

Optimal Hedging Policies

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I. Introduction and Summary

This paper makes contributions in two directions. First, the paper presents a model in which value-maximizing firms pursue active hedging policies. Second, the paper derives optimal hedging policies for risk-averse agents. Whereas the methodology used and the results provided are quite general, this paper deliberately focuses the analysis on hedging foreign exchange exposure through forward contracts on foreign currencies.¹ This emphasis is explained by the fact that hedging foreign currency exposure through forward contracts has been a topic of considerable interest in recent years.²

Shareholders do not decide the hedging policy of the firm, but managers do. Shareholders, however, choose managerial compensation contracts that maximize their wealth and hence maximize the value of the firm. Many papers³ follow Jensen and Meckling [11] and stress the fact that managers choose policies that maximize their expected lifetime utility given their compensation contract and their expectation of the actions shareholders or other potential investors can take to decrease their expected utility. Consequently, in this paper, optimal hedging policies are derived under the assumption that managers maximize their expected lifetime utility and that their income from the firm is an increasing function of the changes in the value of the firm.⁴ It is shown that in such a setting,

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¹ Cox, Ingersoll and Ross [5] compare forward and futures contracts as hedging instruments. They show how holdings of futures contracts together with holdings of other assets can be used to replicate the payoff of a forward contract.

² Jacques [10] and Adler and Dumas [1] provide reviews of the existing literature.

³ See, for instance, [15], [14], [19], [7], [9], and [16]. In this paper, the compensation of a manager is defined only in terms of his pecuniary income. It should be pointed out that the compensation package of managers also incorporates nonpecuniary rewards, as discussed, for instance, in [11].

⁴ Notice that if income is interpreted as the sum of pecuniary income plus the change in the present value of future income from the firm, even compensation contracts that promise fixed payments imply that the compensation of the managers is an increasing function of changes in the value of the firm. The probability of bankruptcy and, hence, the probability that the firm will not fulfill its contract with the manager is a decreasing function of the value of the firm. In general, however,

firms pursue an active hedging policy.⁵ This implies that the optimal hedge in a currency is not, in general, equal in value but of opposite sign to the exposure (or the expected exposure) of the firm in that currency.⁶ In particular, it is shown that the firm can take a position in a forward contract that is larger or smaller in absolute value than the absolute value of its exposure in that currency. The paper demonstrates the crucial role of exchange rate dynamics, hedging costs, and the nature of managerial compensation contracts in the determination of a firm's holdings of forward contracts. It is proved, for instance, that it may turn out that a firm with a nonstochastic currency exposure takes no position in the forward market if changes in the exchange rate offset domestic inflation.

This paper makes contributions to the theory of hedging in general. Most of the literature on hedging deals with two-period models.⁷ In this paper, the whole analysis takes place in a continuous-time model in which the hedge position can be revised as new information is received by the firm. (This paper does not, however, study how the hedge position of the firm evolves through time.) Furthermore, this paper deals explicitly with the issue of hedging state-contingent payments. One advantage of the continuous-time framework is that the value of state-contingent payments can be obtained using the methodology developed by Cox, Ingersoll, and Ross [5].

The plan of the paper is as follows. Section II describes the model and derives the optimal hedging policy under the simplifying assumption that the price of the consumption basket consumed by managers is constant.⁸ In this case, exchange-rate changes affect the expected lifetime utility of managers only through changes in their income from the firm. Section III lets the price of the consumption basket consumed by managers change stochastically through time and discusses how exchange-rate dynamics affect the optimal hedging policy. In particular, the optimal hedging policy when exchange-rate changes offset price-level changes is contrasted with the optimal hedging policy when exchange-rate changes correspond to changes in the terms of trade. Section IV analyzes the implications of holding costs for forward contracts on the hedging policy of the firm. Finally, Section V offers some concluding remarks.

II. The Model

In this section, the problem faced by managers is described and the optimal hedging policy is analyzed. It is assumed that markets are perfect and trading takes place continuously. Before the managers decide on the firm's holdings of forward contracts, managers and shareholders agree on a compensation scheme

compensation contracts involve payments that depend on some measure of the value of the firm. See also [18] for a positive theory of hedging by value-maximizing firms.

⁵ There have been many discussions in the literature about what constitutes hedging and what constitutes speculation. In this paper, a hedging policy is simply a policy whereby the firm holds foreign bonds or forward contracts.

⁶ In the literature on futures markets, exposure is often referred to as the "cash" position.

⁷ See [3] for a two-period model of hedging and for references to the literature.

⁸ It must be noted that this type of exchange rate dynamics has been used often in international finance since first used by Solnik [20].

for managers. The compensation scheme is chosen so that the shareholders' wealth is maximized under the constraint that the managers receive a level of expected utility high enough to induce them to work for the shareholders. Whereas in a more complete model the compensation scheme would be endogenous, it is assumed here, for simplicity, that the compensation scheme is such that the managers' compensation depends on the change in the value of the firm. More precisely, it is assumed that if the instantaneous change in the value of the firm is equal to dV in the absence of a payment to the managers, the managers collectively receive δdV where δ is constant.⁹

For the moment, the firm has only one foreign currency asset. $F(t)$ is the current value in foreign currency of a payment equal to F^* to be made in that currency at time T .¹⁰ The term $e(t)$ is the current exchange rate, i.e., the domestic price of one unit of foreign currency. The domestic-currency value of the promised payment is $e(t)F(t)$. The dynamics for the exchange rate are given by

$$(1) \quad \frac{de}{e} = \mu_e dt + \sigma_e dZ_e$$

where μ_e is the expected rate of change of the exchange rate, σ_e is the instantaneous standard deviation of the rate of change of the exchange rate, and dZ_e is the increment of a Wiener process.¹¹ For the sake of simplicity, it is assumed that μ_e and σ_e are intertemporal constants.

Let the current value in foreign currency of the payment to be made at date T follow a stochastic differential equation given by

$$(2) \quad \frac{dF}{F} = \mu_F dt + \sigma_F dZ_F$$

μ_F and σ_F are not required to be constants. They can be functions of time and of state variables whose dynamics can be represented by Ito processes. For instance, μ_F and σ_F are likely to be functions of the value of the firm, which promises to make the foreign currency payment at date T .

It is possible to invest at home (abroad) in default-free bonds that earn a sure instantaneous rate of return R (R^*). The rates R and R^* are assumed to be constant. As markets are perfect, interest rate parity¹² holds and $f = R - R^*$ is the instantaneous forward premium. As R^* and R are constant, the term structure of forward premiums is flat. Whereas the forward exchange rate can differ from the future expected spot exchange rate in this model, the risk premium incorporated

⁹ If the manager has a logarithmic utility function, all the results of this section and of Section IV hold if δ is a function of a vector of stochastic state variables. Notice also that if the total compensation of the manager is a function of accounting numbers only, V can be interpreted as the accounting value of the firm and managers will want to hedge the accounting value of the firm. Finally, if managers receive $K + \delta dV$, where K is a constant, all the results of this paper hold.

¹⁰ $F(t)$ is chosen to be positive to insure that the value of the firm is positive. The model could be modified so that $F(t)$ may be negative. This additional complication does not add new insights to the present paper.

¹¹ See [5] for references on the techniques used here.

¹² For a useful discussion of interest rate parity and related concepts, see [2].

in the forward exchange rate is constant through time.¹³ Given these assumptions, there is no loss in generality if one assumes that trading takes place only in bonds and forward contracts of instantaneous maturity. As interest rate parity holds, the managers are indifferent between two investment strategies whose dollar returns are perfectly correlated with changes in the exchange rate. The first strategy consists in buying N^* units of foreign currency forward. The instantaneous dollar return of the strategy is $de/e - fdt$. The second strategy consists in buying foreign default-free bonds for N^* units of foreign currency and selling domestic default-free bonds for eN^* . The instantaneous dollar return on the second strategy is $R^*dt + de/e - Rdt = de/e - fdt$. The first strategy is often called a forward market hedge, whereas the second strategy is often called a money market hedge. If the firm pursues either one of these two strategies, the instantaneous change in the value of the firm in the absence of payments to managers is given by

$$(3) \quad dV = d(eF) + eN^* \left(R^*dt + \frac{de}{e} - Rdt \right) + dK$$

where dK is the change in the value K of the firm's other assets. For the moment, it is assumed that $\text{Cov}(dK, d(eF))$ is equal to zero. The problem faced by managers is to choose N^* so that they maximize their collective lifetime expected utility of consumption. All the other decisions of the firm already have been taken. A fraction $(1 - \delta)$ of dV is reinvested in the firm. The firm pays no dividend until date T and has no debt. As, by assumption, shareholders want managers to maximize the value of the firm, they do not care about the manager's choice of N^* as long as hedging is costless and the investment policy is not affected by hedging. In this section, all investment decisions have already been taken and are unaffected by the choice of N^* . Consequently, shareholders do not care about the manager's choice of N^* .¹⁴

To find N^* managers maximize their expected utility of lifetime consumption. To simplify the problem, it is assumed in most of this paper that there is only one manager who lives until time T and has a Bernoulli utility function, so that he or she maximizes

$$(4) \quad E_t \int_t^T e^{-\rho s} \text{Ln } C(s) ds$$

where $C(s)$ is the manager's consumption rate. For the moment, the manager can invest his or her wealth only in the default-free domestic bond. If the manager must pay some transactions costs when he or she purchases foreign default-

¹³ For an analysis of the risk premium incorporated in the forward exchange rate and references, see [22]. The forward exchange rate is a biased predictor of the future spot exchange rate whenever $f \neq \mu_e$.

¹⁴ This does not mean that shareholders do not care about changes in exchange rates; it means that the investment opportunity set of shareholders is such that they want managers to maximize the value of the firm.

free bonds or enters into forward contracts, he or she will choose the cheapest way of hedging and hence will hedge through the firm rather than on his or her personal account. In general, it is realistic to assume the firm has a comparative advantage in trading bonds or forward contracts.

It is well known that the volatility of exchange rates exceeds the volatility of price levels.¹⁵ To emphasize the implications of this fact, this section makes the extreme assumption that the price of the manager's consumption basket is fixed and, for simplicity, equal to one. With this assumption, the manager's budget constraint is given by

$$(5) \quad dW = \delta dV + RWdt - Cdt .$$

Notice that if $\delta = 1$, the manager's optimization problem becomes equivalent to the optimization problem of a risk-averse investor who will receive a (possibly random) payment in foreign currency at a future date. It follows that the solution for N^* applies for any risk-averse agent with relative risk tolerance equal to unity. As the manager maximizes (4) subject to (5), he obtains a solution for N^*

$$(6) \quad eN_0^* = \left(\frac{1}{\delta}\right) \left(\frac{R^* + \mu_e - R}{\sigma_e^2} \right) W - \left(\frac{\sigma_{eF} + \sigma_e^2}{\sigma_e^2} \right) eF(t)$$

where σ_{eF} is the instantaneous covariance between the dynamics for e and the dynamics for F .¹⁶

Equation (6) can be interpreted in the following way. First, suppose that the manager wants to minimize the instantaneous variance of the rate of change of his income. In this case, he or she chooses N^* so that $\text{var}(dV)$ is minimized. If N_m^* is the solution to this problem, then

$$(7) \quad N_m^* = - \left(\frac{\sigma_{eF} + \sigma_e^2}{\sigma_e^2} \right) F(t) .$$

When changes in the current value in foreign currency of the expected payment are uncorrelated with changes in the exchange rate, N_m^* is equal to $-F(t)$; i.e., the firm goes short in the foreign currency for an amount equal to the current value in foreign currency of the expected payment. If the firm holds N_m^* in foreign bonds, the instantaneous covariance between dV/V and de/e is equal to zero; i.e., the firm is not exposed to exchange-rate uncertainty. The current value of the payment appears in equation (7), rather than the promised payment itself.

¹⁵ See, for instance, [8].

¹⁶ If $\text{Cov}(dK/K, de/e)$ is different from zero, equation (6) contains a third term equal to $-(\sigma_{eK}/\sigma_e^2)K(t)$, i.e., the firm uses forward contracts or foreign bonds to hedge against unanticipated changes in the value of foreign bonds. If the manager holds other risky assets, the covariance of the return on these risky assets and changes in the exchange rate matter for the choice of N^* . The higher the covariance of the return to the investor's wealth with changes in the exchange rate, the lower N^* .

To understand this, suppose that it is very likely that the foreign firm that promises to make the payment at time T will be bankrupt at that time. If the foreign firm is bankrupt at time T , one possible outcome is that $F(T) = 0$. In this case, a firm that would have gone short in foreign currency for the amount of the contracted payment would have to settle the forward contract without receiving a payment in foreign currency at time T . If $F(T)$ is equal to zero, the only foreign currency exposure the firm will have at time T is the one created by its holdings of forward contracts. The hedge given by equation (7) takes into account the probability distribution of the random payment to be made at date T . Furthermore, if the probability that the payment will be made increases with $e(T)$, the value of the firm depends more on the exchange rate than if $e(T)$ and the payment made at date T are uncorrelated. In this case, the firm must be short in the foreign currency for an amount larger than $F(t)$.

When holdings of the foreign bond earn a positive expected excess return, i.e., $R^* + \mu_e - R$ is positive, the manager takes a speculative position in the foreign bond. This position is an increasing function of the expected excess return on an investment in the foreign bond and is a decreasing function of the instantaneous variance of this return. The manager behaves as if he is solving the following problem

$$\max E(dV)$$

so that

$$\text{Var}(dV) = A dt$$

where A is a constant. If N_s^* is the solution to this problem, then

$$(8) \quad N_s^* = \left(\frac{1}{\delta} \right) \left(\frac{R^* + \mu_e - R}{\sigma_e^2} \right) W.$$

The coefficient δ appears in the denominator because the firm must take a long position of $(1/\delta)$ dollars for the manager to receive an expected excess return equal to $R^* + \mu_e - R$ per unit of time.

The only restriction made so far on the nature of the promised payment is that its current value must follow an Ito process. As long as it is possible to find $F(t)$ and to find the stochastic differential equation $F(t)$ satisfies, the manager can compute the firm's optimal holdings of foreign bonds. Three useful illustrations can be given. First, if the payment will be made with probability one, then $F(t) = \exp(-R^*(T-t))F^*$ and $N_m^* = -F(t)$. Second, if the foreign firm pays F^* if its value V^* at date T exceeds F^* and V^* otherwise, the current value of the payment can be obtained using the methodology developed by Black and Scholes [4] to price contingent claims, if there exists a traded asset whose value at date T is equal to V^* . Finally, if $F(T)$, i.e., the payment made at date T , depends on the value of nontraded state variables at time T , the methodology developed by Cox, Ingersoll and Ross [5] can be used to value $F(t)$. Notice that the methodology developed in this section can be used to find the firm's optimal

holdings of forward contracts in the case of promised payments resulting from a contract that may or may not be awarded at a future date.

It is important to stress, finally, that the results of this section can be used to find the firm's holdings of foreign bonds when the firm receives a continuous cash flow in foreign currency. In this case, let $F(t)$ be the present value in the foreign currency of the cash flow. ($F(t)$ can be computed using the methodology developed by Cox, Ingersoll, and Ross [5].) If the firm will receive payments at various future dates, the optimal holdings of foreign bonds are given by equation (6) with $F(t)$ being set equal to the sum of the present values of these payments.

III. Exchange Rate Dynamics and Optimal Hedging Policies

The analysis in Section II ignores the fact that changes in exchange rates are often correlated with changes in commodity prices and assumes that the manager has a logarithmic utility function. This section explicitly takes into account the fact that there is a correlation between exchange-rate and price-level changes. The analysis shows that this correlation affects the firm's holdings of foreign bonds when the manager's utility function has a coefficient of relative risk aversion different from one, i.e., when the manager's utility function is not logarithmic.

Define P_i (P_i^*) as the price in domestic (foreign) currency of the i th good and C_i as the number of units of commodity i consumed by the manager. It is assumed that P_i obeys the following stochastic differential equation

$$(9) \quad \frac{dP_i}{P_i} = \mu_{P_i} dt + \sigma_{P_i} dZ_i \quad i = 1, \dots, N.$$

P_i^* follows a similar equation. Furthermore, it is assumed that the manager's expenditure share on the i th good, i.e., $\alpha_i = P_i C_i / \sum_{i=1}^N P_i C_i$, is a constant, for all i 's. The manager maximizes

$$(10) \quad E_t \int_t^T e^{-\rho s} \frac{1}{\gamma} \left(\prod_{i=1}^N C_i(s)^{\alpha_i} \right)^\gamma ds \quad \gamma < 1$$

subject to a budget constraint that can be written as

$$(11) \quad dW = \delta dV + RWdt - \sum_{i=1}^N P_i C_i dt.$$

Notice that dV is given by equation (3). Solving for N^* , the manager obtains

$$(12) \quad eN_I^* = \frac{1}{(1-\gamma)\delta} \left(\frac{R^* - \mu_e - R}{\sigma_e^2} \right) W - \left(\frac{\sigma_{eF} + \sigma_e^2}{\sigma_e^2} \right) eF \\ + \sum_{i=1}^N \sigma_{eP_i} \left(1 - \frac{1}{1-\gamma} \right) \alpha_i \left(\frac{W}{\delta} \right)$$

where σ_{eP_i} is equal to the instantaneous covariance between the dynamics for the exchange rate and the dynamics for the price of the i th commodity in domestic currency. As the manager exhibits constant expenditure shares, it is known (see [15]) that there exists an exact price index P that can be used to deflate nominal consumption expenditures. The price index P is defined so that $dP/P = \sum_{i=1}^N \alpha_i (dP_i/P_i)$. The holdings of foreign bonds in this section differ from those obtained in Section II by

$$(13) \quad \Delta_I^H = eN_I^* - eN_0^* \\ = \left(\frac{TR - 1}{\delta} \right) \left(\frac{R^* - \mu_e - R}{\sigma_e^2} \right) + \left(\frac{\sigma_{eP}}{\sigma_e^2} \right) \left(\frac{1 - TR}{\delta} \right) W$$

where TR is the manager's coefficient of relative risk tolerance, i.e., $1/(1-\gamma)$, and σ_{eP} is the instantaneous covariance between the rate of change of the exchange rate and the rate of change of the price index P . The first term on the right-hand side of equation (13) corresponds to the difference between the speculative holdings of foreign bonds when the coefficient of relative risk tolerance of the manager is TR and when it is one. Not surprisingly, the more risk-tolerant the manager (i.e., the higher the value of γ), the higher the absolute value of the speculative holdings of foreign bonds. The second term on the right-hand side of equation (13), written Δ_I^H in the remainder of this section, corresponds to the holdings of foreign bonds that hedge the manager against unanticipated changes in the price level when the manager's coefficient of relative risk aversion is smaller than one. The manager does not hedge against unanticipated inflation if he has a logarithmic utility function, i.e., $TR = 1$. Notice that if $\sigma_{eP} = 0$, i.e., the exchange rate changes are uncorrelated with price level changes, forward contracts or foreign bonds are useless to hedge against unanticipated changes in the price level.

Suppose now that $\sigma_{eP} \neq 0$. It is useful to look at two polar cases that correspond to well-known models of exchange rate determination. In the first polar case, it is assumed that purchasing power parity (PPP) holds. In the second polar case, it is assumed that exchange rate changes are perfectly correlated with changes in the terms of trade.¹⁷ As empirical studies of exchange rate determination suggest that exchange-rate changes are only partially correlated with

¹⁷ For references on PPP, see [22]. Kouri and de Macedo [12], for instance, have a model in which the exchange rate is perfectly correlated with the terms of trade.

changes in the terms of trade, a realistic model of exchange-rate determination must use some features of the two polar cases studied in this section.

Let P^* be the foreign price index, which is assumed to follow an Ito process. With this notation, PPP holds if $P = eP^*$. If PPP holds, Δ^H can be rewritten as

$$(14) \quad \Delta_I^H = \left(\frac{\sigma_P^2 - \sigma_{PP^*}}{\sigma_P^2 + \sigma_{P^*}^2 + 2\sigma_{PP^*}} \right) \left(\frac{1 - TR}{\delta} \right) W.$$

Equation (14) implies that if there is no unanticipated inflation abroad, the foreign bond is a perfect hedge against unanticipated inflation at home, as changes in the exchange rate are perfectly correlated with changes in the domestic price level. Furthermore, if there is only unanticipated inflation abroad, Δ_I^H is equal to zero, as there can be no unanticipated changes in the price of the manager's consumption.

The second case of interest occurs if exchange-rate changes are perfectly correlated with changes in the terms of trade. Suppose that there are only two goods, good 1 and good 2, and that P_1 and P_2^* are constant. Good 2 is produced abroad only. The law of one price is assumed to hold and it implies that $P_2 = eP_2^*$. In this setting, changes in P_2 are perfectly correlated with changes in the exchange rate. It follows that Δ_I^H is given by

$$(15) \quad \Delta_I^H = \left(\frac{1 - TR}{\delta} \right) \alpha_2 W$$

where α_2 is the expenditure share of good 2 for the manager. Δ_I^H is an increasing function of α_2 provided that the manager's coefficient of relative risk tolerance is smaller than one. In this case, the manager hedges against unanticipated changes in the price of his current consumption and the foreign bond provides a perfect hedge.

In both polar cases analyzed in this section, the existence of a positive correlation between the exchange-rate changes and the changes in the price of the basket of commodities consumed by the manager implies that the firm's holdings of foreign bonds differ from those obtained in Section II, provided that the manager's coefficient of relative risk tolerance, i.e., TR , differs from one. In particular, if TR is smaller than one (which seems to be the case in the aggregate) the firm's holdings of foreign bonds differ in this section from those of Section II, as a result of σ_{eP} being positive, by an additional long investment in the foreign currency. If σ_{eP} is high enough and TR low enough, it is possible that the firm does not take a position to hedge a payment it will receive at a future date. It is even possible that a firm could take a position in the forward market that increases its existing exposure in the foreign currency. In particular, if a firm has to make a payment in foreign currency at a future date, it is likely that the firm will take a long position in that currency on the forward market that exceeds the current value of its exposure.

IV. Hedging Policy When Hedging Is Costly

In earlier sections of this paper, it is assumed that the manager can implement a hedging strategy costlessly. If this assumption is correct, shareholders are not likely to care about the manager's decisions with respect to the firm's holdings of foreign bonds or forward contracts. It must be noticed that, although shareholders could forbid the manager from following a costly hedging policy, this does not imply that the shareholders would be made much better off by doing so. If the manager cannot hedge in a straightforward way, he or she is likely to try to hedge in ways that are harder for the shareholders to detect and reject some projects that have a positive net present value. Furthermore, if the manager is not allowed to hedge, and if this restriction implies that the instantaneous variance of his or her income is higher than otherwise, he or she will require a higher expected income, i.e., a higher δ , to accept employment.

This section is devoted to an analysis of how the fact that hedging is costly affects the hedging policy of the firm. It is assumed that it costs Ω per unit of time to hold one dollar in foreign bonds (either long or short). Let N_+^* (N_-^*) be the firm's holdings of the foreign bond if they are positive (negative). The instantaneous change in the value of the firm before payments to the manager is now given by

$$(16) \quad \begin{aligned} dV = & d(eF) + eN_+^* \left(R^* dt + \frac{de}{e} - Rdt - \Omega dt \right) \\ & + eN_-^* \left(R^* dt + \frac{de}{e} - Rdt + \Omega dt \right) + dK. \end{aligned}$$

The manager's optimization problem can be solved by substituting dV , as given by equation (16), into equation (5). However, the solution to the manager's optimization problem must satisfy the constraints that $N_+^* \geq 0$ and $N_-^* \leq 0$. Solving the manager's optimization problem¹⁸ it follows that if $N_-^* \neq 0$

$$(17) \quad eN_-^* = \frac{1}{\delta} \left(\frac{R^* - \mu_e - R}{\sigma_e^2} \right) W + \frac{1}{\delta} \left(\frac{\Omega}{\sigma_e^2} \right) W - \left(\frac{\sigma_{eF} + \sigma_e^2}{\sigma_e^2} \right) eF.$$

If the holdings of foreign bonds are negative, they differ from those obtained in Section II by

$$(18) \quad eN_-^* - eN_0^* = \Delta_T = \frac{1}{\delta} W \left(\frac{\Omega}{\sigma_e^2} \right) W.$$

Equation (18) implies that the larger the holding cost Ω , the smaller the absolute value of the firm's position in foreign bonds. However, the essential result is that equation (18) does not depend directly on the current value of the

¹⁸ The problem solved here is formally identical to the problem solved in [21].

payment expected in foreign currency, i.e., eF . If $N_-^* < 0$ or $N_+^* > 0$, the optimization problem of the manager is separable in the following way. First, the manager takes a position in the foreign currency that does not depend on Ω but is a function of eF and its dynamics. This position is chosen so that changes in the value of the firm are uncorrelated with changes in the exchange rate. Second, the manager takes a speculative position in the foreign bond, which is a decreasing function of Ω but depends neither on eF nor on its dynamics.

To simplify the argument, it is useful to assume that $R^* + \mu_e - R$ is equal to zero. In this case, it follows that

$$(19) \quad eN_-^* = \frac{1}{\delta} \left(\frac{\Omega}{\sigma_e^2} \right) W - \left(\frac{\sigma_{eF} + \sigma_e^2}{\sigma_e^2} \right) eF$$

for $eN_-^* < 0$. Equation (19) implies that there exists a value Ω' such that if $\Omega = \Omega'$, $eN_-^* = 0$. If, for all values of $\Omega < \Omega'$, $eN_-^* < 0$, it can be shown that for all values of $\Omega > \Omega'$, $eN_-^* = 0$. Whether the firm holds foreign bonds depends crucially on the relationship between ΩW and eF . If ΩW is large compared to eF , eN_-^* is equal to zero. A complete characterization of the solution to the manager's problem shows that the same analysis applies if $eN^* = eN_+^* > 0$, except that in this case the right-hand side of equation (18) is negative.¹⁹

To understand this, notice that Ω/δ is equivalent to an insurance premium the manager pays to reduce the total amount of risk he or she bears. Given his or her wealth and utility function, the manager is willing to leave uninsured a given amount of exposure as he or she saves Ω/δ per unit of exposure that is not insured. For the exposure in excess of the amount the manager leaves uninsured, the gain in expected income per additional unit of exposure left uninsured is not high enough to compensate him or her for the associated increase in the variance of his or her income.

It is interesting to look at the problem the manager faces when the firm expects to receive payments in N currencies. Let $F_i(t)$ be the current value in currency i of a payment F_i^* to be received at a future date, $e_i(t)$ be the current price of one unit of currency i and N_i be the firm's holdings of default-free bonds from country i . To simplify the discussion, the only case examined is the case in which $e_i N_i = D_i < 0$ for all i 's. Let \underline{D} be an $N \times 1$ vector whose i th element is D_i , \underline{V}_{ee} be the $N \times N$ variance-covariance matrix of instantaneous rates of change of exchange rates, and \underline{V}_{eF} be the $N \times N$ covariance matrix between the instantaneous rates of change of exchange rates and the value in foreign currencies of the payments to be received in these currencies. Notice that changes in F_i can be correlated with changes in e_j . In this case, the firm's holdings of foreign bonds are given by

$$(20) \quad \underline{D} = \left(\frac{1}{\delta} \right) \underline{V}_{ee}^{-1} \underline{R} + \left(\frac{1}{\delta} \right) \underline{V}_{ee}^{-1} \Omega \underline{1} - \left(\underline{V}_{ee}^{-1} \underline{V}_{eF} + \underline{I} \right) \underline{E}$$

¹⁹ A complete analysis for the case in which $eN_+^* > 0$ would require the introduction of a valuation equation for the firm which implies that the value of the firm is never negative.

where $\underline{1}$ is an $N \times 1$ vector of ones, \underline{R} is an $N \times 1$ vector whose i th element is equal to $R_i^* + \mu_e - R$, and \underline{E} is an $N \times 1$ vector whose i th element is equal to $e_i F_i$. Two facts need to be noticed. First, $(1/\delta) \underline{V}_{ee}^{-1} \Omega \underline{1}$ is proportional to the minimum-variance portfolio of foreign bonds, i.e., $(\underline{1}' \underline{V}_{ee}^{-1} \underline{1})^{-1} \underline{V}_{ee}^{-1} \underline{1}$. The global minimum-variance portfolio of foreign bonds does not necessarily have positive investments in all currencies. It follows from this that an increase in holding costs for foreign bonds does not always imply a decrease in the absolute value of the firm's holdings of foreign bonds for each currency. To understand this, notice first that an increase in holding costs for foreign bonds increases the value of the firm's holdings of foreign bonds if $D_i < 0$, for all i . The effect of an increase in Ω is to induce the firm to buy a long position in the global minimum-variance portfolio if initially it holds foreign bonds short. If the global minimum-variance portfolio has negative investments in some foreign bonds, the investment of one dollar in the global minimum-variance portfolio decreases the firm's holdings of these foreign bonds. Second, the difference between the vector of holdings of foreign bonds in this section and the vector of holdings of foreign bonds in the absence of holding costs, i.e., $(1/\delta) \underline{V}_{ee}^{-1} \Omega \underline{1}$, does not depend directly on the vector \underline{E} or its dynamics, i.e., on the firm's exposure in foreign currencies.

Equation (20) indicates that if changes in the current value in currency i of an expected payment in that currency, i.e., F_i , are uncorrelated with changes in the exchange rates, then an increase in F_i equal to ΔF_i implies that the firm goes short an additional amount $e_i \Delta F_i$ in the bond denominated in currency i if it already holds short default-free bonds of country i . This result holds even if, at the same time, the current value of the payment expected in currency j , F_j , increases by ΔF_j and changes in currencies i and j are negatively correlated. It follows that equation (20) implies that the firm does not take direct advantage of the variance-covariance matrix of exchange rate changes when it forms a portfolio of bonds that, together with the various F_i 's, is a minimum-variance portfolio. This is not really surprising, as a short position in a bond denominated in currency i exactly offsets a long position in currency i . However, holding costs decrease the total holdings of foreign bonds of the firm. The change in the firm's holdings of foreign bonds in each currency depends on the variance-covariance matrix of the instantaneous changes in exchange rates. Holding costs induce the firm to have a larger exposure in absolute value in some currencies.

V. Concluding Remarks

This paper makes some progress in deriving optimal hedging policies in an intertemporal setting. It is assumed that managers decide which position the firm takes in forward contracts or foreign bonds. The role of managerial compensation contracts, of exchange rate dynamics, and of hedging costs in the determination of the firm's optimal hedging policy are examined. It is shown that firms follow an active hedging policy. This means that one would not expect a firm to take systematically forward positions of opposite sign and equal in value to the promised payment in foreign currency. The paper also derives optimal hedging

policies for risk-averse agents in the presence of uncertainty about future commodity prices and of holding costs for hedge positions.

It must be emphasized that this paper does not present a complete characterization of hedging policies. A number of issues neglected in this paper will have to be addressed before such a characterization becomes available. In particular, throughout the paper, projects that involve foreign currency exposures have already been undertaken by the firm. If hedging is costly, some projects that involve foreign exchange exposure might not be undertaken by the firm. Therefore, it would be useful to make the model used in this paper more realistic by letting managers choose whether or not to undertake projects. Second, the paper considers only the case in which it is costly to keep a hedging position. Explicit transactions costs should be introduced in the model. These costs would raise the interesting issue of how frequently the hedge portfolio ought to be revised. Third, the capital structure of the firm will have an impact on the optimal hedging policy if the costs of financial distress are explicitly taken into account. Finally, the management compensation scheme is taken as given. It would be interesting to show how the choice of management compensation schemes depends on the opportunities managers have to hedge.

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