An Equilibrium Model of Exchange Rate Determination and Asset Pricing with Nontraded Goods and Imperfect Information

René M. Stulz

Ohio State University

This paper presents a two-country model with maximizing households, stochastic production, stochastic money growth, and perfect capital mobility. Because of the presence of nontraded goods, households in different countries consume different goods. Analytic solutions are presented for the nominal exchange rate, the real exchange rate, nominal interest rates, and real interest rates. It is shown that the model is compatible with some important features of the real-world behavior of exchange rates. When households are imperfectly informed about the distribution of money growth, the exchange rate exhibits patterns of overshooting and is more volatile than the ratio of the money stocks.

I. Introduction

This paper studies exchange rate determination and asset pricing in a two-country equilibrium model. The analysis is related to the work of Lucas (1982), Stulz (1984), Stockman and Svensson (1985), and Svensson (1985) in that it focuses on a world economy with infinitely lived maximizing households, stochastic production or endowments, stochastic money growth, and perfect capital mobility. However, con-

I am grateful for useful comments on this paper by Rudi Dornbusch, Robert Flood, Robert Hodrick, George Pennachi, José Scheinkman, Walter Wasserfallen, two anonymous referees, and participants at seminars at Duke, MIT, and Northwestern, at the CESA Conference on International Finance, at the University of Wisconsin Johnson Symposium on Asset Pricing, and at the NBER Conference on International Asset Pricing.

[*Journal of Political Economy*, 1987, vol. 95, no. 5]
 0022-3808/87/9505-0008$1.50

trary to these earlier papers, the present analysis allows households to differ across countries in that they consume different goods. Each country produces a good that is not traded internationally, consumed exclusively by its residents, as well as a good that is traded internationally.

The existence of nontraded goods makes this model useful to address explicitly issues related to the determination of the real exchange rate. In this economy, the logarithm of the real exchange rate is a martingale if the means and variances of the growth rates of the stocks of nontraded goods are constant and identical across countries. It is shown that, in the presence of significant output shocks in the production of nontraded goods, the variance of the real exchange rate can be larger than the variance of the nominal exchange rate. Furthermore, changes in the real exchange rate and the nominal exchange rate are shown to be positively correlated provided that a country's money growth is positively correlated with that country's output of the nontraded good. Even though the model of this paper assumes that markets are perfect internationally, it is capable of capturing some important features of the real-world behavior of real exchange rates.[1]

The issue of the volatility of the nominal exchange rate is analyzed explicitly in the paper. It is shown that if households do not know the true distribution of domestic money growth, the nominal exchange rate overshoots unexpected changes in the domestic money supply so that the nominal exchange rate will seem to be too volatile given the behavior of the money supply. However, this overshooting does not affect the real exchange rate. The model makes it possible to answer explicitly the question of how uncertainty about future money growth affects the level of the exchange rate. Surprisingly, unless households are sufficiently more risk averse than with a logarithmic utility function and money growth is negatively correlated with output, uncertainty about future money growth increases the value of the domestic currency.

In this model, the forward exchange rate is, in general, a biased predictor of the future spot exchange rate. However, paradoxically, the forward premium itself does not depend directly on the risk of a long position in the foreign currency. Conditions are derived under which the risk premium on forward exchange is time varying. It is shown that, when households do not know the true distribution of domestic money growth, the forecasting errors of the forward exchange rate can exhibit serial correlation even in the absence of a risk premium. Finally, in this model, the difference between the domestic

[1] See, e.g., Mussa (1979) for a description of the empirical evidence.

and the foreign real rates of interest is not equal to the expected rate of change of the real exchange rate.

The paper is organized as follows. Section II presents the model. Section III derives the nominal and real exchange rates and compares their volatilities. Section IV discusses the forward premium and the relation among real interest rates across countries. Section V analyzes exchange rate dynamics with imperfect information. Finally, Section VI presents concluding remarks.

II. The Economy

This section describes the two-country economy analyzed in this paper. Throughout the paper, it is assumed that (1) trading takes place continuously, (2) there are neither taxes nor transactions costs, (3) there are neither tariffs nor transportation costs, (4) all households are price takers, (5) contracts are costlessly enforced and negotiated, and (6) all households have the same information available to them.

A. *Production*

In the interest of simplicity, it is assumed that each country produces the same internationally traded commodity and one nontraded commodity. Each commodity is produced with a constant stochastic returns to scale technology. Technologies differ across countries. The only input required to produce a commodity is the commodity itself. An investment of K_{ij} units of the ith good in the jth country follows

$$dK_{ij}(t) = \mu_{ij}(\mathbf{s}, t)K_{ij}dt + \sigma_{ij}(\mathbf{s}, t)K_{ij}dz_{ij}, \quad i = 1, 2; j = 1, 2, \qquad (1)$$

where $\mu_{ij}(\mathbf{s}, t)$ and $\sigma_{ij}(\mathbf{s}, t)$ are bounded functions of the $1 \times s$ vector of state variables $\mathbf{s}$ and time and dz_{ij} is the increment of a standard Wiener process.[2] In each country commodity 1 is chosen to be the traded good. The state variables change stochastically over time and are assumed to follow diffusion processes. Let $\mathbf{K}$ be the 1×4 vector of per capita investments in production processes.[3] It is assumed that $\mathbf{s}$ and the joint process $(\mathbf{K}, \mathbf{s})$ are Markov.

With the assumptions made so far, production is uncertain and the technologies themselves change stochastically over time. If changes in the vectors $\mathbf{K}$ and $\mathbf{s}$ are correlated, it is possible for the output of the various technologies to be correlated over time. Hence, by a proper choice of state variables, one can model both temporary and permanent production shocks.

[2] Cox, Ingersoll, and Ross (1985) provide a description of the methodology used in this paper and references.

[3] In this paper, "per capita" means per household in the world.

B. *Monies*

Each country's money is held only by households of that country.[4] Let M_j be the per capita stock of money of country j. The money stock follows a diffusion process in each country. In each country, the local mean $\mu_{M_j}(\mathbf{s}, t)$ and variance $\sigma^2_{M_j}(\mathbf{s}, t)$ of the growth rate of the money stock are bounded functions of time and of the vector of state variables $\mathbf{s}$. The domestic (foreign) currency price of commodity i in the domestic (foreign) country is defined as P_{i1} (P_{i2}), while the domestic currency price of one unit of foreign currency is written e. Both e and the commodity prices will be solved for as functions of exogenous variables later on.

In each country, the money stock changes through transfers to residents of the country. All residents of a country receive the same transfer.

C. *Description of Financial Assets*

Households can trade $N + 2$ financial assets. The domestic currency price of the ith financial asset is A_i and its local mean rate of return is μ_i. In each country, there is a bond in zero net supply with a safe instantaneous nominal rate of return in that country's currency, $R_j(\mathbf{s}, t)$, $j = 1, 2$, which is allowed to vary stochastically over time with changes in the vector of state variables. For simplicity, the first four financial assets correspond to investments in production. The other financial assets are in zero net supply and are contingent claims whose payoffs are appropriately differentiable functions of the state variables and time. It is assumed that financial markets are sufficiently complete that households can trade claims to future transfers.

D. *The Optimization Problem of Households*

All households in a country are assumed to be the same, and each country has the same number of households. Each household has a logarithmic instantaneous utility function that is time additive and is defined over its consumption of the commodities and its holdings of real balances, respectively, c_{ij} and m_j, for $i = 1, 2$, $j = 1, 2$.[5] Households have constant expenditure shares so that, for each household, there is an exact price index that makes it meaningful to talk about

[4] The results may differ if domestic households hold foreign money to facilitate their purchases abroad.

[5] This assumption is crucial for the following analysis. Without it, I would not be able to show that a representative household exists. Rubinstein (1974) provides general conditions for the existence of a representative household.

real balances.[6] Households all have the same subjective discount rate, but they can have different expenditure shares across countries. Consequently, a household from country j maximizes

$$E_t \int_t^\infty e^{-\rho\gamma}\left[\sum_{i=1}^{2} \alpha_{ij} \ln c_{ij}(\gamma) + \left(1 - \sum_{i=1}^{2} \alpha_{ij}\right)\ln m_j(\gamma)\right]d\gamma, \quad j = 1, 2,$$

where E_t denotes the expectation conditional on information available at time t. All households in a country have the same wealth in terms of the domestic currency, W_j. A household's wealth is equal to its holdings of financial assets and real balances plus the current value of the future transfers it will receive from the monetary authority cum government. Given its wealth, its utility function, the joint distribution of asset returns, and state variables, each household chooses its consumption and portfolio policies.[7]

E. *Characterization of the Equilibrium*

It is well known that, with a logarithmic utility function, consumption and portfolio policies are linear functions of wealth. This property of the logarithmic utility function enables one to aggregate households across countries so that the world economy behaves as if there is one representative investor with expenditure shares that are wealth-weighted averages of the domestic and the foreign households' expenditure shares and wealth equal to world wealth per capita.

The existence of a representative household makes it possible to obtain equilibrium pricing functions for assets and monies that do not depend on the distribution of world wealth across households. In this economy, an equilibrium is a set of pricing functions for assets, a consumption policy, and a portfolio policy, such that the representative household's expected lifetime utility is maximized and (1) the stock of each commodity is invested in production, (2) each money stock is held by households, (3) claims to the present value of all future transfers are held by households, (4) the production of a commodity that is not reinvested is equal to the world's consumption of that commodity, and (5) the net supply of all other assets is zero.

The equilibrium has two features that are crucial for the remainder of the paper. First, it is proved in the Appendix that per capita world

[6] This index is defined in Sec. III.

[7] Harrison and Kreps (1979) point out that, in a model such as the one developed here, some technical conditions must be satisfied by the trading strategies available to households to eliminate free lunches arising from doubling strategies. I restrict the trading strategies available to a household to those that imply that the change in its wealth, over any time period, is the solution of a well-defined Ito integral equation. For a more technical description of the required condition, see Cox et al. (1985, n. 4).

wealth measured in domestic currency is equal to $(1/\alpha_1)P_{11}K_1$, where K_1 is the per capita stock of the traded good and α_1 is the expenditure share of the representative household on the traded commodity.[8] Intuitively, this result holds because households spend a constant fraction ρ of their wealth on consumption expenditures and a constant fraction of their consumption expenditures on each good. The per capita value of the stock of commodity 1 is the present value of the representative household's consumption of commodity 1, but the other consumption expenditures are fixed proportions of the household's expenditures on commodity 1. It follows, therefore, that the value of the other assets whose nonreinvested returns are consumed must be a constant fraction of the value of the stock of commodity 1.

The second important feature of the equilibrium is that the expected return of a risky asset satisfies

$$\mu_i - \sigma_{A_i,P_{11}} - R_1 = \sigma_{A_i,K_1}, \tag{2}$$

where $\sigma_{a,b}$ is the covariance between the rates of growth of a and b. Equation (2), which is derived in the Appendix, is an equilibrium version of the well-known capital asset pricing model. It states that an asset's risk premium in terms of good 1 depends only on the covariance of the asset's return with the growth rate of the stock of commodity 1. In the following, σ_{A_i,K_1} is called the risk of asset i. It stands for the nondiversifiable risk of the capital asset pricing model since, in this model, real wealth is perfectly correlated with changes in K_1. Equation (2) is used to price all assets in this paper.

III. Exchange Rate Determination

In this economy, the law of one price holds for the traded good so that $P_{11} = eP_{12}$. Because it has a logarithmic utility function, the representative household spends a constant fraction α_3 (α_4) of its wealth $(1/\alpha_1)P_{11}K_1$ on the service flow R_1M_1 (R_2M_2) of domestic (foreign) cash balances. Result 1 immediately follows.

Result 1.—The exchange rate is equal to

$$e = \frac{\alpha_4 R_1 M_1}{\alpha_3 R_2 M_2} \tag{3}$$

[8] Note that this result implies that the value of each industry is perfectly correlated, which is an unfortunate drawback of the assumption that households have a logarithmic utility function. (However, this result has no implication for the distribution across countries of invested wealth because the technology to produce the traded good differs across countries and changes over time.)

and depends only on expenditure shares, nominal interest rates, and money supplies.

As with the monetary approach, the exchange rate increases (falls) with the domestic (foreign) money stock and nominal interest rate. However, contrary to this approach, no measure of output or wealth enters the equation for the exchange rate separately from the expenditure shares, nominal interest rates, and money supplies. In this model, capital mobility is perfect and capital markets are complete in the sense that households can hedge against adverse changes in those state variables they care about. Hence, there is no reason for domestic households to bear alone risks that they can share internationally.

Let us now turn to the real exchange rate, eI_2/I_1, where I_1 and I_2 are the price levels at home and abroad. To simplify the analysis, we can make the useful symmetry assumption that $\alpha_{11} = \alpha_{12} = \alpha_1$ and $\alpha_{21} = \alpha_{22} = \alpha_2$. With this assumption, which states that the expenditure shares on traded and nontraded goods are the same across countries, price indices are given by

$$I_i = P_{1i}\left(\frac{P_{2i}}{P_{1i}}\right)^{\alpha_2'}, \quad i = 1, 2, \tag{4}$$

where $\alpha_2' = \alpha_2/(\alpha_1 + \alpha_2)$. When the law of one price is used and P_{Nj} is the relative price of the nontraded good of country j in terms of the traded good, that is, P_{2j}/P_{1j}, the real exchange rate x is equal to $(P_{N2}/P_{N1})^{\alpha_2'}$ and hence changes only with the ratio of the relative prices of the nontraded goods. As households have the same expenditure shares and as the law of one price holds, an increase in the domestic price of the traded good affects all households equally. Therefore, it cannot make one currency more valuable in real terms. However, an increase in the price of the domestic nontraded good relative to the foreign nontraded good means that one unit of the foreign currency buys less of the world consumption basket and hence brings about a real depreciation of the foreign currency. It follows from the discussion in Section II*E* and from the symmetry assumption that the domestic currency value of each country's stock of the nontraded good is $\frac{1}{2}(\alpha_2/\alpha_1)P_{11}K_1$. This implies the following result.

Result 2.—The real exchange rate is equal to

$$x = \left(\frac{K_{21}}{K_{22}}\right)^{\alpha_2'}, \tag{5}$$

and it changes only if the ratio of the stock of the domestic nontraded good to the stock of the foreign nontraded good changes.

The formula for the real exchange rate owes its simplicity to the assumption of constant returns to scale, which implies here that the present value of the future consumption of a commodity in terms of

that commodity is equal to the stock of that commodity. With a logarithmic utility function, consumption expenditures on a commodity do not depend on the investment opportunity set. Hence, the ratio of the present values of the consumption expenditures on two commodities is equal to the ratio of the expenditure shares of these two commodities, so that relative prices depend only on expenditure shares and commodity stocks.

Result 2 implies that in this model relative purchasing power parity holds if the ratio of the stocks of the nontraded goods is constant. The model allows for both temporary and permanent departures from absolute purchasing power parity. For instance, for a given stock of the domestic nontraded good, an unexpected increase in the stock of the foreign nontraded good makes that good cheaper relative to the domestic nontraded good and hence reduces the real value of the foreign currency since one unit of the foreign consumption basket is worth less at home. Whether this change in the real exchange rate is permanent or temporary depends on the time-series properties of output shocks for nontraded goods.[9] If these shocks exhibit negative serial correlation, one expects the real exchange rate to appreciate following an unexpected depreciation. In the absence of serial correlation in the output of nontraded goods, the logarithm of the real exchange rate follows a random walk with a possibly nonzero drift. The model also allows for secular increases or decreases in the real exchange rate. For instance, a secular increase occurs if the foreign country is more productive in producing its nontraded good than the domestic country. Hence, in this model, it is unlikely that the logarithm of the real exchange rate is a martingale. However, in the absence of significant serial correlation in the outputs of nontraded goods, it is likely to be difficult to reject the martingale hypothesis.

The next section shows that nominal interest rates do not depend on the stocks of nontraded goods. Consequently, a doubling of the domestic or foreign money supply has no impact on the real exchange rate, while a doubling of the domestic or foreign stock of nontraded goods has no impact on the nominal exchange rate. Furthermore, an increase in the expected growth rate of the money stock in any country does not affect the real exchange rate. However, now it can be shown that, in spite of this, the behavior of the nominal and the real exchange rates is compatible with two important stylized facts: that the nominal exchange rate and the real exchange rate move together

[9] The view that the real exchange rate returns to one in the long run is generally associated with the notion that purchasing power parity holds in the long run. See Dornbusch (1976) for an example of such a model. Roll (1979) argues that the real exchange rate is a martingale in efficient markets.

and that the variance of the rate of change of the real exchange rate is not much smaller than the variance of the rate of change of the nominal exchange rate.

To show that the model can yield realistic nominal and real exchange rate dynamics, let us consider the special case of a stationary investment opportunity set so that nominal rates of interest are constant.[10] In this case, the nominal exchange rate changes with the ratio of the money stocks while the real exchange rate changes with the ratio of stocks of nontraded goods. Hence, if changes in the money stock are correlated with changes in the stock of the nontraded good in each country, changes in the real and nominal exchange rates are correlated. Furthermore, if the variance of output shocks in the production of nontraded goods is of the same order of magnitude as the variance of money shocks, the variance of the rate of change of the real exchange rate is of the same order of magnitude as the variance of the rate of change of the nominal exchange rate. Not surprisingly, if the assumption of a constant investment opportunity set is relaxed, it is possible to construct many different scenarios that imply a positive correlation of changes in the real exchange rate with changes in the nominal exchange rate.

IV. Asset Returns, Exchange Rate Dynamics, and International Investment

Solving for the nominal rate of interest at home and abroad, we get

$$R_j = \mu_{M_j} + \mu_{R_j} + \rho - \sigma^2_{M_j} - \sigma^2_{R_j} - \sigma_{R_j,M_j}, \quad j = 1, 2. \tag{6}$$

The representative household's subjective rate of time preference enters equation (6) because the higher that rate, the lower the rate of growth of real wealth and the higher the expected rate of inflation. A nominal bond is risky in real terms so that one would expect its real return to incorporate a risk premium that is equal to the covariance of its real return with the rate of growth of the stock of the traded good. However, equation (6) does not depend on the distribution of the growth rate of the stock of the traded good. It is therefore possible to change the risk of the nominal bond without changing its nominal return because such an increase in the risk of the nominal bond increases its expected real return just enough to make up for the increase in its risk. With a logarithmic utility function, the risk premium is a covariance term and hence is of the same order of mag-

[10] It can be shown that R_1 and R_2 are constant when the joint distribution of the money stocks is constant.

nitude as the convexity term because of the convexity of the relation between the real value of a bond and the price of the traded good.

We can now solve for the forward premium f. With our assumptions, interest rate parity holds so that f is equal to the domestic nominal rate of interest minus the foreign nominal rate of interest. Straightforward manipulations yield an expression for f as a function of the expected rate of change of the exchange rate μ_e.

Result 3.—The forward premium f is equal to

$$f = \mu_e - \sigma_{M_2,R_2} + \sigma_{M_1R_1,M_2R_2} - \sigma^2_{M_1} - \sigma^2_{R_1} - 2\sigma_{R_1,M_1} \qquad (7)$$

so that the mean forecasting error of the forward exchange rate depends only on the joint distribution of money growth and interest rates.

Result 3 is paradoxical. The risk of a long forward position in the foreign currency is equal to the covariance of the rate of change of the exchange rate with the growth rate of the stock of the traded good, σ_{e,K_1}. However, inspection of the equation for the forward premium shows that the forward premium can stay constant while σ_{e,K_1} changes. There is a simple explanation for this paradox. An increase in σ_{e,K_1} increases the expected return of a long forward position in terms of the traded good by exactly the amount of the increase in σ_{e,K_1}. Hence, changes in the risk of a long forward position affect the expected real return but not the nominal expected return of such a position. Empirically, therefore, one may not find a relation between the nominal forecast error of the forward exchange rate and the risk of a forward position, while one may find such a relation when one focuses on the real forecast error. Note, however, that if the representative household has a degree of relative risk tolerance that differs from one, the risk of a long forward position explicitly enters the expression for the forward premium. Another point worth making is that if the representative household has a logarithmic utility function, the risk premium incorporated in the forward premium can change only because the covariance of the rate of change of the spot exchange rate and of world wealth per capita changes over time. Finally, it is useful to note that in this model, because households capitalize future transfers, the risk of a long position in a foreign currency does not depend on the supply of outside assets as it does in the popular model of Frankel (1979).[11]

Now, let us turn to a comparison of the expected real rate of return of domestic and foreign nominal bonds. Let r_1 (r_2) be the expected

[11] Many models have been offered to explain the existence of a risk premium on forward exchange. These models are generally partial equilibrium models and do not formulate the risk premium in terms of exogenous variables. See Adler and Dumas (1983) for a discussion of the literature.

real rate of return in terms of the domestic (foreign) consumption basket of a bond that earns R_1 (R_2) in domestic (foreign) currency over the next instant. Following the empirical literature, we call r_1 and r_2 real rates of interest. The Appendix shows the following result.

Result 4.—The relation between the real rates of interest at home and abroad is given by

$$r_1 - r_2 = -\sigma_{e,K_1} + \mu_x - (\alpha_2')^2(\sigma^2_{P_{21}} - \sigma_{P_{22},P_{21}} - 2\sigma_{P_{11},P_{21}} + 2\sigma_{P_{11},P_{22}}) + (\alpha_2')(\sigma_{P_{21},P_{22}} - \sigma_{P_{11},P_{22}}) \quad (8)$$

so that the difference between the domestic and the foreign real rates of interest increases with the expected rate of change of the real exchange rate and falls with the risk of a long position in foreign currency.

As the nominal rate of interest does not depend on the expected rate of change of the relative price of nontraded goods, it is not surprising that the difference between the domestic and foreign real rates of interest increases with the expected rate of change of the real exchange rate. If μ_x is positive, expected inflation is higher abroad (adjusting for the expected rate of change of the exchange rate) so that the expected real rate of return of the foreign bond must be lower. A long position in the domestic bond financed by a short position in the foreign bond is equivalent to a short forward position in the foreign currency. Hence, the expected real return of such a position must fall with the risk of a long forward position, which explains why $r_1 - r_2$ falls as σ_{e,K_1} increases. Notice finally that in a deterministic world the difference in the expected real rates of return of nominal bonds is equal to the expected rate of change of the real exchange rate.

In this model, savings by domestic and foreign households are perfectly correlated and do not depend on real or nominal interest rates. Hence, capital flows can come about only because investment differs across countries. The fraction of world savings invested in a particular country can change over time as technologies change over time.[12] However, in this model, there is no necessary relation between changes in the fraction of world wealth invested in the domestic country and domestic nominal or real interest rates. To see this, suppose that the technology for producing the traded good improves at home and worsens abroad so that the expected output from investing the stock of the traded good is not affected in equilibrium. In this case, inspection of equations (6) and (8) shows that both the nominal and

[12] The importance of shocks to technologies for capital movements is emphasized, e.g., by Sachs (1983).

the real interest rates are unchanged at home, yet a larger fraction of world saving is invested in the home country.

V. Imperfect Information

It is often argued that exchange rates are too volatile given the behavior of the economic variables that affect them. In this section, it is shown that this phenomenon can take place in the model if one assumes that households are imperfectly informed about a country's monetary policy.[13] The introduction of this assumption also serves to illustrate some limits of the model.

To focus on the main points of the analysis, assume that the true joint distribution of the output rates of the technologies and of the growth rates of the money stocks is constant but that households do not know the true mean growth rate of the money stock for the domestic country. For instance, there could be some uncertainty about the true target growth rate of the money stock pursued by the monetary authority. Following Stulz (1986), assume that households substitute their predictive distribution over money growth, with mean $\mu_M^e(t)$ and variance $\Omega(t)\sigma_M^2$, for the unknown true distribution. When households use only the time series of money growth to compute $\mu_M^e(t)$ and $\Omega(t)$, they obtain

$$\mu_M^e(t) = \tfrac{1}{2}\sigma_M^2 + \frac{1}{t}\log\frac{M(t)}{M(0)} \tag{9a}$$

and

$$\Omega(t) = \frac{t+1}{t}. \tag{9b}$$

In the following, assume that households have no other useful information available to compute their predictive distribution over money growth than the time series of money growth. The assumption of continuous trading together with the assumption that the money stock follows a lognormal diffusion process imply that σ_M^2 is observable by the households.

Using the households' predictive distribution over money growth, we can solve for the exchange rate when households do not know the true mean growth rate of the money stock:

$$e = \frac{(\mu_{M_1}^e + \mu_{R_1} + \rho - \Omega^2\sigma_{M_1}^2 - \sigma_{R_1}^2 - \Omega\sigma_{R_1,M_1})M_1}{(\mu_{M_2} + \rho - \sigma_{M_2}^2)M_2}. \tag{10}$$

[13] Mussa (1976) analyzes some implications of imperfect information about monetary policy in a small-country monetary model.

Inspection of the formula for the exchange rate together with the formula for $\mu_M^e(t)$ shows that a 1 percent unexpected increase in the domestic money stock increases the spot exchange rate by more than 1 percent. In other words, the spot exchange rate overshoots changes in the money stock. The reason for this phenomenon is that an unexpected increase in the money stock leads households to expect greater money growth. Hence, unexpected money growth brings about an increase in the nominal rate of interest, which depreciates the value of the domestic currency. Notice, however, that, because the households' expected growth rate of the money stock is increased by unexpected money growth while the true mean growth rate of the money stock is unchanged, it is now more likely that money growth will be accompanied by a less than proportional change in the exchange rate. This is caused by the fact that the probability that money growth will be below the households' expected value has increased. Suppose, however, that the true mean of money growth changes. Without going through the rather complicated derivations, it will now be more likely that actual money growth exceeds the households' forecast. Hence, when the mean of money growth changes, the model predicts persistent overshooting until the households' expectation of money growth catches up with the true mean of money growth. This type of overshooting has been obtained in other models (see Kouri 1976; Calvo and Rodríguez 1977; Flood 1979; Obstfeld 1981). However, here money is superneutral so that the overshooting has no real effects. While most other models made ad hoc assumptions about the behavior of households, Obstfeld (1981) derived that behavior by maximizing the households' lifetime utility. His results differ from ours since in his model money is not superneutral. This difference is explained by the fact that, in his model, households do not include the present value of future transfers in their wealth as they do here.[14] As the real exchange rate does not depend on nominal variables, it is not affected by unexpected money growth. Hence, in this model, for given stocks of nontraded goods, overshooting of the nominal exchange rate cannot cause a change in the real exchange rate. Furthermore, the real exchange rate can never display any "excess" variability since it depends on observable stocks of commodities.

A surprising result implied by equation (10) is that an increase in uncertainty about the domestic country's monetary policy leads to an appreciation of the domestic currency. With the distributional assumptions made in this section, an increase in uncertainty about mon-

[14] Note that deterministic models, such as Obstfeld (1981), assume that households do not know that expected money growth can change and hence do not make trades across countries to hedge against such changes.

etary policy leads to an increase in the expected real payoff of the domestic nominal bond because the real value of the nominal bond is a convex function of the money supply. However, equation (8) shows that the relation between the real rates of interest does not depend on the mean growth rates of the price levels. Hence, the nominal rate of interest must fall to keep constant the expected real return on the domestic nominal bond. It follows from this that money demand increases at home[15] and the domestic currency appreciates when uncertainty about domestic monetary policy increases. Note, however, that if the representative household is sufficiently more risk averse than with a logarithmic utility function, it could be possible for the domestic currency to depreciate when $\Omega(t)$ increases because in this case the risk premium term might dominate the convexity term.

The variance of the growth rate of the nominal exchange rate is now

$$\sigma_e^2 = \sigma_{R_1}^2 + \Omega^2\sigma_{M_1}^2 + 2\Omega\sigma_{R_1,M_1} - 2\sigma_{R_1,M_2} - 2\Omega\sigma_{M_1,M_2} + \sigma_{M_2}^2. \quad (11)$$

Note that, with the assumptions made in this section, the domestic nominal rate of interest is constant only if the households know the true distribution of the growth rate of the domestic money stock. If households do not know the true value of μ_{M_1} but learn about it over time, changes in the nominal rate of interest are positively correlated with changes in the domestic money supply. Consequently, unless the covariance between the domestic and the foreign money stocks is extremely large, monetary policy uncertainty increases the variance of the rate of change of the nominal exchange rate. Provided that σ_{R_1,M_2} and σ_{M_1,M_2} are not too large, the spot exchange rate is more volatile than the ratio of the money supplies.

The model of this section implies that the forecasting errors of the forward exchange rate are likely to exhibit serial correlation so that one could be led to believe that there is a time-varying risk premium even when there is none. To see this, consider a high unexpected increase in the domestic money stock when $\mu_{M_1}^e(t)$ is equal to μ_{M_1}. In this case, expected money growth increases and households consistently overpredict changes in the money supply until expected money growth falls back to its previous level. This point is particularly relevant since empirical research on risk premia focuses on the time-series properties of the forecasting errors of the forward exchange rate.

[15] This result has been shown in other models. Brock and Scheinkman (1980) show that it holds for models in which real balances do not enter the utility function.

VI. Conclusion

This paper solves simultaneously for nominal and real exchange rates and for expected real returns on risky assets using a two-country model with stochastic production, stochastic money growth, and nontraded goods. The main limitations of the model are that it uses the assumption that all households have a logarithmic utility function and that it leaves little room for a more serious modeling of the role of money. These drawbacks may be the price one has to pay if one wants to derive analytic results in terms of exogenous variables.

Besides having a methodological interest, this paper also shows that models with perfect markets can produce fairly realistic dynamics for real and nominal exchange rates when the joint distribution of outputs and money stocks satisfies some critical requirements. By introducing considerations of risk in the analysis of exchange rate dynamics, the model sheds new light on the issue of exchange rate volatility. It shows that when households do not know the true distribution of money growth, the nominal exchange rate will seem to be "too volatile" given the time-series behavior of the money supplies.

Appendix

Each household's holdings of risky assets are given by

$$\mathbf{n} = \mathbf{V}^{-1}(\boldsymbol{\mu} - \mathbf{R} \cdot \mathbf{1}), \quad j = 1, 2, \tag{A1}$$

where $\mathbf{n}$ is the $1 \times (N + 1)$ vector of investment proportions, $\mathbf{V}$ is the $(N + 1) \times (N + 1)$ variance-covariance matrix of domestic currency risky asset returns, and $\boldsymbol{\mu}$ is the $(N + 1) \times (N + 1)$ vector of expected domestic currency returns for assets risky in domestic currency. Hence, all households have the same portfolio, and, as each household spends on consumption the same constant fraction of its wealth, the rate of growth of wealth is the same for all households. This result enables us to use a representative household. Premultiplying (A1) by $\mathbf{V}$ and using the fact that world wealth per capita is $(1/\alpha_1)P_{11}K_1$ yields equation (2).

To obtain the result that world wealth per capita is $(1/\alpha_1)P_{11}K_1$, let T_1 be the present value in domestic currency of the transfers that accrue to the representative household from the domestic monetary authority. Let $Z_1 = T_1 + M_1$ be the household's domestic monetary wealth and α_3 (α_4) be the fraction of its consumption expenditures it spends on the services of domestic (foreign) money. By construction, if W is the domestic currency wealth of the representative household, $R_1M_1 = \rho\alpha_3 W$, and monetary services correspond to a constant fraction of wealth when wealth includes the present value of future transfers. The term R_1M_1 has the interpretation of the dividend of Z_1 (see Jones 1980; Stulz 1984). As R_1M_1 is equal to $(\alpha_3/\alpha_1)P_{11}C_1$, where C_1 is the representative household's consumption of the traded good, it follows that $Z_1 = (\alpha_3/\alpha_1)P_{11}K_1$ is the present value of the future consumption of the traded good. If all the components of the wealth of the representative household are valued in the same way, it follows that $W = (1/\alpha_1)P_{11}K_1$. Now it can be shown how each of our results is derived.

Result 1.—To derive P_{1j}, note that $R_jM_j = \rho(\alpha_{2+j}/\alpha_1)P_{1j}K_1, j = 1, 2$. Using this result and the law of one price yields e.

Result 2.—Note that the price index in country j is $I_j = P_{1j}^{\alpha_1}P_{2j}^{\alpha_2} = P_{1j}P_{Nj}^{\alpha_2}$. To derive P_{N1}, note that $P_{12}K_{12} = \frac{1}{2}(\alpha_2/\alpha_1)P_{11}K_1$, so that $P_{N1} = P_{12}/P_{11} = \frac{1}{2}(\alpha_2/\alpha_1)K_1/K_{12}$. The same approach yields P_{N2}. By substitution, x follows.

Result 3.—To derive R_1, use the asset pricing equation with the investment in the production of the traded good as the risky asset. Substitute out for the expected rate of change and the variance of the rate of change of P_{11}. Proceed in the same way to obtain R_2 noting that the return on the foreign bond in terms of the traded good in the domestic country is $R_2dt + (de/e)$. Result 3 follows by substituting in the expression for $R_1 - R_2$ the expected rate of change of e obtained by taking the expectation of $1/e$ times the Ito differential of e.

Result 4.—Note that

$$r_j = R_j - \mu_{I_j} + \sigma_{I_j}^2, \quad j = 1, 2, \tag{A2}$$

where μ_{I_j} is $E(dI_j/I_j)/dt$. To obtain dI_j/I_j, differentiate (5) using Ito's lemma. To obtain the expression for $r_1 - r_2$, substitute μ_x in $r_1 - r_2$ obtained from (A2), where μ_x is obtained by taking the expectation of $1/x$ times the Ito differential of x.

References

Adler, Michael, and Dumas, Bernard. "International Portfolio Choice and Corporation Finance: A Synthesis." *J. Finance* 38 (June 1983): 925–84.

Brock, William A., and Scheinkman, José A. "Some Remarks on Monetary Policy in an Overlapping Generations Model." In *Models of Monetary Economies*, edited by John H. Kareken and Neil Wallace. Minneapolis: Fed. Reserve Bank, 1980.

Calvo, Guillermo A., and Rodríguez, Carlos Alfredo. "A Model of Exchange Rate Determination under Currency Substitution and Rational Expectations." *J.P.E.* 85 (June 1977): 617–25.

Cox, John C.; Ingersoll, Jonathan E., Jr.; and Ross, Stephen A. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica* 53 (March 1985): 363–84.

Dornbusch, Rudiger. "Expectations and Exchange Rate Dynamics." *J.P.E.* 84 (December 1976): 1161–76.

Flood, Robert P. "An Example of Exchange Rate Overshooting." *Southern Econ. J.* 46 (July 1979): 168–78.

Frankel, Jeffrey A. "The Diversifiability of Exchange Risk." *J. Internat. Econ.* 9 (August 1979): 379–93.

Harrison, J. Michael, and Kreps, David M. "Martingales and Arbitrage in Multiperiod Securities Markets." *J. Econ. Theory* 20 (June 1979): 381–408.

Jones, E. P. "Intertemporal Financial and Monetary Equilibrium." Ph.D. dissertation, Massachusetts Inst. Tech., 1980.

Kouri, Pentti J. K. "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach." *Scandinavian J. Econ.* 78, no. 2 (1976): 280–304.

Lucas, Robert E., Jr. "Interest Rates and Currency Prices in a Two-Country World." *J. Monetary Econ.* 10 (November 1982): 335–59.

Mussa, Michael. "The Exchange Rate, the Balance of Payments and Monetary and Fiscal Policy under a Regime of Controlled Floating." *Scandinavian J. Econ.* 78, no. 2 (1976): 229–48.

———. "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market." *Carnegie-Rochester Conf. Ser. Public Policy* 11 (1979): 9–57.

Obstfeld, Maurice. "Macroeconomic Policy, Exchange-Rate Dynamics, and Optimal Asset Accumulation." *J.P.E.* 89 (December 1981): 1142–61.

Roll, Richard. "Violations of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets." In *International Finance and Trade*, vol. 1, edited by Marshall Sarnat and Giorgio P. Szegö. Cambridge, Mass.: Ballinger, 1979.

Rubinstein, Mark. "An Aggregation Theorem for Securities Markets." *J. Financial Econ.* 1 (September 1974): 225–44.

Sachs, Jeffrey. "Aspects of the Current Account Behavior of OECD Economies." In *Recent Issues in the Theory of Flexible Exchange Rates*, edited by Emil Claassen and Pascal Salin. Amsterdam: North-Holland, 1983.

Stockman, Alan C., and Svensson, Lars E. O. "Capital Flows, Investment and Exchange Rates." Mimeographed. Rochester, N.Y.: Univ. Rochester, 1985.

Stulz, René M. "Currency Preferences, Purchasing Power Risks, and the Determination of Exchange Rates in an Optimizing Model." *J. Money, Credit and Banking* 16 (August 1984): 302–16.

———. "Interest Rates and Monetary Policy Uncertainty." *J. Monetary Econ.* 17 (May 1986): 331–47.

Svensson, Lars E. O. "Currency Prices, Terms of Trade, and Interest Rates: A General Equilibrium Asset-pricing Cash-in-Advance Approach." *J. Internat. Econ.* 18 (February 1985): 17–41.