1209	0.9637
	(0.5040)	(0.5281)	(0.6004)	(0.6115)	(0.3720)	(0.3642)

This table presents the yearly averages of estimates of the exposure of the dollar return of the Nikkei to the rate of change of the dollar price of the yen. The estimates are obtained using all daily returns within a month. Columns 1 and 2 report estimates using the dollar price of a three-month euro-yen deposit and the dollar price of the yen reported respectively in Tokyo and in New York. Columns 3 and 4 are similar to columns 1 and 2, except that the estimates are Dimson-Fowler betas. Columns 5 and 6 report estimates using the dollar price of the yen reported, respectively, in Tokyo and in New York. The standard deviations reported in parentheses are computed using each year's observations. The last two rows report averages across years of the variables.

Interestingly, exposures measured using exchange rates observed in Tokyo are higher than are exposures measured using exchange rates observed in New York.[10] The explanation for this result is most likely that the exchange rate returns observed in Tokyo are more contemporaneous with the index returns than are the exchange rate returns observed in New York. A lack of synchronization of index and exchange rate returns is equivalent to measuring the exchange rate returns imprecisely and biases the regression coefficient downward.[11] One way to reduce the problem caused by a lack of synchronization is to estimate the regression coefficients by including leads and lags of the exchange rate return.[12] The third and fourth columns in Table 1 provide estimates of the regression coefficient obtained by regressing the index return on the lagged, the contemporaneous, and the leading exchange rate return. In this case, the reported coefficient is obtained by adding up the three estimated regression coefficients. It turns out that if this procedure is used, the estimated exposures are much less sensitive to where the exchange rate returns are observed.

In our procedure for estimating the relevant exposures, we assume that hedging is undertaken by borrowing at the three-month euro-yen offering rate and repaying the amount borrowed on the next day. Hence, the exposures we are measuring are really exposures to the dollar return of a yen bond with a maturity of three months. Volatility in the yen interest rate affects the measured exposure. To the extent that hedging takes place by borrowing at that rate, however, we are measuring the appropriate exposure. If, alternatively, investors borrow at an overnight rate so that they bear no interest rate risk, the appropriate measure of the exposure is the return on the spot exchange rate overnight. This measure is provided in the fifth and sixth columns of Table 1. It does not differ significantly from the one obtained using the three-month borrowing rate.

In this paper, we do not try to forecast exposures using variables other than past exposures. Hence, using exchange rate forecasts to forecast exposures is left for further work. However, using past exposures to predict future exposures, we find that exposures follow an ARIMA(0,1,1) process.

So far, we have estimated exposures in a variety of ways. We found that exposures are volatile and that there is no statistical evidence that they differ systematically from one when using daily data. We also found that exposures are somewhat predictable using a time-series model. In Table 2, we investigate whether the alternative ways of measuring exposures using daily data make a difference in the performance of minimum-variances hedges. For each way of measuring the exposure, we report the return of the hedged portfolio, the ratio of the volatility of the hedged portfolio, and the volatility of the yen return of the Nikkei. Overall, it turns out that the volatility of a hedged portfolio of the Nikkei 225 is about the same regardless of how exposures are measured.

In Table 2, the daily volatility of the Nikkei 225 is 0.62 percent in yen and 0.79 percent in dollars, using the exchange rate observed in New York. Through hedging, an American investor can hold the Nikkei 225 bearing the same volatility as a Japanese investor. The hedged returns have the interpretation of excess returns. Comparing the excess return on the unhedged portfolio with the excess return on hedged portfolios, we find that hedging would have reduced the excess return by about 0.014 percent per day. Using a minimum-variance hedge, an investor would have reduced the dollar volatility of his Nikkei investment by slightly more than 20 percent at the cost of reducing the return by slightly more than 15 percent. The cost of hedging is consistent with the existence of a positive risk premium for bearing yen risk over the sample period.

RETURNS OF HEDGED PORTFOLIOS

It is clear from Table 2 that using an estimated beta as opposed to assuming that beta is one has no impact on hedging performance. The volatility of the

Table 2
RETURNS OF HEDGED PORTFOLIOS

Year	1	2	3	4	5	6	7
1982	1.09	−2.13	−4.75	−0.83	−2.44	−2.10	−2.08
	(62.54)	(59.64)	(92.71)	(62.63)	(63.82)	(65.19)	(65.20)
1983	8.80	−1.38	4.71	6.96	6.65	6.74	6.55
	(41.57)	(35.36)	(56.55)	(41.58)	(41.60)	(41.60)	(41.60)
1984	5.81	−1.02	1.75	4.05	3.69	3.78	3.79
	(61.25)	(36.62)	(74.80)	(61.19)	(61.18)	(61.18)	(61.18)
1985	5.37	8.07	11.11	3.49	3.86	3.83	3.83
	(44.38)	(41.95)	(57.00)	(44.40)	(44.43)	(44.41)	(44.41)
1986	16.57	7.64	22.32	15.18	15.06	15.20	15.20
	(78.27)	(57.38)	(97.00)	(78.22)	(78.23)	(78.23)	(78.23)
1987	12.00	10.30	20.31	10.78	11.18	11.02	11.01
	(95.10)	(52.51)	(102.56)	(95.52)	(95.46)	(95.47)	(95.49)
1988	10.03	1.41	9.23	8.76	9.11	9.02	9.01
	(52.86)	(46.40)	(65.89)	(52.90)	(52.97)	(52.94)	(52.93)
1989	7.88	−4.81	0.49	6.37	6.46	6.43	6.43
	(38.92)	(52.55)	(67.43)	(38.90)	(38.90)	(38.90)	(38.90)
All	8.46	2.23	8.14	6.86	6.79	6.89	6.90
	(62.05)	(48.74)	(78.90)	(62.14)	(62.30)	(62.43)	(62.43)

Note:

The returns in percent are multiplied by 100. The statistics in column 1 are in yen; all other statistics are in dollars. The first four portfolios are (1) a long position in the Nikkei; (2) a long position in yen; (3) a long position in the Nikkei financed in dollars at the three-month euro-dollar rate; (4) a long position in the Nikkei financed in yen at the 3-month euro-yen rate. The last three portfolios all involve a long position in yen, an amount borrowed at the three-month euro-yen rate equal to β, with the excess invested or borrowed at the three-month euro-dollar rate. Portfolio (5) uses the daily returns within a month to compute β and uses that estimated β every day for the following month. Portfolio (6) uses a rolling regression using 30 days of data to compute β and holds the hedge for a day. Portfolio (7) uses β estimated from an ARIMA (0,1,1) model that uses all β's estimated to construct portfolio (5). In all cases, we use the dollar/yen exchange rate observed in New York.

hedged portfolio is the same whether the exchange rate is observed in the US or in Tokyo. Further, this volatility is the same whether betas are estimated with leads and lags, using a thirty-day rolling regression, or forecasted using an ARIMA(0,1,1) model.

One might argue that our results are due to an estimation period that is too short. In Table 3, we show that estimating exposures over four months leads to

Table 3
EXPOSURE ESTIMATES USING LONGER ESTIMATION PERIODS

Year	1	2
1982	1.1233	1.0959
	(0.2241)	(0.1147)
1983	1.1706	1.1686
	0.2766	(0.1287)
1984	1.1948	1.1689
	(0.3732)	(0.1802)
1985	0.7903	0.8863
	(0.3084)	(0.1414)
1986	1.0671	0.9873
	(0.3014)	(0.1317)
1987	0.6533	0.8812
	(0.4840)	(0.1998)
1988	0.7020	0.7046
	(0.3261)	(0.1627)
1989	0.9086	0.9600
	(0.2023)	(0.0906)
All	0.9494	0.9766
	(0.3129)	(0.1450)

Note:
This table presents the yearly averages of estimates of the exposure of the dollar return of the Nikkei to the rate of change of the dollar price of the yen. The estimates for each month are obtained using (1) all returns within the month and (2) all returns within the month and within the three previous months. The estimates measure the exposure with respect to the dollar price of a three-month euro-yen deposit with the New York dollar price of the yen. The standard deviations are averages of the standard deviations of the estimates.

substantially more precise estimates of the exposures. However, the greater precision of exposures does not lead to a noticeable improvement in the performance of the hedged portfolio. In Table 4, we compare the performance of a hedged portfolio with a hedge ratio of −1 to portfolios that use hedge ratios estimated using, respectively, 4 months of daily data and 20 months of monthly data. The volatility of the hedged portfolio is compared to the volatility of a hedged portfolio with a hedge ratio of −1. The standard deviations of the various hedged portfolios are indistinguishable.

The conclusion from our analysis is that the yen return of the Nikkei is not exposed to the dollar/yen exchange rate, so that the dollar return of the Nikkei has a beta with respect to the dollar/yen exchange rate of 1. Although we want

Table 4
THE PERFORMANCE OF HEDGED PORTFOLIOS USING LONGER ESTIMATION PERIODS FOR EXPOSURES

Year	1	2	3
1982–1983	0.99	1.00	1.01
	(19)	(65)	(5)
1984–1985	0.99	1.00	1.00
	(24)	(95)	(24)
1986–1987	1.00	1.00	1.00
	(24)	(94)	(24)
1988–1989	1.01	1.00	0.99
	(24)	(96)	(24)
All	1.00	1.00	1.00
	(81)	(350)	(77)

Note:

All portfolios have a long position in yen, an amount borrowed at the three-month euro-yen rate equal to β, with the excess invested or borrowed at the three-month euro-dollar rate. Portfolio (1) uses four months of daily data to compute β. Portfolio (2) uses a rolling regression of 20 weeks of data. Portfolio (3) uses monthly data for 20 months. For portfolios (1) and (3), we assume that the hedge is maintained for a month, whereas for (2) we assume it to be held for a week. All returns assume that the hedge is maintained for a month. In all cases, we use the dollar/yen exchange rate observed in New York. We report the standard deviation of the portfolio divided by the standard deviation of a portfolio that uses a hedge ratio of 1 and is held for the same period of time. We report this ratio for two-year averages; the number of observations differs across cells and is reported in parentheses.

to be cautious in generalizing from this example, it suggests that little is to be gained by investigating the own currency exposure of market indices. After all, the gain from exploiting own currency exposures of market indices other than the Nikkei is likely to be extremely small if such exposures are nontrivial because these indices are unlikely to be a large component of internationally diversified portfolios.

Optimal hedge ratios.

Given our previous evidence, we can restate the optimal investment in yen in excess of the amount borrowed in yen to finance the investment in the Nikkei as

$$w_Y = (\alpha - R_1)(\mu \mathbf{V}^{-1}\mu)\mu_f \sigma_Y^{-2} \qquad (5'')$$

since β^* (in this case, the exposure of an investment in the Nikkei 225 financed in yen) is effectively zero for the Nikkei 225. w_Y is the investor's optimal yen holdings in excess of the yen position required to finance his investment in the Nikkei if the investor can invest in the dollar risk-free asset, the yen risk-free asset, and the Nikkei. Hence, if it is optimal for the investor to invest half his wealth in the Nikkei, the optimal yen holdings would be $w_Y - 0.5$ since the investor would have to borrow the equivalent of half his wealth in Japan to finance his investment in the Nikkei. The resulting optimal hedge ratio is $(w_Y - w_N)/w_N$, where w_N is the fraction of wealth invested in yen to finance the zero net investment portfolio long in the Nikkei. If $\mu_f = 0$, the investor invests only in a zero net foreign investment portfolio long in the Nikkei; and the optimal hedge ratio is -1. In Table 5, we report optimal hedges constructed using equation (5″) assuming that, for each year, the expected returns and the variances of long positions in a zero net foreign investment portfolio long in the Nikkei and in a euro-yen deposit are equal to their **ex post** value. We choose as the investor's target return the expected return on an unhedged position in the Nikkei, assuming that the expected rate of change of the dollar/yen exchange rate and its

Table 5
COMPOSITION AND PERFORMANCE OF MEAN-VARIANCE EFFICIENT PORTFOLIO

Estimation Period	Nikkei weight	Yen weight	Hedge ratio	Var. ratio
1982	0.38	0.75	0.99	0.38
1983	0.53	−0.80	−2.49	0.53
1984	0.12	−0.67	−6.48	0.12
1985	0.60	0.66	0.10	0.60
1986	0.77	0.71	−0.09	0.77
1987	0.54	0.97	0.79	0.54
1988	0.99	0.10	−0.90	0.99
1989	0.00	−0.08	−20.76	0.01
1982–1989	0.94	0.39	−0.59	0.93

Note:
Optimal portfolio and performance relative to portfolio fully invested in Nikkei with hedge ratio of −1. The position in the Nikkei is fully hedged against exchange rate risk using a hedge ratio of −1. The position in yen is a position in a three-month euro-yen deposit and ignores the amount borrowed to hold the Nikkei. The hedge ratio is defined as (euro-yen deposit weight—Nikkei weight)/(Nikkei weight). The var. ratio is the ratio of the variance of the optimally hedged portfolio and of the unhedged portfolio. The hedged portfolio is constructed assuming that the investor uses as the ex ante forecasts of means, variances, and covariances their ex post values during the estimation period.

volatility equal their **ex post** values. Hence, the table provides a mean-variance efficient portfolio invested in a hedged position in the Nikkei and a currency position that yields the same expected return as an unhedged position in the Nikkei.

It is clear from Table 5 that the optimal hedge changes substantially over time. However, the hedge ratios used assume for some years that the expected return on a hedged position in the Nikkei will be negative, since **ex post** the return was sometimes negative. One interpretation of the variance ratios is that they show the decrease in variance that could have been achieved through investment in yen assuming that otherwise the investor would have held a position in the Nikkei unhedged. One might choose a longer period of time to measure the means and variances to form expectations about the future than one year to gain another perspective on the benefits of hedging. If one uses the whole sample period to form estimates of means and variances, the optimal hedge ratio would be −0.59, which is substantially smaller than one in absolute value. However, the variance reduction brought about by selecting that hedge ratio instead of a hedge ratio of −1 is remarkably small—about 7 percent. On average, investments in a euro-yen deposit would have had a small excess return relative to investing in a euro-dollar deposit. This implies that reducing the hedged investment in the Nikkei and increasing the investment in the euro-yen deposit by equivalent amounts reduces the portfolio's expected return. To keep the expected return of the portfolio constant, one has to increase investment in the euro-yen deposit substantially more than one has to decrease the hedged investment in the Nikkei. Consequently, even a substantial departure of the hedge ratio from −1 has only a moderate impact on the performance of the portfolio.

Rather than using the **ex post** cost of hedging to construct hedge ratios, we could use a measure of the **ex ante** cost derived from a model for the pricing of forward contracts. For instance, suppose that the appropriate world market portfolio is simply the world market portfolio of equities with returns measured in dollars. In this case, if the Nikkei is 40 percent of the world market portfolio, our evidence suggests that the beta of the yen would be 0.4. Hence, if the market portfolio earns a risk premium of 10%, the ex ante cost of hedging is 4%. Since such a hedging cost turns out to be not very different from the whole sample **ex post** hedging cost, using this **ex ante** measure yields a similar hedge ratio as the whole sample hedge ratio obtained in Table 5.

Whereas **ex post** the yen forward rate is on average lower than the spot rate at maturity of the forward contract, it is also clear that the forward price is sometimes higher than the realized spot rate for a substantial period of time. Hence, the extent to which one can construct a portfolio with currency positions that dominate a portfolio of investments in hedged equity investments depends crucially on how well one can forecast exchange rates relative to the forward rate. If one can forecast that the spot exchange rate will differ from the forward rate with some degree of precision, then a hedge ratio of −1 is not optimal. Recent empirical literature on the risk premium embedded in forward prices suggests that this risk premium can be forecasted.[13]

CONCLUSION

This paper provides a simple formula for mean-variance efficient hedging and applies it to an investment in the Nikkei. We find that there is no gain from trying to get the best possible estimate of the Nikkei's exposure to the dollar/yen exchange rate. In particular, the standard deviation of the return of hedged portfolios is generally lowest when the exposure is assumed to be one rather than when the exposure is estimated from historical data. If some gain can be achieved through an optimal choice of currency positions, it is through better estimates of the cost of hedging. The expected return cost of hedging is the difference between the expected dollar return from holding a euro-yen deposit and the dollar return from holding a euro-dollar deposit. If this expected return difference is positive, there is a positive reward for bearing exchange rate risk and hedging is costly.

NOTES

1. See, for instance, Arnott and Henrikkson [3], Black [5], Celebuski, Hill and Kilgannon [7], Jorion [11], Perold and Shulman L [14] and Kaplanis and Schaefer [12].
2. For instance, Perold and Schulman [14] argue that exposures are equal to one whereas Adler and Simon [1] and Kaplanis and Schaefer [12] find exposures significantly greater than one using weekly data for a subsample at the beginning of the nineteen eighties.
3. See Stulz [16] for more details.
4. See Stulz [16] and Kaplanis and Schaefer [12].
5. This result applies only to assets that are nominally risky, however.One cannot construct zero net foreign investment portfolios in the nominally riskless asset, i.e., the return on a long investment in the riskless asset of country j financed by a short investment in that asset is identically equal to zero.
6. This is the model used by Black [5]. For a detailed analysis of the conditions under which universal hedging ratios obtain, see Adler and Prasad [2].
7. The formula we obtain holds for any numeraire with some obvious adjustments.
8. See Stulz [16].
9. See Adler and Simon [1] and Kaplanis and Schaefer [12] for evidence on the exposure of own currency returns to non-dollar exchange rates to using weekly data.
10. In addition to the difference in mean betas, 17 betas using exchange rates observed in Tokyo are significantly greater than one, whereas only three betas using exchange rates observed in New York are significantly greater than one.
11. Bailey and Stulz [4] show that the lack of synchronization between US and Japanese index returns biases the correlation between these returns downward.

12. See Dimson [8] and Fowler and Rorke [9].
13. See, for instance, Cumby [7], Engel and Hamilton [10] and Mark [13].

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